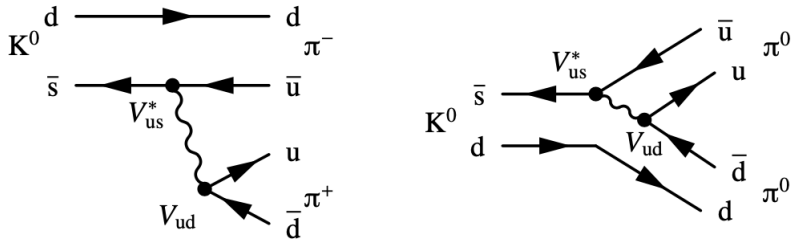
Particle Physics 2 : Exercise 6

1) Kaon decays

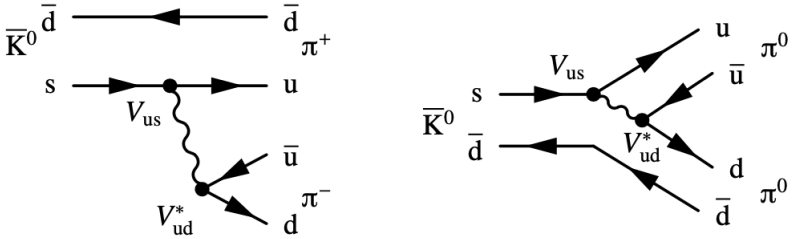
The two lowest-order Feynman diagrams for the $K^0 \to \ \pi^+\pi^-$ and $K^0 \to \ \pi^0\pi^0$ decays are sketched below.



In the two flavour approximation, the matrix element for both diagrams is proportional to :

$$\mathcal{M} \propto |V_{us}||V_{ud}| \approx \sin\theta_C \cos\theta_C.$$

The two corresponding lowest-order Feynman diagrams for the $\bar{K}^0 \to \ \pi^+\pi^-$, and, $\bar{K}^0 \to \ \pi^0\pi^0$ decays are sketched below.

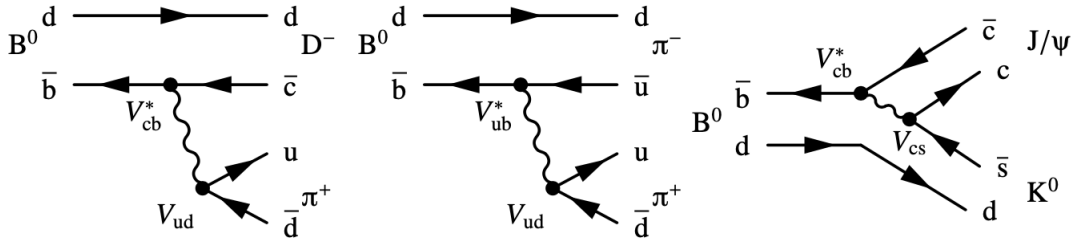


Again, for both diagrams :

$$\mathcal{M} \propto |V_{us}||V_{ud}| \approx \sin\theta_C \cos\theta_C.$$

2) B^0 decays

The lowest-order Feynman diagrams for the given decays are sketched below.



From consideration of the CKM elements alone the matrix elements scale as

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$$\mathcal{M}(B^0 \to\ D^-\pi^+) : \mathcal{M}(B^0 \to\ \pi^+\pi^-) : \mathcal{M}(B^0 \to\ J/\psi K^0)$$
$$= |V_{cb}||V_{ud}| : |V_{ub}||V_{ud}| : |V_{cb}||V_{cs}|$$

