Astrophysics V **Observational Cosmology**

Sheet 1: Solutions

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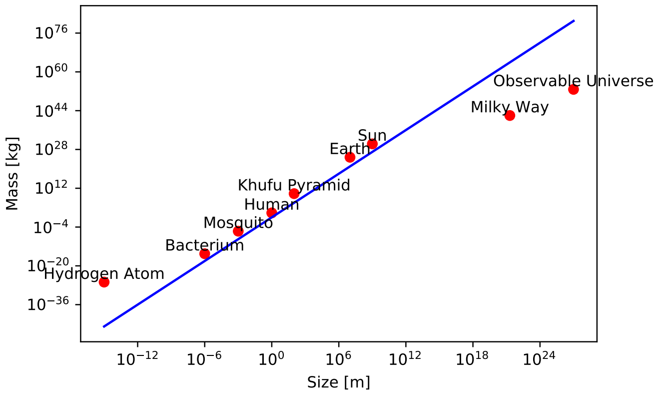
Exercise 1 : Hubble parameter and expansion rate

In [1]:
```
import matplotlib.pyplot as plt
%matplotlib inline
import numpy as np

# Main references:
# https://en.wikipedia.org/wiki/Orders_of_magnitude_(length)
# https://en.wikipedia.org/wiki/Orders_of_magnitude_(mass)
name = ['Observable Universe', 'Milky Way', 'Sun', \
        'Earth', 'Khufu Pyramid', 'Human', 'Mosquito', \
        'Bacterium', 'Hydrogen Atom']
size = [1e27, 2e21, 1e9, 1e7, 1e2, 1, 1e-3, 1e-6, 1e-15]
mass = [6e52, 1e42, 2e30, 6e24, 6e9, 70, 2e-6, 1e-15, 2e-27]

# Plot the masses as function of sizes.
plt.plot(size, mass, 'or')
for nm, sz, ms in zip(name, size, mass):
    plt.annotate(nm, (sz, ms), ha='center', va='bottom')

# Mass-size relation with the water density.
size_water = np.logspace(-15, 27)
mass_water = size_water**3
plt.plot(size_water, mass_water, 'b')

plt.xlabel('Size [m]')
```

```
plt.ylabel('Mass [kg]')
plt.xscale('log')
plt.yscale('log')
```



Exercise 2 : The Hubble law

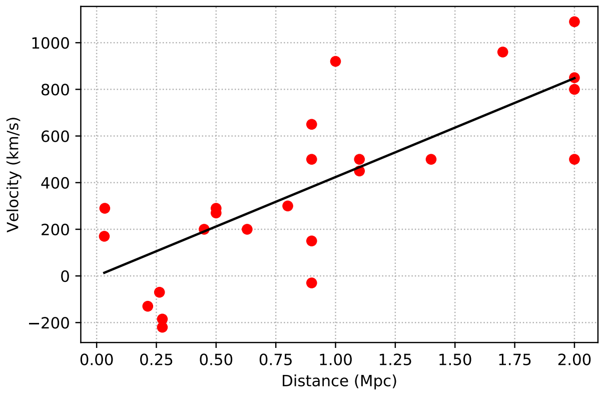
A negative radial velocity means that the observed galaxy is moving towards us; this can happen for close-by galaxies, which are leaving in the same dark matter halos than us; the relative motion inside the halos are not sensitive to the overall expansion of the Universe.

```
In [1]: import matplotlib.pyplot as plt
        %matplotlib inline
```

```
In [2]: # Velocity - distance relation from Hubble (1929).
        import numpy as np
        from scipy.optimize import leastsq

        D = [0.032, 0.034, 0.214, 0.263, 0.275, 0.275, 0.45, 0.5, \
            0.5, 0.63, 0.8, 0.9, 0.9, 0.9, 0.9, 1.0, \
            1.1, 1.1, 1.4, 1.7, 2.0, 2.0, 2.0, 2.0]
        V = [170, 290, -130, -70, -185, -220, 200, 290, \
            270, 200, 300, -30, 650, 150, 500, 920, \
            450, 500, 500, 960, 500, 850, 800, 1090]
        D = np.array(D)
```

```
V = np.array(V)

plt.plot(D, V, 'ro')

# Least-square fit
def fitfunc(par, x):
    return par[0] * x

def errfunc(par, x, y):
    return y - fitfunc(par, x)

pfit, pcov = leastsq(errfunc, [0], args=(D, V))

H0 = pfit[0]
res_sq = (errfunc(pfit, D, V)**2).sum() / (len(V)-len(pfit))
H0_err = np.sqrt(pcov * res_sq)

# Unit conversion
km_per_Mpc = 3.09e19
sec_per_Gyr = 3.15e16
H0_in_Gyr = H0 * sec_per_Gyr / km_per_Mpc

print('H0 = {:g} +- {:g} km/s/Mpc'.format(H0, H0_err))
print('1/H0 = {:g} Gyr'.format(1.0/H0_in_Gyr))

# Plot best-fit curve
x = np.linspace(D.min(),D.max())
y = fitfunc(pfit, x)
plt.plot(x, y, 'k-')

plt.grid(ls=':')
plt.xlabel('Distance (Mpc)')
plt.ylabel('Velocity (km/s)')
plt.show()
```

```
H0 = 423.937 +- 323.89 km/s/Mpc
1/H0 = 2.31391 Gyr
```



