Astrophysics V Observational Cosmology

Sheet 9: Solutions

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Exercise 1 : Cosmological horizons

Hubble radius

The proper distance of an emitter at redshift z is

$$D(t) = a(t)\chi(z)\,.$$

Differentiating this yields the radial velocity :

$$\begin{aligned} v_{\text{rad}} &= \dot{a}(t)\chi(z) + a(t)\dot{\chi}(z) \\ &= H(t)D(t) + a(t)\dot{\chi}(z) \\ &\equiv v_{\text{rec}} + v_{\text{pec}}\,, \end{aligned}$$

where $v_{\text{rec}} = H(t)D(t)$ is essentially the Hubble's law, while v_{pec} is the peculiar velocity of the emitter.

Hubble radius D_H is defined as the radius at which the recession velocity is equal to the speed of light :

$$D_H = c/H$$

Past light-cone

The light travelling distance fulfils

$$c\,dt = a(t)\,d\chi\,.$$

So the comoving distance to the past light cone is

$$\chi_{\rm lc}(t_{\rm em}) = c\int_{t_{\rm em}}^{t_0}\frac{dt'}{a(t')} = c\int_0^{z_{\rm em}}\frac{dz'}{H(z')}\,,$$

where $t_{\rm em}$ ($z_{\rm em}$) is the time (redshift) when photons are emitted.

Event horizon

The event horizon is the distance light can travel from a given time t to the end of the time ($t_{\rm end}$) :

$$\chi_{\rm eh}(t) = c\int_t^{t_{\rm end}}\frac{dt'}{a(t')} = c\int_{-1}^{z}\frac{dz'}{H(z')}\,,$$

where $t_{\rm end} = \infty$ in eternally expanding models or the time of the big crunch in recollapsing models. The event horizon characterise the most distant objects from which we will ever be able to receive information about the present day.

Particle horizon

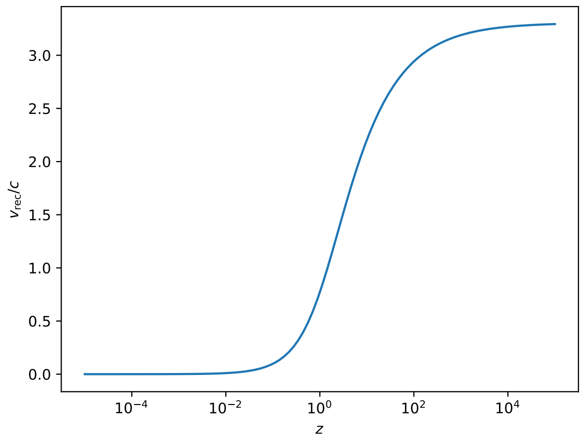
The particle horizon is the distance light can have travelled from the beginning of the Universe ($t = 0$), to a given time t :

$$\chi_{\rm ph}(t) = c\int_0^{t}\frac{dt'}{a(t')} = c\int_{z}^{\infty}\frac{dz'}{H(z')}\,.$$

Thus, it is the radius of the observable Universe.
Ref.: Davis & Lineweaver, Common misconceptions in cosmology

In [1]:
```
# Current recession velocity of objects
import matplotlib.pyplot as plt
%matplotlib inline
from astropy.cosmology import FlatLambdaCDM
import numpy as np
import astropy.constants as const

H0, Om, OL = [70., 0.3, 0.7]
z = np.logspace(-5, 5, 100)

cosmo = FlatLambdaCDM(H0=H0, Om0=Om)
z0 = 0.0
a0 = cosmo.scale_factor(z0)
```

```
hubble0 = cosmo.H(z0)

prop_dist = cosmo.comoving_distance(z) * a0
v_rec = hubble0 * prop_dist
v_rec = (v_rec / const.c).decompose()
plt.plot(z, v_rec)

plt.xscale('log')
plt.xlabel(r'$z$')
plt.ylabel(r'$v_{\rm rec} / c$')
plt.show()
```

