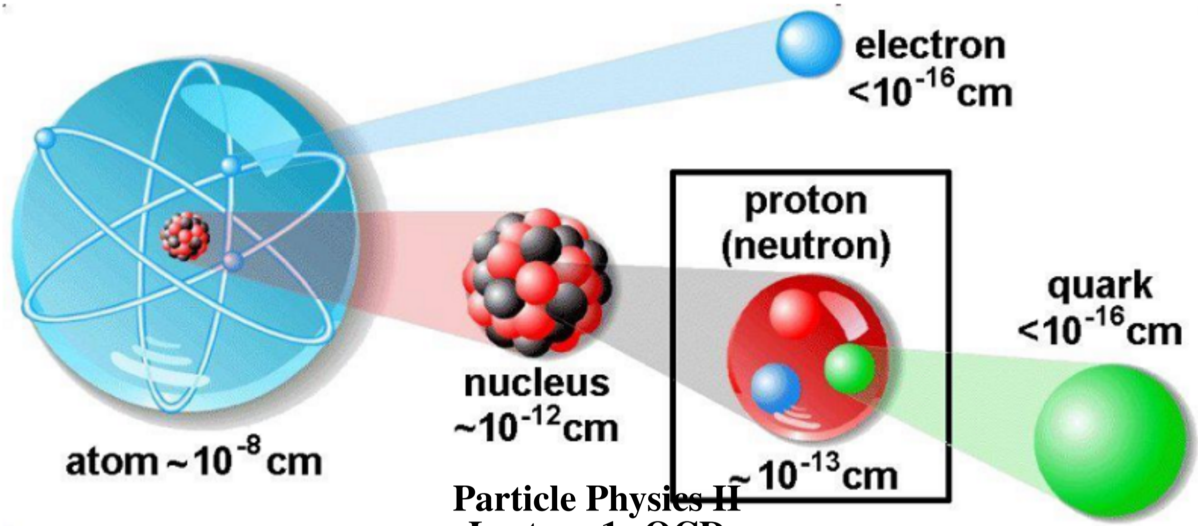

Particle Physics II
Lecture 1: QCD

Lesya Shchutska

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Practical information

Professor: Lesya Shchutska

web: https://people.epfl.ch/lesya.shchutska

mail: lesya.shchutska@epfl.ch

Teaching assistant: Raphaël van Laak

web: https://people.epfl.ch/raphael.vanlaak

mail: raphael.vanlaak@epfl.ch

Course material:

moodle: https://moodle.epfl.ch/course/view.php?id=15032

contains slides, pointers to old video recordings, exercises, and solutions

forum: https://edstem.org/eu/courses/1217/discussion/

you can post questions and post/get answers (also anonymously)

books: Mark Thomson “Modern Particle Physics” (library)

Particle Data Group “The Review of Particle Physics” http://pdg.lbl.gov/

Course structure

1. **Particle physics I (fall semester)** (PHYS-415: moodle link)
   - Introduction, detectors, accelerators
   - The Klein-Gordon and Dirac equations and spin
   - Interaction by particle exchange
   - QED: quantum electrodynamics
   - Symmetries and quark model
2. **Particle physics II (spring semester)** ← this course
   - QCD: quantum chromodynamics
   - The weak interaction
   - Electroweak unification and the W and Z bosons
   - The Higgs boson: theory and discovery
   - Beyond the standard model

Quantum Chromodynamics: main theory concepts

first principles: particle-wave function invariance under SU(3) local phase transformations

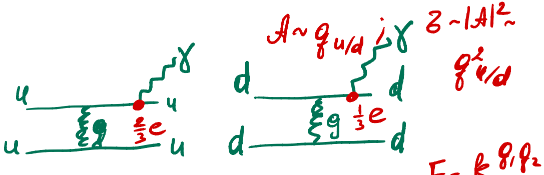
QED: emerges from U(1) invariance

coupling constant: $\alpha_s \sim 0.1 \ldots 1$

QED: $\alpha \equiv \dfrac{e^2}{4\pi} \sim \dfrac{1}{137} \ll 1$

3 charges: called colors (red, green, blue)

QED: 1 electric charge



force carriers: 8 massless color-charged gluons carrying color-anticolor (e.g. red-antiblue)

QED: 1 massless zero-electric charge photon

color confinement: only color-neutral states exist as free particles

in QED we have plenty of free charged particles

The local gauge principle

All the interactions between fermions and spin-1 *gauge* bosons in the SM are emerging from the principle of local *gauge* invariance:

