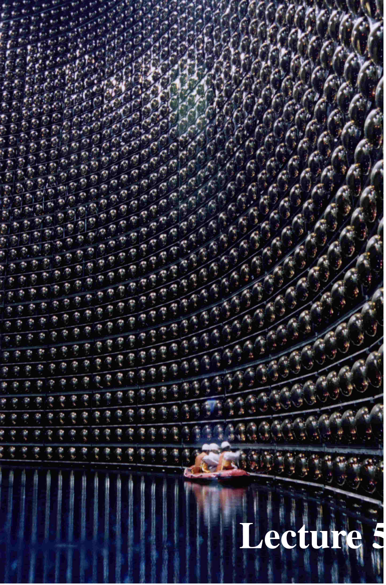

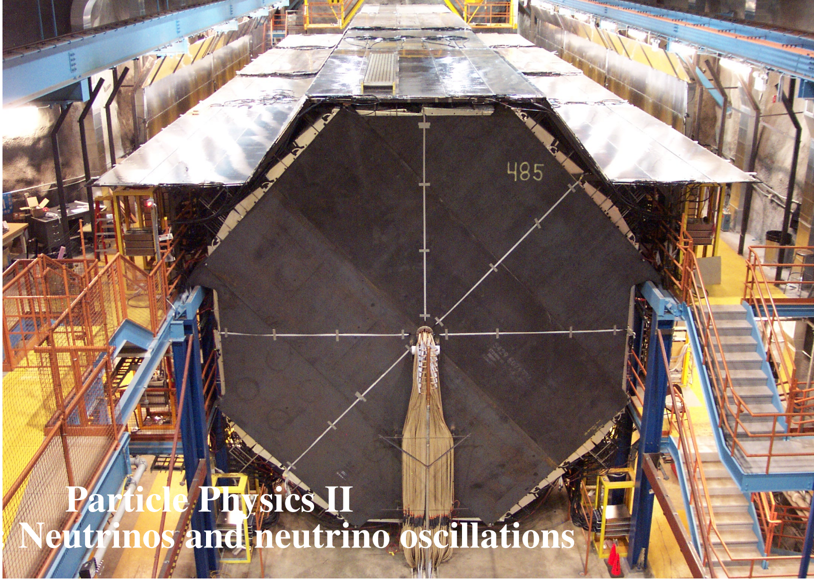

Particle Physics II
Lecture 5: Neutrinos and neutrino oscillations

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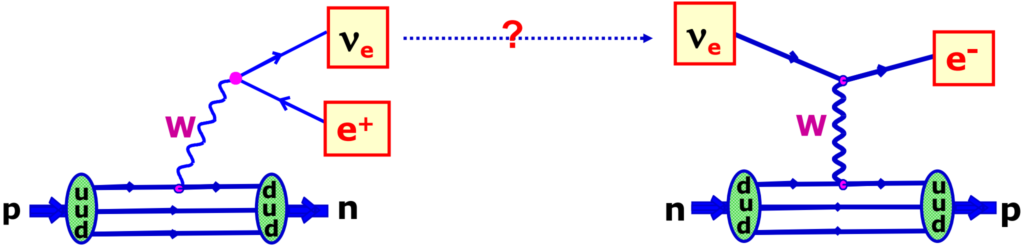
Neutrino flavors

- ν are never observed directly but detected via their weak interactions
- by definition, ν_e is the ν state produced along with e^+
- and vice versa: charged current weak interactions of the state ν_e produce e^-

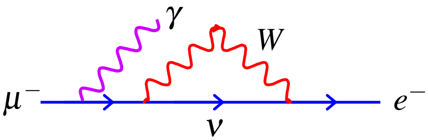
$$\nu_e, \nu_\mu, \nu_\tau \text{ – weak eigenstates}$$

- was assumed for many years: ν_e, ν_μ, ν_τ – massless fundamental particles

Experimental evidence: ν produced along with e^+ always lead to e^- in CC weak interactions, etc:

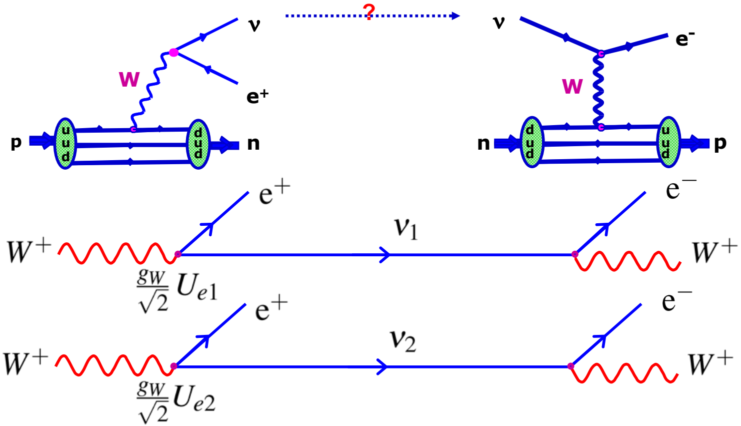


Experimental evidence: absence of $\mu^- \to e^-\gamma$ suggests that ν_e and ν_μ are distinct particles
$\mathcal{B}(\mu^- \to e^-\gamma) < 10^{-11}$



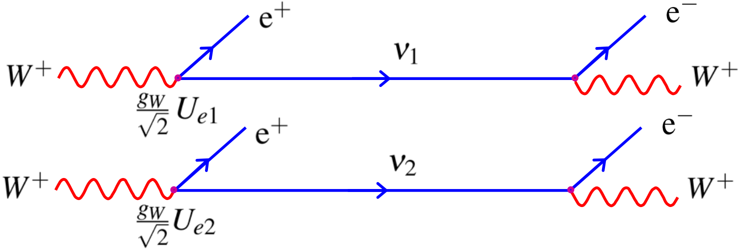
Mass eigenstates and weak eigenstates

- the essential feature in understanding the physics of neutrino oscillations is to understand what is meant by weak eigenstates and mass eigenstates ν_1, ν_2
- suppose the process below proceeds via two fundamental particle states



Mass eigenstates and weak eigenstates

- can't know which mass eigenstate (ν_1, ν_2) was involved



- in QM treat as a coherent state $\psi = \nu_e = U_{e1}\nu_1 + U_{e2}\nu_2$

- ν_e represents the wave-function of the coherent state produced along with e^+ in the weak interaction, i.e. the weak eigenstate

- neutrinos are produced and interact as weak eigenstates, ν_e, ν_μ
- they are coherent linear combinations of the fundamental "mass eigenstates" ν_1, ν_2
- the mass eigenstates are the free particle solutions to the wave equation and will be taken to propagate as plane waves:

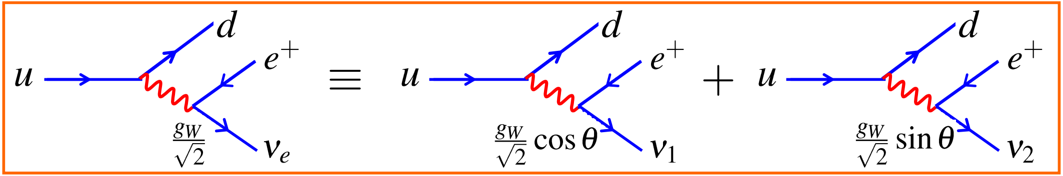
$$|\nu_1(t)\rangle = |\nu_1\rangle \, e^{i\vec{p}_1\cdot\vec{x}-iE_1 t}$$

$$|\nu_2(t)\rangle = |\nu_2\rangle \, e^{i\vec{p}_2\cdot\vec{x}-iE_2 t}$$

Neutrino oscillations for two flavors

- the weak and mass eigenstates are related by the unitary 2×2 matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$



- equation inversion leads to:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}$$

Neutrino oscillations for two flavors

- suppose at time $t = 0$ a neutrino is produced in a pure ν_e state, e.g. in a decay $u \to d e^+ \nu_e$:

$$|\psi(0)\rangle = |\nu_e\rangle = \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle$$

- take the z-axis to be along the neutrino direction

- the wave-function evolves according to the time evolution of the mass eigenstates (free particle solutions to the wave equation)

$$|\psi(t)\rangle = \cos\theta\,|\nu_1\rangle\,e^{-ip_1\cdot x} + \sin\theta\,|\nu_2\rangle\,e^{-ip_2\cdot x},$$

where $p_i \cdot x = E_i t - \vec{p}_i \cdot \vec{x} = E_i t - |\vec{p}_i| z$

Neutrino oscillations for two flavors

- suppose the neutrino interacts in a detector at a distance L and a time T
  $\phi_i = p_i \cdot x = E_i T - |\vec{p}_i| L$, giving:

$$|\psi(L,T)\rangle = \cos\theta\, |\nu_1\rangle\, e^{-i\phi_1} + \sin\theta\, |\nu_2\rangle\, e^{-i\phi_2}$$

- expressing the mass eigenstates, $|\nu_1\rangle$, $|\nu_2\rangle$, in terms of weak eigenstates:

$$|\psi(L,T)\rangle = \cos\theta\Big[\cos\theta\,|\nu_e\rangle - \sin\theta\,|\nu_\mu\rangle\Big]e^{-i\phi_1} + \sin\theta\Big[\sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\mu\rangle\Big]e^{-i\phi_2}$$

$$|\psi(L,T)\rangle = |\nu_e\rangle\left[\cos^2\theta e^{-i\phi_1} + \sin^2\theta e^{-i\phi_2}\right] + |\nu_\mu\rangle \sin\theta\cos\theta\Big[-e^{-i\phi_1} + e^{-i\phi_2}\Big]$$

- if the masses of $|\nu_1\rangle$, $|\nu_2\rangle$ are the same, the mass eigenstates remain in phase, $\phi_1 = \phi_2$, and the state remains the linear combination corresponding to $|\nu_e\rangle$, and in a weak interaction will produce an electron

- if the masses are different, the wave-function is no longer a pure $|\nu_e\rangle$:

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L,T) \rangle|^2 \quad (1)$$
$$= \cos^2\theta \sin^2\theta \left(-e^{-i\phi_1} + e^{-i\phi_2}\right)\left(-e^{+i\phi_1} + e^{+i\phi_2}\right) \quad (2)$$
$$= \frac{1}{4}\sin^2 2\theta (2 - 2\cos(\phi_1 - \phi_2)) = \sin^2 2\theta \sin^2\left(\frac{\phi_1 - \phi_2}{2}\right) \quad (3)$$

Neutrino oscillations for two flavors

- let's look at the phase difference:

$$\Delta\phi_{12} = \phi_1 - \phi_2 = (E_1 - E_2)T - (|p_1| - |p_2|)L$$

- can assume that $|p_1| = |p_2| = p$ in which case

$$\Delta\phi_{12} = (E_1 - E_2)T = \left[\left(p^2 + m_1^2\right)^{1/2} - \left(p^2 + m_2^2\right)^{1/2}\right]L, \text{ as } L \approx (c)T$$

$$\Delta\phi_{12} = p\left[\left(1 + \frac{m_1^2}{p^2}\right)^{1/2} - \left(1 + \frac{m_2^2}{p^2}\right)^{1/2}\right]L \approx \frac{m_1^2 - m_2^2}{2p}L$$

- here we neglected that for the same momentum, different mass eigenstates propagate at different velocities and are observed at different times

- the full derivation requires a wave-packet treatment and gives the same result

- the phase difference can be written as:

$$\Delta\phi_{12} = (E_1 - E_2)T - \left(\frac{|p_1|^2 - |p_2|^2}{|p_1| + |p_2|}\right)L$$

$$\Delta\phi_{12} = (E_1 - E_2)\left[T - \left(\frac{E_1 + E_2}{|p_1| + |p_2|}\right)L\right] + \left(\frac{m_1^2 - m_2^2}{|p_1| + |p_2|}\right)L$$

- the first term of the RHS vanishes if we assume $E_1 = E_2$ or $\beta_1 = \beta_2$
- therefore in all cases:

$$\Delta\phi_{12} = \frac{m_1^2 - m_2^2}{2p}L = \frac{\Delta m^2}{2E}L$$

- hence the two-flavor oscillation probability is:

