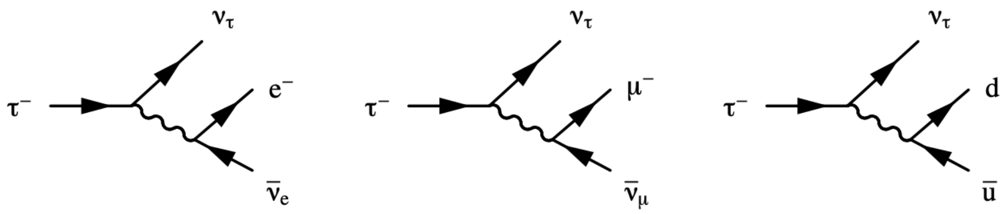
Particle Physics 2 : Exercise 4

1) τ lepton branching ratios

The Feynman diagrams for the main decay modes are shown below :



In the case of the hadronic decays the dū system can form a π^- with $J^P = 0^-$, a ρ^- with $J^P = 1^-$, or system of light mesons produced through an intermediate mesonic state or through the hadronisation of the dū system. Assuming a universal strength for the weak interaction vertex (i.e. putting aside the discussion of the CKM matrix for now), the amplitudes for the three decays shown would be expected to be roughly equal. Thus, remembering that the diagram with quarks is repeated for the three separate colour combinations $r\bar{r}$, $b\bar{b}$ and $g\bar{g}$ one would expect :

$$Br(\tau^- \rightarrow\ e^-\nu_\tau\bar{\nu_e}) : Br(\tau^- \rightarrow\ \mu^-\nu_\tau\bar{\nu_\mu}) : Br(\tau^- \rightarrow\ \nu_\tau + \text{hadrons}) \approx 1 : 1 : 3.$$

In reality, the branching ratios are :

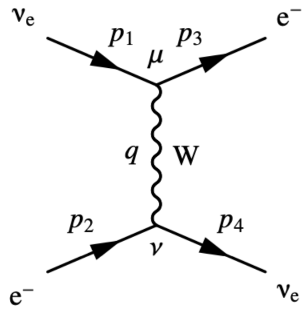
$$Br(\tau^- \rightarrow\ e^-\nu_\tau\bar{\nu_e}) : Br(\tau^- \rightarrow\ \mu^-\nu_\tau\bar{\nu_\mu}) : Br(\tau^- \rightarrow\ \nu_\tau + \text{hadrons})$$

$$\approx 17.83\% : 17.41\% : 62.65\%$$

with the additional 2% being due to $\tau^- \rightarrow\ e^-\nu_\tau\bar{\nu}_e\gamma$ and $\tau^- \rightarrow\ \mu^-\nu_\tau\bar{\nu}_\mu\gamma$. The BR to muons is slightly lower than that to electrons, due to mass of the muon reducing the available phase space. Furthermore, the branching ratio to hadrons is slightly higher than the naive prediction due to enhancements from QCD corrections (for example the diagram with an additional gluon in the final state).

2) Weak charged-current interaction

The Feynman diagram for the CC $\nu_e e^- \to e^- \nu_e$ process is shown below. Neglecting the q^2



dependence of the propagator, the corresponding matrix element is

$$
\begin{aligned}
-i\mathcal{M} &= \left[-i\frac{g_W}{\sqrt{2}}\bar{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_1)\right]\frac{ig_{\mu\nu}}{m_W^2}\left[-i\frac{g_W}{\sqrt{2}}\bar{u}(p_4)\gamma^\nu\frac{1}{2}(1-\gamma^5)u(p_2)\right] \\
\mathcal{M} &= \frac{g_W^2}{2m_W^2}g_{\mu\nu}\left[\bar{u}(p_3)\gamma^\mu\frac{1}{2}(1-\gamma^5)u(p_1)\right]\left[\bar{u}(p_4)\gamma^\nu\frac{1}{2}(1-\gamma^5)u(p_2)\right] \\
&= \frac{g_W^2}{2m_W^2}g_{\mu\nu}[\bar{u}_\downarrow(p_3)\gamma^\mu u_\downarrow(p_1)][\bar{u}_\downarrow(p_4)\gamma^\nu u_\downarrow(p_2)],
\end{aligned} \tag{1}
$$