1. U(1) $\equiv$ local phase transformation of particle wave-functions $\Longrightarrow$ QED
2. SU(2) $\equiv$ local SU(2) phase transformation $\Longrightarrow$ weak interaction
3. SU(3) $\equiv$ local SU(3) phase transformation $\Longrightarrow$ QCD

The local gauge principle: how it works in QED (reminder)

- require physics to be invariant under the local phase transformation of particle wave-functions:

$$\psi \quad \longrightarrow \quad \psi' = \psi e^{iq\xi(x)} \tag{1}$$

$\psi(p,x)$

$|\psi|^2$ – probability

where the change of phase $\xi(x) \equiv \xi(t, \vec{x})$ can vary with the space-time coordinate

Example: $\psi = \sin(x) \cdot e^{-\frac{x}{\lambda}}$

- plugging this transformation into the Dirac equation, we see that it changes as:

$e^{iq\xi}$ $e^{iq\xi}$

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0 \quad \longrightarrow \quad i\gamma^\mu(\partial_\mu + iq\partial_\mu \xi)\psi - m\psi = 0 \tag{2}$$

The local gauge principle: how it works in QED (reminder)

- to make "physics", i.e. the Dirac equation, invariant under this local phase transformation, we are **forced** to introduce a **massless gauge boson**, A_μ

- and we modify the Dirac equation by including this field:

$$i\gamma^\mu(\partial_\mu - qA_\mu)\psi - m\psi = 0 \tag{3}$$

- the modified Dirac equation is invariant under local phase transformations if there is **gauge invariance**, i.e. A_μ field transforms as

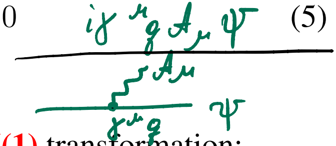
$$A_\mu \longrightarrow A'_\mu = A_\mu - \partial_\mu \xi \tag{4}$$

- physics is unchanged $\equiv$ physical predictions are unchanged for this transformation

The local gauge principle: how it works in QED (reminder)

- this principle of invariance under local phase transformations completely specifies the interaction between a charged fermion and the gauge boson – photon:

$$i\gamma^{\mu}(\partial_{\mu}\psi - qA_{\mu})\psi - m\psi = 0 \qquad (5)$$



$\Longrightarrow$ interactions vertex is $i\gamma^{\mu}qA_{\mu}$ $\Longrightarrow$ **QED**

- the local phase transformation of QED is a unitary **U(1)** transformation:

$$\psi \longrightarrow \psi' = \hat{U}\psi \qquad (6)$$

i.e.

$$\psi \longrightarrow \psi' = \psi e^{iq\xi(x)} \text{ with } U^{\dagger}U = 1 \qquad (7)$$

- suppose there is another fundamental symmetry of the universe, say

  “invariance under SU(3) local phase transformations”

  i.e. require invariance under $\psi \quad \longrightarrow \quad \psi' = \psi e^{ig_s \vec{\lambda}\cdot\vec{\theta}(x)}$, where

  $\vec{\lambda}$ are the eight 3×3 Gell-Mann matrices

  $\vec{\theta}(x)$ are eight functions taking different values at each point in space-time

  $\Longrightarrow$ 8 spin-1 gauge bosons

  $$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$ is a wave-function which is a vector in **color space**

  $\Longrightarrow$ QCD!

$e^{\alpha} \approx 1+\alpha$

$\psi(1 + i g_s \vec{\lambda}\cdot\vec{\Theta}(x))$

$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix}$

From QED to QCD

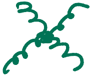
- QCD is fully specified by requiring invariance under SU(3) local phase transformations:

  $\Longrightarrow$ corresponds to rotating states in color space around an axis whose direction is different at every space-time point

  $\Longrightarrow$ interaction vertex: $-\frac{1}{2} i g_s \lambda^a \gamma^\mu$

- it predicts 8 massless gauge bosons – the gluons (one for each λ matrix)

- also predicts exact form for interactions between gluons, i.e. the 3- and 4-gluon vertices (which we will not consider here)

Color in Quantum Chromodynamics

The theory of the strong interaction, QCD is very similar to QED but with 3 conserved "color" charges:

In QED:

- the electron carries one unit of charge $-e$
- the antielectron carries one unit of anticharge $+e$
- the force is mediated by a massless neutral "gauge boson" – the photon

In QCD:

- quarks carry color charge r, g, b
- antiquarks carry anticharge $\bar{r}, \bar{g}, \bar{b}$
- the force is mediated by massless color-charged gluons

Color in QCD

In QCD, the strong interaction is invariant under rotations in color space:

$$r \longleftrightarrow b; \quad r \longleftrightarrow g; \quad b \longleftrightarrow g$$

i.e. the same force for all three colors $\implies$ SU(3) color symmetry

This is an exact symmetry, unlike the approximate u-d-s flavor symmetry (leading to similar properties of hadrons composed of these quarks)

Color in QCD

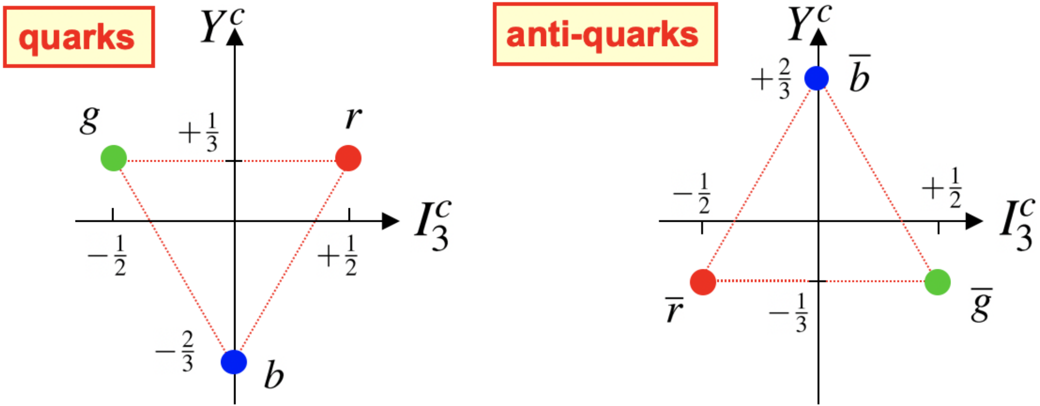
- we can represent r, g, b SU(3) color states by:

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}; \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

- color states can be labeled by two quantum numbers:
  - I_3^C color isospin
  - Y^C color hypercharge

Exactly analagous to labeling u, d, s flavor states by I_3 and Y

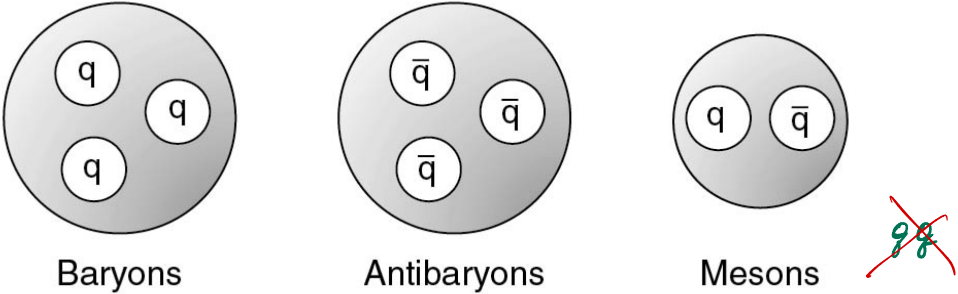
Each quark (antiquark) can have the following color quantum numbers:



Color in QCD

Question:

Why we have only mesons ($q\bar{q}'$), baryons ($qq'q''$), and antibaryons ($\bar{q}\bar{q}'\bar{q}''$) states in nature?



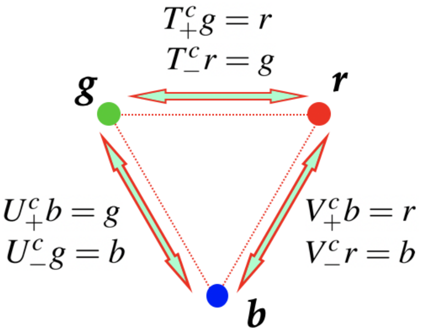
And no other quark combinations?

