Particle Physics 2 : Exercise 5

1) KamLAND data

Use the data of Figure 1 to obtain estimates of $\sin^2(2\theta_{12})$ and $|\Delta m^2_{21}|$. In the computation, use the value $\cos^4\theta_{13} \simeq 0.95$, derived from short-baseline reactor experiments.

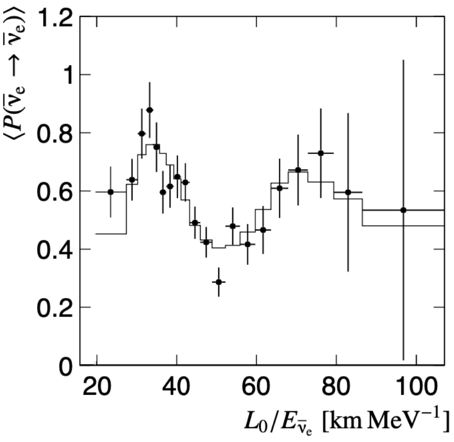

Figure 1 – KamLAND data showing the measured mean survival probability as a function of the measured neutrino energy divided by the flux-weighted mean distance to the reactors, L_0. The histogram shows the expected distribution for the oscillation parameters that best describe the data.

2) MINOS data

Use the data of Figure 2 to obtain estimates of $\sin^2(2\theta_{23})$ and $|\Delta m^2_{32}|$.

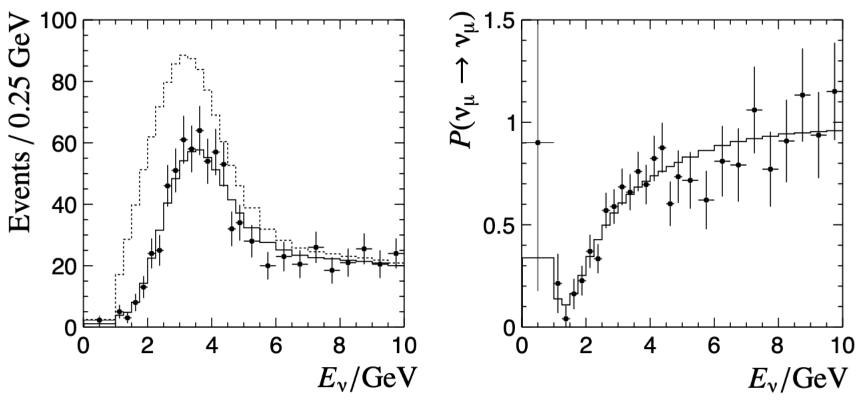

Figure 2 – The MINOS far detector energy spectrum compared to the unoscillated prediction and the oscillation probability as measured from the ratio of the far detector data to the unoscillated prediction.

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3) The T2K experiment

The T2K experiment uses an off-axis ν_μ beam produced from $\pi^+ \to \mu^+\nu_\mu$ decays. Consider the case where the pion has velocity β along the z-direction in the laboratory frame and a neutrino with energy E^* is produced at an angle θ^* with respect to the z'-axis in the π^+ rest frame.

1. Show that the neutrino energy in the pion rest frame is $p^* = (m_\pi^2 - m_\mu^2)/2m_\pi$.
2. Using a Lorentz transformation, show that the energy E and angle of production θ of the neutrino in the laboratory frame are

   $$E = \gamma E^*(1 + \beta\cos\theta^*) \quad \text{and} \quad E\cos\theta = \gamma E^*(cos\theta^* + \beta),$$

   where $\gamma = E_\pi/m_\pi$.
3. Using the expressions for E^* and θ^* in terms of E and θ, show that

   $$\gamma^2(1 - \beta\cos\theta)(1 + \beta\cos\theta^*) = 1.$$

4. In the limit $\theta << 1$, show that

   $$E \approx 0.43E_\pi \frac{1}{1 + \beta\gamma^2\theta^2}$$

   and therefore on-axis ($\theta = 0$) the neutrino energy spectrum follows that of the pions. Assume that $E_\nu \gg m_\pi$, such that $\gamma \gg 1$.
5. Assuming that the pions have a flat energy spectrum in the range $1 - 5$ GeV, sketch the form of the resulting neutrino energy spectrum at the T2K far detector (Super-Kamiokande), which is off-axis at $\theta = 2.5°$. Given that the Super-Kamiokande detector is 295 km from the beam, explain why this angle was chosen.