with the spinors

$$
u_\downarrow(p_1) = \sqrt{E}\begin{pmatrix}0\\1\\0\\-1\end{pmatrix}, u_\downarrow(p_2) = \sqrt{E}\begin{pmatrix}-1\\0\\1\\0\end{pmatrix}, u_\downarrow(p_3) = \sqrt{E}\begin{pmatrix}-s\\c\\s\\-c\end{pmatrix}, u_\downarrow(p_4) = \sqrt{E}\begin{pmatrix}-c\\-s\\c\\s\end{pmatrix},
$$

where $c = \cos\frac{\theta^*}{2}$ and $s = \sin\frac{\theta^*}{2}$. If this were the only Feynman diagram contributing to the process $\nu_e e^- \to e^- \nu_e$, following the derivation for the neutrino-quark scattering, one would obtain :

$$
\sigma_{CC}(\nu_e e^- \to e^- \nu_e) = \frac{G_F^2 s}{\pi}
$$

It should be noted that this neglects the NC Z-exchange diagram and that $\mathcal{M} \to \mathcal{M}_{CC} + \mathcal{M}_{NC}$, which has the effect of reducing the $\nu_e e^- \to e^- \nu_e$ cross section through negative interference. Carrying on regardless, the centre-of-mass energy is

$$
s = (p_1 + p_2)^2 = (E_\nu + m_e)^2 - E_\nu^2 = 2m_e E_\nu + m_e^2 \approx 2m_e E_\nu,
$$

and therefore

$$
\sigma_{CC}(\nu_e e^- \to e^- \nu_e) = \frac{2m_e E_\nu G_F^2}{\pi}.
$$

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3) Solar neutrinos

From the previous question, the "CC cross section" for a 10 MeV electron neutrino is given by

$$\begin{aligned}\sigma_{CC}(\nu_e e^- \to\ e^- \nu_e) &= \frac{2m_e E_\nu G_F^2}{\pi} \\ &= 2 \cdot (511 \times 10^{-6}\ \text{GeV}) \cdot (10^{-2}\ \text{GeV}) \cdot (1.16638 \times 10^{-5}\ \text{GeV}^{-2})^2/\pi \\ &= 4.43 \times 10^{-16}\ \text{GeV}^{-2}.\end{aligned}$$

In order to compute the cross section in cm^2, one needs to multiply the above value by the square of $\hbar c = 197\ \text{MeV} \cdot \text{fm}$:

$$\sigma_{CC}(\nu_e e^- \to\ e^- \nu_e) = (4.43 \times 10^{-16}\ \text{GeV}^{-2}) \cdot (197\ \text{MeV} \cdot \text{fm})^2 = 1.7 \times 10^{-43}\ \text{cm}^2.$$

A neutrino traversing the Earth passes through 12800 km of rock. Consider a cylinder of this length and area 1 cm^2. The number of nucleons contained in this cylinder will be :

$$n_N = \frac{\rho V}{m_N} = \frac{(5520\ \text{kg} \cdot \text{m}^{-3}) \cdot (1 \times 10^{-4}\ \text{m}^2) \cdot (12.8 \times 10^6\ \text{m})}{1.67 \times 10^{-27}\ \text{kg}} \simeq 4.2 \times 10^{33}$$

Assuming half the nucleons are protons (for which there will be an equal number of electrons), the number of electrons is therefore

$$n_e \simeq 2.1 \times 10^{33}\ \text{cm}^{-2}.$$

Hence the probability of interaction is

$$P = \sigma_{CC}(\nu_e e^- \to\ e^- \nu_e)[\text{cm}^2] \times n_e[\text{cm}^{-2}] \sim 10^{-10}$$