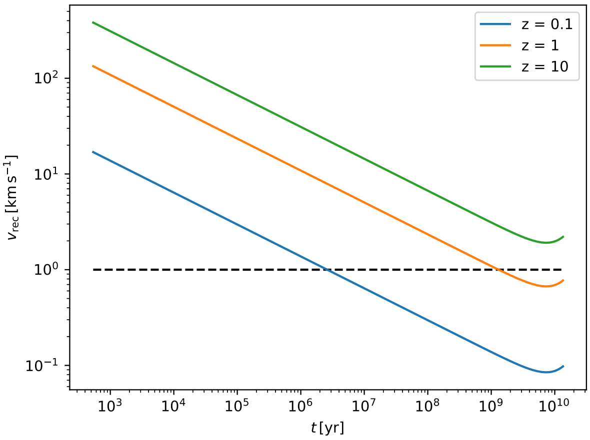


In [2]:
```
# Redshift at which the recession velocity is c
from scipy.optimize import fsolve

def func_vrec_c(z):
    xi = cosmo.comoving_distance(z)
    prop_dist = xi * a0
    v_rec = hubble0 * prop_dist
    v_rec = (v_rec / const.c).decompose()
    return v_rec - 1

init_guess = 0.0
zc = fsolve(func_vrec_c, init_guess)[0]
```

```
        print('Our current Hubble radius:')
        print(' z = {0:.2f}'.format(zc))
```

```
Our current Hubble radius:
 z = 1.46
```

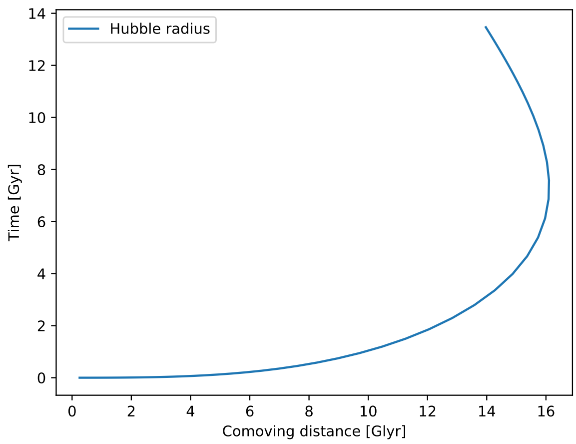
```
In [3]: # Recession velocity as a function of time
        import astropy.units as u

        z_emitter = [0.1, 1, 10]
        z = np.logspace(-5, 5, 100)

        t = cosmo.age(z)
        t = t.to(u.yr)
        a = cosmo.scale_factor(z)
        hubble = cosmo.H(z)

        tv = t.value
        plt.hlines(1, tv.min(), tv.max(), linestyles='dashed')

        for ze in z_emitter:
            prop_dist = cosmo.comoving_distance(ze) * a
            v_rec = hubble * prop_dist
            v_rec = (v_rec / const.c).decompose()
            plt.plot(t, v_rec, label='z = {}'.format(ze))

        plt.xscale('log')
        plt.yscale('log')
        plt.xlabel(r'$t\,[{\rm yr}]$')
        plt.ylabel(r'$v_{\rm rec}\,[{\rm km}\,{\rm s}^{-1}]$')
        plt.legend()
        plt.show()
```



In [4]:
```
# Recession velocity of last scattering surface

z_cmb = 1000.0
a_cmb = cosmo.scale_factor(z_cmb)

chi = cosmo.comoving_distance(z_cmb)
prop_dist_cmb = chi * a_cmb
prop_dist0 = chi * a0
hubble_cmb = cosmo.H(z_cmb)

v_rec_cmb = hubble_cmb * prop_dist_cmb
v_rec_cmb = (v_rec_cmb / const.c).decompose()
v_rec0 = hubble0 * prop_dist0
v_rec0 = (v_rec0 / const.c).decompose()

print('Recession velocity of CMB emitter:')
print(' at z=1000: {0:.2f} c'.format(v_rec_cmb))
print(' at z=0: {0:.2f} c'.format(v_rec0))
```

