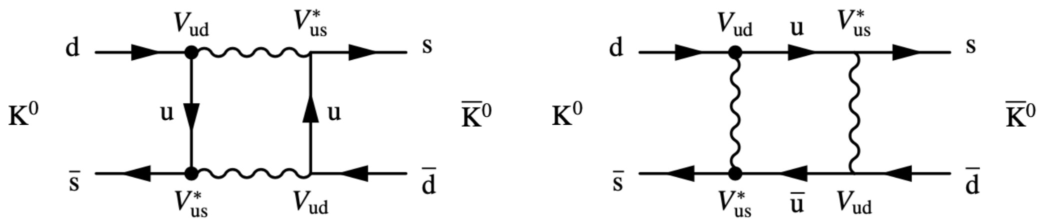
Particle Physics 2 : Exercise 7

1) K_S - K_L mass difference

The relative importance of the contributions from the u, c and t quarks in the box diagrams (shown below) depends on $V_{\mathrm{qd}}V^*_{\mathrm{qs}}m_{\mathrm{q}}$.



Taking $m_{\mathrm{u}} \approx 0.3$ GeV, $m_{\mathrm{c}} \approx 1.5$ GeV and $m_{\mathrm{t}} \approx 175$ GeV the relative contributions are in the ratio :

$$V_{\mathrm{ud}}V^*_{\mathrm{us}}m_{\mathrm{u}} : V_{\mathrm{cd}}V^*_{\mathrm{cs}}m_{\mathrm{c}} : V_{\mathrm{td}}V^*_{\mathrm{ts}}m_{\mathrm{t}} = 0.07 \text{ GeV} : 0.33 \text{ GeV} : 0.06 \text{ GeV},$$

and, therefore, the largest contribution to the mass difference comes from the box diagrams with two charm quarks. Taking $f_K \sim 0.1$ GeV, the numerical value for the contribution from the diagrams involving the charm quarks alone is

$$\Delta m = m(K_L) - m(K_s) = \frac{G_F^2}{3\pi^2} f_K^2 m_K |V_{\mathrm{cd}}V^*_{\mathrm{cs}}|^2 m_c^2 = 2.3 \times 10^{-15} \text{ GeV}$$

2) B^0 - $\bar{B}^0$ mass difference

From the question above, the mass difference of the B_H and B_L mass eigenstates can be expressed as

$$\Delta m = m(B_H) - m(B_L) \approx \sum_{q,q'} \frac{G_F^2}{3\pi^2} f_B^2 m_B |V_{\mathrm{qd}}V^*_{\mathrm{qb}}V_{\mathrm{q'd}}V^*_{\mathrm{q'b}}| m_{\mathrm{q}} m_{\mathrm{q'}}$$

where q and q' are the quark flavours appearing in the box diagram. Taking $m_{\mathrm{u}} \approx 0.3$ GeV, $m_{\mathrm{c}} \approx 1.5$ GeV and $m_{\mathrm{t}} \approx 175$ GeV the relative contributions are in the ratio :

$$V_{\mathrm{ud}}V^*_{\mathrm{ub}}m_{\mathrm{u}} : V_{\mathrm{cd}}V^*_{\mathrm{cb}}m_{\mathrm{c}} : V_{\mathrm{td}}V^*_{\mathrm{tb}}m_{\mathrm{t}} = 0.001 \text{ GeV} : 0.014 \text{ GeV} : 1.57 \text{ GeV},$$

and the contributions from the top quarks in the box diagrams clearly dominate.

3) Asymmetric b-factories

a) The masses of the particles are

$$m(\Upsilon(4S)) = 10.579 \text{ GeV} \quad \text{and} \quad m(B^0) = m(\bar{B}^0) = 5.279 \text{ GeV},$$

and for the decay at rest the conservation of energies gives

$$E(B^0)^* = E(\bar{B}^0)^* = m(\Upsilon(4S))/2 = 5.289 \text{ GeV}.$$

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The momenta of the daughter particles are

$$p^* = (E^{*2} - m^2)^{\frac{1}{2}} = 0.333 \text{ GeV},$$

and thus the velocities are

$$\beta^* = \frac{p^*}{E^*} = \frac{\gamma m \beta}{\gamma m} = 0.063.$$

b) The electron and positron energies are chosen such that $\sqrt{s} = m(\Upsilon(4S))$ and here the $\Upsilon(4S)$ is produced with momentum 4.5 GeV in the laboratory frame, corresponding to a velocity of

$$\beta_\Upsilon = \frac{p_\Upsilon}{E_\Upsilon} = \frac{4.5}{11.5} = 0.39 \Rightarrow \gamma_\Upsilon = 1.086.$$