- it is believed (and not yet proven) that all observed free particles are **"colorless"**:
  - i.e. never observe a free quark (which would carry color charge)
  - quarks are always found in bound states – colorless hadrons
- **Color confinement hypothesis:** only **color singlet** states can exist as free particles
- all hadrons must be "colorless", i.e. color **singlets**

$q\bar{q}$; qqq ; $\bar{q}\bar{q}\bar{q}$

- to construct color wave-functions for hadrons can apply results for SU(3) flavor symmetry to SU(3) color with replacing quark flavors (u, d, s) by quark colors (r, g, b)

- in the same way can define color ladder operators:



- let's look what is meant by a singlet state
- as a reminder, consider spin states obtained with two spin-1/2 particles:
  - four spin combinations: $\uparrow\uparrow$, $\uparrow\downarrow$, $\downarrow\uparrow$, $\downarrow\downarrow$
  - gives four eigenstates of $\hat{S}^2$, $\hat{S}_z$ as $(2 \otimes 2 = 3 \oplus 1)$:

$$\begin{aligned} |1,+1\rangle &= \uparrow\uparrow \\ |1,0\rangle &= \tfrac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \\ |1,-1\rangle &= \downarrow\downarrow \end{aligned}$$

spin-1 triplet

$$\oplus \quad |0,0\rangle = \tfrac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

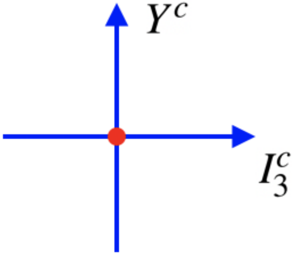
spin-0 singlet

- the singlet state is "spinless": it has 0 angular momentum, is invariant under SU(2) spin transformations and spin ladder operators yield 0:

$$S_{\pm}\,|0,0\rangle = 0$$

$S_+(\uparrow\downarrow + \downarrow\uparrow) = (\uparrow\uparrow + \uparrow\uparrow)\frac{1}{\sqrt{2}} = \uparrow\uparrow$ | $S_+(\uparrow\downarrow - \downarrow\uparrow) = \uparrow\uparrow - \uparrow\uparrow = 0$

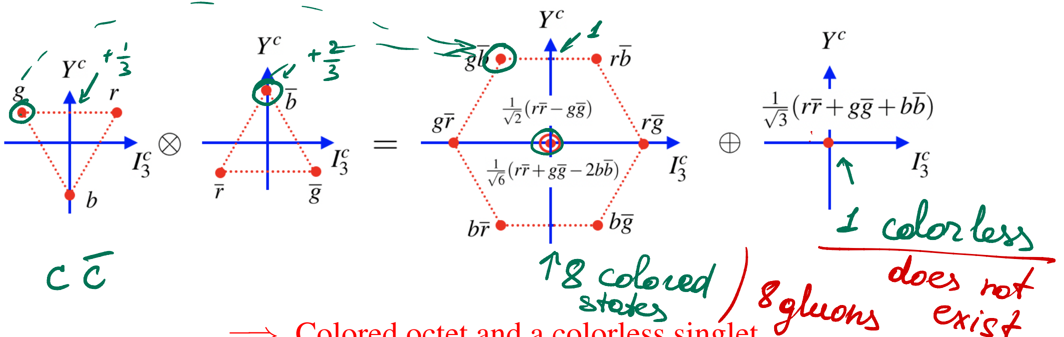
- in the same way color singlets are "colorless" combinations:
  - they have 0 color quantum numbers $I_3^C = 0, Y^C = 0$
  - invariant under SU(3) color transformations
  - ladder operators yield 0



- it is not sufficient to have $I_3^C = 0, Y^C = 0$: does not mean that such a state is a singlet

Meson color wave-function

- let's consider color wave-functions for $q\bar{q}$
- the combination of color with anticolor is mathematically identical to construction of meson wave-function with uds flavor symmetry:



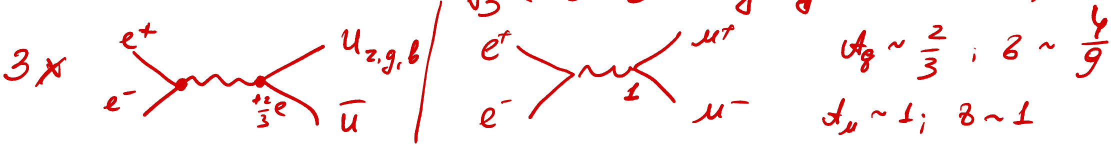
$\Longrightarrow$ Colored octet and a colorless singlet

