Astrophysics V Observational Cosmology

Sheet 7: Assignments

Prof. Jean-Paul Kneib

Teaching assistants: Dr. Rafaela Gsponer, Dr. Antoine Rocher,
Mathilde Guitton, Shengyu He, Ashutosh Mishra & Aurélien Verdier

Laboratoire d'astrophysique http://lastro.epfl.ch
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Exercise 1 : Large scale galaxy spectroscopic surveys

The SDSS-III's Baryon Oscillation Spectroscopic Survey (BOSS) was designed to measure the expansion rate of the Universe. It aimed at measuring the spatial distribution of luminous red galaxies (LRGs) and quasars, in order to detect the characteristic scale imprinted by baryon acoustic oscillations in the early universe. The final data release (DR12) of BOSS contains ~ 1.2 million LRGs, in the redshift range $0.2 < z < 0.75$, with a sky coverage of $9329\,\mathrm{deg}^2$.

a) Calculate the volume of the data set, as well as the mean number density of LRGs in the comoving space, assuming that we are living in a flat ΛCDM Universe, with the cosmological parameters being the most probable ones obtained through the CMB measurements of the Planck satellite in 2015 ($H_0 = 67.7\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_\mathrm{m} = 0.307$).

b) To estimate the covariance matrix of BOSS LRG clustering (e.g, 2-point correlation function and power spectrum), a commonly used way is to simulate a large number of universes with the same set of input cosmological parameters. If one wants to run a simulation in a cubic volume covering the BOSS LRG sample, estimate the minimum side length of the cubic box. (The BOSS data is actually separated into two galactic caps. For simplicity, we assume that the angular distribution of BOSS LRGs is elliptical, with the largest angular separation between galaxies being $\sim 150\,\mathrm{deg}$.)

Exercise 2 : Hubble parameter and expansion rate

The following statement reveals a common misconception about the time evolution of H : "The Hubble parameter measures the expansion rate of the

Universe, and since the Universe is accelerating now, H must have been smaller in the past and will in the future become greater than its present value."

Considering the period from $z = 5$, to the time that the Universe will be 100 times as large as the current size, plot $H(z)$ using the `cosmology` class of the `nbodykit` package [1], and explain what you observe. (Suppose that we are living in a flat ΛCDM Universe suggested by the results of the Planck satellite in 2015, i.e., $H_0 = 67.7\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_\mathrm{m} = 0.307$).

What is the parameter that quantifies the expansion rate of the Universe ? Plot its evolution in the same period as above. Is there a transition point ? Explain what you see with the evolution of density parameters.

Exercise 3 : Rotation curve of spiral galaxies

A commonly used way to measure the mass distribution inside a galaxy using the rotation curve is the decomposition method, in which the galaxy is represented by the superposition of several mass components, and the rotation curve is fitted by searching for the best-fitting parameters of those components.

It is often assumed that the mass components are the central black hole, a bulge, a disk, and a dark halo. The rotation velosity is then expressed by

$$V(R) = \sqrt{V_\mathrm{BH}^2(R) + V_\mathrm{b}^2(R) + V_\mathrm{d}^2(R) + V_\mathrm{h}^2(R)}\,.$$

Here, subscript BH, b, d, and h represent black hole, bulge, disk, and dark halo, respectively.

For simplicity, we consider only the disk and dark matter halo components here.

Exponential disk

The galactic disk is normally represented by an exponential disk, where the surface mass density is

$$\Sigma_\mathrm{d}(R) = \Sigma_0 \exp(-R/R_\mathrm{d})\,.$$

Here, Σ_0 is the central value and R_d is the scale radius. The rotation speed of a thin exponential disk is then expressed by

$$V_\mathrm{d}^2(R) = 4\pi G \Sigma_0 R_\mathrm{d} y^2 [I_0(y)K_0(y) - I_1(y)K_1(y)]\,,$$

where $y = R/(2R_\mathrm{d})$, and I_i and K_i are the modified Bessel functions.

1. https://nbodykit.readthedocs.io/en/latest/getting-started/cosmology.html

NFW dark matter halo

The most popular dark matter halo density profile is the Navarro-Frenk-White (NFW) model empirically obtained from numerical simulations in the cold dark matter scenario of galaxy formation :

$$\rho_{\rm NFW}(R) = \frac{\rho_0}{(R/R_s)(1+R/R_s)^2} .$$

Here, ρ_0 is the characteristic density, and R_s is the scale radius. The NFW profile leads to the following circular velocity :

$$V_{\rm h}^2(R) = V_{200}^2 \frac{1}{x} \frac{\ln(1+cx) - (cx)/(1+cx)}{\ln(1+c) - c/(1+c)} ,$$

where V_{200} is the circular velocity at R_{200}, i.e., the virial radius for which $\langle\rho\rangle = 200\rho_{\rm crit}$. The concentration parameter c is R_{200}/R_s, and x is R/R_{200}.

a) For a dark matter halo with $R_s = R_{200} = 50\,$kpc, and $V_{200} = 120\,{\rm km\,s^{-1}}$, suppose that there is a spiral galaxy in its centre, with the central surface mass density of the disk of $2 \times 10^8\,M_\odot\,{\rm kpc}^{-2}$, and the scale radius of the disk is 5 kpc. Plot the rotation curve of the dark matter halo, the galactic disk, and the overall system. ($G = 4.302 \times 10^{-6}\,{\rm kpc\,km^2\,s^{-2}}\,M_\odot^{-1}$)

b) Plot the rotation curve (with error bars) of NGC 3198 using the data in the table below [2]. And discuss if the galactic disk alone can explain this curve.

