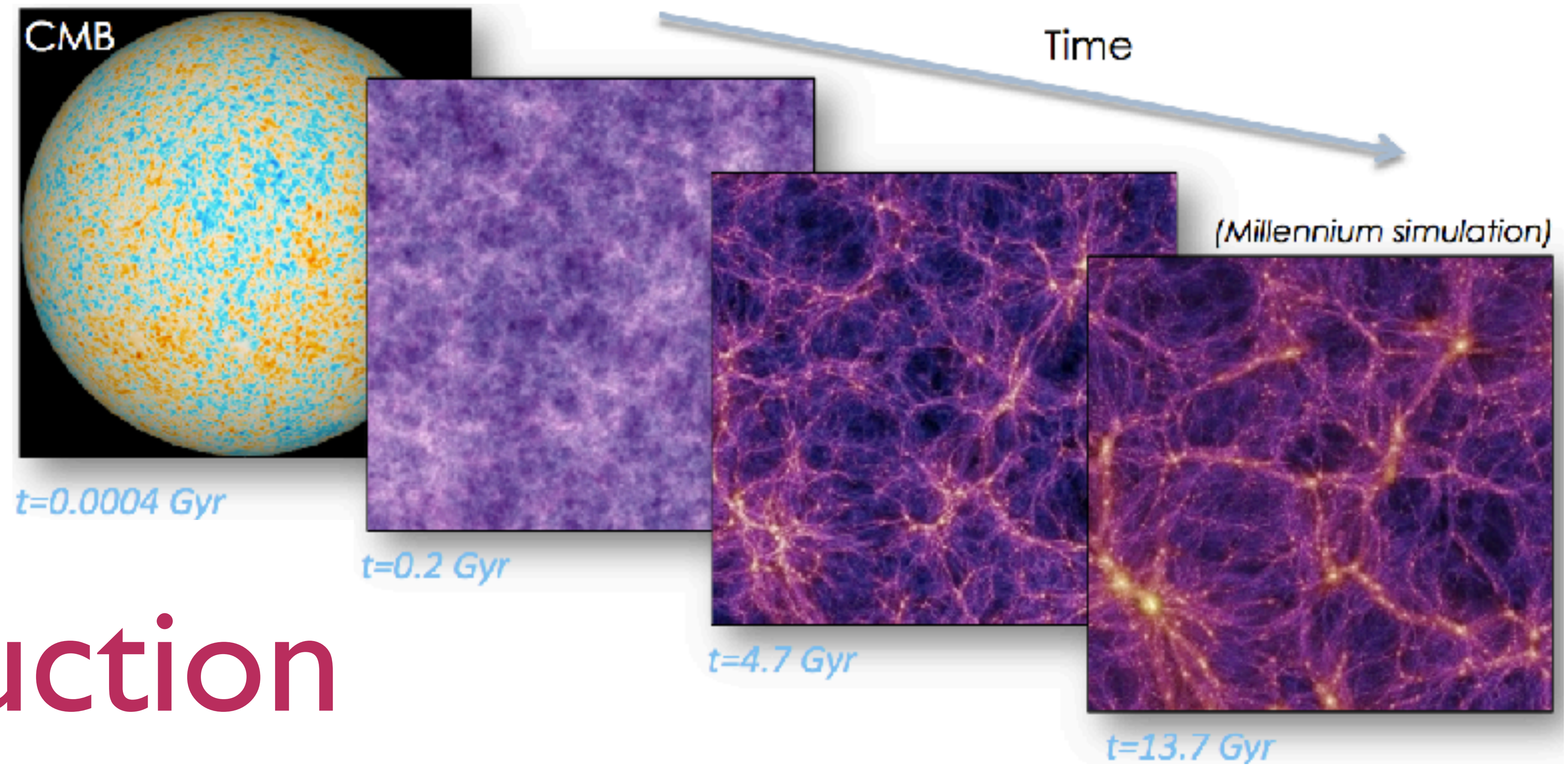
Inhomogeneities in the Universe

Jean-Paul KNEIB

Quiz

- What is the Inter-Galactic-Medium ?
- How can we probe it?
- What is the Lyman-alpha forest?
- What is a Damped-Lyman-alpha (DLA)?
- What is the Gunn-Peterson effect?
- What is the Epoch of Re-ionisation?
- What is the largest Radio Telescope?



