Astrophysics V Observational Cosmology

Sheet 3: Assignments

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Exercise 1 : CMB radiation density

The Stefan–Boltzmann law implies that the energy density of photons radiated by a black body of temperature T is :

$$U = \frac{4}{c} \cdot \sigma T^4 \tag{1}$$

This is often written as $\rho_r c^2 = U = aT^4$, where $a \approx 7.56 \cdot 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$.

a) Calculate the CMB energy density of today.

b) Assuming $\rho_{\mathrm{crit},0} = \frac{3H_0^2}{8\pi G} \approx 1.88 \cdot 10^{-26}\, h^2$ kg m^{-3} at t_0, calculate $\Omega_{\mathrm{CMB},0} = \rho_{\mathrm{CMB},0}/\rho_{\mathrm{crit},0}$, the present day fractional contribution of the CMB radiation energy density to the total density. $h = 100 \times H_0$, you can leave it as a unit.

c) Give a rough estimate of the present day mean number density of CMB photons. The total energy density $U_\nu = n \cdot h\nu$ where n is the number density of photons. The typical energy of a photon in a black-body distribution of temperature T is roughly $3kT$, where $k = 1.38 \cdot 10^{-23}\ J \cdot K^{-1}$.

d) Calculate the present day baryon number density, provided that $\Omega_{b,0} = \rho_{\mathrm{b},0}/\rho_{\mathrm{crit},0} \approx 0.022h^{-2}$ and that $m_{\mathrm{proton}} \approx 1.7 \cdot 10^{-27}$ kg. Are baryons in the whole universe as compact as on the earth ?

Exercise 2 : Redshifting Planck's law

The CMB photons did have a black body spectrum ($T \sim 3000$ K) when they were emitted, as they were before in equilibrium with the baryonic matter. But how come we still observe them today with a black body spectrum ($T \sim 2.7$K)? The goal of this exercie is to show that the despite the Universe's expansion, the CMB spectrum remains a black body.

The spectral energy density of black body photons (i.e., energy per unit volume and frequency interval), is :

$$u_\nu = \frac{4\pi}{c} \cdot I_\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp\left(h\nu/kT\right) - 1} \tag{2}$$

a) How does the CMB photons total energy density $U = n * h\nu$ vary between $t_1 < t_2$? We remind the following definitions :
- the scale radius $R \equiv R(t)$: it quantifies the expansion of the Universe at a time t; at two different epochs $t_1 < t_2$, two objects following the expansion flow will be separated by $d(t_1) < d(t_2)$, with $d(t_2)/d(t_1) = R(t_2)/R(t_1)$.
- $z \equiv z(t)$, the redshift : it is a relativistic observational effect due to the expansion of the Universe; it is related to the scale factor : $R/R_0 = 1/(1+z)$.

b) Show that the despite the Universe's expansion CMB spectrum remains black body.

c) In the jupyter notebook, you will fit the temperature of the Universe with a CMB spectrum measured by Cosmic Background Explorer (COBE) Satellite in 2003. Is your measurement consistent with the number presented in the lecture? Why?

d) Compare the CMB spectrum at the time of emission and today in the second exercise of the jupyter notebook.

Exercise 3 : Clustering of galaxies

One method to study the clustering of galaxies is to compute their 2-point correlation function (2PCF) $\xi(\mathbf{r}_1, \mathbf{r}_2)$. This function shows the surplus ($\xi > 0$) or the deficit of probability ($\xi < 0$) (with respect to randomly distributed galaxies) to find one galaxy at position $\mathbf{r}_1$ and the other at position $\mathbf{r}_2$. For a homogeneous and isotropic Universe, we have $\xi(\mathbf{r}_1, \mathbf{r}_2) = \xi(r)$, where $r = |\mathbf{r}_2 - \mathbf{r}_1|$. In this case, the correlation function shows the excess or the deficit of probability that two galaxies are separated by a distance r, with respect to randomly distributed galaxies :

$$\xi(r) = \langle \delta(\mathbf{x}) \delta(\mathbf{x} + \mathbf{r}) \rangle, \tag{3}$$

