

Astrophysics V Observational Cosmology

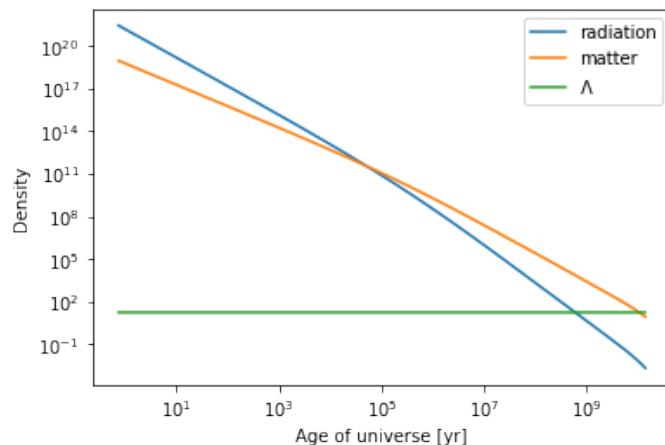
Sheet 5: Solutions

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Exercise 1 : Evolution of the Universe



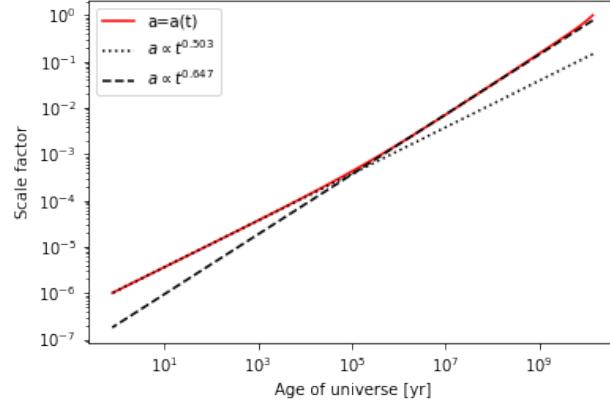
The evolution in time of matter, radiation and cosmological constant energy densities.

In figure 1, one can observe that the density of dark energy is constant in time, while the energy densities of radiation and matter is continuously decreasing.

One can observe that the radiation density decreases faster than the matter density and it was dominant in the early universe. After some time, the matter density became dominant. The moment when the two densities were equal is represented by the intersection of the blue and orange curves.

Today, one can observe that the cosmological constant (or Dark Energy) is

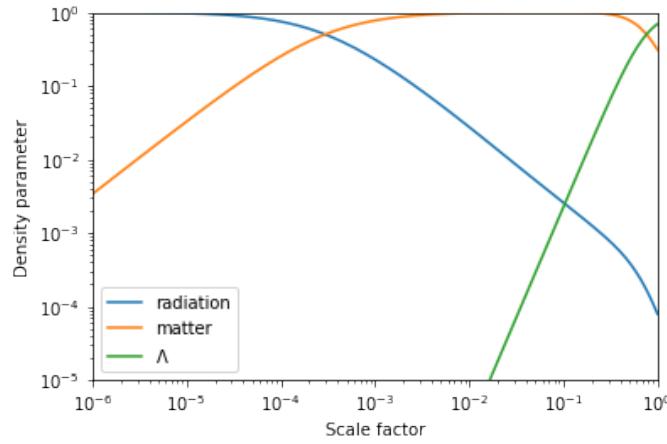
dominant, and the radiation is negligible. In figure 2, one can observe the evo-



The time evolution of the scale factor and two best fit curves for $t \in [10, 1000]$ years and $t \in [10^6, 10^8]$ years, respectively.

lution of the scale factor with the age of the universe. As you already know, the Universe is expanding, thus the scale factor increases in time.

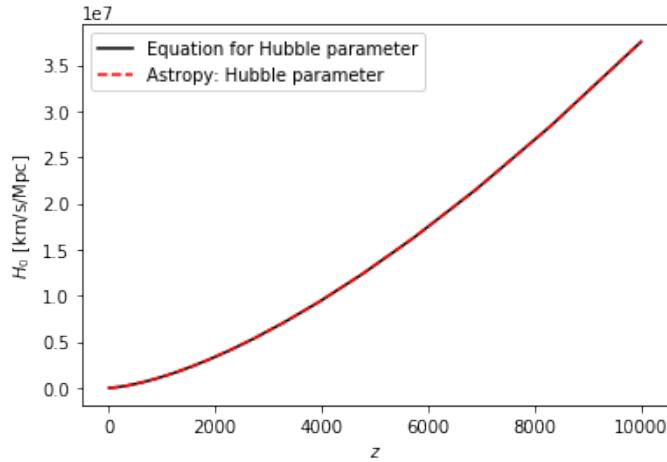
It is interesting to observe that at early time scales, the scale factor increases at a different rate compared to later time scales. This is related to the fact that during the early universe, the expansion was dominated by the radiation, which would imply that $a \propto t^{2/3}$, while for the second part, the "expansion" was dominated by the matter implying $a \propto t^{1/2}$. The last figure 3, reaffirms the previous