Introduction

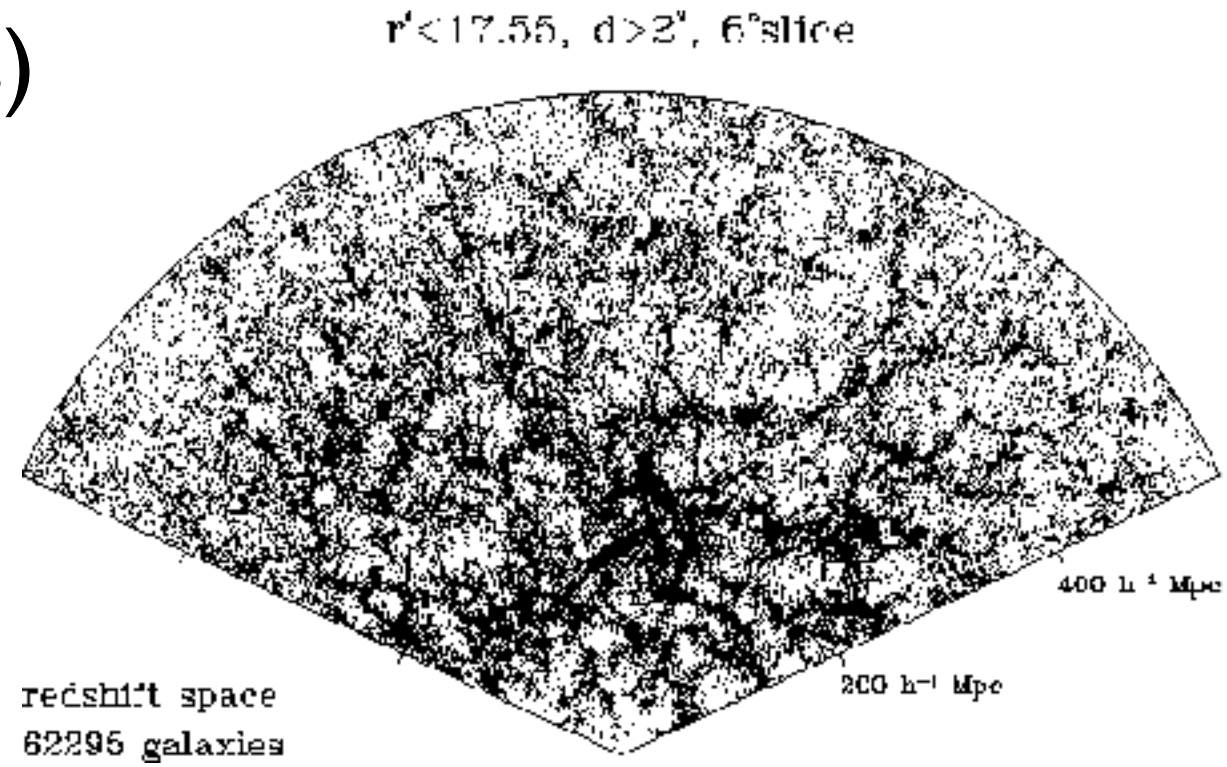
- CMB light display anisotropies of $\Delta T/T \sim 10^{-5}$ in the temperature at $z \sim 1100$
- These anisotropies will translate into inhomogeneities in the mass distribution
- **Goal of today:** how do we quantify the inhomogeneities in the Universe? How structures grow with time?

Outline

- Gravitational instability
- Description of the density fluctuation
- Evolution of the density fluctuation
- Non-Linear structure evolution
- Baryonic Acoustic Oscillation & Redshift Space Distortion

Some facts

- Evidence of structures (filaments) over $R_S \sim 100\ h^{-1}$ Mpc
- Size of the universe (Hubble radius): $R_H = c/H_0 \sim 3\ h^{-1}$ Gpc
- $R_S << R_H$
- $(R_H/R_S)^3 \sim (30)^3 \sim 30000$ volume elements exists per Hubble Volume
- *Overall the Universe can be consider homogeneous on large scale*



SDSS galaxy survey (2005)

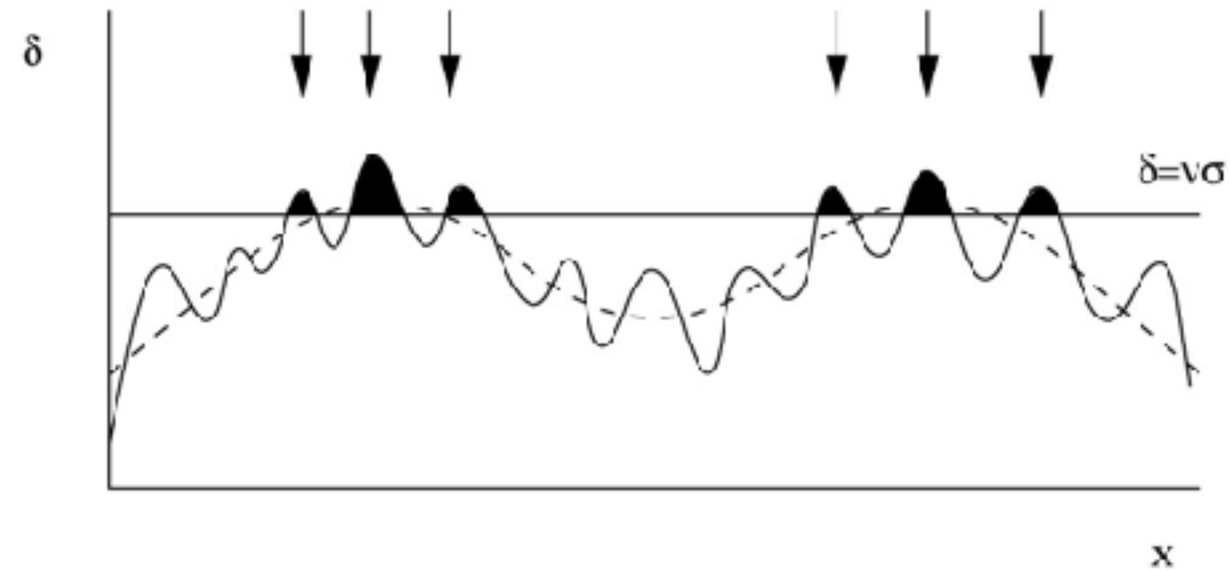
Gravitational instability

- **Definition:** relative density contrast

$$\delta(\mathbf{r},t)=\frac{\rho(\mathbf{r},t)-\overline{\rho}(t)}{\overline{\rho}(t)}$$

$$\delta \geq -1$$

- At z~1100 (CMB) $\delta \ll 1$
- Universe evolution depends on the mean density $\overline{\rho}(t)$
- A density fluctuation $\Delta\rho(\mathbf{r},t)=\rho(\mathbf{r},t)-\overline{\rho}(t)$ generates a gravitational field that will evolve with time.



