Astrophysics V Observational Cosmology

Sheet 2: Solutions

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Exercise 1 : Colours

The object with the redder colour will be the one with the larger F_2/F_1 flux ratio. The fluxes are related to the magnitudes by :

$$m_1 = -2.5 \times \log_{10}(F_1) + \mathrm{cst}_1 \quad m_2 = -2.5 \times \log_{10}(F_2) + \mathrm{cst}_2 \tag{1}$$

Therefore,

$$m_1 - m_2 = -2.5 \log_{10}(F_1/F_2) + (\mathrm{cst}_1 - \mathrm{cst}_2) \tag{2}$$

and thus

$$(F_2/F_1) \propto 10^{(m_1-m_2)/2.5} \tag{3}$$

The larger the value of $m_1 - m_2$, the larger the value of F_2/F_1. Therefore $m_1 - m_2 = 1$ is redder than $m_1 - m_2 = 0$.

Exercise 2 : Surface of the sky

By definition, an angle of 1 radian intercepts on a circle of radius R an arc of length R. So $2 \times \pi$ radians intercept the full circle, thus is equal to $360°$: 1 deg = π / 180 rad.

By analogy, one steradian intercepts on a sphere of radius R an area of R^2. So $4 \times \pi$ steradians intercept the whole sphere. By analogy with the degree-radian relation, we define a square degree with : 1 $\mathrm{deg}^2 = (\pi/180)^2$ sr.

The apparent surface of the celestial sphere is : 4π sr $= 4\pi\ (180/\pi)^2\ \mathrm{deg}^2 \sim 41,253\ \mathrm{deg}^2$.

Exercise 3 : Pencil-beam vs. wide-field

Let be S_o and S_s the object and sky photon counts per pixel per second. If we observe during an exposure time t, the number of photons coming from the object is $n_{\mathrm{pix}} \cdot S_o \cdot t$; the number of photons coming from the sky is $n_{\mathrm{pix}} \cdot S_s \cdot t$. If we assume that the sky has

Poissonian noise, the noise coming from the sky is : $\sqrt{n_{\rm pix} \cdot S_s \cdot t}$. Hence the signal-to-noise ratio is : $S/N = (n_{\rm pix} \cdot S_o \cdot t)/\sqrt{n_{\rm pix} \cdot S_s \cdot t} = \sqrt{n_{\rm pix}} \cdot \sqrt{t} \cdot S_o/\sqrt{S_s}$. An object being at the flux limit of a chosen S/N thus has a flux : $S_o = S/N \cdot \sqrt{S_s}/(\sqrt{n_{\rm pix}} \cdot \sqrt{t})$.

Suppose that your camera or detector covers an area A on the sky. Let's say that you invest all your time in a pencil-beam survey, and it reaches a flux $F_{\rm pencil}$. If the number counts are Euclidian, $N(F > F_{\rm pencil}) = k \cdot F_{\rm pencil}^{-3/2}$, where k is some constant. Therefore the number of galaxies seen in the pencil-beam survey is

$$n_{\rm pencil} = A \cdot N(F > F_{\rm pencil}) = A \cdot k \cdot F_{\rm pencil}^{-3/2}. \tag{4}$$

Now suppose that instead of doing a pencil-beam survey, you spread your integration time over M fields of view, each of which has an area A. The total area that you cover is $M \times A$, but the images would be shallower by a factor of $\sqrt{M}$. Thus the flux limit of the wide survey is : $F_{\rm wide} = \sqrt{M} \cdot F_{\rm pencil}$. Hence, the total number of galaxies in the wide-field survey would be :

$$n_{\rm wide} = M \cdot A \cdot N(F > F_{\rm wide}) = M \cdot A \cdot N(F > \sqrt{M} \cdot F_{\rm pencil}) \tag{5}$$

$$n_{\rm wide} = M \cdot A \cdot k \cdot M^{-3/4} F_{\rm pencil}^{-3/2} = M^{1/4} \cdot A \cdot k \cdot F_{\rm pencil}^{-3/2}. \tag{6}$$

Comparing to $n_{\rm pencil}$, we see that $n_{\rm wide} = M^{1/4} \cdot n_{\rm pencil}$, so the wide-field survey finds $M^{1/4}$ more galaxies.

