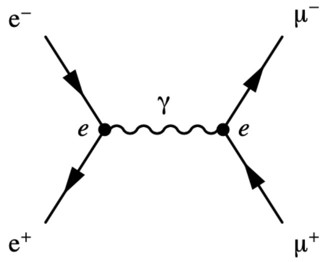
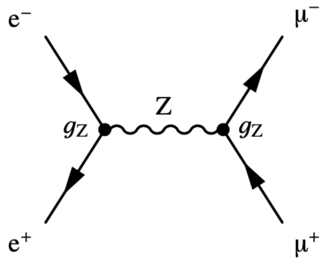
Particle Physics 2 : Exercise 8

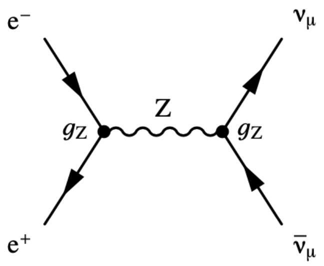
1) Feynman diagrams

In all cases the Higgs exchange diagrams will give negligible contributions and are ignored. The two possible lowest-order diagrams for $e^+e^- \rightarrow \mu^+\mu^-$ are :

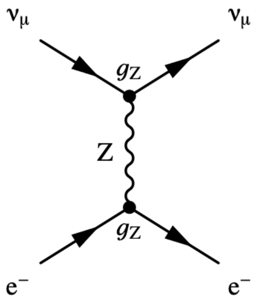




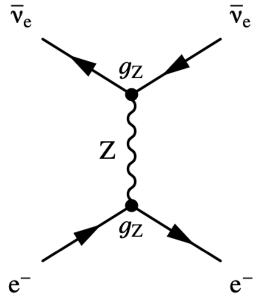
Since neutrinos are neutral, there is no QED diagram for $e^+e^- \rightarrow \nu_\mu\bar{\nu_\mu}$ but a the Z-exchange diagram is still present.

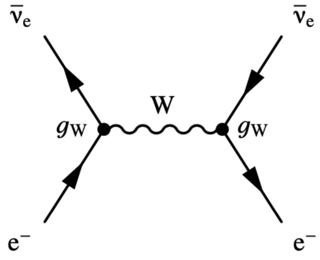


For $\nu_\mu e^- \rightarrow \nu_\mu e^-$ only the neutral current weak interaction contributes at lowest order.



Finally, for $\bar{\nu_e}e^- \rightarrow \bar{\nu_e}e^-$ there are weak charged-current and weak neutral current diagrams.





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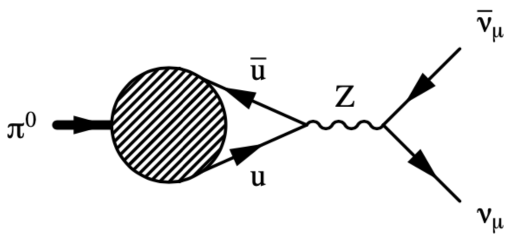
2) π^0 decays

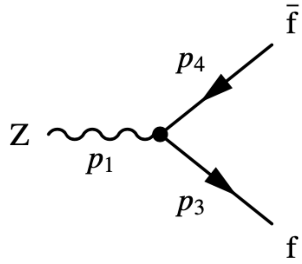
The lowest-order Feynman diagram is shown below. The general form of the neutral current vertex, $\gamma^\mu(c_V - c_A\gamma^5)$, reduces to a $V - A$ form for the coupling at $Z\nu_\mu\bar{\nu_\mu}$ vertex, and thus neutrino is produced in a LH chiral state and anti-neutrino is produced in a RH chiral state. Because neutrinos are almost massless ($E >> m$) the chiral states effectively correspond to helicity states and thus the decay would result in a $J = 1$ final state, violating conservation of angular momentum.