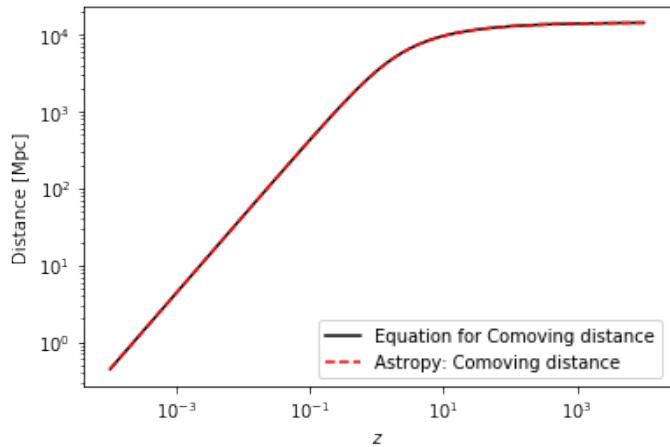
The evolution of the Ω parameters as function of the scale factor.

discussion. However, in this terms, the dark energy density Ω_Λ is not constant (compared to ρ_Λ which was constant). This is caused by the fact that the critical density depends on time.

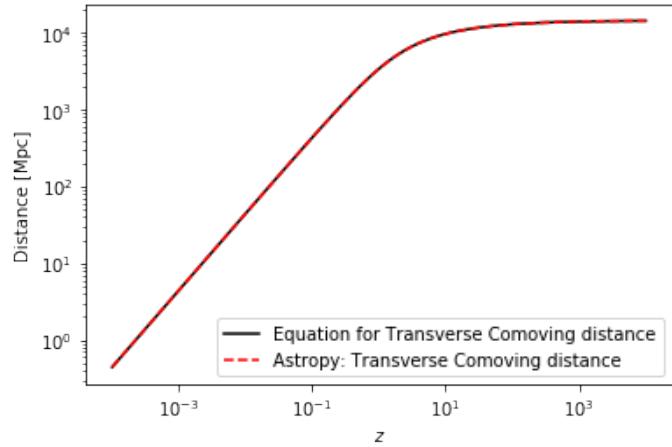
Exercise 2 : Distances in the Universe



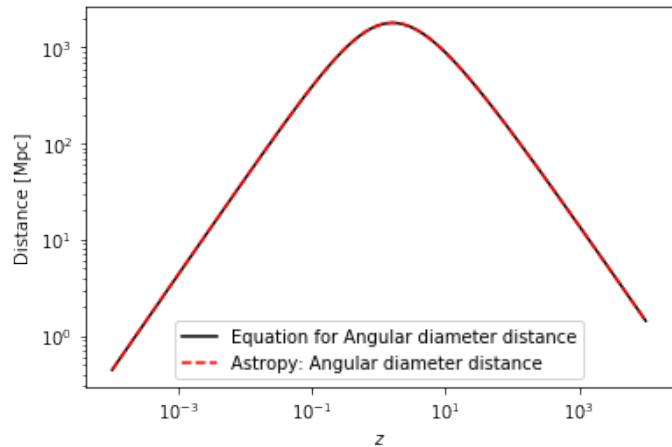
The Hubble parameter as function of redshift for the Flat Λ CMD cosmological model with $H_0=67.7 \text{ km/s/Mpc}$, $\Omega_\Lambda = 0.693$ and $\Omega_{\text{mat}} = 0.307$.



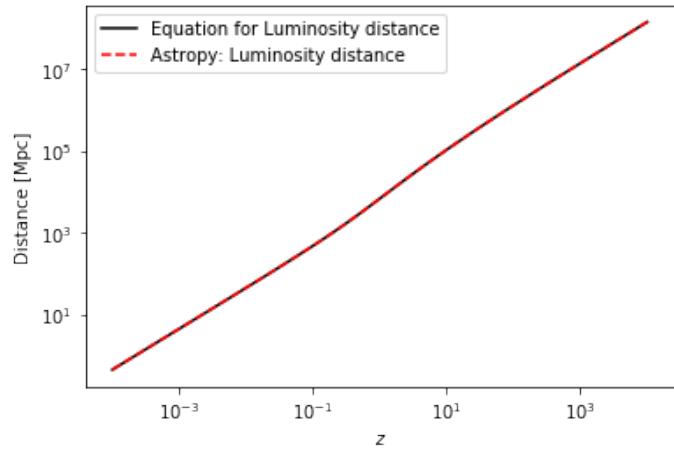
The comoving distance as function of redshift for the Flat Λ CMD cosmological model with $H_0=67.7 \text{ km/s/Mpc}$, $\Omega_\Lambda = 0.693$ and $\Omega_{\text{mat}} = 0.307$.