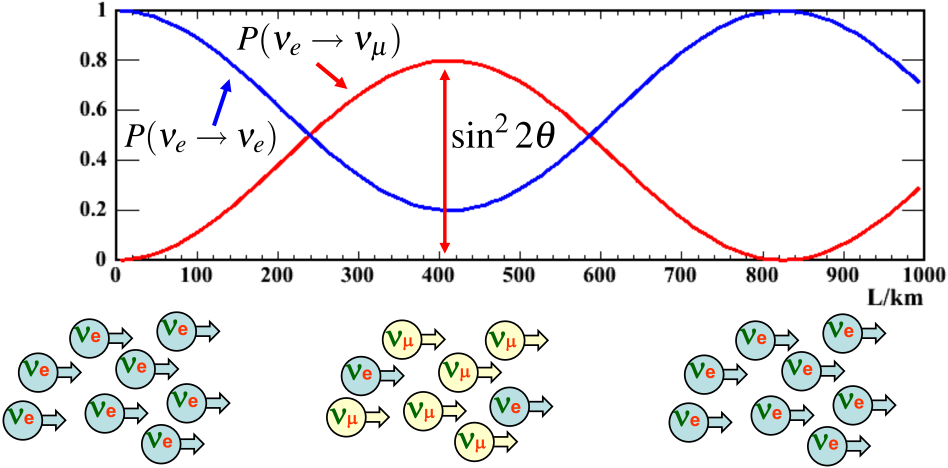
$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right), \text{ with } \Delta m^2_{21} = m^2_2 - m^2_1$$

- the corresponding two-flavor survival probability is:

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right)$$

Neutrino oscillations for two flavors

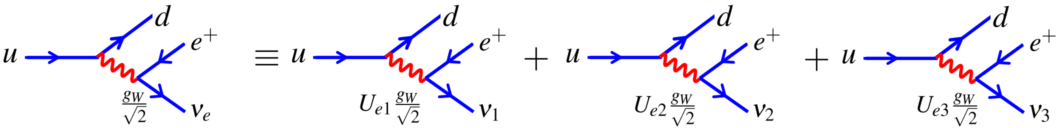
- e.g. $\Delta m^2 = 0.003\ \text{eV}^2$, $\sin^2 2\theta = 0.8$, $E_\nu = 1$ GeV, $\lambda_{\text{osc}} = \dfrac{4\pi E}{\Delta m^2}$



Neutrino oscillations for three flavors

- it is straightforward to extend this treatment to three generations of neutrinos
- in this case we have:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



- the 3×3 Unitary matrix U is known as the Pontecorvo-Maki-Nakagawa-Sakata matrix, usually abbreviated PMNS

- it has to be unitary to conserve probability

- using $U^\dagger U = 1 \implies U^{-1} = U^\dagger = (U^*)^T$ gives

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U^*_{e1} & U^*_{\mu 1} & U^*_{\tau 1} \\ U^*_{e2} & U^*_{\mu 2} & U^*_{\tau 2} \\ U^*_{e3} & U^*_{\mu 3} & U^*_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

Unitarity relations

- the unitarity of the PMNS matrix gives several useful relations:
  $UU^\dagger = I \implies$

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} U_{e1}^* & U_{\mu 1}^* & U_{\tau 1}^* \\ U_{e2}^* & U_{\mu 2}^* & U_{\tau 2}^* \\ U_{e3}^* & U_{\mu 3}^* & U_{\tau 3}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^* = 1 \quad (4)$$

$$U_{\mu 1}U_{\mu 1}^* + U_{\mu 2}U_{\mu 2}^* + U_{\mu 3}U_{\mu 3}^* = 1 \quad (5)$$

$$U_{\tau 1}U_{\tau 1}^* + U_{\tau 2}U_{\tau 2}^* + U_{\tau 3}U_{\tau 3}^* = 1 \quad (6)$$

$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0 \quad (7)$$

$$U_{e1}U_{\tau 1}^* + U_{e2}U_{\tau 2}^* + U_{e3}U_{\tau 3}^* = 0 \quad (8)$$

$$U_{\mu 1}U_{\tau 1}^* + U_{\mu 2}U_{\tau 2}^* + U_{\mu 3}U_{\tau 3}^* = 0 \quad (9)$$

- to calculate the oscillation probability we can proceed as before

3-flavor oscillation probability

- consider a state produced at $t = 0$ as $|\nu_e\rangle$:

$$|\psi(t=0)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle + U_{e2}\,|\nu_2\rangle + U_{e3}\,|\nu_3\rangle$$

- the wave-function evolves as:

$$|\psi(t)\rangle = |\nu_e\rangle = U_{e1}\,|\nu_1\rangle\,e^{-ip_1\cdot x} + U_{e2}\,|\nu_2\rangle\,e^{-ip_2\cdot x} + U_{e3}\,|\nu_3\rangle\,e^{-ip_3\cdot x},$$

where $p_i \cdot x = E_i t - \vec{p}_i \cdot \vec{x} = E_i t - |\vec{p}|z$ (z-axis in direction of propagation)

- after traveling a distance L:

$$|\psi(L)\rangle = U_{e1}\,|\nu_1\rangle\,e^{-i\phi_1} + U_{e2}\,|\nu_2\rangle\,e^{-i\phi_2} + U_{e3}\,|\nu_3\rangle\,e^{-i\phi_3},$$

where $\phi_i = p_i \cdot x = E_i t - |p|L = (E_i - |p|_i)L$

- as before we can approximate $\phi_i \approx \dfrac{m_i^2}{2E_i}L$

3-flavor oscillation probability

- expressing the mass eigenstates in terms of the weak eigenstates:

$$|\psi(L)\rangle = U_{e1}\Big[U_{e1}^* |\nu_e\rangle + U_{\mu 1}^* |\nu_\mu\rangle + U_{\tau 1}^* |\nu_\tau\rangle\Big] e^{-i\phi_1} \quad (10)$$

$$+ U_{e2}\Big[U_{e2}^* |\nu_e\rangle + U_{\mu 2}^* |\nu_\mu\rangle + U_{\tau 2}^* |\nu_\tau\rangle\Big] e^{-i\phi_2} \quad (11)$$

$$+ U_{e3}\Big[U_{e3}^* |\nu_e\rangle + U_{\mu 3}^* |\nu_\mu\rangle + U_{\tau 3}^* |\nu_\tau\rangle\Big] e^{-i\phi_3} \quad (12)$$

- which can be rearranged to give:

$$|\psi(L)\rangle = \Big[U_{e1}U_{e1}^* e^{-i\phi_1} + U_{e2}U_{e2}^* e^{-i\phi_2} + U_{e3}U_{e3}^* e^{-i\phi_3}\Big] |\nu_e\rangle \quad (13)$$

$$+ \Big[U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3}\Big] |\nu_\mu\rangle \quad (14)$$

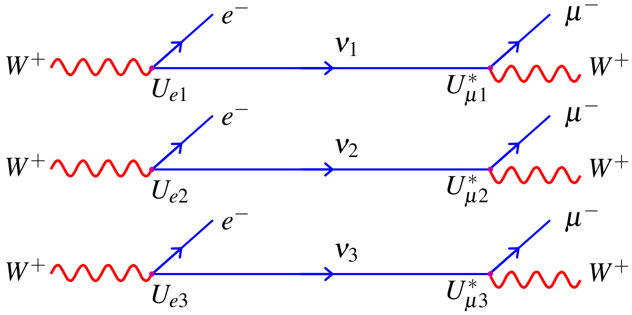
$$+ \Big[U_{e1}U_{\tau 1}^* e^{-i\phi_1} + U_{e2}U_{\tau 2}^* e^{-i\phi_2} + U_{e3}U_{\tau 3}^* e^{-i\phi_3}\Big] |\nu_\tau\rangle \quad (15)$$

- from there:

$$P(\nu_e \to \nu_\mu) = |\langle \nu_\mu | \psi(L) \rangle|^2 \tag{16}$$

$$= \left| U_{e1} U_{\mu 1}^* e^{-i\phi_1} + U_{e2} U_{\mu 2}^* e^{-i\phi_2} + U_{e3} U_{\mu 3}^* e^{-i\phi_3} \right|^2 \tag{17}$$

- the terms in this expression can be represented as:



- because of the unitarity of the PMNS matrix we have:

$$U_{e1}U^*_{\mu 1} + U_{e2}U^*_{\mu 2} + U_{e2}U^*_{\mu 2} = 0 :$$

unless the phases of the different components are different, the sum of these three diagrams is 0, i.e., **need different ν_i masses for oscillation**

- evaluate

$$P(\nu_e \to \nu_\mu) = \left| U_{e1}U_{\mu 1}^* e^{-i\phi_1} + U_{e2}U_{\mu 2}^* e^{-i\phi_2} + U_{e3}U_{\mu 3}^* e^{-i\phi_3} \right|^2$$

using $|z_1 + z_2 + z_3|^2 \equiv |z_1|^2 + |z_2|^2 + |z_3|^2 + 2\mathcal{R}(z_1 z_2^* + z_1 z_3^* + z_2 z_3^*)$

$$P(\nu_e \to \nu_\mu) = \left|U_{e1}U_{\mu 1}^*\right|^2 + \left|U_{e2}U_{\mu 2}^*\right|^2 + \left|U_{e3}U_{\mu 3}^*\right|^2 + \tag{18}$$

$$2\mathcal{R}(U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2} e^{-i(\phi_1-\phi_2)} + U_{e1}U_{\mu 1}^* U_{e3}^* U_{\mu 3} e^{-i(\phi_1-\phi_3)} + \tag{19}$$

$$U_{e2}U_{\mu 2}^* U_{e3}^* U_{\mu 3} e^{-i(\phi_2-\phi_3)}) \tag{20}$$

- can simplify the expression using $\left|U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^*\right|^2 = 0$

$$\implies \left|U_{e1}U_{\mu 1}^*\right|^2 + \left|U_{e2}U_{\mu 2}^*\right|^2 + \left|U_{e3}U_{\mu 3}^*\right|^2 =$$

$$= -2\mathcal{R}\left(U_{e1}U_{\mu 1}^* U_{e2}^* U_{\mu 2} + U_{e1}U_{\mu 1}^* U_{e3}^* U_{\mu 3} + U_{e2}U_{\mu 2}^* U_{e3}^* U_{\mu 3}\right)$$

- substituting the last expression into Eq. 18:

$$P(\nu_e \to \nu_\mu) = 2\mathcal{R}\Big\{U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}\big[e^{-i(\phi_1-\phi_2)} - 1\big]\Big\} \tag{21}$$

$$+ 2\mathcal{R}\Big\{U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}\big[e^{-i(\phi_1-\phi_3)} - 1\big]\Big\} \tag{22}$$

$$+ 2\mathcal{R}\Big\{U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}\big[e^{-i(\phi_2-\phi_3)} - 1\big]\Big\} \tag{23}$$

- for electron survival probability:

$$P(\nu_e \to \nu_e) = |\langle \nu_e|\psi(L)\rangle|^2 \tag{24}$$

$$= \Big|U_{e1}U^*_{e1}e^{-i\phi_1} + U_{e2}U^*_{e2}e^{-i\phi_2} + U_{e3}U^*_{e3}e^{-i\phi_3}\Big|^2 \tag{25}$$

3-flavor oscillation probability

- using for it U unitarity $U_{e1}U^*_{e1} + U_{e2}U^*_{e2} + U_{e3}U^*_{e3} = 1$ get:

$$P(\nu_e \to \nu_e) = 1 + 2|U_{e1}|^2|U_{e2}|^2\mathcal{R}\Big\{e^{-i(\phi_1-\phi_2)} - 1\Big\} \quad (26)$$

$$+ 2|U_{e1}|^2|U_{e3}|^2\mathcal{R}\Big\{e^{-i(\phi_1-\phi_3)} - 1\Big\} \quad (27)$$

$$+ 2|U_{e2}|^2|U_{e3}|^2\mathcal{R}\Big\{e^{-i(\phi_2-\phi_3)} - 1\Big\} \quad (28)$$