In the laboratory frame, the energy of the boosted decay B^0 is given by

$$E' = \gamma_\Upsilon E^* + \gamma_\Upsilon \beta_\Upsilon p^* \cos\theta^*,$$

where θ^* is the polar angle of the B^0 relative to the z-axis along the direction of the boost and p^* and E^* are the rest frame decay momentum and energy, calculated in the previous question. Since $p^* = \beta^* E^*$, the energy of the decay B^0 in the laboratory frame

$$E' = \gamma_\Upsilon E^*(1 + \beta_\Upsilon \beta^* \cos\theta^*) = \gamma_\Upsilon E^*(1 + 0.024\cos\theta^*)$$

does not depend strongly on the decay angle. Taking into account time dilation, the mean distance the B^0 will travel is

$$d = \gamma\tau\beta c = \frac{p'}{m}\tau c$$

Taking $E' \approx \gamma_\Upsilon E^* = 5.74$ GeV, the momentum of the B^0 is approximately $p' = 2.23$, and therefore $\beta\gamma = p'/m = 0.43$. The mean distance the B^0 travels in the laboratory frame is :

$$d = \frac{p'}{m}\tau c = 197\ \mu\text{m}$$

4) **Unitary triangle**

The length of the shortest side of the unitarity triangle shown in the figure is

$$x = \left| \frac{|V^*_{\text{ub}}||V_{\text{ud}}|}{|V_{\text{cd}}||V^*_{\text{cb}}|} \right| = \frac{|V^*_{\text{ub}}||V_{\text{ud}}|}{|V_{\text{cd}}||V^*_{\text{cb}}|} = 0.43 \pm 0.06$$

where the uncertainty can be found from error propagation, i.e. the error on a function $f(x, y, z, \dots)$ is

$$\sigma_f = \sqrt{\left(\sigma_x \frac{\partial f}{\partial x}\right)^2 + \left(\sigma_y \frac{\partial f}{\partial y}\right)^2 + \left(\sigma_z \frac{\partial f}{\partial z}\right)^2 + \dots}.$$

So in this case

$$\sigma_x = \sqrt{\left(\sigma_{V_{ub}} \frac{|V_{ud}|}{|V_{cd}||V_{cb}|}\right)^2 + \left(\sigma_{V_{ud}} \frac{|V_{ub}|}{|V_{cd}||V_{cb}|}\right)^2 + \left(\sigma_{V_{cd}} \frac{|V_{ub}||V_{ud}|}{|V_{cd}|^2|V_{cb}|}\right)^2 + \left(\sigma_{V_{cb}} \frac{|V_{ub}||V_{ud}|}{|V_{cd}||V_{cb}|^2}\right)^2}.$$

To find the values of (ρ, η) and the errors thereof, we use the measured value

$$\sin(2\beta) = 0.685 \pm 0.032$$

which can be found in Thomson. This equation can be inserted to find β. Using the rule of sines and defining α as the angle in the unitary triangle opposite to the $\eta = 0$ line, we find

$$\frac{\sin\beta}{x} = \frac{\sin\alpha}{1} \Rightarrow \alpha = \arcsin\left(\frac{\sin\beta}{x}\right).$$

From this we can find the remaining angle of the triangle

$$\gamma = 180^\circ - \beta - \alpha,$$

which will give us

$$\rho = x\cos\gamma,$$
$$\eta = x\sin\gamma.$$

Calculating the values of $\beta, \alpha, \gamma, \rho, \eta$ and performing the corresponding error propagations given the equations above (software could be used to simplify this — e.g. the uncertainties package in python) gives

$$\beta = 21.6^\circ \pm 1.3^\circ,$$
$$\alpha = 57^\circ \pm 12^\circ,$$
$$\gamma = 101^\circ \pm 13^\circ,$$
$$\rho = -0.09 \pm 0.11,$$
$$\eta = 0.43 \pm 0.04.$$