To find out what the source count slope should be for there to be more galaxies in wide field survey than a pencil survey, we first assume the general form :

$$N(F > F_{\rm th}) \propto F_{\rm th}^{-p} \tag{7}$$

A similar calculation shows that :

$$n_{\rm pencil} = A \cdot k \cdot F_{\rm pencil}^{-p}, \tag{8}$$

and

$$n_{\rm wide} = M^{1-p/2} \cdot A \cdot k \cdot F_{\rm pencil}^{-p}. \tag{9}$$

We find that $n_{\rm pencil} > n_{\rm wide}$ if $p > 2$, i.e. if the source counts are steeper than $N(F > F_{\rm th}) \propto S^{-2}$.

However, only rarely are source counts that steep. In the vast majority of cases, wide-field surveys find more objects in a given observing time than pencil-beam surveys. In practice, though, there's often a limit to how wide you can make a survey, because the time spent simply moving the telescope or reading out the detector becomes significant. Another effect is that the total area of the sky available for extragalactic studies in limited by obscuration from our own Milky Way.

Exercise 4 : The Tully-Fisher relation

The balance between the centrifugal force and gravitation is :

$$m\frac{V_{max}^2}{R} = \frac{\mathrm{G}M}{R^2}m \qquad \Leftrightarrow \qquad M = \frac{V_{max}^2\,R}{\mathrm{G}}$$

With the assumption that all spiral galaxies have the same mass-to-luminosity ratio $M/L = \Upsilon$:

$$L = \frac{M}{\Upsilon} = \frac{V_{max}^2\,R}{\Upsilon\,\mathrm{G}} \qquad \Leftrightarrow \qquad R = \frac{L\,\Upsilon\,\mathrm{G}}{V_{max}^2}$$

With the assumption that all spiral galaxies have the same surface brightness $\mu = L/(\pi R^2)$:

$$L = \mu\,\pi R^2 = \mu\,\pi\frac{L^2\,\Upsilon^2\,\mathrm{G}^2}{V_{max}^4} \qquad \Leftrightarrow \qquad L = \frac{V_{max}^4}{\mu\,\pi\,\Upsilon^2\,\mathrm{G}^2}$$

With the definition of the absolute magnitude : $\mathcal{M} - \mathcal{M}_\odot = -2.5\log\left(L/L_\odot\right)$, one can obtain the Tully-Fisher relation :

$$\mathcal{M} = \mathcal{M}_\odot - 2.5\log\left(\frac{V_{max}^4}{\mu\,\pi\,\Upsilon^2\,\mathrm{G}^2 L_\odot}\right) = -10\log V_{max} + \mathrm{cte}$$

For Sa galaxies, we observe : $\mathcal{M}_B = -9.95\log V_{max} + 3.15$, with V_{max} in km.s^{-1}. The NGC 2639 galaxy has $V_{max} = 324$ km.s^{-1}, hence an absolute magnitude of $\mathcal{M}_B = -21.8$. Its apparent magnitude being $B = 11.5$, one can deduce its distance modulus :

$$B - \mathcal{M}_B = -2.5\log\left(\frac{F}{F_{10\,\mathrm{pc}}}\right) = -2.5\log\left(\frac{L}{4\pi D^2}\frac{4\pi 10^2}{L}\right) = 5\log\left(D[\mathrm{pc}]\right) - 5$$

Its distance is : $D = 10^{(B-\mathcal{M}_B+5)/5} = 46$ Mpc. Its luminosity is given by :

$$\mathcal{M}_B - \mathcal{M}_{B,\odot} = -2.5\log\left(\frac{L_B}{L_{B,\odot}}\right) \qquad \Leftrightarrow \qquad L_B = 10^{\frac{\mathcal{M}_{B,\odot}-\mathcal{M}_B}{2.5}}\,L_{B,\odot} = 8.1\cdot 10^{10}\,L_{B,\odot},$$

where $\mathcal{M}_{B,\odot} = 5.47$. The mass enclosed in a $R = 30$ kpc radius is :

$$M = \frac{V_{max}^2\,R}{\mathrm{G}} = 7.3\cdot 10^{11}\ \mathrm{M}_\odot,$$

where $\mathrm{G} = 4.303\cdot 10^{-6}$ km^2 s^{-2} kpc $\mathrm{M}_\odot^{-1}$. Eventually, NGC 2639 has : $M/L_B \simeq 9\ \mathrm{M}_\odot/L_{B,\odot}$.