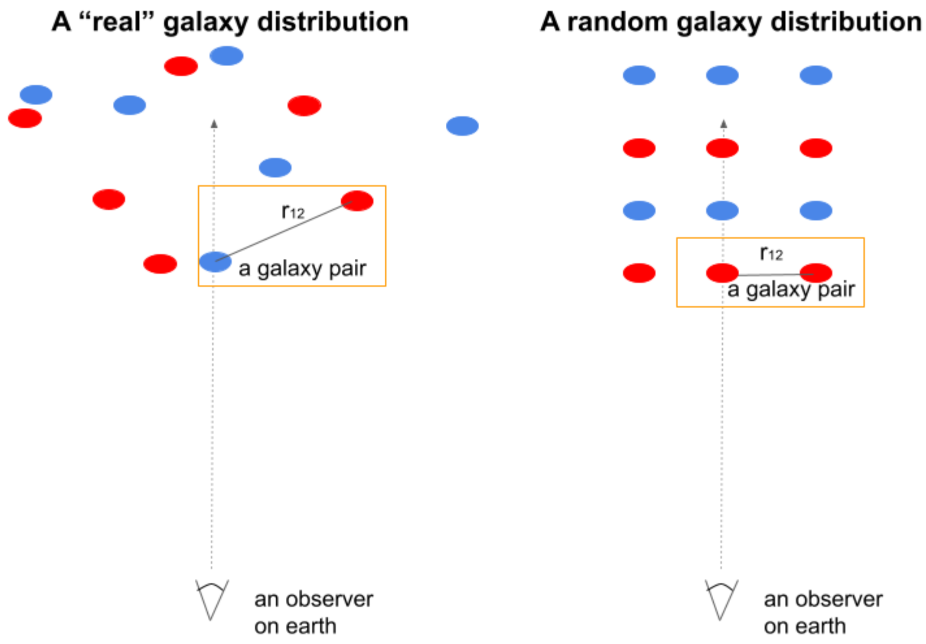

Figure 1. The galaxy-pair distance histogram (the solid line in the upper right panel) of a given galaxy distribution (the lower panel). The dashed line in the distance histogram is for the random distribution, where ρ_0 is the number density of randomly-distributed particles.

where $\delta(\mathbf{x})$ is the density contrast or the excess of mass density :

$$\delta(\mathbf{x}) = \frac{\rho(\mathbf{x}) - \overline{\rho}}{\overline{\rho}}. \tag{4}$$

In the last formula, $\rho(\mathbf{x})$ is the mass density at each point in the Universe, while $\overline{\rho}$ is the average mass density in the Universe. These formulas can be used in the case of continuously distributed matter, which is not the case in reality. In practice, one obtains the 3D positions of galaxies and use an estimator for the correlation function.

The form of the simplest estimator is the following :

$$\xi(r) = \frac{N_{\text{obs}}(r)}{N_{\text{random}}(r)} - 1. \tag{5}$$

In this case, $N_{obs}(r)$ and $N_{random}(r)$ are numbers of galaxy pairs separated by a distance r and normalized by the total number of pairs of galaxies. The first one is obtained from the observed data, while the second one is obtained from a random catalog (i.e. a catalog where the points are uniformly and randomly distributed). Figure 1 shows an example of obtaining $N_{obs}(r)$ for a given galaxy distribution and calculating $N_{random}(r)$ analytically.

In this exercise, you will use data from a simulated Universe that has a shape of a box with a side length of 2500 h^{-1}Mpc and contains 5468756 galaxies. You are provided the number of pair counts for a set of distances r and $\Delta r = 2.5h^{-1}$Mpc. You will have to use two python packages : `numpy` for numerical computation and `matplotlib` to plot the results.

a) Compute the total number of pairs and then compute the N_{obs} term. Plot N_{obs} as function r;

b) Obtain an analytical formula for the N_{random} term, starting from the fact that the random points should be uniformly distributed. Plot N_{random} and N_{obs} as functions of r on the same figure. What do you expect to see ? Are the resulting curves as you expected them to be ? Hint : Assume periodic boundary conditions.

c) Compute and plot the 2PCF $\xi(r)$ as function of r from equation 5.