Meson color wave-function

- color confinement implies that hadrons exist in color singlet states
- hence the color wave-function for mesons is the one corresponding to

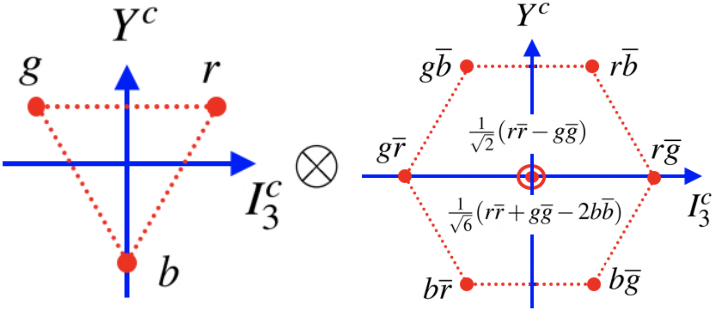
$$\psi_C^{q\bar{q}} = \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right) \tag{8}$$

$$\pi^+ : u\bar{d} = \frac{1}{\sqrt{3}}\left(u_r\bar{d}_r + u_g\bar{d}_g + u_b\bar{d}_b\right)$$



Question: can we have a $qq\bar{q}$ state?

I.e. by adding a quark to the above octet can we form a state with $Y^C = 0$, $I_3^C = 0$?



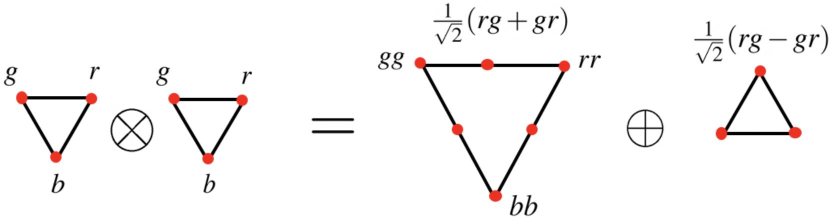
$\Longrightarrow$ a singlet?

The answer is a clear no.

$\Longrightarrow$ $qq\bar{q}$ bound states do not exist in nature.

Baryon color wave-function

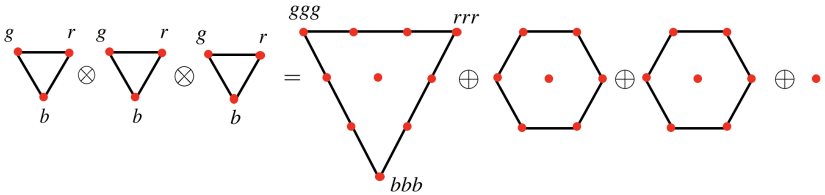
- do qq bound states exist? This is equivalent to asking whether it is possible to form a color singlet from two color triplets
- following the discussion of construction of baryon wave-functions in SU(3) flavor symmetry obtain:



- no qq color singlet state
- color confinement $\Longrightarrow$ bound states of qq do not exist

Baryon color wave-function

- but combination of three quarks (three color triplets) gives a color singlet state:



- the singlet color wave-function is:

$$\psi_C^{qqq} = \frac{1}{\sqrt{6}}(rgb - rbg + gbr - grb + brg - bgr) \quad (9)$$

$\Longrightarrow$ antisymmetric color wave-function (you can check yourselves if it's a color singlet: what are the conditions to check?)

- colorless singlet $\Longrightarrow$ qqq bound states exist!

Allowed hadrons, i.e. possible color singlet states:

$q\bar{q}$, qqq, $\bar{q}\bar{q}\bar{q}$: mesons, baryons and antibaryons

$q\bar{q}q\bar{q}$, $qqqq\bar{q}$: tetraquarks and pentaquarks

The exotic hadrons (pentaquarks and tetraquarks) were observed for the first time rather recently!