3-flavor oscillation probability

- can simplify this expression using:

$$\mathcal{R}\{e^{-i(\phi_1-\phi_2)}-1\} = \cos(\phi_2-\phi_1)-1 = -2\sin^2\left(\frac{\phi_2-\phi_1}{2}\right) =$$

$$= -2\sin^2\left(\frac{(m_2^2-m_1^2)L}{4E}\right) \text{ with } \phi_i \approx \frac{m_i^2}{2E}L$$

- define

$$\Delta_{21} = \frac{(m_2^2-m_1^2)L}{4E} = \frac{\Delta m_{21}^2 L}{4E} \text{ with } \Delta m_{21}^2 = m_2^2 - m_1^2$$

note that $\Delta_{21} = (\phi_2-\phi_1)/2$ is a phase difference, i.e. dimensionless

- which gives electron neutrino survival probability:

$$P(\nu_e \to \nu_e) = 1 - 4|U_{e1}|^2|U_{e2}|^2\sin^2\Delta_{21} \quad (29)$$

$$- 4|U_{e1}|^2|U_{e3}|^2\sin^2\Delta_{31} \quad (30)$$

$$- 4|U_{e2}|^2|U_{e3}|^2\sin^2\Delta_{32} \quad (31)$$

3-flavor oscillation probability

- similar expressions can be obtained for the muon and tau neutrino survival probabilities
- note that since we only have three neutrino generations there are only two independent mass-squared differences, i.e.:

$$m_3^2 - m_1^2 = (m_3^2 - m_2^2) + (m_2^2 - m_1^2)$$

  and in the above equations only two of the Δ_{ij} are independent
- all expressions are in natural units
- converting to more practical units:

$$\Delta_{21} = 1.27 \frac{\Delta m_{21}^2(\mathrm{eV}^2) L(\mathrm{km})}{E(\mathrm{GeV})} \text{ and } \lambda_{\mathrm{osc}}(\mathrm{km}) = 2.47 \frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)}$$

CP and CPT in the weak interaction

- there are three important discrete symmetries:

$$\boxed{\text{Parity}} \qquad \hat{P}: \quad \vec{r} \to -\vec{r} \tag{32}$$

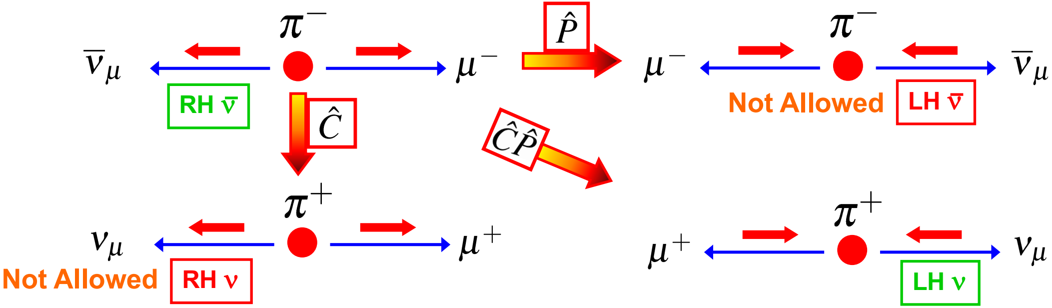
$$\boxed{\text{Time reversal}} \qquad \hat{T}: \quad t \to -t \tag{33}$$

$$\boxed{\text{Charge conjugation}} \qquad \hat{C}: \quad \text{Particle} \to \text{Antiparticle} \tag{34}$$

- the weak interaction violates parity conservation P, but also C

CP and CPT in the weak interaction

- consider pion decay remembering that the neutrino is ultra-relativistic and only left-handed ν and right-handed $\bar{\nu}$ participate in WI:



- hence weak interaction also violates charge conjugation symmetry but appears to be invariant under combined effect of C and P

CP and CPT in the weak interaction

CP transforms:

RH particles $\leftrightarrow$ LH antiparticles

LH particles $\leftrightarrow$ RH antiparticles

- if the weak interaction were invariant under CP, expect:

$$\Gamma\left(\pi^{+} \rightarrow \mu^{+} \nu_{\mu}\right)=\Gamma\left(\pi^{-} \rightarrow \mu^{-} \bar{\nu}_{\mu}\right)$$

- all Lorentz-invariant quantum field theories are invariant under $CPT \implies$ particles and antiparticles have identical mass, lifetime, magnetic moments etc

  Best current experimental test: $m_{K^0} - m_{\bar{K}^0} < 6 \times 10^{-19} m_{K^0}$
- since CPT holds:

  if CP invariance holds $\implies$ time reversal symmetry

  if CP is violated $\implies$ time reversal symmetry violated
- to account for the small excess of matter over antimatter that must have existed early in the universe, require CP violation in particle physics
- CP violation can arise in the weak interaction

CP and T violation in neutrino oscillations

- previously derived the oscillation probability for $\nu_e \to \nu_\mu$:

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) = 2\mathcal{R}\Big\{ & U_{e1}U^*_{\mu 1}U^*_{e2}U_{\mu 2}\big[e^{-i(\phi_1-\phi_2)} - 1\big]\Big\} && (35)\\
+ 2\mathcal{R}\Big\{ & U_{e1}U^*_{\mu 1}U^*_{e3}U_{\mu 3}\big[e^{-i(\phi_1-\phi_3)} - 1\big]\Big\} && (36)\\
+ 2\mathcal{R}\Big\{ & U_{e2}U^*_{\mu 2}U^*_{e3}U_{\mu 3}\big[e^{-i(\phi_2-\phi_3)} - 1\big]\Big\} && (37)
\end{aligned}
$$

- the oscillation probability for $\nu_\mu \to \nu_e$ can be obtained in the same manner or by simply exchanging the labels $(e) \leftrightarrow (\mu)$:

$$
\begin{aligned}
P(\nu_\mu \to \nu_e) = 2\mathcal{R}\Big\{ & U_{\mu 1}U^*_{e1}U^*_{\mu 2}U_{e2}\big[e^{-i(\phi_1-\phi_2)} - 1\big]\Big\} && (38)\\
+ 2\mathcal{R}\Big\{ & U_{\mu 1}U^*_{e1}U^*_{\mu 3}U_{e3}\big[e^{-i(\phi_1-\phi_3)} - 1\big]\Big\} && (39)\\
+ 2\mathcal{R}\Big\{ & U_{\mu 2}U^*_{e2}U^*_{\mu 3}U_{e3}\big[e^{-i(\phi_2-\phi_3)} - 1\big]\Big\} && (40)
\end{aligned}
$$

- unless the elements of the PMNS matrix are real $P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$

CP and T violation in neutrino oscillations

- unless the elements of the PMNS matrix are real $P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$
- if any of the elements of the PMNS matrix are complex, neutrino oscillations are not invariant under time reversal $t \to -t$

CP and T violation in neutrino oscillations

- consider the effects of T, CP, and CPT on neutrino oscillations:

$$\boxed{\text{T}} \quad \nu_e \to \nu_\mu \quad \xrightarrow{\hat{T}} \quad \nu_\mu \to \nu_e \tag{41}$$

$$\boxed{\text{CP}} \quad \nu_e \to \nu_\mu \quad \xrightarrow{\hat{C}\hat{P}} \quad \bar{\nu}_e \to \bar{\nu}_\mu \tag{42}$$

$$\boxed{\text{CPT}} \quad \nu_e \to \nu_\mu \quad \xrightarrow{\hat{C}\hat{P}\hat{T}} \quad \bar{\nu}_\mu \to \bar{\nu}_e \tag{43}$$

  Note that C alone is not sufficient in Eq. 42 as it transforms LH neutrinos into LH antineutrinos (not involved in weak interaction)

- if the weak interactions are invariant under CPT:

  $P(\nu_e \to \nu_\mu) = P(\bar{\nu}_\mu \to \bar{\nu}_e)$

  $P(\nu_\mu \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_\mu)$

- if the PMNS matrix is not purely real then

  $P(\nu_e \to \nu_\mu) \neq P(\nu_\mu \to \nu_e)$

  From above: $P(\nu_e \to \nu_\mu) \neq P(\bar{\nu}_e \to \bar{\nu}_\mu) \implies$ CP is violated in neutrino oscillations!

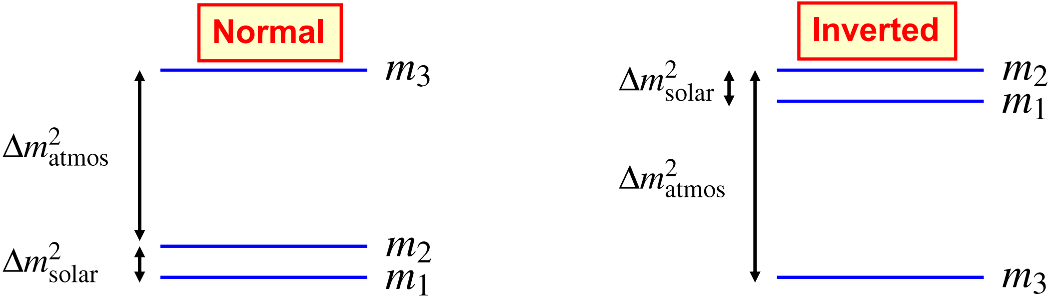
- to date, results on neutrino oscillations only determine

$$\left|\Delta m_{ij}^2\right| = \left|m_j^2 - m_i^2\right|$$

- there are two distinct and very different mass scales:
  - atmospheric neutrino oscillations: $\left|\Delta m^2\right|_{\text{atmos}} \sim 2.5 \times 10^{-3}\text{eV}^2$
  - solar neutrino oscillations: $\left|\Delta m^2\right|_{\text{solar}} \sim 8 \times 10^{-5}\text{eV}^2$

Neutrino mass hierarchy

- two possible assignments of mass hierarchy:



- in both cases: $\Delta m^2_{21} \sim 8 \times 10^{-5} \text{eV}^2$ (solar)
  $|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3} \text{eV}^2$ (atmospheric)
- hence we can approximate $\Delta m^2_{31} \approx \Delta m^2_{32}$

3-flavor oscillations neglecting CP violation

- neglecting CP violation considerably simplifies the algebra of 3-flavor neutrino oscillations
- taking the PMNS matrix to be real:

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) = & -4U_{e1}U_{\mu1}U_{e2}U_{\mu2}\sin^2\Delta_{21} && (44)\\
& -4U_{e1}U_{\mu1}U_{e3}U_{\mu3}\sin^2\Delta_{31} && (45)\\
& -4U_{e2}U_{\mu2}U_{e3}U_{\mu3}\sin^2\Delta_{32} && (46)
\end{aligned}
$$

with $\Delta_{ij} = \frac{(m_j^2 - m_i^2)L}{4E} = \frac{\Delta m_{ij}^2 L}{4E}$

- using $\Delta_{31} \approx \Delta_{32}$

$$
\begin{aligned}
P(\nu_e \to \nu_\mu) = & -4U_{e1}U_{\mu1}U_{e2}U_{\mu2}\sin^2\Delta_{21} && (47)\\
& -4(U_{e1}U_{\mu1} + U_{e2}U_{\mu2})U_{e3}U_{\mu3}\sin^2\Delta_{32} && (48)
\end{aligned}
$$

- using $U_{e1}U_{\mu1}^* + U_{e2}U_{\mu2}^* + U_{e3}U_{\mu3}^* = 0$

  $P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu1}U_{e2}U_{\mu2}\sin^2\Delta_{21} + 4U_{e3}^2U_{\mu3}^2\sin^2\Delta_{32}$

