Astrophysics V Observational Cosmology

Sheet 9: Assignments

Prof. Jean-Paul Kneib

Teaching assistants: Dr. Rafaela Gsponer, Dr. Antoine Rocher, Mathilde Guitton, Shengyu He, Ashutosh Mishra & Aurélien Verdier

Laboratoire d'astrophysique http://lastro.epfl.ch
Ecole Polytechnique Fédérale de Lausanne, Spring Semester 2025

Exercise 1 : Cosmological horizons

In the framework of the standard cosmological model (GR + ΛCDM), discuss whether the following propositions are correct :

(1) Nothing can recede faster than the speed of light, so the Hubble's law ($v_{\text{vec}} = HD$) needs special relativistic corrections when the recession velocities (v_{vec}) approaches the speed of light (c).

(2) Inflation causes superluminal expansion of the Universe, but the normal expansion of the Universe does not.

(3) Objects with recession velocities exceeding the speed of light are not observable.

(4) We can receive photons from objects only if their recession velocities were lower than the speed of light, when the photons were emitted.

(5) After inflation, the recession velocity of an object is a monotonically increasing function of time.

(6) If the recession velocity of an object is equal to the speed of light, then we will never receive photons that are emitted by it at present time.

(7) The age of the Universe is 13.8 billion years, so the radius of the observable Universe now is 13.9 billion light years.

In order to clarify these issues, let us consider an emitter at redshift z, with the radial comoving distance $\chi(z)$ in the observationally favoured flat ΛCDM cosmology : $\Omega_{\text{m}} = 0.3, \Omega_\Lambda = 0.7$, and $H_0 = 70\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$. The radial proper distance of the emitter at time t is

$$D(t) = a(t)\chi(z)\,,$$

where $a(t)$ is the scale factor.

(a) Express the radial velocity v_{rad} of the emitter. Is it consistent with the Hubble's law ?

(b) The difference between the radial velocity and the recession velocity expressed by the Hubble's law is indeed the peculiar velocity v_{pec}. Can v_{pec} be larger than the speed of light ? Under what circumstances can galaxies be blueshifted in an expanding universe ?

(c) Assume the peculiar velocities are negligible, plot the current recession velocity of emitters as a function of z, using the `cosmology` class of the `astropy` package [1].

(d) Using the equation solver `fsolve` in the `scipy` package [2], find the redshift at which the recession velocity of an object without peculiar motion is equal to the speed of light.

(e) Assume that the redshift of 3 emitters at present time are 0.1, 1, and 10, respectively, plot their recession velocities as the function of time (age of the Universe).

(f) Calculate the recession velocity of the last scattering surface now, and at the time when the CMB photons were emitted. (Suppose that the redshift of the last scattering surface is ~ 1000.)

(g) Plot the Hubble radius in comoving coordinates (the distance at which the recession speed is equal to the speed of light, horizontal axis, in Giga-lightyear) at different time (age of the Universe, vertical axis, in Giga-year). (Hint : try to express the Hubble radius directly, instead of using an equation solver.)

(h) Over-plot our past light cone in the Hubble radius diagram in (g), and explain why we can see objects outside the Hubble radius. (Hint : past light cone is defined as the radial distance to an object at given time, that emitted the light we see today.)

(i) The cosmic event horizon describes the most distant objects from which we will ever be able to receive photons emitted at present time. Express the event horizon in comoving distance as an integration function, and compute the redshift of the current event horizon using the `quad` routine for integration in the `scipy` package [3]. (Hint : one can interpolate comoving

1. http://docs.astropy.org/en/stable/cosmology/index.html
2. https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.fsolve.html
3. https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.quad.html

distance vs. redshift, to get the redshift of a given comoving distance, using the `interp1d` routine of the `scipy` package [4].)

(j) Plot the event horizon in the diagram you got in (h).

(k) The radius of the observable Universe is in fact the distance light can have travelled from the beginning of the Universe, to a given time (it is also called the particle horizon). Calculate the current radius of the observable Universe. What is the redshift at that distance ?

(l) Plot the particle horizon in the diagram you got in (j)

4. https://docs.scipy.org/doc/scipy/reference/generated/scipy.interpolate.interp1d.html