Density fluctuation can exists at different scale

Gravitational instability

- Density fluctuation grows over time depending on their self gravity:

$$\overline{\rho}(t)=(1+z)^3\rho_0=\frac{\rho_0}{a^3(t)}$$

  - over-dense regions (peaks) increase their density contrast over time

$\Delta\rho>0$ Stronger gravitational field, slower expansion, density contrast will increase

  - under-dense regions (voids) decrease their contrast over time

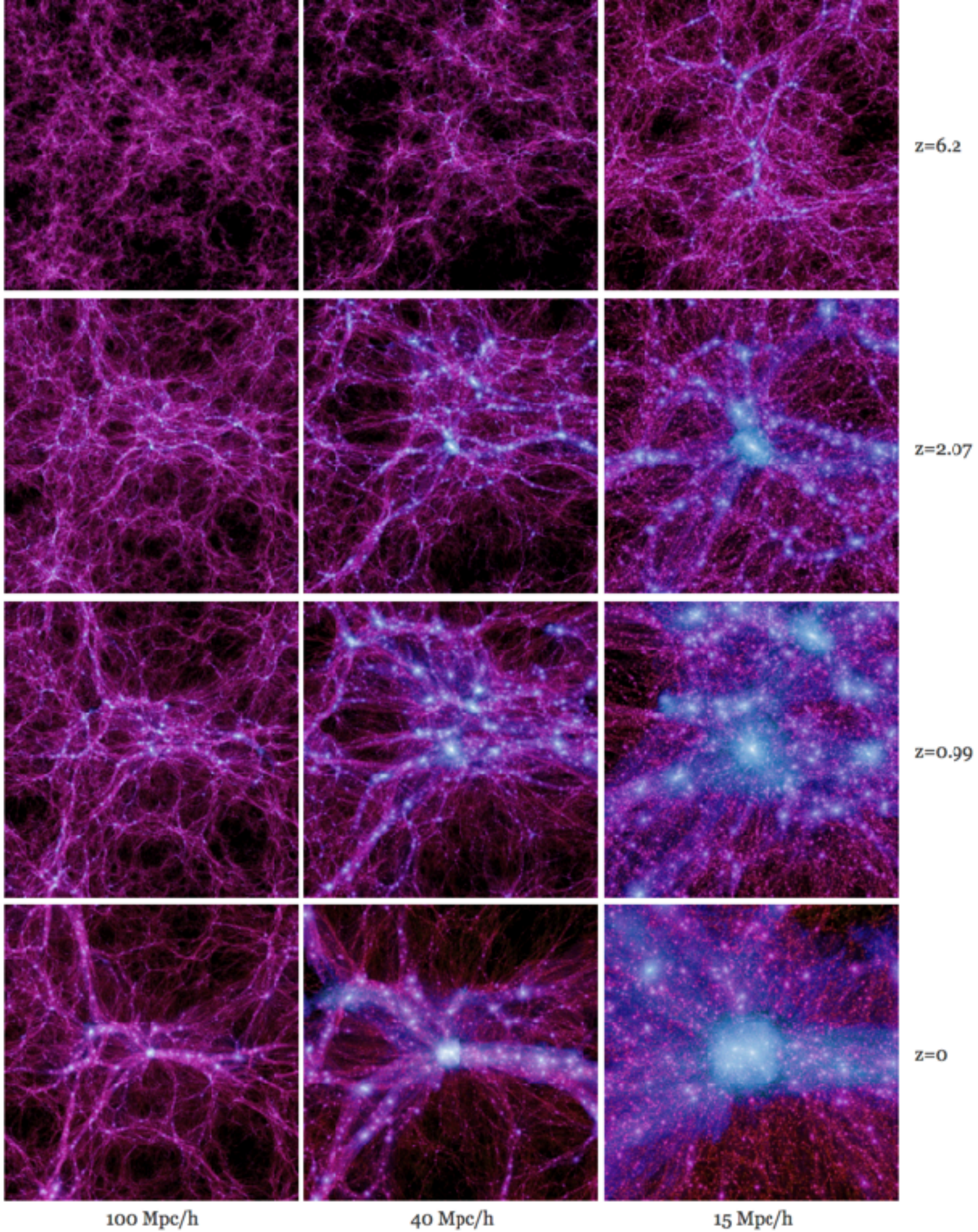
$\Delta\rho<0$ Weaker gravitational field, faster expansion, density contrast will decrease

- Evolution of structures is described by the model of gravitational instabilities

Time evolution of structures

Dark Matter only cosmological simulation by Volker Springer

http://wwwmpa.mpa-garching.mpg.de/galform/millennium-II/



Gravitational instability

- Consider a matter (pressure free) dominated universe with density: $\rho(\mathbf{r},t)$
- Evolution of density is described in the framework of Newtonian gravity (for scale smaller than the Hubble scale) using the fluid approximation with velocity: $\mathbf{v}(\mathbf{r},t)$
- The equations of motion are:
  - The continuity equation (matter conservation)
    $$\frac{\partial\rho}{\partial t}+\nabla\rho v=0$$
  - Euler equation (conservation of momentum)
    $$\frac{\partial v}{\partial t}+(v.\nabla)v=-\frac{\nabla P}{\rho}-\nabla\Phi$$
  - Poisson equation
    $$\Delta\Phi=4\pi G\rho$$

Gravitational instability

- For the homogeneous universe, the velocity field is described by the Hubble law.

$$\boldsymbol{v}(\boldsymbol{r},t)=H(t)\boldsymbol{r}=\frac{\dot{a}}{a}\boldsymbol{r}$$

Hubble Flow

- We will use comoving coordinates ($\boldsymbol{x}$)

$$\boldsymbol{r}=a(t)\boldsymbol{x}$$

- For small fluctuation in density the deviation of the velocity should be small

$$\boldsymbol{v}(\boldsymbol{r},t)=\frac{\dot{a}}{a}\boldsymbol{r}+\boldsymbol{u}\left(\frac{\boldsymbol{r}}{a},t\right)$$