3-flavor oscillations neglecting CP violation

- can apply $\Delta_{31} \approx \Delta_{32}$ to the expression for ν_e survival probability:

$$P(\nu_e \to \nu_e) = 1 - 4U_{e1}^2 U_{e2}^2 \sin^2 \Delta_{21} - 4U_{e1}^2 U_{e3}^2 \sin^2 \Delta_{31} - 4U_{e2}^2 U_{e3}^2 \sin^2 \Delta_{32} \tag{49}$$

$$\approx 1 - 4U_{e1}^2 U_{e2}^2 \sin^2 \Delta_{21} - 4\big(U_{e1}^2 + U_{e2}^2\big) U_{e3}^2 \sin^2 \Delta_{32} \tag{50}$$

- which can be simplified using $U_{e1}U_{e1}^* + U_{e2}U_{e2}^* + U_{e3}U_{e3}^* = 1$:

$\implies$ $P(\nu_e \to \nu_\mu) \approx -4U_{e1}U_{\mu 1}U_{e2}U_{\mu 2} \sin^2 \Delta_{21} + 4U_{e3}^2 U_{\mu 3}^2 \sin_{32}^2$

3-flavor oscillations neglecting CP violation

- neglecting CP violation and taking $\left|\Delta m^2_{31}\right| \approx \left|\Delta m^2_{32}\right|$ obtain the following expressions for neutrino oscillation probabilities:

$$P(\nu_e \to \nu_e) \approx 1 - 4U^2_{e1}U^2_{e2}\sin^2\Delta_{21} - 4\big(1 - U^2_{e3}\big)U^2_{e3}\sin^2\Delta_{32} \tag{51}$$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - 4U^2_{\mu1}U^2_{\mu2}\sin^2\Delta_{21} - 4\big(1 - U^2_{\mu3}\big)U^2_{\mu3}\sin^2\Delta_{32} \tag{52}$$

$$P(\nu_\tau \to \nu_\tau) \approx 1 - 4U^2_{\tau1}U^2_{\tau2}\sin^2\Delta_{21} - 4\big(1 - U^2_{\tau3}\big)U^2_{\tau3}\sin^2\Delta_{32} \tag{53}$$

$$P(\nu_e \to \nu_\mu) = P(\nu_\mu \to \nu_e) \approx -4U_{e1}U_{\mu1}U_{e2}U_{\mu2}\sin^2\Delta_{21} + 4U^2_{e3}U^2_{\mu3}\sin^2\Delta_{32} \tag{54}$$

$$P(\nu_e \to \nu_\tau) = P(\nu_\tau \to \nu_e) \approx -4U_{e1}U_{\tau1}U_{e2}U_{\tau2}\sin^2\Delta_{21} + 4U^2_{e3}U^2_{\tau3}\sin^2\Delta_{32} \tag{55}$$

$$P(\nu_\mu \to \nu_\tau) = P(\nu_\tau \to \nu_\mu) \approx -4U_{\mu1}U_{\tau1}U_{\mu2}U_{\tau2}\sin^2\Delta_{21} + 4U^2_{\mu3}U^2_{\tau3}\sin^2\Delta_{32} \tag{56}$$

3-flavor oscillations neglecting CP violation

- the wavelength associated with $\sin^2\Delta_{21}$ and $\sin^2\Delta_{32}$ are:

"SOLAR"

$$\lambda_{21} = \frac{4\pi E}{\Delta m^2_{21}}$$

"Long"-Wavelength

and

$$\lambda_{32} = \frac{4\pi E}{\Delta m^2_{32}}$$

"ATMOSPHERIC"

"Short"-Wavelength

- PMNS matrix is expressed in terms of 3 rotation angles θ_{12}, θ_{23}, θ_{13}, and a complex phase δ, using the notation $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$:

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{"Atmospheric"}} \times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{"Solar"}}$$

Dominates: "Atmospheric" ... "Solar"

PMNS matrix

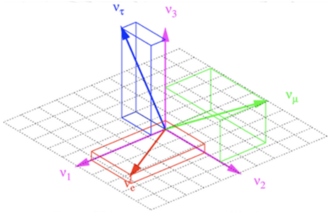
- writing out in full:

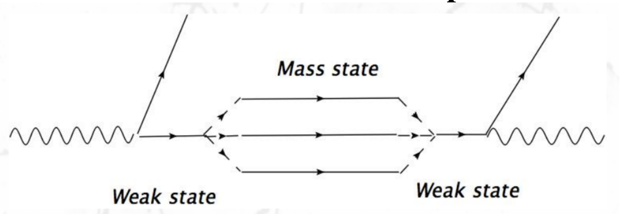
$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- there are six SM parameters that can be be measured in ν oscillation experiments

$\lvert\Delta m_{21}\rvert^2 = \lvert m_2^2 - m_1^2\rvert$	θ_{12}	**Solar and reactor neutrino experiments**
$\lvert\Delta m_{32}\rvert^2 = \lvert m_3^2 - m_2^2\rvert$	θ_{23}	**Atmospheric and beam neutrino experiments**
	θ_{13}	**Reactor neutrino experiments + future beam**
	δ	**Future beam experiments**

PMNS matrix: current picture





ν oscillation:

Atmospheric — Reactor — Solar

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\theta_{23} \approx 45°$ — *Atmospheric exp.*

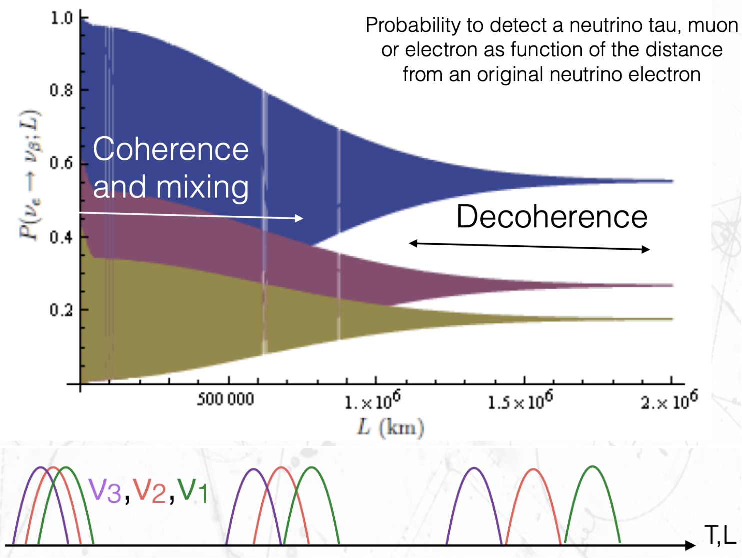
$\theta_{13} \approx 10°$ — *Reactor*

$\theta_{12} \approx 35°$ — *Solar exp.*, *Reactor LBL*

Accelerator LBL

LBL = Long BaseLine – new generation of the long baseline experiments

Quantum (de)coherence



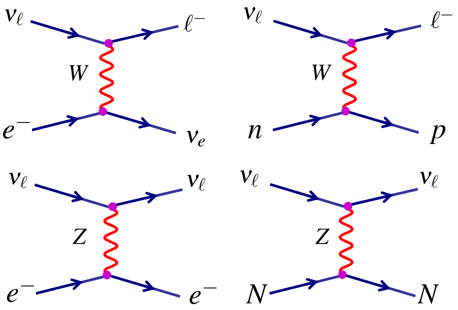
Neutrino experiments

Before discussing current experimental data, need to consider how ν interact in matter:

- two processes
  - charged current (CC) interactions (via a W boson) $\Longrightarrow$ charged lepton
  - neutral current (NC) interactions (via a Z boson)
- two possible "targets": can have neutrino interactions with
  - atomic electrons
  - nucleons within the nucleus

CHARGED CURRENT

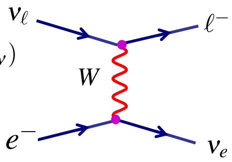
NEUTRAL CURRENT



Neutrino interaction thresholds

- neutrino detection method depends on the neutrino energy and (weak) flavor
  - neutrinos from the sun and nuclear reactions have $E_\nu \sim 1$ MeV
  - atmospheric neutrinos have $E_\nu \sim 1$ GeV
- these energies are relatively low and not all interactions are kinematically allowed, i.e. there is a threshold energy before an interaction can occur $\implies$ need sufficient energy in the centre-of-mass frame to produce final state particles

1. Charged current interactions on atomic electrons (in lab. frame)



$$p_\nu = (E_\nu, 0, 0, E_\nu)$$
$$p_e = (m_e, 0, 0, 0)$$

$$s = (p_\nu + p_e)^2 = (E_\nu + m_e)^2 - E_\nu^2$$

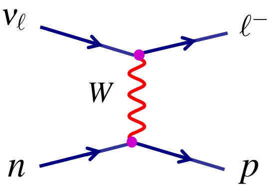
Require: $s > m_\ell^2$

$$\Rightarrow \quad E_\nu > \left[\left(\frac{m_\ell}{m_e}\right)^2 - 1\right]\frac{m_e}{2}$$

- putting in the numbers, for CC interactions with atomic electrons require: $E_{\nu_e} > 0$, $E_{\nu_\mu} > 11$ GeV, $E_{\nu_\tau} > 3090$ GeV
- for ν_μ, ν_τ high energy thresholds compared to typical energies considered here

Neutrino interaction thresholds

2 Charged current interactions on nucleons (in lab. frame):



$$s = (p_\nu + p_n)^2 = (E_\nu + m_n)^2 - E_\nu^2$$

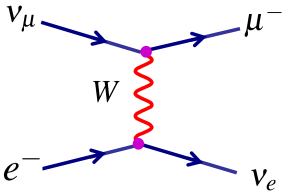
Require: $s > (m_\ell + m_p)^2$

$$\Rightarrow \boxed{E_\nu > \frac{(m_p^2 - m_n^2) + m_\ell^2 + 2m_p m_\ell}{2m_n}}$$

- for CC interactions with neutrons: $E_{\nu_e} > 0$, $E_{\nu_\mu} > 110$ MeV, $E_{\nu_\tau} > 3.5$ GeV
- ν_e from the sun and nuclear reactors $E_\nu \sim 1$ MeV which oscillate into ν_μ and ν_τ cannot interact via charged current interactions: "they effectively disappear"
- atmospheric ν_μ $E_\nu \sim 1$ GeV which oscillate into ν_τ cannot interact via charged current interactions: "disappear"
- to date, most experimental signatures for neutrino oscillation are a deficit of neutrino interactions (with the exception of SNO and OPERA) because below threshold for producing lepton of different flavor from original neutrino

Neutrino interaction thresholds

- previously derived CC νq cross sections in ultra-relativistic limit (neglecting $m(\nu/q)$)
- for high energy ν_μ can directly use previous results:



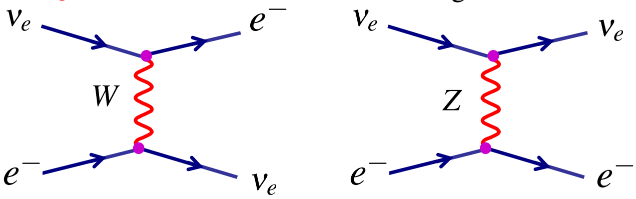
$$\sigma_{\nu_\mu e^-} = \frac{G_F^2 s}{\pi}$$

with $\quad s = (E_\nu + m_e)^2 - E_\nu^2 \approx 2m_e E_\nu$

$$\boxed{\sigma_{\nu_\mu e^-} = \frac{2m_e G_F^2 E_\nu}{\pi}}$$

Cross section increases linearly with lab. frame neutrino energy

- for ν_e there is another lowest order diagram with the same final state:

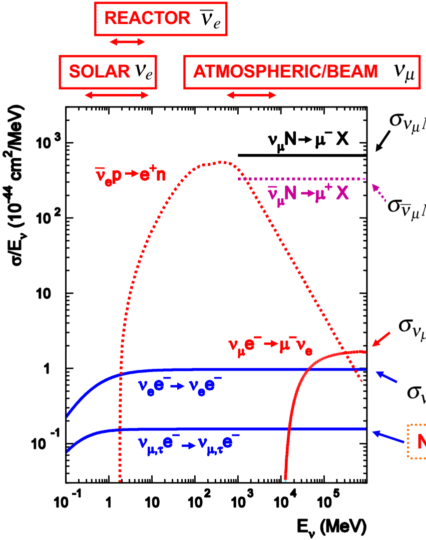


  Total cross section is lower than the pure CC cross section due to negative interference $|M_{CC} + M_{NC}|^2 < |M_{CC}|^2$: $\sigma_{\nu_e e} \approx 0.6\sigma_{\nu_e e}^{CC}$