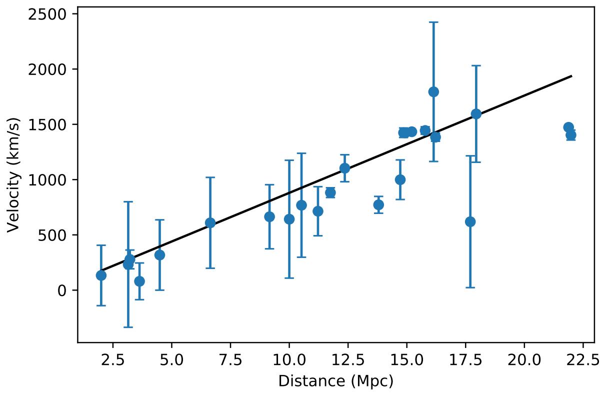
When expressed in km.s^{-1}.Mpc^{-1}, H_0 is typical speed at which two galaxies distant from 1 Mpc will be receding one from each other.

The time that has elapsed since the two galaxies were in contact is: $t = d/v = d/(H_0 \times d) = 1/H_0$. This time is refered as the Hubble time; it provides an estimate of the age of the Universe.

```
In [3]: # Velocity - distance relation from Freedman et al. (2001).

        D = [2.00, 9.16, 16.14, 17.95, 21.88, 3.22, 11.22, 11.75, \
            3.63, 13.80, 10.00, 10.52, 6.64, 15.21, 17.70, 14.86, \
            16.22, 15.78, 14.93, 21.98, 12.36, 4.49, 3.15, 14.72]
        V = [133, 664, 1794, 1594, 1473, 278, 714, 882, \
            80, 772, 642, 768, 609, 1433, 619, 1424, \
            1384, 1444, 1423, 1403, 1103, 318, 232, 999]
        dV = [273, 290, 630, 437, 8, 85, 222, 44, \
             166, 76, 533, 470, 411, 3, 596, 43, \
             37, 34, 25, 45, 122, 318, 568, 179]
        D = np.array(D)
        V = np.array(V)
        dV = np.array(dV)

        plt.errorbar(D, V, dV, marker='o', ls='none', capsize=3)

        # Least-square fit with error bars
        def fitfunc(par, x):
            return par[0] * x
```

```
def errfunc(par, x, y, err):
    return (y - fitfunc(par, x)) / err

pfit, pcov = leastsq(errfunc, [0], args=(D, V, dV))

H0 = pfit[0]
res_sq = (errfunc(pfit, D, V, dV)**2).sum() \
            / (len(V)-len(pfit))
H0_err = np.sqrt(pcov * res_sq)
H0_in_Gyr = H0 * sec_per_Gyr / km_per_Mpc

print('H0 = {:g} +- {:g} km/s/Mpc'.format(H0, H0_err))
print('1/H0 = {:g} Gyr'.format(1.0/H0_in_Gyr))

x = np.linspace(D.min(),D.max())
y = fitfunc(pfit, x)
plt.plot(x, y, 'k-')

plt.xlabel('Distance (Mpc)')
plt.ylabel('Velocity (km/s)')
plt.show()
```

```
H0 = 88.0236 +- 19.5971 km/s/Mpc
1/H0 = 11.1442 Gyr
```