Peculiar velocity due to local motion because of inhomogeneities

Gravitational instability

- The continuity equation in an expanding universe reads:

$$\frac{\partial \rho}{\partial t} + \frac{3\dot{a}}{a}\rho + \frac{1}{a}\nabla \rho \boldsymbol{u} = 0$$

$$\rho = \overline{\rho}\,(1+\delta)$$

- Expressing the continuity as a function of the density contrast:

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla(1+\delta)\boldsymbol{u} = 0$$

- The Euler equation for density contrast is:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\boldsymbol{u}.\nabla}{a}\boldsymbol{u} + \frac{\dot{a}}{a}\boldsymbol{u} = -\cancel{\frac{\nabla P}{\overline{\rho}\,a}} - \frac{\nabla \phi}{a}$$

- The Poisson equation reads:

$$\nabla^2 \phi(\boldsymbol{x},t) = 4\pi G a^2 \overline{\rho}\,\delta(\boldsymbol{x},t)$$

Gravitational instability

- Let's consider only the first order terms (density contrast, peculiar velocity)

$$\delta \ll 1 \qquad u \ll 1$$

- The continuity equation is then:

$$\frac{\partial \delta}{\partial t} + \frac{1}{a} \nabla \boldsymbol{u} = 0$$

- The Euler equation is:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \frac{\dot{a}}{a} \boldsymbol{u} = - \frac{\nabla \phi}{a}$$

- Taking the derivative of the continuity equation and replacing in the Euler+Poisson equations gives a second-order differential equation:

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2\dot{a}}{a} \frac{\partial \delta}{\partial t} = 4\pi G \bar{\rho} \delta$$

Gravitational instability

- This equation has no derivatives in the spatial coordinates, nor the coefficient depends on the spatial coordinates
- We can thus separate time and spatial coordinates
- We thus have an equation of the **growth factor** *D(t)*

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial \delta}{\partial t} = 4\pi G \bar{\rho}\,\delta$$

$$\frac{\partial^2 \delta}{\partial t^2} + \frac{2\dot{a}}{a}\frac{\partial \delta}{\partial t} = \frac{3}{2} H_0^2 \Omega_m \frac{\delta}{a^3}$$

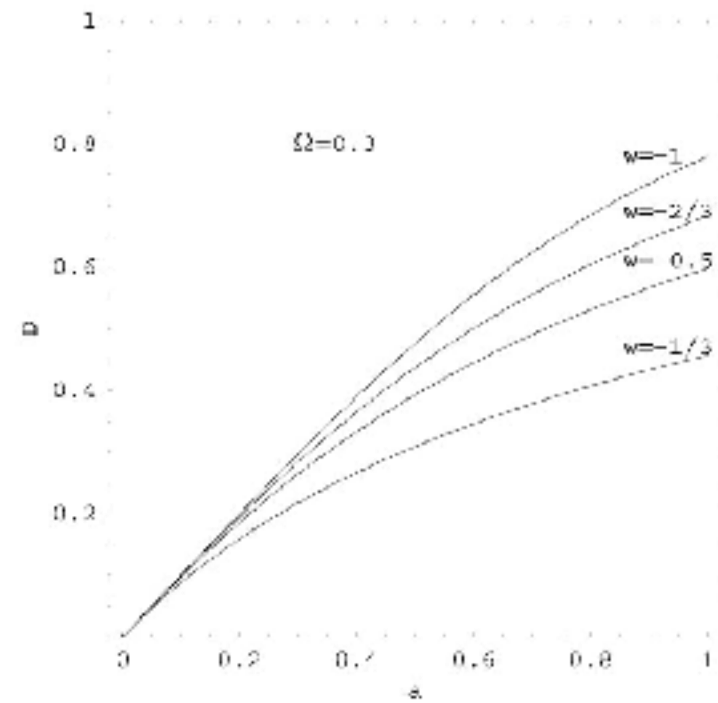
$$\delta(\boldsymbol{x},t) = D(t)\,\tilde{\delta}(\boldsymbol{x})$$

$$\ddot{D} + \frac{2\dot{a}}{a}\dot{D} = 4\pi G \bar{\rho}\, D$$

Gravitational instability

$$\ddot{D} + \frac{2\dot{a}}{a}\dot{D} = 4\pi G\bar{\rho}D$$

- The equation of the growth factor has 2 solutions: one increasing with time, one decreasing with time. Only the one increasing with time is relevant.



- One can show that the solution for any cosmological model has the form:

$$D_+(a) \propto \frac{H(a)}{H_0}\int_0^a \frac{da}{[\Omega_m/a + \Omega_\Lambda a^2 - (\Omega_m + \Omega_\Lambda - 1)]^{3/2}}$$