- in the high energy limit, the CC νN cross sections are larger due to the higher center-of-mass energy: $s = (E_\nu + m_n)^2 - E_\nu^2 \approx 2m_n E_\nu$

Neutrino detection

The detector technology/interaction process depends on neutrino type and energy:



$$\sigma_{\nu_\mu N} = 0.22\frac{2m_p G_F^2 E_\nu}{\pi}$$

$$\sigma_{\overline{\nu}_\mu N} = 0.33\frac{2m_p G_F^2 E_\nu}{3\pi}$$

Deep Inelastic Scattering (p. 332)

$$\sigma_{\nu_\mu e^-} = \frac{2m_e G_F^2 E_\nu}{\pi}$$

CC only Threshold 11 GeV

$$\sigma_{\nu_e e^-} \approx 0.6\frac{2m_e G_F^2 E_\nu}{\pi}$$

NC + CC

NC only

Atmospheric/beam neutrinos

ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$: $E_\nu > 1$ GeV

1. water Cherenkov: e.g. Super Kamiokande
2. Iron Calorimeters: e.g. MINOS, CDHS

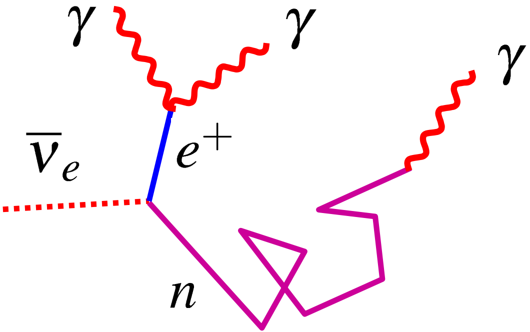
Produce high energy charged lepton: relatively easy to detect

ν_e: $E_\nu < 20$ MeV

1. water Cherenkov: e.g. Super Kamiokande
   - detect Cherenkov light from electron produced in $\nu_e + e^- \to \nu_e + e^-$
   - because of background from natural radioactivity limited to $E_\nu > 5$ MeV
   - because Oxygen is a doubly magic nucleus don't get $\nu_e + n \to e^- + p$
2. Radio-Chemical: e.g. Homestake, SAGE, GALLEX
   - use inverse β-decay process, e.g. $\nu_e +^{71}\text{Ga} \to e^- +^{71}\text{Ge}$
   - chemically extract produced isotope and count decays (only gives a rate)

$\bar{\nu}_e$: $E_{\bar{\nu}} < 5$ MeV

1. liquid scintillator: e.g. KamLAND
   - low energies $\implies$ large radioactive background
   - dominant interaction: $\bar{\nu}_e + p \to e^+ + n$
   - prompt positron annihilation signal + delayed signal from n (space/time correlation reduces background)
   - electrons produced by photons excite the scintillator which produces light



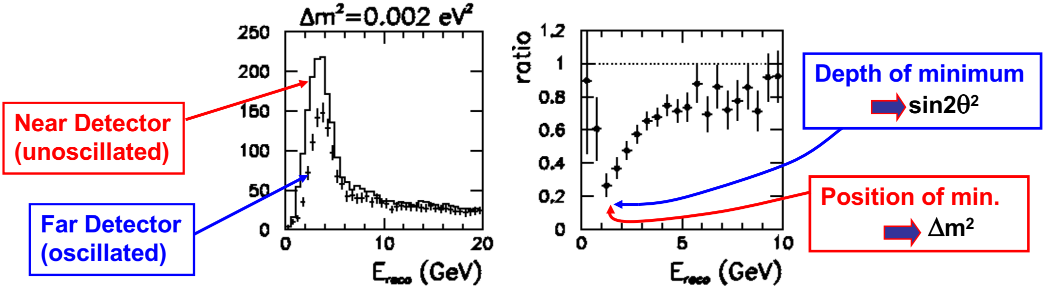
Long baseline neutrino experiments

- initial studies of ν oscillations done with atmospheric and solar ν
- now the emphasis is on neutrino beam experiments
- allows to take control: design an experiment with specific goals
- many long baseline ν oscillation experiments were taking data:
  - K2K in Japan, MINOS in the US, CNGS in Europe
- and currently taking data:
  - T2K in Japan, NOvA in the US
- new ultimate long baseline experiments are currently under construction:
  - HyperK in Japan and DUNE in the US

Long baseline neutrino experiments

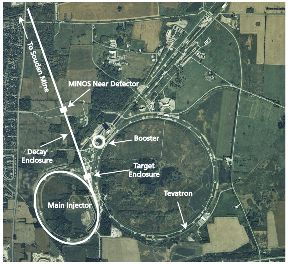
Basic idea:

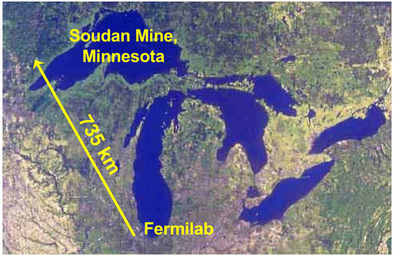
- intense ν beam
- two detectors: one close to beam, the other hundreds of km away
- measure ratio of the neutrino energy spectrum in the far detector (oscillated) to that in the near detector (unoscillated)
- partial cancellation of systematic biases



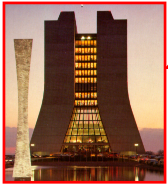
MINOS (2005 – 2016)

- 120 GeV protons extracted from the Main injector at Fermilab
- 2.5×10^{13} p/pulse hit target $\Longrightarrow$ very intense beam 0.3 MW on target





Two detectors:

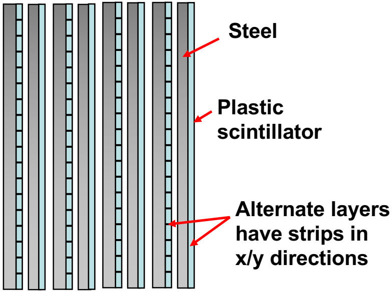


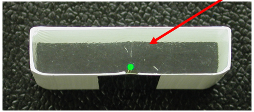
- ★ 1000 ton, NEAR Detector at Fermilab : 1 km from beam
- ★ 5400 ton FAR Detector, 720m underground in Soudan mine, N. Minnesota: 735 km from beam

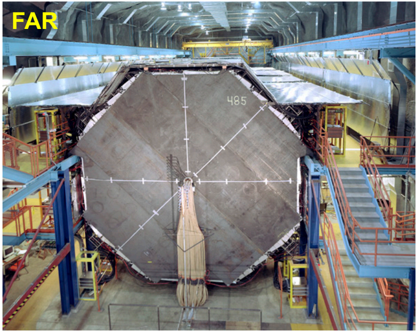
https://www-numi.fnal.gov/

The MINOS Detectors:

- Dealing with high energy neutrinos $E_\nu > 1\,\text{GeV}$
- The muons produced by ν_μ interactions travel several metres
- Steel-Scintillator sampling calorimeter
- Each plane: 2.54 cm steel +1 cm scintillator
- A charged particle crossing the scintillator produces light – detect with PMTs

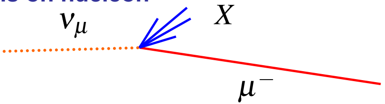




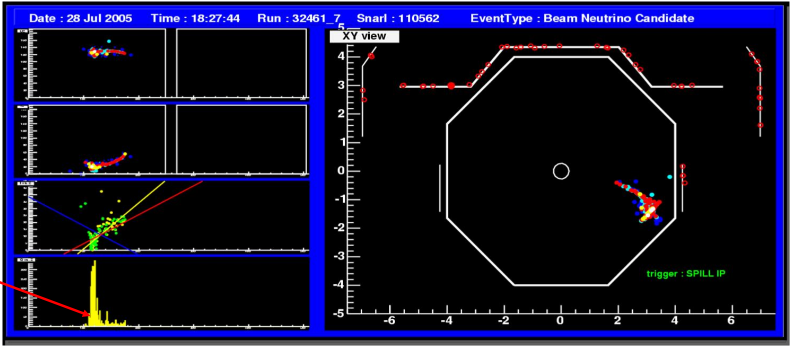


- **Neutrino detection via CC interactions on nucleon**

$$\nu_\mu + N \rightarrow \mu^- + X$$



Example event:



Signal from hadronic shower

- **The main feature of the MINOS detector is the very good neutrino energy resolution**

$$E_\nu = E_\mu + E_X$$

- **Muon energy from range/curvature in B-field**
- **Hadronic energy from amount of light observed**

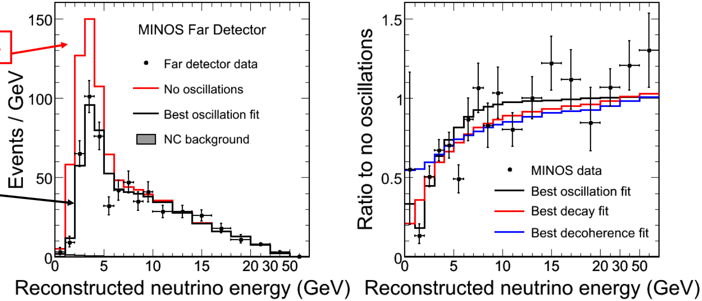
MINOS results

- for the MINOS experiment, L is fixed and observe oscillations as function of E_ν
- for $\left|\Delta m^2_{32}\right| \sim 2.5 \times 10^{-3}$ eV^2 first oscillation minimum at $E_\nu = 1.5$ GeV
- to a very good approximation can use 2-flavor case as oscillations corresponding to $\left|\Delta m^2_{21}\right| \sim 8 \times 10^{-5}$ eV^2 occur at $E_\nu = 50$ MeV, beam contains very few neutrinos at this energy + well below detection threshold

MINOS Collaboration, Phys. Rev. Lett. 101, 131802, 2008

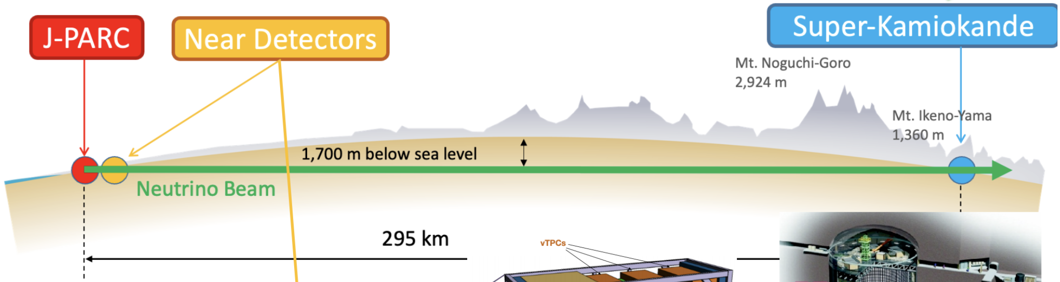
no oscillations

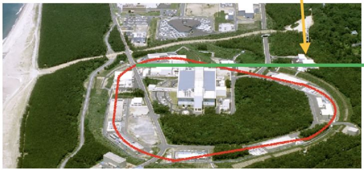
best fit osc

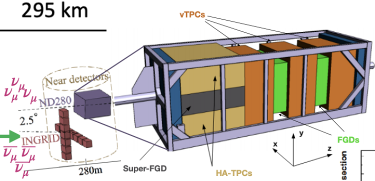


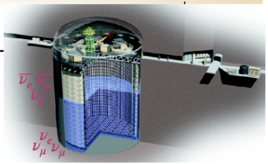
$$|\Delta m^2_{32}| = (2.43 \pm 0.12) \times 10^{-3}\,\text{eV}^2$$

