Particle Physics 2 : Exercise 1

1) Symmetries of the $\Omega^-(sss)$

In the absence of colour, the overall wavefunction has the following degrees of freedom:

$$\psi = \phi_{\text{flavour}}\chi_{\text{spin}}\eta_{\text{space}} \tag{1}$$

The overall wavefunction must be anti-symmetric under the interchange of any two quarks (since they are fermions). For the state with zero orbital angular momentum (l = 0), the spatial wavefunction is symmetric. The flavour wavefunction sss is clearly symmetric under the interchange of any two quarks. Therefore, the required overall anti-symmetric wavefunction would imply a totally anti-symmetric spin wavefunction. However, there is no totally anti-symmetric spin wavefunction for the combination of three spin-half particles. Hence, without an additional degree of freedom, in this case colour, the Ω^- would not exist.

2) Running couplings

The strong coupling constant runs according to:

$$\alpha_S(Q^2) = \frac{\alpha_S(Q_0^2)}{1 + B\alpha_S(Q_0^2)\ln(\frac{Q^2}{Q_0^2})} \tag{2}$$

where $B = (11N_c - 2N_f)/(12\pi)$. For $N_c = 3$ colours and $N_f = 3$ (valid at low values of $|q^2|$), $B = 27/(12\pi)$.

The strong coupling constant appears to become infinite when

$$\alpha_S(Q_0^2)\ln\Big(\frac{Q_\infty^2}{Q_0^2}\Big) = -\frac{12\pi}{27} \tag{3}$$

At $\sqrt{|Q_0^2|} = 10$ GeV, $\alpha_S \simeq 0.18$, and therefore α_S becomes infinite when

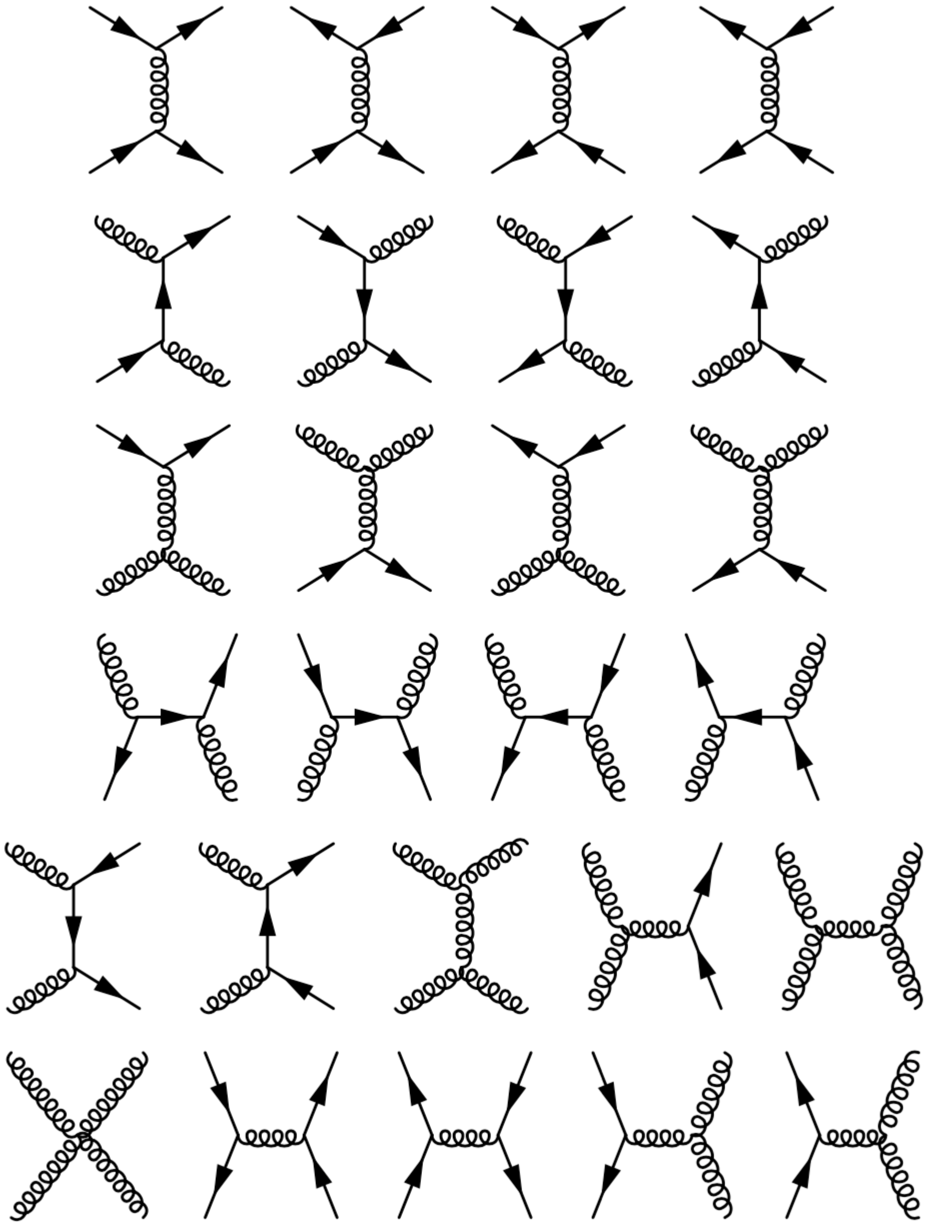
$$\begin{aligned} 0.18\ln\Big(\frac{Q_\infty^2}{100}\Big) &= -\frac{12\pi}{27} \\ \rightarrow \frac{Q_\infty^2}{100} &= 4.3\times 10^{-4} \\ \rightarrow Q_\infty &\approx 200 \text{ MeV} \end{aligned} \tag{4}$$