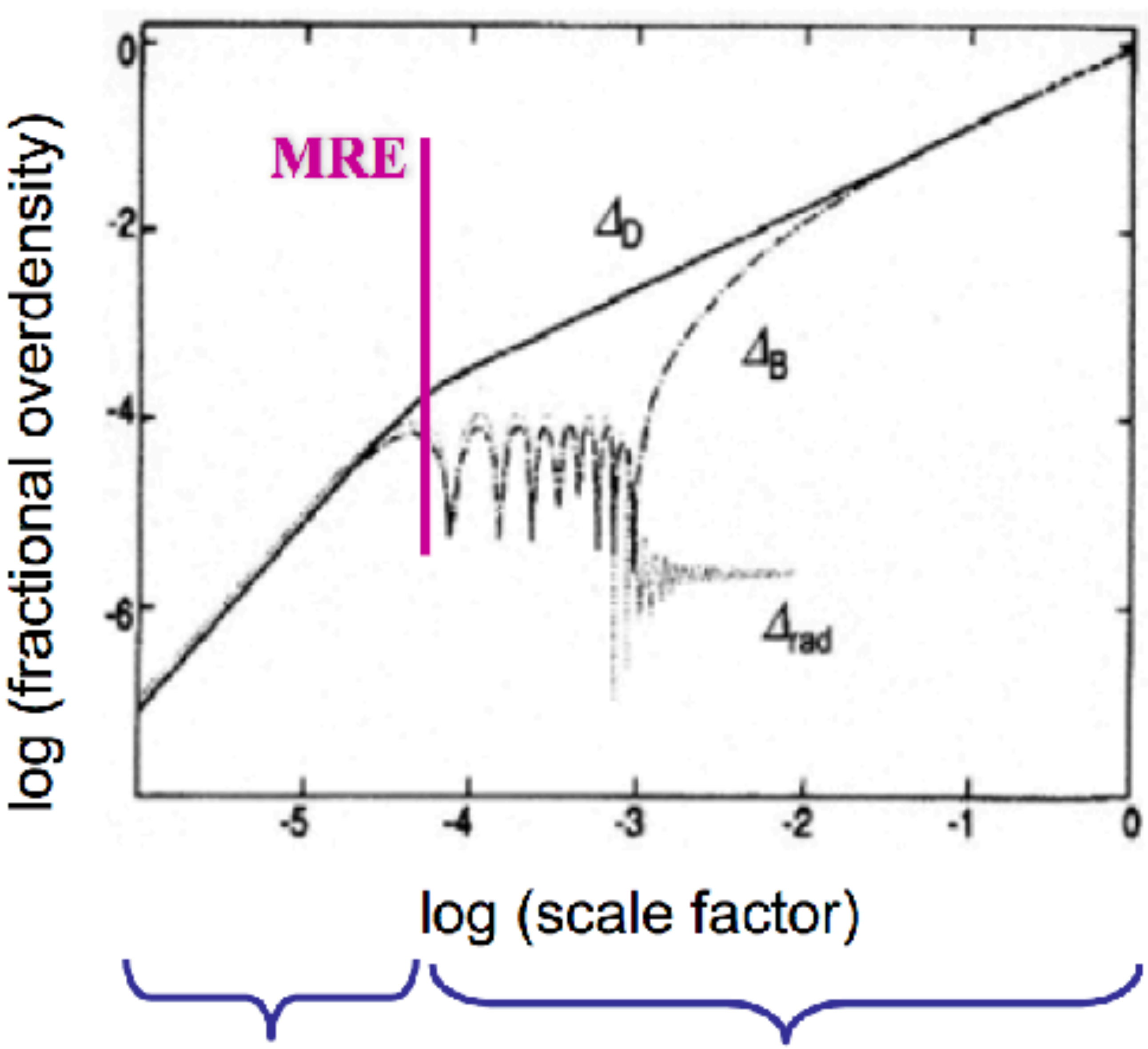
- This calculation does not hold for large fluctuation: $\delta \sim 1$ (non-linear fluctuation) and requires numerical computation/simulations

Evidence of Dark Matter

- At present the density contrast is:
  - $\delta >> 1$ on cluster scale ~1Mpc
  - $\delta \sim 1$ on supercluster scale ~20 Mpc
- For Einstein de Sitter model (flat, $\Omega_m = 1$) the growth factor is proportional to the scale factor ***a***.
- Thus $\delta \sim 1$ today corresponds to fluctuation of 10^{-3} at CMB
- But we observe 10^{-5} fluctuation in the CMB
- To solve this, we require that Dark Matter as a higher density contrast (x100) than baryon at z~1100