T2K experiment



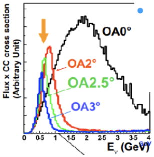






- **Beam:** 30 GeV protons on a graphite target, producing Kaons and Pions

- **ND site:** *on and off axis detectors:*
  - Beam monitoring
  - Tune flux & interaction models predictions f
  - ν cross-sections
  - Magnetized 0.2T
  - Upgraded ND280 completed



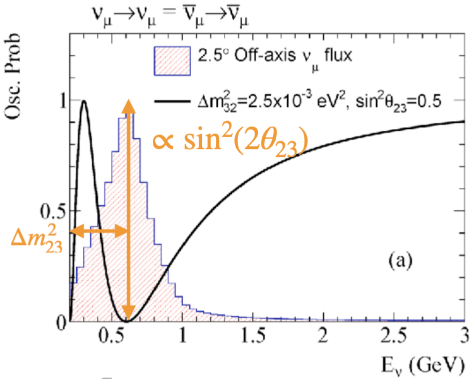
- **FD site:**
  - Super-Kamiokande
  - 50kton water Cherenkov detector

Oscillation at long baseline

Main idea: produce $\nu_\mu/\bar{\nu}_\mu$ beam and perform the measurement of rate, energy and flavor before and after oscillation

- Disappearance channel

$$P(\nu_\mu \to \nu_\mu) = P\left(\frac{L}{E}, \theta_{23}, \Delta m^2_{23}\right)$$

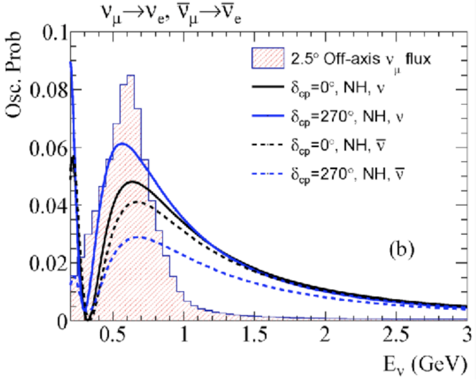


Oscillation at long baseline

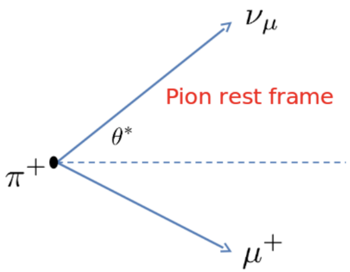
Main idea: produce $\nu_\mu/\bar{\nu}_\mu$ beam and perform the measurement of rate, energy and flavor before and after oscillation

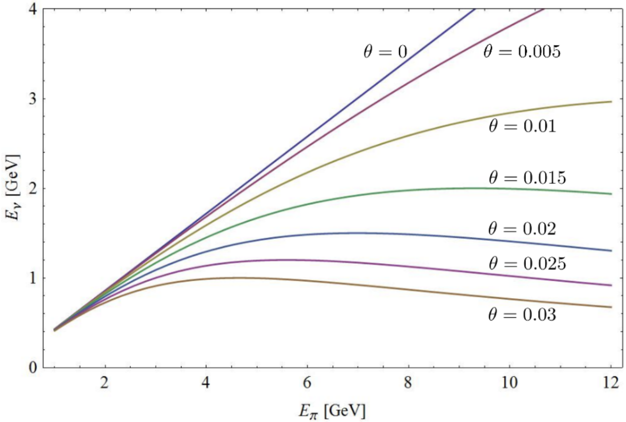
- Appearance channel

$$P(\nu_\mu \to \nu_e/\bar{\nu}_\mu \to \bar{\nu}_e) = P\left(\frac{L}{E}, \theta_{23}, \theta_{13}, \Delta m^2_{21}, \Delta m^2_{23}, \pm \sin\delta_{CP}\right)$$

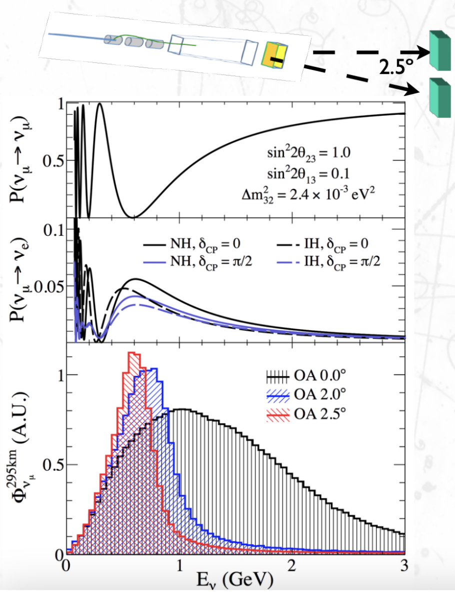


Off(On)-axis beam





Off(On)-axis beam



Off-axis

- off-axis optimises the flux at the maximum of the oscillation.
- Only one oscillation maximum can be measured at a fixed distance.
- Narrow beam less dependent on beam uncertainties but more on beam pointing.
- Lower energies achieved.

On-axis

- on-axis optimises the total integrated flux.
- Spectrum with higher neutrino energy (longer oscillation distances)
- If broad enough, more than one oscillation maximum can be measured at a fixed distance.

T2K-only oscillation results

First presented at Neutrino 2024 : https://doi.org/10.5281/zenodo.12704703

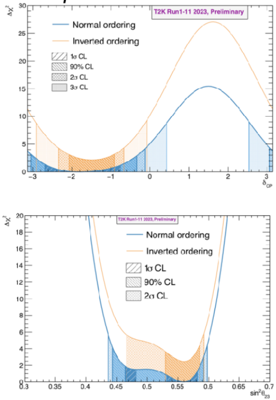
δ_{CP}:

- Preference for $\delta_{CP} \approx -\frac{\pi}{2}$
- Jarlskog-invariant gives a parametrized independent way to measure CP violation
- CP conservation excluded at >2σ in case of IO and <2σ for NO

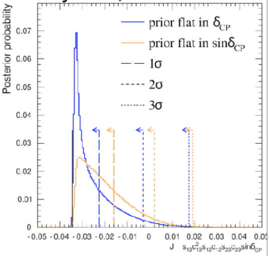
θ_{23} and mass ordering:

- Preference for NO and upper octant but not significant

Frequentist



Bayesian, both MO



Best fit:

	Normal ordering	Inverted ordering
$\sin^2(\theta_{13})/10^{-3}$	$(21.9^{+0.9}_{-0.5})$	$(22.0^{+1.0}_{-0.4})$
δ_{CP}	$-2.08^{+1.33}_{-0.61}$	$-1.41^{+0.64}_{-0.82}$
Δm^2_{32} (NO)/Δm^2_{31} (IO)	$(2.521^{+0.037}_{-0.050})10^{-3}\mathrm{eV}^2/\mathrm{c}^4$	$(-2.486^{+0.043}_{-0.044})10^{-3}\mathrm{eV}^2/\mathrm{c}^4$
$\sin^2(\theta_{23})$	$0.568^{+0.024}_{-0.125}$ (90%)	$0.567^{+0.021}_{-0.048}$ (90%)

13

T2K-NOvA joint fit

Challenges :

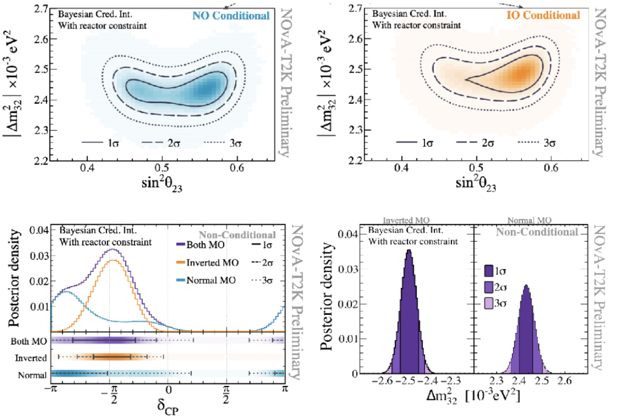
- Main source of correlations: Cross-section model
  - Studied artificial scenarios to see possible correlations
  - Evaluate the robustness of the fit against various models
  - Cross-experiment models after ND constraint

θ_{23} and $\left|\Delta m^2_{32}\right|$:

- Results still consistent with maximal mixing of θ_{23}

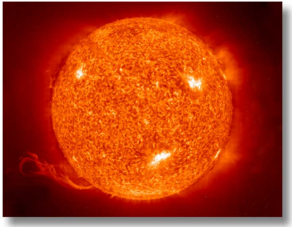
δ_{CP}:

- $\delta_{CP} = \frac{\pi}{2}$ excluded at 3σ for both mass ordering
- In case of IO, CP-conservation is excluded at 3σ



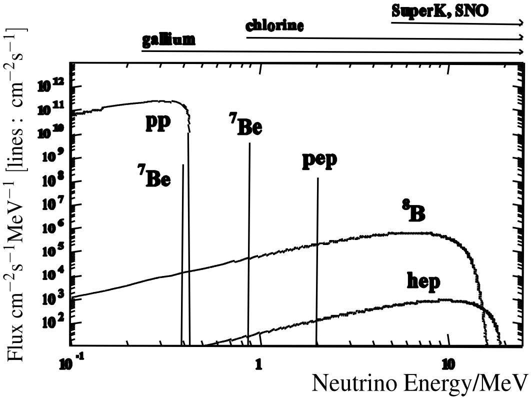
First presented at Fermilab on February, 16th, 2024 : https://indico.fnal.gov/event/62062/contributions/279004/attachments/175258/237774/021624_NOvAT2K_JointFitResults_ZV.pdf

Solar neutrinos



The Sun is powered by the weak interaction: producing a very large flux of ν_e

$2 \times 10^{38} \nu_e \mathrm{s}^{-1}$



- different nuclear reactions in the sun $\implies$ complex E_ν spectrum

  $p + p \to d + e^+ + \nu_e$ $E_\nu < 0.5$ MeV

  ${}^8B \to {}^8Be^* + e^+ + \nu_e$ $E_\nu \sim 5$ MeV

  $p + e^- + p \to d + \nu_e$

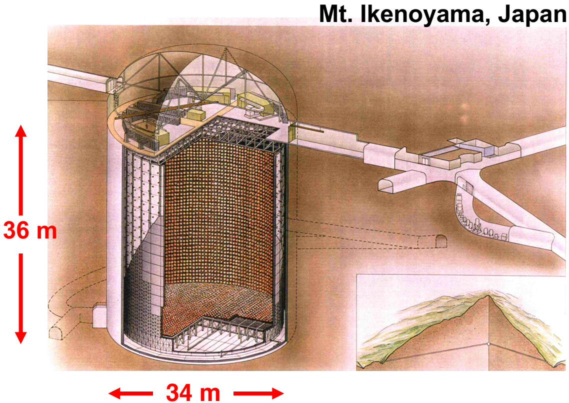
  ${}^7Be + e^- \to {}^7Li + \nu_e$

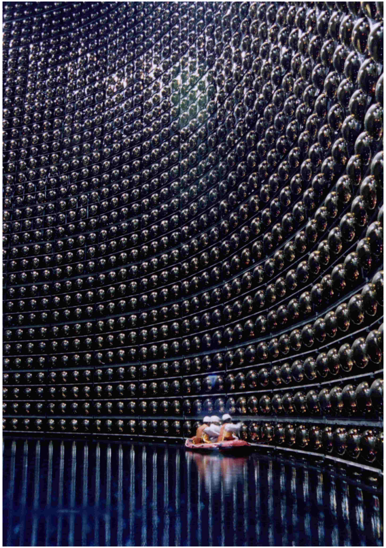
  ${}^3He + p \to {}^4He + e^+ + \nu_e$

- all experiments saw a deficit of ν_e compared to prediction: the solar neutrino problem