To first order the CKM matrix elements depend on the number of generations changed at the vertex. Thus the matrix elements for the decays $\mathcal{M}(B^0 \to\ D^-\pi^+)$ and $\mathcal{M}(B^0 \to\ J/\psi K^0)$, which both have one vertex at which there is no change of generation and one vertex where there is one change of generation $\bar{b} \to\ \bar{c}$, will be larger than that for $\mathcal{M}(B^0 \to\ \pi^+\pi^-)$, which has one vertex at which there are two changes of generation $\bar{b} \to\ \bar{u}$. Being more quantitative, from consideration of the CKM matrix alone,

$$Br(B^0 \to\ D^-\pi^+) : Br(B^0 \to\ \pi^+\pi^-) : Br(B^0 \to\ J/\psi K^0)$$
$$= |V_{cb}|^2|V_{ud}|^2 : |V_{ub}|^2|V_{ud}|^2 : |V_{cb}|^2|V_{cs}|^2$$
$$= 1.6 \times 10^{-3} : 1.5 \times 10^{-5} : 1.6 \times 10^{-3}.$$

From consideration of the CKM matrix alone, one would expect the $Br(B^0 \to\ \pi^+\pi^-)$ to be about 100 times smaller than for the other two decays. The observed BRs are :

$$Br^{obs}(B^0 \to\ D^-\pi^+) : Br^{obs}(B^0 \to\ \pi^+\pi^-) : Br^{obs}(B^0 \to\ J/\psi K^0)$$
$$= 2.7 \times 10^{-3} : 5.1 \times 10^{-6} : 8.7 \times 10^{-4}.$$

in reasonable agreement with the dependence from CKM matrix elements alone. A more sophisticated estimate of the BRs would account for the differences in phase space for the decays, where

$$\Gamma \propto p^* \propto \{[m_B^2 - (m_1 + m_2)^2][m_B^2 - (m_1 - m_2)^2]\}^{\frac{1}{2}}$$

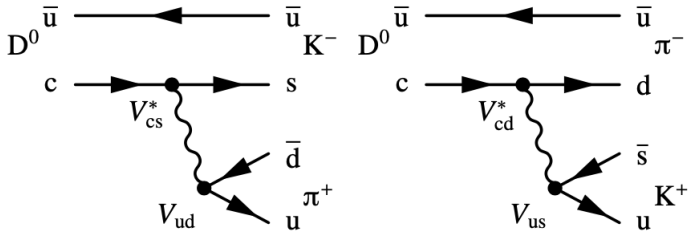
Taking account the different phase space factors and scaling to the observed BR for $B^0 \to D^-\pi^+$

$$Br(B^0 \to\ D^-\pi^+) : Br(B^0 \to\ \pi^+\pi^-) : Br(B^0 \to\ J/\psi K^0)$$
$$= |V_{cb}|^2|V_{ud}|^2 p^* : |V_{ub}|^2|V_{ud}|^2 p^* : |V_{cb}|^2|V_{cs}|^2 p^*$$
$$= 2.7 \times 10^{-3} : 2.9 \times 10^{-5} : 2.0 \times 10^{-3}.$$

The remaining differences, compared to the observed values, can be attributed to other differences in the matrix element, which includes factors accounting for the formation of the final-state mesons.

3) D^0 decays

The lowest-order Feynman diagrams for the given decays are are sketched below.



On the basis of the CKM matrix alone, one would expect

$$\frac{\Gamma(D^0 \to\ K^+\pi^-)}{\Gamma(D^0 \to\ K^-\pi^+)} \approx \frac{|V_{cd}|^2|V_{us}|^2}{|V_{ud}|^2|V_{cs}|^2} = \frac{0.225^2 \cdot 0.225^2}{0.974^2 \cdot 0.973^2} = 3 \times 10^{-3}$$

explaining most of the difference in the observed decay rates.

4) **T meson**

In each case the decay of the quark paired with the $\bar{u}$ decays according to the largest CKM matrix element, in this case

$$t \to\ b \to\ c \to\ s \to\ u$$

and mesons can be identified as

$$W = B^-(b\bar{u}), \quad X = \bar{D}^0(c\bar{u}), \quad Y = K^-(s\bar{u}) \quad Z = \pi^0(u\bar{u})$$

The direct decay to $\pi^0\pi^0$ final state would involve a $t \to\ d$ decay with the the CKM element $|V_{td}|^2 \sim 10^{-4}$.

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