Why CMB implies dark matter

Evolution of amplitude of a single k-mode



MRE: Matter-Radiation Equilibrium

From D. Webb

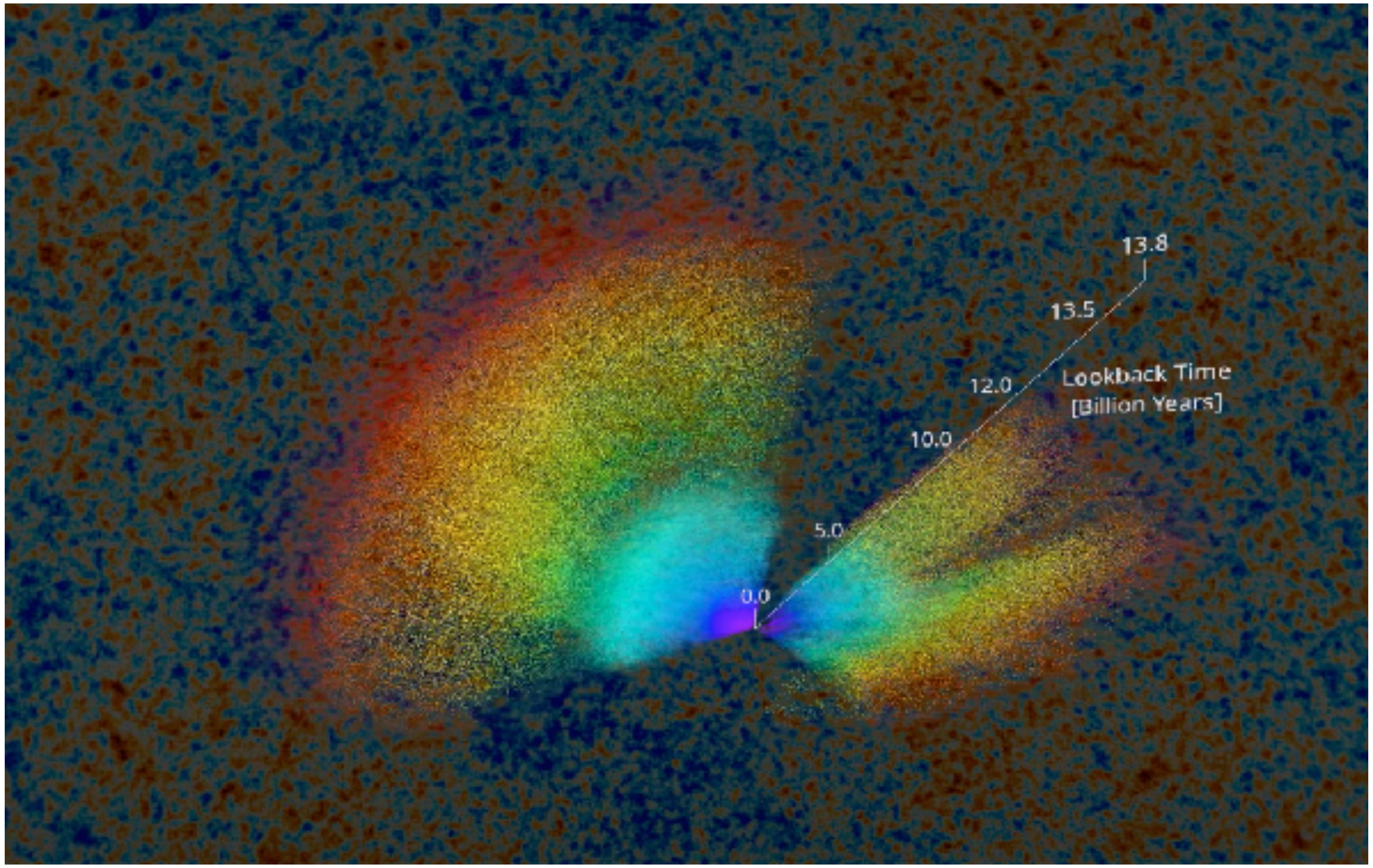
Dark matter is not coupled to photons and baryons, so its fluctuations can grow independently. DM fractional overdensities are larger at recombination (but we do not see them directly)

Baryon-photon fluid oscillates in the potential wells of DM, but fluctuation amplitude is small – this is what we see as dT/T ~ 1 part in 10^5.

After recombination baryons are let go from photons, and fall into the potential wells of DM.

Radiation is free-streaming after recombination.

The eBOSS 3D map of the Universe SDSS compilation until 2020



https://www.youtube.com/watch?v=KJJXbcf8kxA&t=3s

Description of the density fluctuation

- The probability to find a galaxy in a volume dV is given by:

$$P = \overline{n}\, dV$$

- The probability to find 2 galaxies at position $\boldsymbol{x}$ and $\boldsymbol{x}+\boldsymbol{r}$ in a volume element dV is given by:

$$P = (\overline{n}\, dV)^2 [1 + \xi(\boldsymbol{r})]$$

- Where we have defined the 2-point correlation function:

$$\xi(\boldsymbol{r}) = <\delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r})>$$

Description of the density fluctuation

- The Fourier analogue of the correlation function is the **Power Spectrum** $P(k)$. It describes the level of structure as a function of the length scale $L=2\pi/k$. The larger $P(k)$, the larger the amplitude of the fluctuation at scale $2\pi/k$.

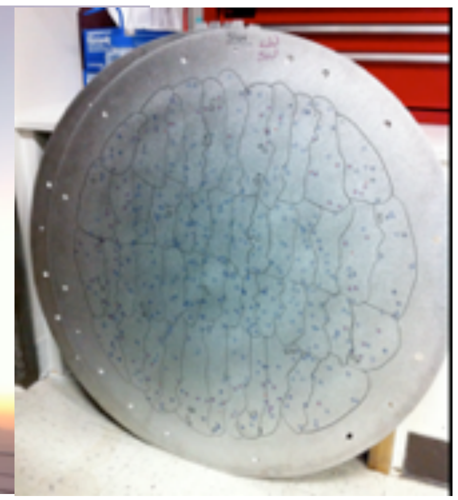
$$P(k) = 2\pi \int_0^\infty \xi(r) \frac{\sin kr}{kr} r^2 dr$$