3) Partial decay rate

The matrix element for the $Z \to \mathrm{f\bar{f}}$ decay, shown below is

$$\mathcal{M}_{fi} = g_Z \epsilon_\mu^\lambda(p_1)\bar{u}(p_3)\gamma^\mu \frac{1}{2}(c_V - c_A\gamma^5)v(p_4),$$



or equivalently

$$\mathcal{M}_{fi} = g_Z \epsilon_\mu^\lambda(p_1)\bar{u}(p_3)\gamma^\mu \Big[c_L\frac{1}{2}(1-\gamma^5) + c_R\frac{1}{2}(1+\gamma^5)\Big]v(p_4),$$

where $c_V = c_L + c_R$ and $c_A = c_L - c_R$. Written in this form it should be clear that only two chiral combinations give non-zero matrix elements, and in the limit where the final-state fermions are ultra-relativistic, only two helicity combinations give non-zero matrix elements :

$$\mathcal{M}_{LR} = g_Z c_L \epsilon_\mu^\lambda(p_1)\bar{u}_\downarrow(p_3)\gamma^\mu v_\uparrow(p_4) \quad \text{and} \quad \mathcal{M}_{RL} = g_Z c_R \epsilon_\mu^\lambda(p_1)\bar{u}_\uparrow(p_3)\gamma^\mu v_\downarrow(p_4).$$

The leptonic currents with $E = m_Z/2$, are

$$j^\mu_{LR} = m_Z(0, -\cos\theta, -i, \sin\theta) \quad \text{and} \quad j^\mu_{RL} = m_Z(0, -\cos\theta, +i, \sin\theta).$$

Without loss of generality, we are free to choose the polarisation state of the Z. Here take the Z to be at rest and to be longitudinally polarised, such that

$$\epsilon_\mu = \epsilon_L^\mu = (0, 0, 0, 1).$$

In this case, the matrix elements reduce to

$$\mathcal{M}_{LR} = -g_Z c_L j^3_{LR} = -g_Z c_L m_Z \sin\theta \quad \text{and} \quad \mathcal{M}_{RL} = -g_Z c_R j^3_{RL} = -g_Z c_R m_Z \sin\theta$$

The total decay rate is determined by the summed matrix element squared (no need to average since we have chosen a particular initial state polarisation)

$$\langle|\mathcal{M}|^2\rangle = |\mathcal{M}|^2_{LR} + |\mathcal{M}|^2_{RL} = -g_Z^2 m_Z^2 (c_L^2 + c_R^2)\sin^2\theta.$$

Substituting this into the decay rate formula

$$\begin{aligned}\Gamma(Z \to \mathrm{f\bar{f}}) &= \frac{p^*}{32\pi^2 m_Z^2}\int\langle|\mathcal{M}|^2\rangle d\Omega^* \\ &= \frac{2\pi p^*}{32\pi^2 m_Z^2} g_Z^2 m_Z^2 (c_L^2 + c_R^2)\int_{-1}^{1}\sin^2\theta d(\cos\theta) \\ &= \frac{p^*}{16\pi} g_Z^2 (c_L^2 + c_R^2)\int_{-1}^{1}(1 - x^2)d(x) \\ &= \frac{p^*}{12\pi} g_Z^2 (c_L^2 + c_R^2),\end{aligned} \tag{1}$$

where p is the momentum of the final state fermion in the centre-of-mass frame. If the masses of the final-state particles are neglected, $p^* = m_Z/2$, and therefore the $Z \to \mathrm{f\bar{f}}$ decay rate is given by

$$\Gamma(Z \to \mathrm{f\bar{f}}) = \frac{g_Z^2 m_Z}{24\pi}(c_L^2 + c_R^2) = \frac{g_Z^2 m_Z}{48}(c_V^2 + c_A^2),$$

The partial decay widths therefore depend on the sum of the squares of the vector and axial-vector couplings of the Z to the fermions. Taking into account the three colours and that the Z cannot decay to top quarks, the ratio

$$\begin{aligned}R_\mu &= \frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to \text{hadrons})} \\ &= \frac{\Gamma(Z \to \mu^+\mu^-)}{9\Gamma(Z \to \mathrm{d\bar{d}}) + 6\Gamma(Z \to \mathrm{u\bar{u}})}.\end{aligned} \tag{2}$$

The individual partial decay widths are proportional to :

$$\mu : c_V^2 + c_A^2 = 0.2516, \quad \mathrm{d} : c_V^2 + c_A^2 = 0.3725, \quad \mathrm{u} : c_V^2 + c_A^2 = 0.2861,$$

and therefore

$$\begin{aligned}R_\mu &= \frac{\Gamma(Z \to \mu^+\mu^-)}{\Gamma(Z \to \text{hadrons})} \\ &= \frac{0.2516}{9\cdot 0.3725 + 6\cdot 0.2861} = 0.496 \approx \frac{1}{20}.\end{aligned} \tag{3}$$