Pentaquark observation at LHCb in 2015 (#2)

All LHCb papers ranked by citation number as of February 2024 (link):

2,737 results | cite all   Citation Summary   Most Cited

The LHCb Detector at the LHC #1
LHCb Collaboration • A.Augusto Alves, Jr. (Rio de Janeiro, CBPF) et al. (Aug 14, 2008)
Published in: *JINST* 3 (2008) S08005
DOI cite claim   reference search 4,546 citations

Observation of $J/\psi p$ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \to J/\psi K^- p$ Decays #2
LHCb Collaboration • Roel Aaij (CERN) et al. (Jul 13, 2015)
Published in: *Phys.Rev.Lett.* 115 (2015) 072001 • e-Print: 1507.03414 [hep-ex]
pdf links DOI cite claim   reference search 1,686 citations

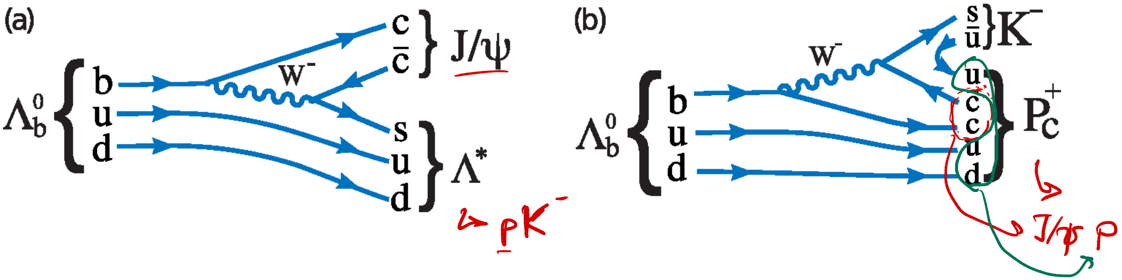
Test of lepton universality using $B^+ \to K^+\ell^+\ell^-$ decays #3
LHCb Collaboration • Roel Aaij (NIKHEF, Amsterdam) et al. (Jun 25, 2014)
Published in: *Phys.Rev.Lett.* 113 (2014) 151601 • e-Print: 1406.6482 [hep-ex]
pdf DOI cite claim   reference search 1,330 citations

Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays #4
LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)
Published in: *JHEP* 08 (2017) 055 • e-Print: 1705.05802 [hep-ex]
pdf links DOI cite datasets claim   reference search 1,291 citations

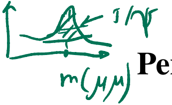
Pentaquark observation at LHCb

http://lhcb-public.web.cern.ch/lhcb-public/Welcome.html#Penta

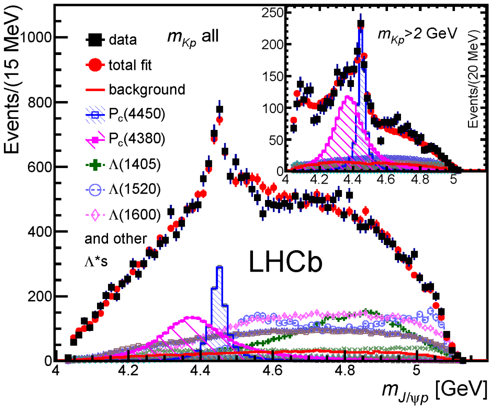
- analyzed a sample of about 26k $\Lambda_b^0 \to J/\psi p K^-$ decays:
  - a decay via known particles
  - a decay with a new pentaquark P_c^+ in the chain, $P_c^+ \to J/\psi p$