At this scale, known as Λ_{QCD}, QCD is clearly non-perturbative.

Updated: February 23, 2023

3) QCD Feynman diagrams lowest-order

There are diagrams involving: i) the scattering of quarks and antiquarks, ii) the scattering of a quark/antiquark and a gluon and iii) the scattering of gluons, where the antiquarks/quarks can either be from the valence or sea content of the proton and antiproton.



4) Drell-Yan production

The PDFs for the $\pi^+(\mathrm{u\bar{d}})$ can be written in terms of valence and sea quark distributions:

$$
\begin{aligned}
u^{\pi^+}(x) &= u_V^{\pi^+}(x) + S^{\pi^+}(x) \equiv u_V^{\pi}(x) + S^{\pi}(x) \\
\bar{d}^{\pi^+}(x) &= \bar{d}_V^{\pi^+}(x) + S^{\pi^+}(x) \equiv \bar{d}_V^{\pi}(x) + S^{\pi}(x) \\
d^{\pi^+}(x) &= S^{\pi^+}(x) \equiv S^{\pi}(x) \\
\bar{u}^{\pi^+}(x) &= S^{\pi^+}(x) \equiv S^{\pi}(x)
\end{aligned}
\tag{5}
$$

where the symbols with a superscript π implicitly refer to the PDFs for the π^+. Assuming isospin symmetry, e.g. the down-quark PDF in the $\pi^-(\mathrm{d\bar{u}})$ will be identical to the up-quark PDF in the π^+, the PDFs for the π^- are

$$
\begin{aligned}
u^{\pi^-}(x) &= S^{\pi^-}(x) \equiv S^{\pi}(x) \\
\bar{d}^{\pi^-}(x) &= S^{\pi^-}(x) \equiv S^{\pi}(x) \\
d^{\pi^-}(x) &= d_V^{\pi^-}(x) + S^{\pi^-}(x) \equiv u_V^{\pi}(x) + S^{\pi}(x) \\
\bar{u}^{\pi^-}(x) &= \bar{u}_V^{\pi^-}(x) + S^{\pi^-}(x) \equiv \bar{d}_V^{\pi}(x) + S^{\pi}(x)
\end{aligned}
\tag{6}
$$

For low Q^2 Drell-Yan production annihilation of sea quarks will dominate and therefore

$$
\frac{\sigma(\pi^+ C \to \mu^+\mu^- C)}{\sigma(\pi^- C \to \mu^+\mu^- C)} \to 1 \qquad \text{as } Q^2 \to 0 \tag{7}
$$

As $Q^2 \to s$, both x_1 and x_2 tend to unity and the annihilation of valance quarks will dominate. Since $\bar{d}^{\pi^+}(x) = \bar{u}^{\pi^-}(x)$ and carbon contains an equal number of valance up- and down-quarks

$$
\frac{\sigma(\pi^+ C \to \mu^+\mu^- C)}{\sigma(\pi^- C \to \mu^+\mu^- C)} \to \frac{Q_d^2 \bar{d}^{\pi^+}(1) d^C(1)}{Q_u^2 \bar{u}^{\pi^-}(1) u^C(1)} \to \frac{1}{4} \qquad \text{as } Q^2 \to s \tag{8}
$$

Updated: February 23, 2023