Solar neutrinos I: Super Kamiokande

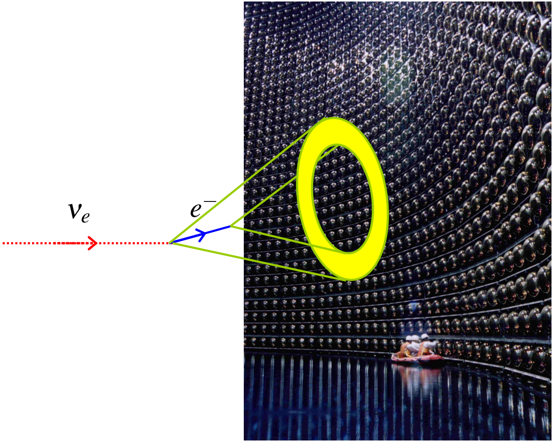
- 50 kton water Cherenkov detector
- water viewed by 11146 PMT
- deep underground to filter out cosmic rays
  otherwise too much background

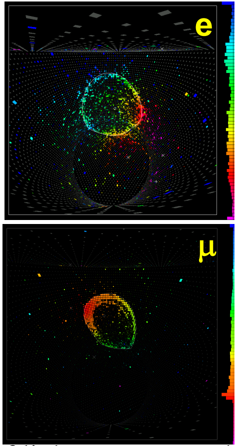




Solar neutrinos I: Super Kamiokande

- detect neutrinos by observing Cherenkov radiation from charged particles which travel faster than speed of light in water c/n



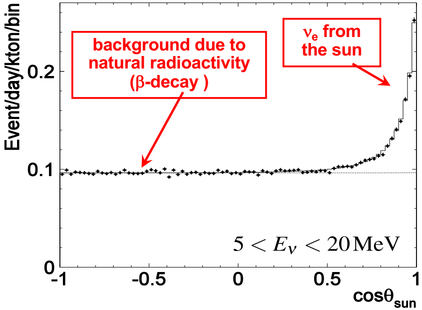


- can distinguish electrons from muons from pattern of light: muons produce clean rings whereas electrons produce more diffuse “fuzzy” rings

Solar neutrinos I: Super Kamiokande

- sensitive to solar neutrinos with $E_\nu > 5$ MeV
- for lower energies too much background from natural radioactivity (β-decays)
- hence detect mostly neutrinos from ${}^8B \rightarrow {}^8 Be^* + e^+ + \nu_e$
- detect electron Cherenkov rings from $\nu_e + e^- \rightarrow \nu_e + e^-$
- in lab frame the e^- is produced preferentially along the ν_e direction

S.Fukada et al., Phys. Rev. Lett. 86 5651-5655, 2001



Results:

- clear signal of ν from the sun
- too few neutrinos:

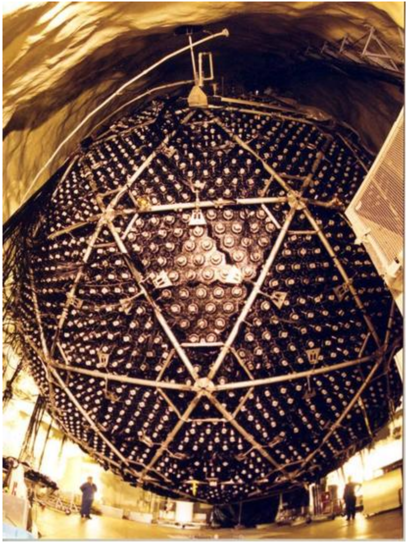
$$\text{Data/SSM} = 0.45 \pm 0.02$$

SSM="Standard Solar Model" prediciton

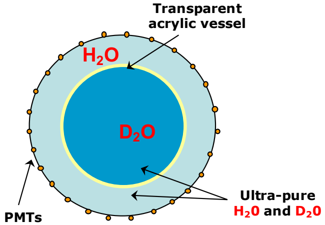
The solar Neutrino "Problem"

Solar neutrinos II: SNO

Sudbury Neutrino Observatory located in a deep mine in Ontario, Canada



- 1 kton heavy water (D_2O) Cherenkov detector
- D_2O inside a 12m diameter acrylic vessel
- surrounded by 3 kton of H_2O
- main experimental challenge: need for very low background from radioactivity
- ultra-pure H_2O and D_2O
- surrounded by 9546 PMTs



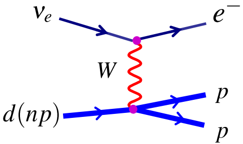
Solar neutrinos II: SNO

Detect Cherenkov light from three different reactions:

CHARGE CURRENT

- Detect Čerenkov light from electron
- Only sensitive to ν_e (thresholds)
- Gives a measure of ν_e flux

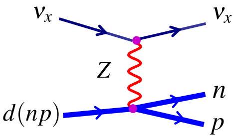
$$\text{CC Rate} \propto \phi(\nu_e)$$



NEUTRAL CURRENT

- Neutron capture on a deuteron gives **6.25 MeV**
- Detect Čerenkov light from electrons scattered by γ
- Measures total neutrino flux

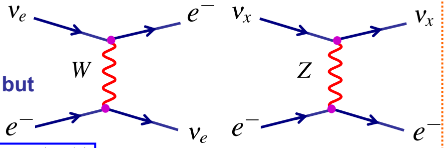
$$\text{NC Rate} \propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$$



ELASTIC SCATTERING

- Detect Čerenkov light from electron
- Sensitive to all neutrinos (NC part) – but larger cross section for ν_e

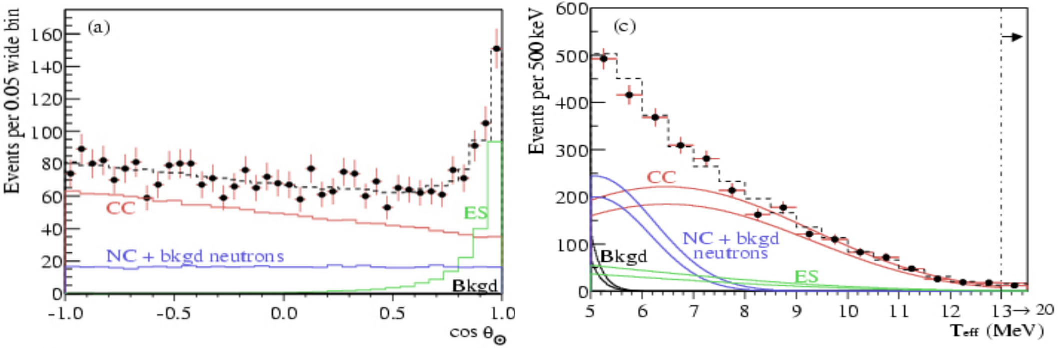
$$\text{ES Rate} \propto \phi(\nu_e) + 0.154(\phi(\nu_\mu) + \phi(\nu_\tau))$$



Solar neutrinos II: SNO

- experimentally can determine rates for different interactions from:
  - angle with respect to sun: electrons from ES point back to sun
  - energy: NC events have lower energy – 6.25 MeV γ from n capture
  - radius from center of detector: gives a measure of background from neutrinos

SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89:011301, 2002

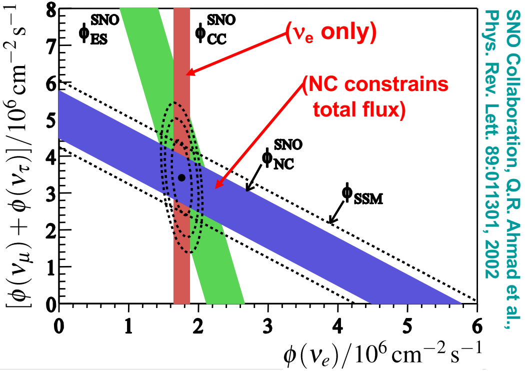


- using different distributions measure number of events of each type:
  - CC: $1968 \pm 61 \propto \phi(\nu_e)$
  - ES: $264 \pm 26 \propto \phi(\nu_e) + 0.154[\phi(\nu_\mu) + \phi(\nu_\tau)]$
  - NC: $576 \pm 50 \propto \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau)$

$\Longrightarrow$ Measure of electron neutrino flux + total flux!

Solar neutrinos II: SNO

- using known cross sections can convert observed number of events into fluxes
- the different processes impose different constraints
- where constraints meet gives separate measurements of ν_e and $\nu_\mu + \nu_\tau$ fluxes



SNO results:

$\phi(\nu_e) = (1.8 \pm 0.1) \times 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

$\phi(\nu_\mu) + \phi(\nu_\tau) = (3.4 \pm 0.6) \times 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

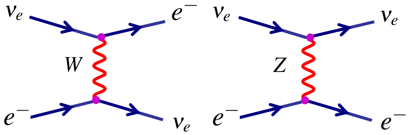
SSM prediction:

$\phi(\nu_e) = 5.1 \times 10^{-6}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$

- clear evidence for a flux of ν_μ and/or ν_τ from the sun
- total neutrino flux is consistent with expectation from SSM
- clear evidence of $\nu_e \to \nu_\mu$ and/or $\nu_e \to \nu_\tau$ neutrino transitions

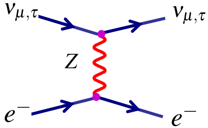
Interpretation of solar neutrino data

- the interpretation of the solar neutrino data is complicated by matter effects
  - the quantitative treatment is nontrivial and is not discussed
  - basic idea is that as ν leaves the sun it crosses a region of high electron density
  - the coherent forward scattering process ($\nu_e \rightarrow \nu_e$) for an electron neutrino



CC + NC

is different to that for a muon or tau neutrino



NC

  - can enhance oscillations - “MSW effect”
- a combined analysis of all solar neutrino data gives:

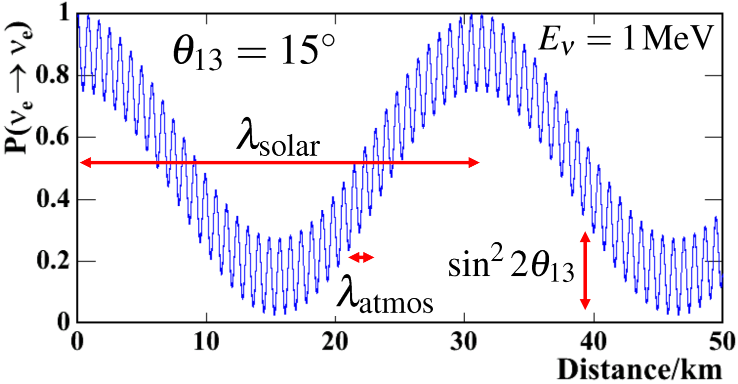
$$\Delta m^2_{\text{solar}} \approx 8 \times 10^{-5} \text{eV}^2, \sin^2 2\theta_{\text{solar}} \approx 0.85$$

- to explain reactor neutrino experiments we need full three neutrino expression for the electron neutrino survival probability which depends on U_{e1}, U_{e2}, U_{e3}
- substituting these PMNS matrix elements in the expression:

$$
\begin{aligned}
P(\nu_e \to \nu_e) &\approx 1 - 4U_{e1}^2 U_{e2}^2 \sin^2 \Delta_{21} - 4\big(1 - U_{e3}^2\big) U_{e3}^2 \sin^2 \Delta_{32} && (57)\\
&= 1 - 4(c_{12}c_{13})^2 (s_{12}c_{13})^2 \sin^2 \Delta_{21} - 4(1 - s_{13}^2) s_{13}^2 \sin^2 \Delta_{32} && (58)\\
&= 1 - c_{13}^4 (2s_{12}c_{12})^2 \sin^2 \Delta_{21} - (2c_{13}s_{13})^2 \sin^2 \Delta_{32} && (59)\\
&= 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{32} && (60)
\end{aligned}
$$

- contributions with short (atmospheric) and long (solar) wavelengths

For a 1 MeV neutrino:



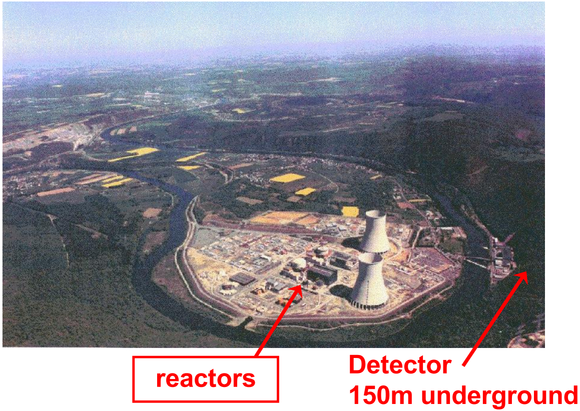
$$\lambda_{osc}(\mathrm{km}) = 2.47\frac{E(\mathrm{GeV})}{\Delta m^2(\mathrm{eV}^2)} \implies \lambda_{21} = 30.0\mathrm{km},\ \lambda_{32} = 0.8\mathrm{km}$$

Amplitude of short wavelength oscillations given by $\sin^2 2\theta_{13}$

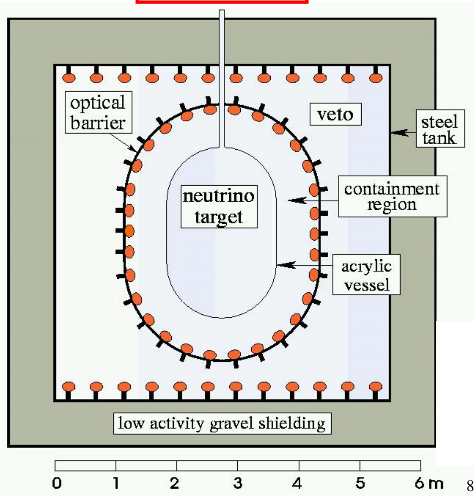
Reactor experiments I: CHOOZ

France

- two nuclear reactors, each 4.2 GW
- detector is 1 km from reactor cores
- reactors produce intense flux of $\bar{\nu}_e$



Detector

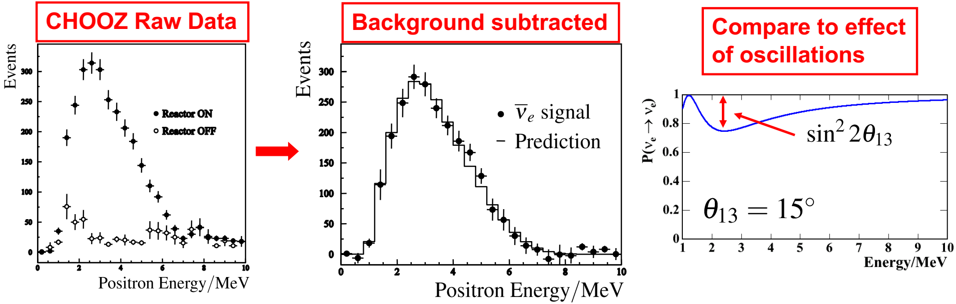


Reactor experiments I: CHOOZ

- antineutrinos interact via inverse β-decay: $\bar{\nu}_e + p \to e^+ + n$
- detector: liquid scintillator with Gd (large n capture cross section)
- detect γ from e^+ annihilation and a delayed signal from γ from n capture on Gd: $e^+ + e^- \to \gamma + \gamma, n + Gd \to Gd^* \to Gd + \gamma + \gamma + \dots$

- at 1km and energies > 1 MeV, only short wavelength component matters:

$$P(\nu_e \to \nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32} \approx 1 - \sin^2 2\theta_{13}\sin^2\Delta_{32}$$



- data agree with unoscillated prediction both in terms of rate and energy spectrum:

$$N_{\text{data}}/N_{\text{expect}} = 1.01 \pm 0.04$$

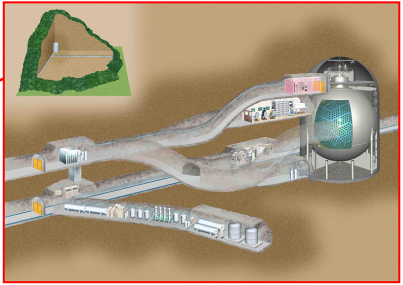
Reactor experiments I: CHOOZ

- hence $\sin^2 2\theta_{13}$ must be small: $\Longrightarrow$ $\sin^2 2\theta_{13} < 0.12 - 0.2$ (exact limit depends on $\left|\Delta m^2_{32}\right|$)
- from atmospheric neutrinos can exclude $\theta_{13} \sim \dfrac{\pi}{2}$
- hence the CHOOZ limit $\sin^2 2\theta_{13} < 0.2$ can be interpreted as $\sin^2 \theta_{13} < 0.05$

Reactor experiments II: KamLAND

- 70 GW from nuclear power (7% of World total) from reactors within 130-240 km

•Detector located in same mine as Super Kamiokande



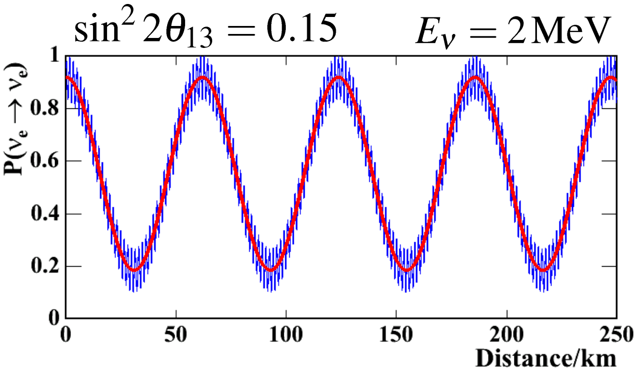
18m

- liquid scintillator detector, 1789 PMTs
- detection via inverse β-decay: $\nu_e + p \to e^+ + n$ followed by

$e^+ = e^- \to \gamma\gamma$ – prompt

$n + p \to d + \gamma(2.2\ \mathrm{MeV})$ – delayed

Reactor experiments II: KamLAND



- for MeV neutrinos at a distance of 130-240 km oscillations due to Δm^2_{32} are very rapid
- experimentally only see average effect $\langle \sin^2 \Delta_{32} \rangle = 0.5$

Reactor experiments II: KamLAND

$$P(\nu_e \to \nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{32} \quad (61)$$

$$\approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13} \quad (62)$$

$$= \cos^4\theta_{13} + \sin^4\theta_{13} - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \quad (63)$$

$$\approx \cos^4\theta_{13}(1 - \sin^2 2\theta_{12}\sin^2\Delta_{21}) \quad (64)$$

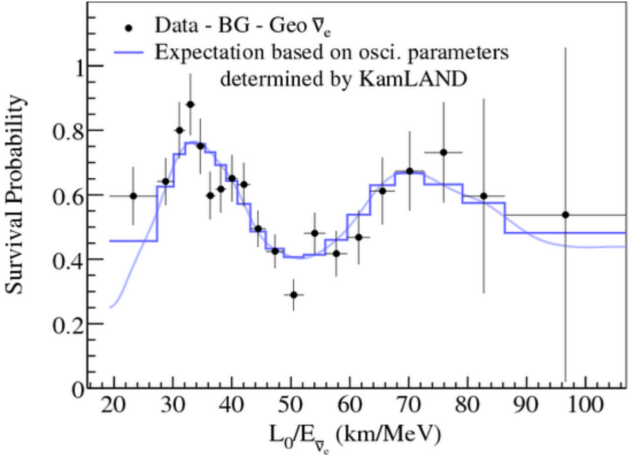
- got two-flavor oscillation formula multiplied by $\cos^4\theta_{13}$
- from CHOOZ $\cos^4\theta_{13} > 0.9$

Reactor experiments II: KamLAND results

Observed: 1609 events

Expected: 2179 $\pm$ 89 events (if no oscillations)

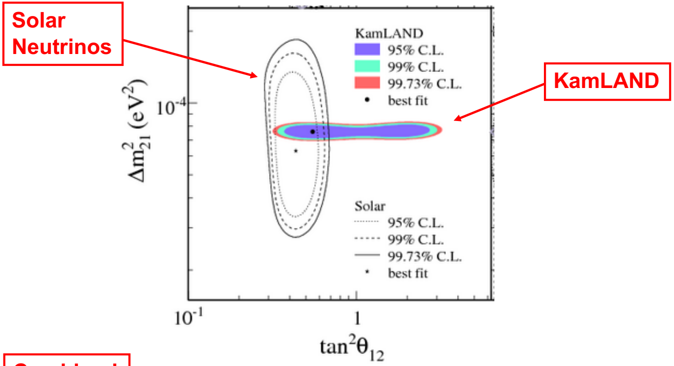
KamLAND Collaboration, Phys. Rev. Lett., 221803, 2008



- clear evidence of $\bar{\nu}_e$ oscillations consistent with the results from solar neutrinos
- oscillatory structure clearly visible
- compare data with expectations for different osc. parameters and perform χ^2 fit to extract measurement

Combined solar neutrino and KamLAND results

- KamLAND data provide strong constraints on $|\Delta m^2_{21}|$
- solar neutrino data (especially SNO) provide a strong constraint on θ_{12}



Combined

$$|\Delta m^2_{21}| = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$$

$$\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$$

Summary of current knowledge

Solar neutrinos/KamLAND

- KamLAND + Solar: $\left|\Delta m^2_{21}\right| \approx (7.6 \pm 0.2) \times 10^{-5}\ \text{eV}^2$
- SNO + KamLAND + Solar: $\tan^2\theta_{12} \approx 0.47 \pm 0.05$

  $\Longrightarrow\ \sin\theta_{12} \approx 0.56; \cos\theta_{12} \approx 0.82$

Atmospheric neutrinos/Long baseline experiments

- MINOS: $\left|\Delta m^2_{32}\right| \approx (2.4 \pm 0.1) \times 10^{-3}\text{eV}^2$
- Super Kamiokande: $\sin^2\theta_{23} \approx 0.512^{+0.019}_{-0.022}$
- Super Kamiokande: $\delta_{CP} = 1.37^{+0.18}_{-0.16}$

DayaBay, CHOOZ + atmospheric

- $\sin^2\theta_{13} \approx (2.18 \pm 0.07) \times 10^{-2}$

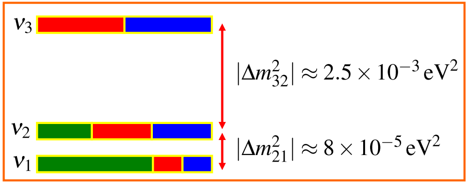
Summary of current knowledge

- have approximate expressions for mass eigenstates in terms of weak eigenstates:

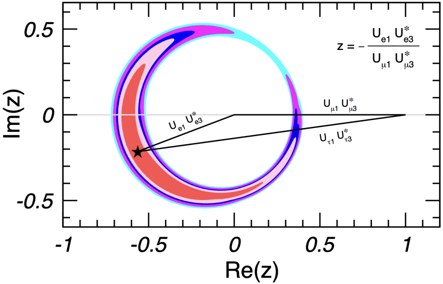
$$|\nu_3\rangle \approx \frac{1}{\sqrt{2}}(|\nu_\mu\rangle + |\nu_\tau\rangle)$$

$$|\nu_2\rangle \approx 0.53|\nu_e\rangle + 0.60(|\nu_\mu\rangle - |\nu_\tau\rangle)$$

$$|\nu_1\rangle \approx 0.85|\nu_e\rangle - 0.37(|\nu_\mu\rangle - |\nu_\tau\rangle)$$



- graphic representation of the mixing matrix measurements (unitarity triangle):



Neutrino masses

- neutrino oscillations require non-zero neutrino masses
- but only determine mass-squared differences – not the masses themselves
- no direct measure of neutrino mass – only mass limits:

  $m_\nu(e) < 0.8$ eV, $m_\nu(\mu) < 0.17$ MeV, $m_\nu(\tau) < 18.2$ MeV

  Note that e, μ, τ refer to charged lepton flavor in the experiment, e.g. $m_\nu(e) < 2$ eV refers to the limit from tritium β-decay
- also from cosmological evolution infer that the sum

  $\sum_i m_{\nu_i} <$ few eV
- 20 years ago: assumed massless neutrinos + hints that neutrinos might oscillate
- now, know a lot about massive neutrinos
- but many unknowns: mass hierarchy, absolute values of neutrino masses
- measurement of these SM parameters is the focus of the next generation of experiments