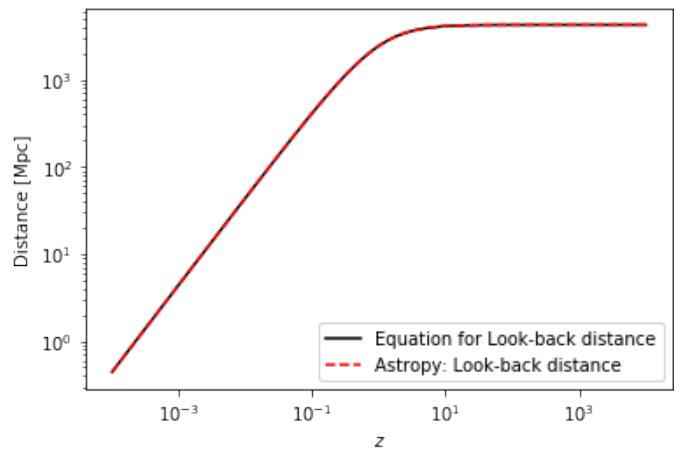
The transverse comoving distance as function of redshift for the Flat Λ CMD cosmological model with $H_0=67.7$ km/s/Mpc, $\Omega_\Lambda = 0.693$ and $\Omega_{\text{mat}} = 0.307$.



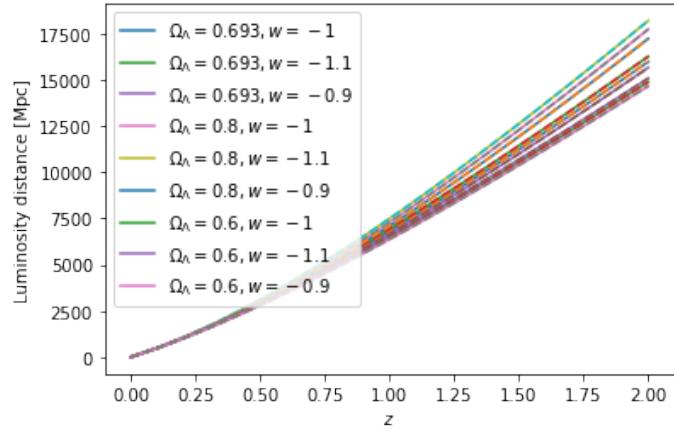
The angular diameter distance as function of redshift for the Flat Λ CMD cosmological model with $H_0=67.7$ km/s/Mpc, $\Omega_\Lambda = 0.693$ and $\Omega_{\text{mat}} = 0.307$.



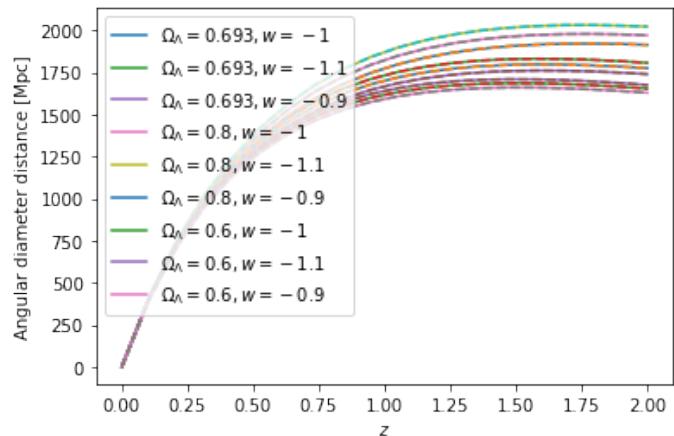
The luminosity distance as function of redshift for the Flat Λ CMD cosmological model with $H_0=67.7$ km/s/Mpc, $\Omega_\Lambda = 0.693$ and $\Omega_{\text{mat}} = 0.307$.



The look-back distance as function of redshift for the Flat Λ CMD cosmological model with $H_0=67.7$ km/s/Mpc, $\Omega_\Lambda = 0.693$ and $\Omega_{\text{mat}} = 0.307$.



The luminosity distance as function of redshift for the Flat w CMD cosmological model with $H_0=67.7$ km/s/Mpc and $\Omega_{\text{mat}} = 1 - \Omega_{\Lambda}$.



The angular diameter distance as function of redshift for the Flat w CMD cosmological model with $H_0=67.7$ km/s/Mpc and $\Omega_{\text{mat}} = 1 - \Omega_\Lambda$.