Exercise 3 : Parsec

a) Parallax is the difference in apparent position of an object when viewed from different angles or lines of sight. The stellar parallax is the shift in apparent positions of nearby stars with respect to distant stars depending on the time of the year we observe them in. (This is different from the fact that we can see different stars at different times of the year).

b) A parsec stands for 'distance corresponding to a parallax of one arcsecond, as viewed from the earth, i.e. for a change in line of sight corresponding to 1AU, the distance of the earth to the sun. See Figure 1.

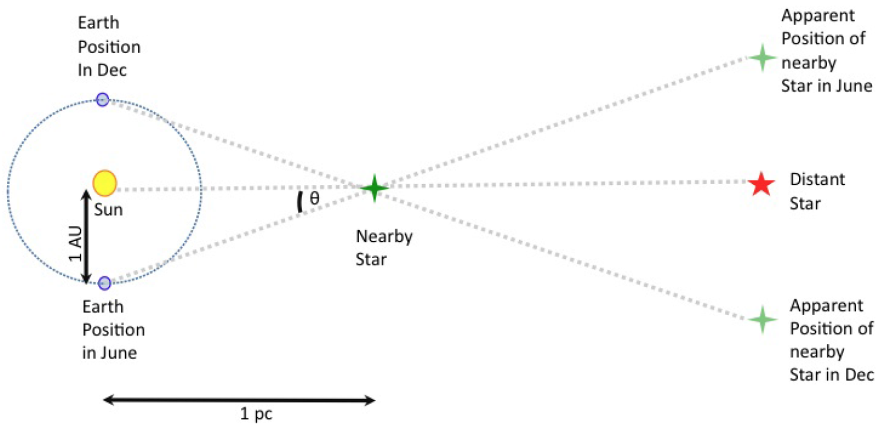


Illustration of stellar parallax (not to scale). The distance to the nearby star is 1 parsec if the angle $\theta = 1''$.

c) The distance can be determined by :

$$d = \frac{1\text{A.U.}}{\tan(1'')} \simeq \frac{1\text{A.U.}}{1''} = \frac{1\text{A.U.}}{\frac{1}{3600}\frac{\pi}{180}} \simeq 2.06265 \times 10^5 AU \tag{1}$$

where we've used tan $(1'') \simeq 1''$ since the angles are very small.

d) Since 1 A.U. $=1.49597871 \times 10^{11}$ m, this gives 1 pc $\simeq 3.0857 \times 10^{16}$ m.

e) 1 pc $\sim$ 3.26 light-years.

Exercise 4 : Introduction to redshift

Let us assume that M31 has a radial velocity v and that the observed redshift $z = -0.001$ is only due to Doppler effect.

Let λ_{em} be the emitted wavelength and λ_{obs} the observed wavelength.

The corresponding frequencies are $f = c/\lambda$, where c is the speed of light.

If the wavefront is emitted from M31 at t_0, we will receive it at $t_0 + d_0/c$, where d_0 is the distance of M31 at t_0.

The next wavefront will be emitted at $t_0 + 1/f_{\text{em}}$ and we will receive it at $t_0 + 1/f_{\text{em}} + (d_0 + v/f_{\text{em}})/c$.

Hence the observed frequency will be: $1/f_{\text{obs}} = 1/f_{\text{em}}(1 + v/c)$.

And the observed wavelength: $\lambda_{\text{obs}} = \lambda_{\text{em}}(1 + v/c)$.

We thus obtain: $z = (\lambda_{\text{obs}} - \lambda_{\text{em}})/\lambda_{\text{em}} \sim v/c$.

The observed $z = -0.001$ corresponds to a velocity of -300 km.s^{-1}.