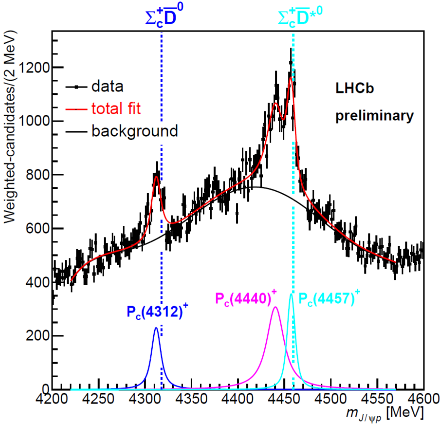


More information: https://arxiv.org/abs/1507.03414



Pentaquark observation at LHCb: 2015 and 2019

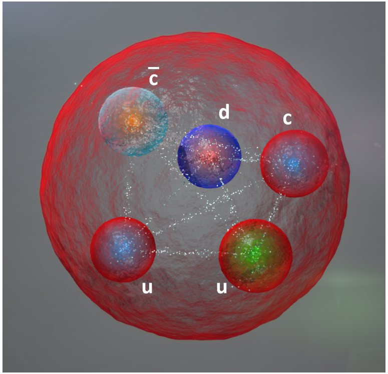


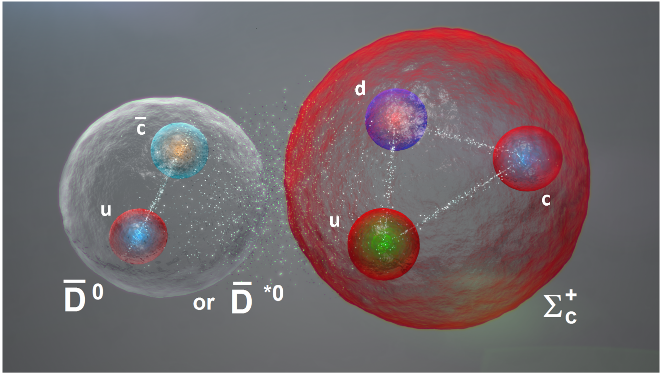


- $P_c(4450)^+$ state seen in 2015 (left) is zoomed into two peaks, $P_c(4440)^+$ and $P_c(4457)^+$ in 2019 (right)

More information: https://arxiv.org/abs/1904.03947

Pentaquark possible models

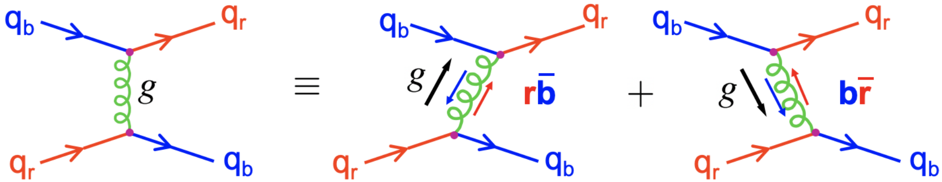




The color of the central part of each quark is related to the strong interaction color charge, while the external part shows its electric charge. The quarks could be tightly bound, or they could also be loosely bound in meson-baryon molecule, in which color-neutral meson and baryon feel a residual strong force similar to the one that binds nucleons together within nuclei.
To be studied by the LHCb in the future.

Gluons

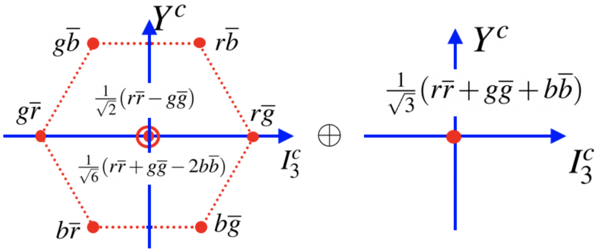
- in QCD quarks interact by exchanging virtual massless gluons, e.g.:



- gluons carry color and anticolor, e.g.:

Gluons

- gluon color wave-functions (color + anticolor) are the same as those obtained for mesons (also color + anticolor) $\Longrightarrow$ we get an octet + "colorless" singlet



- so we might expect 9 physical gluons:

octet: $r\bar{g}, r\bar{b}, g\bar{b}, b\bar{r}, b\bar{g}, \dfrac{1}{\sqrt{2}}(r\bar{r} - g\bar{g}), \dfrac{1}{\sqrt{6}}\left(r\bar{r} + g\bar{g} - 2b\bar{b}\right)$

singlet: $\dfrac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$

- but remember color confinement hypothesis:

only colour singlet states can exist as free particles ⇨ Colour singlet gluon would be unconfined. It would behave like a strongly interacting photon ➡ infinite range Strong force.

- empirically, the strong force is short range
- therefore we know that physical gluons are confined
- $\Longrightarrow$ the color singlet state does not exist in nature!

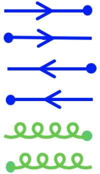
- **note:** this is not entirely ad hoc. In the context of gauge field theory the strong interaction arises from a fundamental SU(3) symmetry. The gluons arise from the generators of the symmetry group (the Gell-Mann λ matrices). There are 8 such matrices $\implies$ 8 gluons. Had nature "chosen" a U(3) symmetry, would have 9 gluons, the additional gluon would be the colour singlet state and QCD would be an unconfined long-range force.
- **note:** the "gauge symmetry" determines the exact nature of the interaction $\implies$ Feynman rules

Feynman rules for QCD

- **External Lines**

spin 1/2
- incoming quark $u(p)$
- outgoing quark $\overline{u}(p)$
- incoming anti-quark $\overline{v}(p)$
- outgoing anti-quark $v(p)$

spin 1
- incoming gluon $\varepsilon^{\mu}(p)$
- outgoing gluon $\varepsilon^{\mu}(p)^*$



