Particle Physics 2 : Exercise 9

1) Z decay rates

After correcting for QED effects, including initial-state radiation, the measured $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to$ hadrons cross sections at the peak of the Z resonance give

$$\sigma^0(e^+e^- \to Z \to \mu^+\mu^-) = 1.9993 \text{ nb} \quad \text{and} \quad \sigma^0(e^+e^- \to Z \to \text{hadrons}) = 41.476 \text{ nb}$$

a) Assuming lepton universality, determine $\Gamma_{\ell\ell}/\Gamma_Z$ and $\Gamma_{\text{hadrons}}/\Gamma_Z$.

b) Hence, using the measured value of $\Gamma_Z = 2.4952 \pm 0.0023$ GeV and of $\Gamma_{\nu\nu} = 167$ MeV, obtain an estimate of the number of light neutrino flavours.

2) Forward-backward asymmetry

Show that $e^+e^- \to Z \to \mu^+\mu^-$ differential cross section can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto (1 + \cos^2\theta) + \frac{8}{3} A_{\text{FB}} \cos\theta$$

where A_{FB} is the forward-backward asymmetry :

$$A_{\text{FB}} = \frac{N_F - N_B}{N_F + N_B}.$$

Hint : start from the expression of the differential cross section given in the lecture

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = k \cdot [a(1 + \cos^2\theta) + 2b\cos\theta]$$

(where a and b are constants related to the couplings of electron and muon to the Z and k is a normalisation factor) and derive N_F and N_B by integrating over the angle.

3) Muon asymmetry

From the measurement of the muon asymmetry parameter,

$$\mathcal{A}_\mu = 0.1456 \pm 0.0091,$$

determine the corresponding value of $\sin^2\theta_W$.

4) Feynman diagrams

There are ten possible lowest-order Feynman diagrams for the process $e^+e^- \to \mu^-\bar{\nu}_\mu \mathrm{u}\bar{\mathrm{d}}$, of which only three involve a $\mathrm{W}^+\mathrm{W}^-$ intermediate state. Draw the other seven diagrams (they are all s-channel processes involving a single virtual W).

5) Top quark decay

Verify that the momenta of the final-state particles in the decay $\mathrm{t} \to W^+\mathrm{b}$ in the centre-of-mass frame of the top quark are

$$p^* = \frac{m_\mathrm{t}^2 - m_\mathrm{W}^2}{2m_\mathrm{t}}.$$

Use the fact that $m_b \ll m_W$ to approximate p^*.

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6) τ polarisation

The average tau polarisation in the process $e^+e^- \to Z \to \tau^+\tau^-$ can be determined from the energy distribution of π^- in the decay $\tau^- \to \pi^-\nu_\tau$. In the τ^- rest frame, the π^- four-momentum can be written $p = (E^*, p^*\sin\theta^*, 0, p^*\cos\theta^*)$ where θ^* is the angle with respect to the τ^- spin, and the differential partial decay width is

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta^*} \propto \frac{(p^*)^2}{m_\tau}(1+\cos\theta^*).$$

a) Without explicit calculation, explain this angular dependence.

b) For the case where the τ^- is right-handed, show that the observed energy distribution of the π^- *in the laboratory frame* is

$$\frac{\mathrm{d}\Gamma_{\tau^-_\uparrow}}{\mathrm{d}E_{\pi^-}} \propto x$$

where $x = E_\pi/E_\tau$.
Hint : Using the appropriate Lorentz transformation, show that

$$\frac{\mathrm{d}\Gamma_{\tau^-_\uparrow}}{\mathrm{d}E_{\pi^-}} = \frac{1}{\beta_\tau^2\gamma_\tau E_\tau}(E_\pi + \gamma_\tau\beta_\tau p^* - \gamma_\tau E^*).$$

Then, using $E_\tau \approx m_Z/2$, show that the additional terms in the parenthesis are negligible.

c) What is the corresponding π^- energy distribution for the decay of a LH helicity τ^-.

d) If the observed pion energy distribution is consistent with

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = 1.14 - 0.28x \equiv 0.86x + 1.14(1-x),$$

determine $\mathcal{A}_\tau = [(c_L^\tau)^2 - (c_R^\tau)^2]/[(c_L^\tau)^2 + (c_R^\tau)^2]$ and the corresponding value of $\sin^2\theta_\mathrm{W}$.
Hint : $\mathcal{A}_\tau$ can be measured as

$$\mathcal{A}_\tau = \frac{N_\downarrow - N_\uparrow}{N_\downarrow + N_\uparrow}$$

where $N_\downarrow$ and $N_\uparrow$ are the number of produced left- and right-handed taus respectively.