R (kpc)	2.0	3.0	4.0	5.5	6.0	7.0	8.0	9.0
V (km s^{-1})	79.0	97.8	118.0	139.4	144.2	143.3	150.3	149.9
ΔV (km s^{-1})	7.0	5.0	5.6	4.3	4.3	4.5	4.3	4.3
R (kpc)	10.1	11.0	12.1	14.1	16.1	18.1	20.1	22.1
V (km s^{-1})	152.1	151.1	156.2	161.0	155.3	148.7	149.1	148.4
ΔV (km s^{-1})	4.3	4.5	4.3	4.3	4.3	4.3	4.3	4.3
R (kpc)	24.1	26.1	28.1	30.2	32.2	34.2	36.2	38.2
V (km s^{-1})	146.2	145.5	147.3	146.5	148.4	149.3	149.9	149.3
ΔV (km s^{-1})	4.3	4.3	4.3	4.3	4.3	5.0	4.3	4.3
R (kpc)	40.2	42.1	44.2	46.2	48.2			
V (km s^{-1})	150.0	147.6	149.8	151.5	151.9			
ΔV (km s^{-1})	4.6	7.0	4.3	4.3	7.7			

c) Assume that the error on the circular velocity of NGC 3198 at different radii are independent, and neglecting the error on the radius, apply a minimum χ^2 fitting of the rotation curve using the `minimize` routine in the

2. taken from arXiv:1503.04049

`scipy.optimize` package[3]. Then, compare the best-fitting rotation curve with the observational data, and plot the circular velocity of the disk and dark matter components using the best-fitting parameters. (Hint : $\chi^2 \equiv \sum(\text{data} - \text{model})^2/\text{error}^2$)

Exercise 4 : Cosmology with Type Ia supernovae

A type Ia supernova is a type of supernova that occurs in binary systems (two stars orbiting one another) in which one of the stars is a white dwarf. The other star can be anything from a giant star to an even smaller white dwarf.

Physically, carbon-oxygen white dwarfs with a low rate of rotation are limited to below 1.44 solar masses. Beyond this, they reignite and in some cases trigger a supernova explosion. Therefore, type Ia supernovae produce consistent peak luminosity because of the uniform mass of white dwarfs that explode via the accretion mechanism. The stability of this value allows these explosions to be used as standard candles to measure the luminosity distance to their host galaxies because the visual magnitude of the supernovae depends primarily on the distance.

a) The measured redshift and B band apparent magnitude of a small sample of type Ia supernovae are listed in the table below[4]. Assume that the B band absolute magnitude for type Ia supernovae is $M_B = -19.3$ for all cases, plot the distance modulus (μ) of type Ia supernovae as a function of redshift ($\mu = m_B - M_B$).

3. `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html`
4. taken from `https://archive.stsci.edu/prepds/ps1cosmo/scolnic_datatable.html`, the Pantheon type Ia supernova database

z	0.17331	0.21946	0.26862	0.2836	0.30158	0.31156
m_B	20.15345	20.8469	21.1762	21.40915	21.64245	21.7417
Δm_B	0.09925	0.1023	0.0993	0.1028	0.0939	0.10825
z	0.32454	0.34052	0.3507	0.36791	0.37386	0.40455
m_B	21.79365	21.9463	21.9623	22.24455	22.3047	22.44625
Δm_B	0.10225	0.10555	0.10725	0.1019	0.0854	0.1075
z	0.43544	0.46156	0.47572	0.48881	0.51961	0.53532
m_B	22.36675	22.76265	22.6952	22.8653	23.03865	23.06945
Δm_B	0.109	0.11115	0.109	0.1136	0.109	0.11085
z	0.58375	0.62063	0.68196	0.74	0.78904	0.81773
m_B	23.2686	23.5114	23.59755	23.8446	24.2188	24.3431
Δm_B	0.09915	0.1131	0.1484	0.12605	0.11585	0.12085
z	0.85073	0.92624	0.99781	1.12	1.23	1.33
m_B	24.31865	24.7687	24.6122	25.0639	25.4985	25.6034
Δm_B	0.1304	0.10935	0.0953	0.1649	0.17085	0.1484

b) Using this data and the `cosmology` class of the `astropy` package[5], determine which of the following three ΛCDM cosmological models is the most likely (assume that $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$) :

i. $\Omega_\mathrm{m} = 0.3, \Omega_\Lambda = 0.7$

ii. $\Omega_\mathrm{m} = 0.3, \Omega_\Lambda = 0.0$

iii. $\Omega_\mathrm{m} = 1.0, \Omega_\Lambda = 0.0$

c) Assume that the error on the redshifts are negligible, apply a minimum χ^2 fitting to the $\mu(z)$ data using the `minimize` routine of the `scipy.optimize` package[6], in the framework of ΛCDM cosmology, and find the best-fitting H_0, Ω_m, and Ω_Λ. Are the results consistent with a flat Universe ? (Hint : $\chi^2 \equiv \sum(\mathrm{data} - \mathrm{model})^2/\mathrm{error}^2$)

d) Considering now a flat ΛCDM cosmology, apply a Markov Chain Monte Carlo (MCMC) sampling of the parameters H_0 and Ω_m, using the `emecc` package[7], by assuming a Gaussian likelihood ($L = \exp(-\chi^2/2)$). Then, given flat priors of $H_0 \in [50, 100]\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and $\Omega_\mathrm{m} \in [0, 1]$, plot the posterior probability distribution of the parameters, and estimate the mean value and error of H_0 and Ω_m. (Hint : please consult the example[8] in the documentation of `emcee`. You may use 50 walkers and 500 sampling steps for this exercise.)

5. `http://docs.astropy.org/en/stable/cosmology/index.html`
6. `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.minimize.html`
7. `https://emcee.readthedocs.io/en/stable/`
8. `https://emcee.readthedocs.io/en/stable/user/line.html`