- **Internal Lines (propagators)**

spin 1 gluon

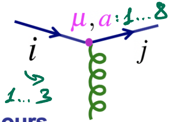
$$\boxed{\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}}$$

a, b = 1,2,…,8 are gluon colour indices

- **Vertex Factors**

spin 1/2 quark

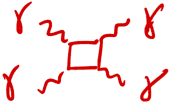
$$\boxed{-ig_s\frac{1}{2}\lambda^a_{ji}\gamma^{\mu}}$$



i, j = 1,2,3 are quark colours,

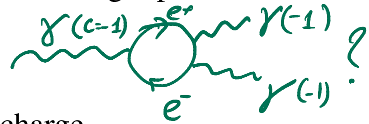
λ^a a = 1,2,..8 are the Gell-Mann SU(3) matrices

- **+ 3 gluon and 4 gluon interaction vertices**
- **Matrix Element** $-iM$ = **product of all factors**



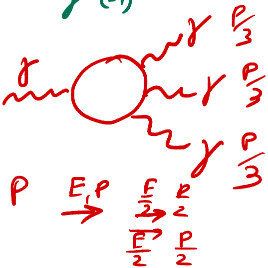
Gluon-gluon interaction

- in QED, photon does not carry EM interaction charge: photons are electrically neutral



- in contrast, in QCD, gluons do carry color charge

$\Longrightarrow$ **gluon self-interactions**



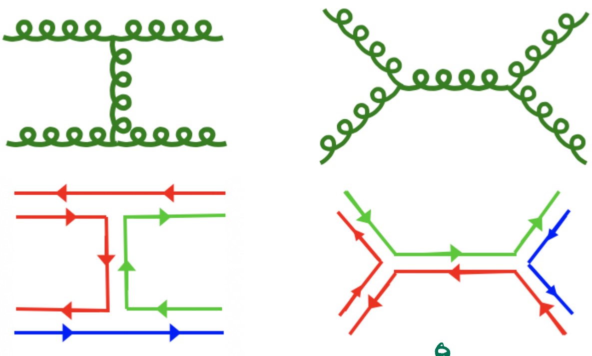
- two new vertices appear (no QED analogues):

triple-gluon vertex

quartic-gluon vertex

Gluon-gluon interaction

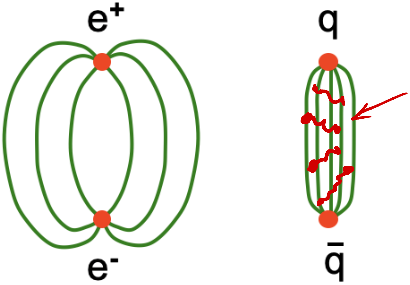
- in addition to quark-quark scattering, can have gluon-gluon scattering:



a possible color-flow is shown in the bottom

Gluon self-interaction and confinement

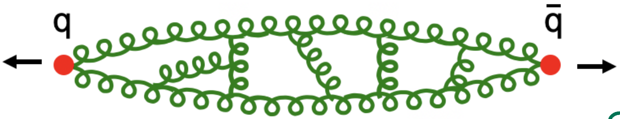
- gluon self-interactions are believed to give rise to color confinement
- qualitative picture:
  - compare QED with QCD:

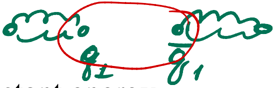


  - in QCD, “gluon self-interactions squeeze lines of force into a flux tube”

Gluon self-interaction and confinement

What happens when try to separate two colored objects, e.g. $q\bar{q}$:





- form a flux tube of interacting gluons of approximately constant energy density ~ 1 GeV/fm $\Longrightarrow V(r) \sim \lambda r$ $\left(V_{QED}(r) \sim \frac{k}{r} \right)$
- hence require infinite energy to separate colored objects to infinity
- colored quarks and gluons are always confined within colorless states
- in this way QCD provides a plausible explanation of confinement – but not yet proven

Summary of the first part

- superficially QCD is very similar to QED
- but gluon self-interactions are believed to result in color confinement
- all hadrons are colour singlets which explains why we only observe mesons or baryons (and tetra-, pentaquarks)
- there are also hypotyhesized sexaquarks which can be stable and provide a dark matter candidate: arXiv:1708.08951 uds uds
  - necessary condition: $m_S < 2(m_p + m_e) \implies$ decay is forbidden
- LHC experiments are looking for them: no public results yet