Particle Physics 2 : Exercise 11

1) Higgs potential

Explain why the Higgs potential can only contain terms with even powers of the field ϕ.

2) Covariant derivative

Show that the Lagrangian for a complex scalar field ϕ,

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi),$$

with the covariant derivative $D_\mu = \partial_\mu + igB_\mu$, is invariant under local U(1) gauge transformations,

$$\phi(x) \to \phi'(x) = e^{ig\chi(x)}\phi(x),$$

provided the gauge field transforms as

$$B_\mu \to B'_\mu = B_\mu - \partial_\mu \chi(x).$$

3) Z and γ fields

From the mass matrix and its eigenvalues, show that the eigenstates in the diagonal basis are

$$A_\mu = \frac{g' W_\mu^{(3)} + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \quad \text{and} \quad Z_\mu = \frac{g_W W_\mu^{(3)} - g' B_\mu}{\sqrt{g_W^2 + g'^2}},$$

where A_μ and Z_μ correspond to the physical fields for the photon and Z.

4) HZZ coupling

By considering the interaction terms in

$$\begin{aligned}(D_\mu \phi)^\dagger (D^\mu \phi) = \frac{1}{2}(\partial_\mu h \partial^\mu h) &+ \frac{1}{8} g_W^2 (W_\mu^{(1)} + iW_\mu^{(2)})(W^{\mu(1)} - iW^{\mu(2)})(v+h)^2 \\ &+ \frac{1}{8}(g_W W_\mu^{(3)} - g' B^\mu)(g_W W^{\mu(3)} - g' B_\mu)(v+h)^2,\end{aligned} \tag{1}$$

show that the HZZ coupling is given by

$$g_{HZZ} = \frac{1}{2}\frac{g_W}{\cos\theta_W} m_Z.$$

5) H $\to$ WW

For a Higgs boson with $m_H > 2m_W$, the dominant decay mode is into two on-shell W bosons, $H \to W^+W^-$. The matrix element for this decay can be written

$$\mathcal{M} = -g_W m_W g_{\mu\nu} \xi^\mu(p_2)^* \xi^\nu(p_3)^*,$$

where p_2 and p_3 are respectively the four-momenta of the W^- and W^+.

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a) Taking p_2 to lie in the positive z-direction, consider the nine possible polarisation states of the W^+ and W^- and show that the matrix element is only non-zero when both W bosons are left-handed ($\mathcal{M}_{\downarrow\downarrow}$), both W bosons are ($M_{\uparrow\uparrow}$), or both are longitudinally polarised (M_{LL}).

b) Show that

$$\mathcal{M}_{\uparrow\uparrow} = \mathcal{M}_{\downarrow\downarrow} = -g_{\mathrm{W}} m_{\mathrm{W}} \quad \text{and} \quad \mathcal{M}_{LL} = \frac{g_{\mathrm{W}}}{m_{\mathrm{W}}}\left(\frac{1}{2}m_{\mathrm{H}}^2 - m_{\mathrm{W}}^2\right).$$

c) Hence show that

$$\Gamma(\mathrm{H} \to \ \mathrm{W}^+\mathrm{W}^-) = \frac{G_{\mathrm{F}} m_{\mathrm{H}}^3}{8\pi\sqrt{2}}\sqrt{1-4\lambda^2}(1-4\lambda^2+12\lambda^4),$$

where $\lambda = m_{\mathrm{W}}/m_{\mathrm{H}}$.