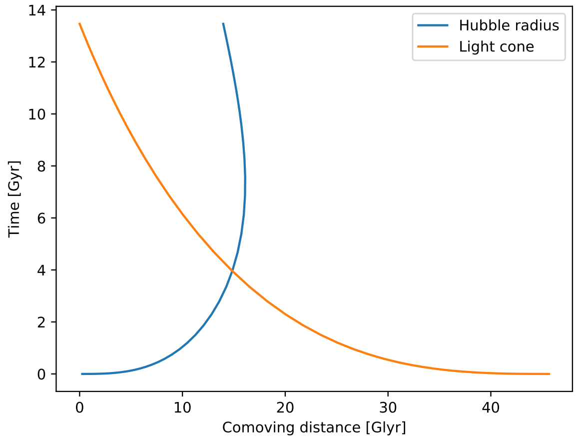
```
Recession velocity of CMB emitter:
 at z=1000: 55.27 c
 at z=0: 3.19 c
```

```
In [5]: # Evolution of Hubble radius

        z = np.logspace(-4, 4, 100)
        a = cosmo.scale_factor(z)
        t = cosmo.age(z)
        hubble = cosmo.H(z)

        d_hubble = const.c / hubble / a
        d_hubble = d_hubble.decompose().to(u.Glyr)

        plt.plot(d_hubble, t, label='Hubble radius')
        plt.xlabel('Comoving distance [Glyr]')
        plt.ylabel('Time [Gyr]')
        plt.legend()
        plt.show()
```




```
In [6]: # Past light cone

        d_lc = cosmo.comoving_distance(z)
        d_lc = d_lc.to(u.Glyr)

        plt.plot(d_hubble, t, label='Hubble radius')
        plt.plot(d_lc, t, label='Light cone')
        plt.xlabel('Comoving distance [Glyr]')
```

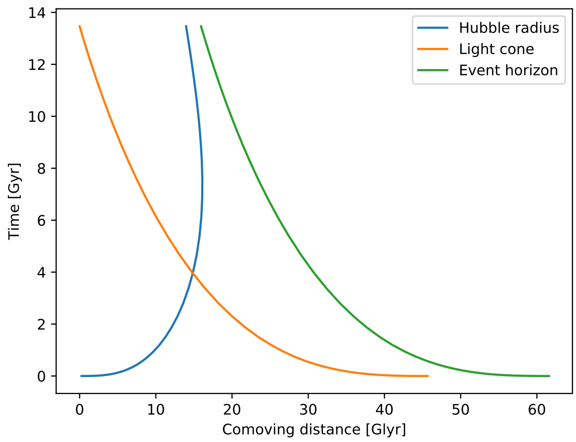
```
plt.ylabel('Time [Gyr]')
plt.legend()
plt.show()
```




```
In [7]: # Event horizon at z=0
        from scipy.integrate import quad
        from scipy.interpolate import interp1d

        def light_path(z):
            hubble = cosmo.H(z)
            lp = (const.c / hubble).decompose()
            return lp.to(u.Glyr).value

        d_eh0 = quad(light_path, -1, 0)[0]

        cd = cosmo.comoving_distance(z)
        cd = cd.to(u.Glyr)

        z_of_cd = interp1d(cd, z, kind='cubic')
        z_eh0 = z_of_cd(d_eh0)

        print('Our current event horizon:')
        print(' z = {0:.2f}'.format(z_eh0))
```

```
Our current event horizon:
 z = 1.81
```

In [8]:
```python
# Evolution of event horizon

num_z = len(z)
d_eh = np.zeros(num_z)

for i in range(num_z):
    d_eh[i] = quad(light_path, -1, z[i])[0]

plt.plot(d_hubble, t, label='Hubble radius')
plt.plot(d_lc, t, label='Light cone')
plt.plot(d_eh, t, label='Event horizon')
plt.xlabel('Comoving distance [Glyr]')
plt.ylabel('Time [Gyr]')
plt.legend()
plt.show()
```



In [9]:
```python
# Particle horizon at z=0

z0 = 0.0
d_ph0 = quad(light_path, 0, np.inf)[0]
```

```
        print('Current radius of the observable Universe:')
        print(' Dobs = {0:.2f} Glyr'.format(d_ph0))
```

```
Current radius of the observable Universe:
 Dobs = 46.17 Glyr
```

```
In [10]: # Evolution of particle horizon
         from scipy.integrate import quad_explain

         num_z = len(z)
         d_ph = np.zeros(num_z)

         for i in range(num_z):
             d_ph[i] = quad(light_path, z[i], np.inf)[0]

         plt.plot(d_hubble, t, label='Hubble radius')
         plt.plot(d_lc, t, label='Light cone')
         plt.plot(d_eh, t, label='Event horizon')
         plt.plot(d_ph, t, label='Particle horizon')
         plt.xlabel('Comoving distance [Glyr]')
         plt.ylabel('Time [Gyr]')
         plt.legend()
         plt.show()
```