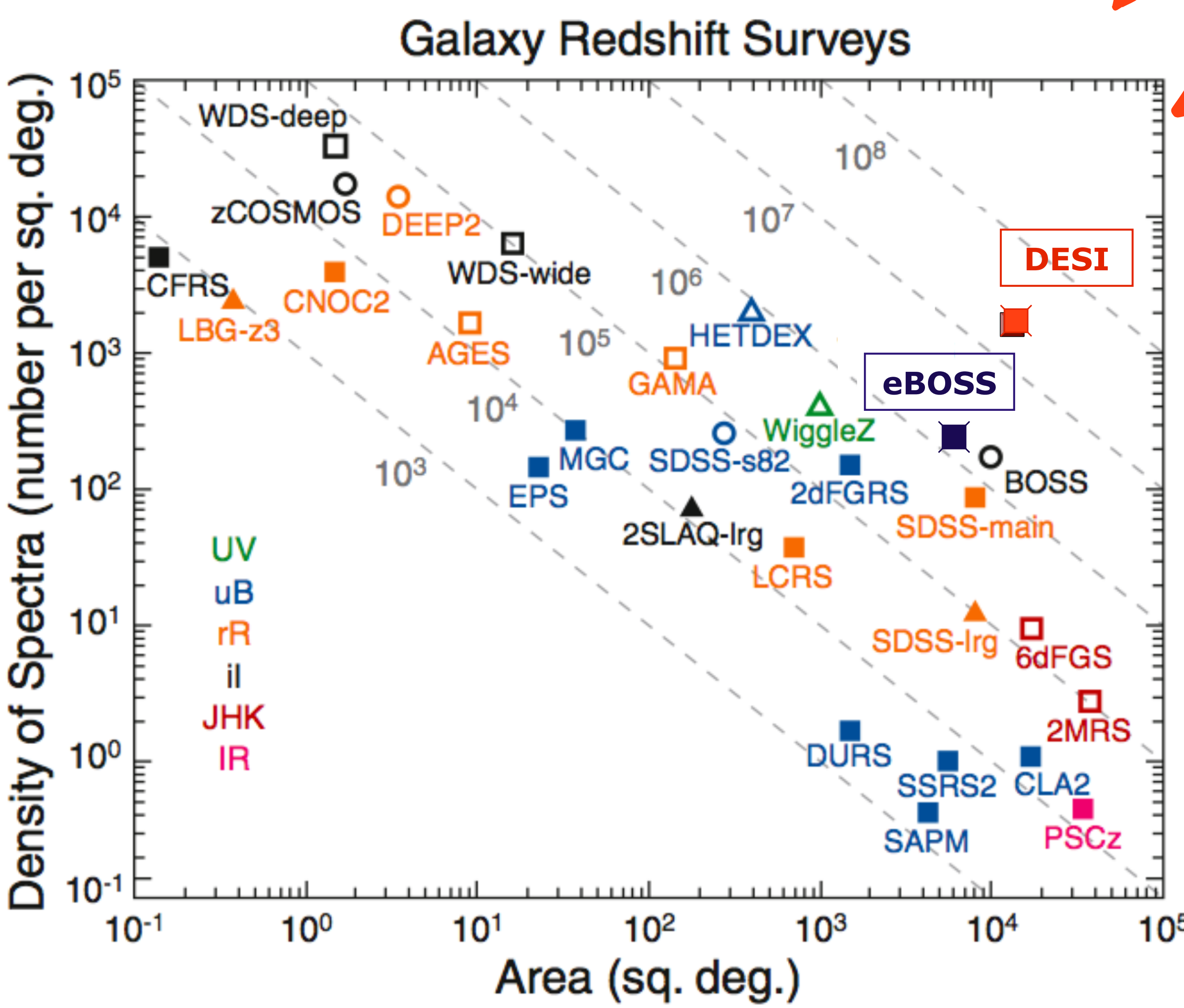
- k is the *wave number.*

- Equivalently the correlation function can be deduced from the Power Spectrum:

$$\xi(r) = \frac{1}{2\pi^2} \int_0^\infty P(k) \frac{\sin kr}{kr} k^2 dk$$

Massive galaxy redshift surveys

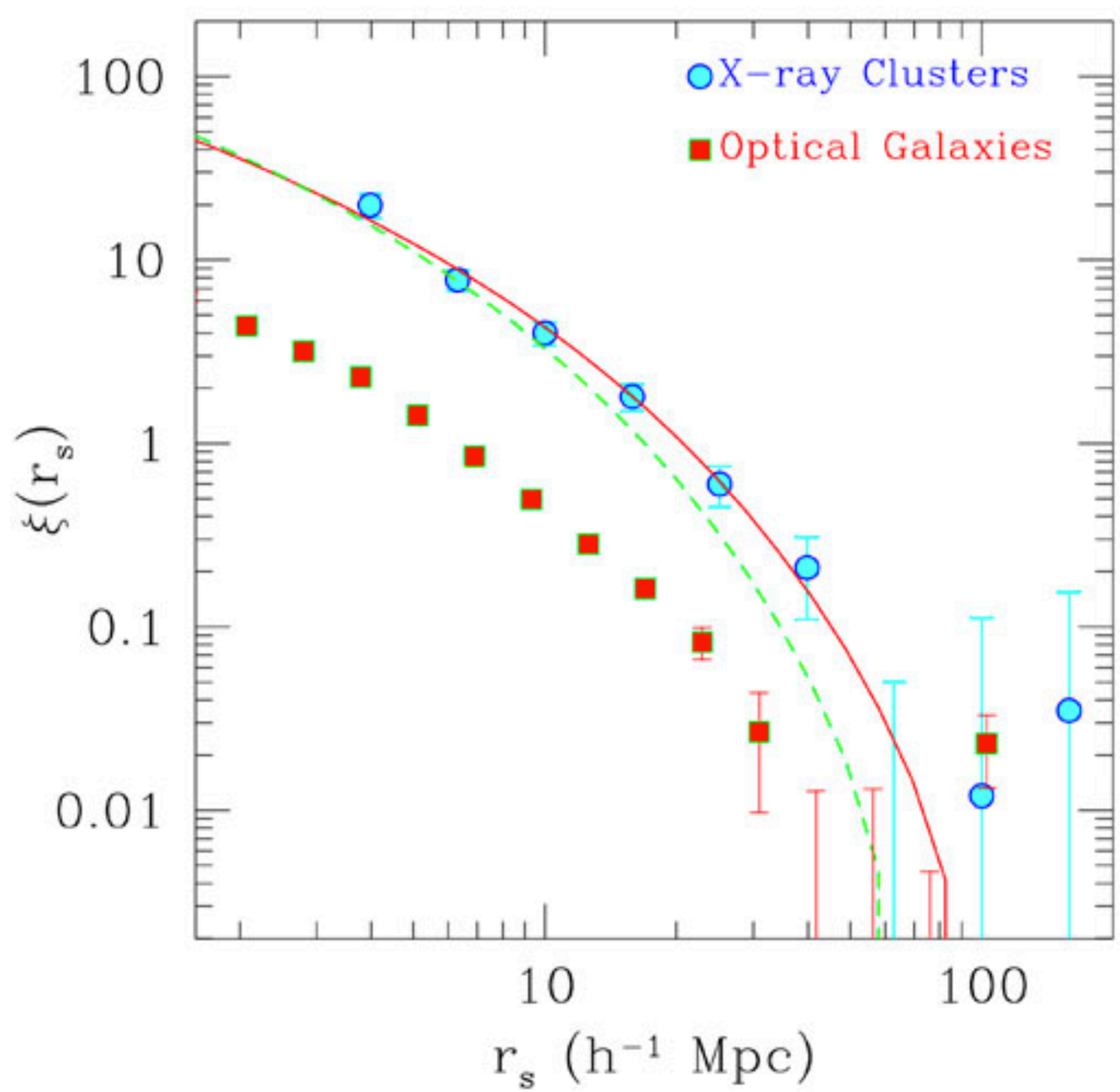




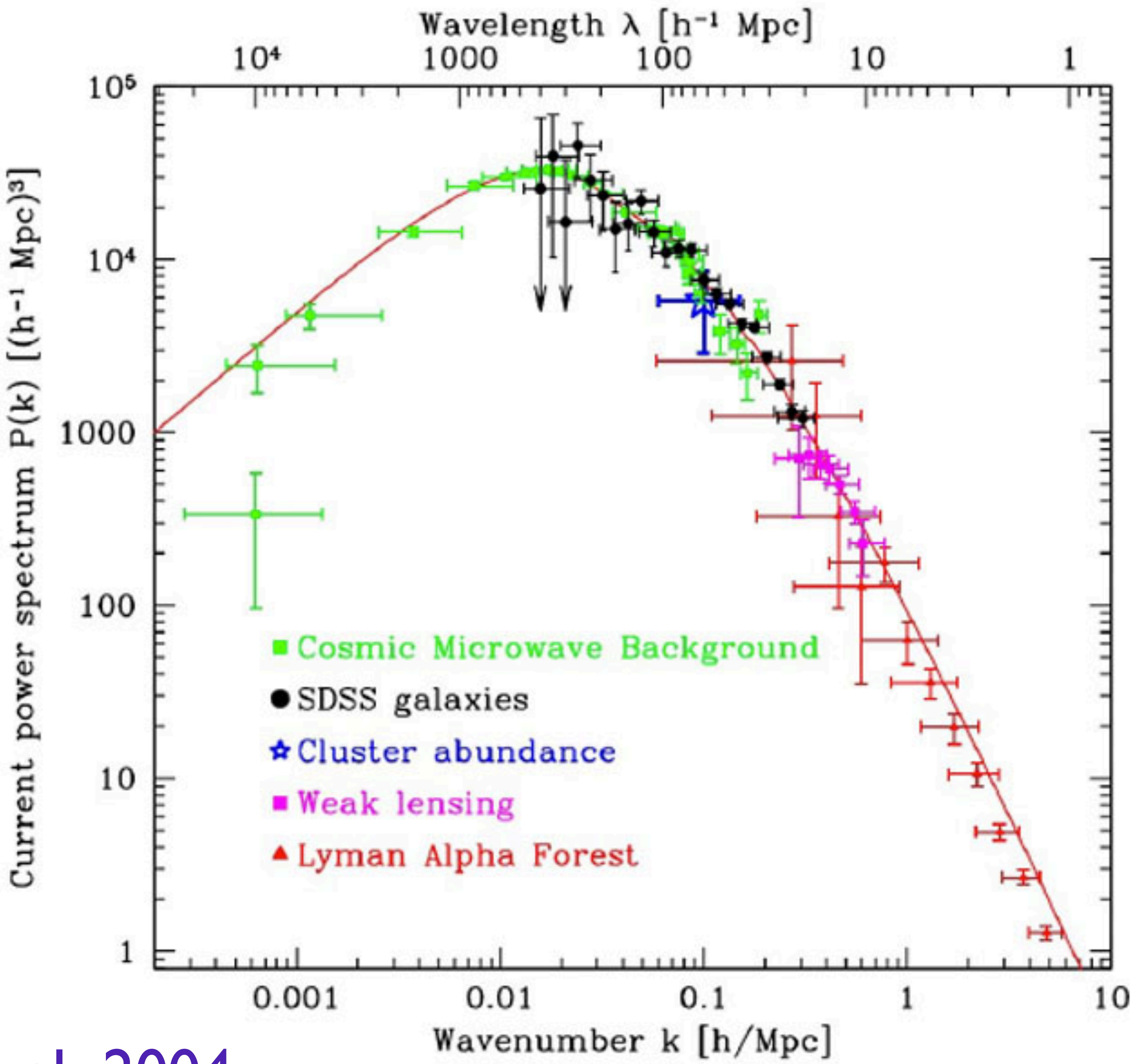
- **Hubble (1930):** expanding Universe
- **CfA Redshift Survey (1985):** first large scale structures (wall, filaments)
- **2dF (~2000):** 1500 sqdeg
- **SDSS (~2002):** 5700 sqdeg
- **VVDS/DEEP2 (~2004):** deep Universe ~1 sqdeg
- **WiggleZ (2011)**: 800 sqdeg (BAO)
- **SDSS-III/BOSS (2014)**: 10,000 sqdeg BAO/LSS (BAO)
- **e-BOSS (2014-2020):** BAO/LSS: 7,500 sqdeg w/ LRG+QSO & 1,500 sqdeg of ELGs (BAO)

- **DESI** (2020) Optical Telescope
- **Euclid** (2023) Space mission
- **SKA** (2027) Radio Telescope

Galaxy correlation function



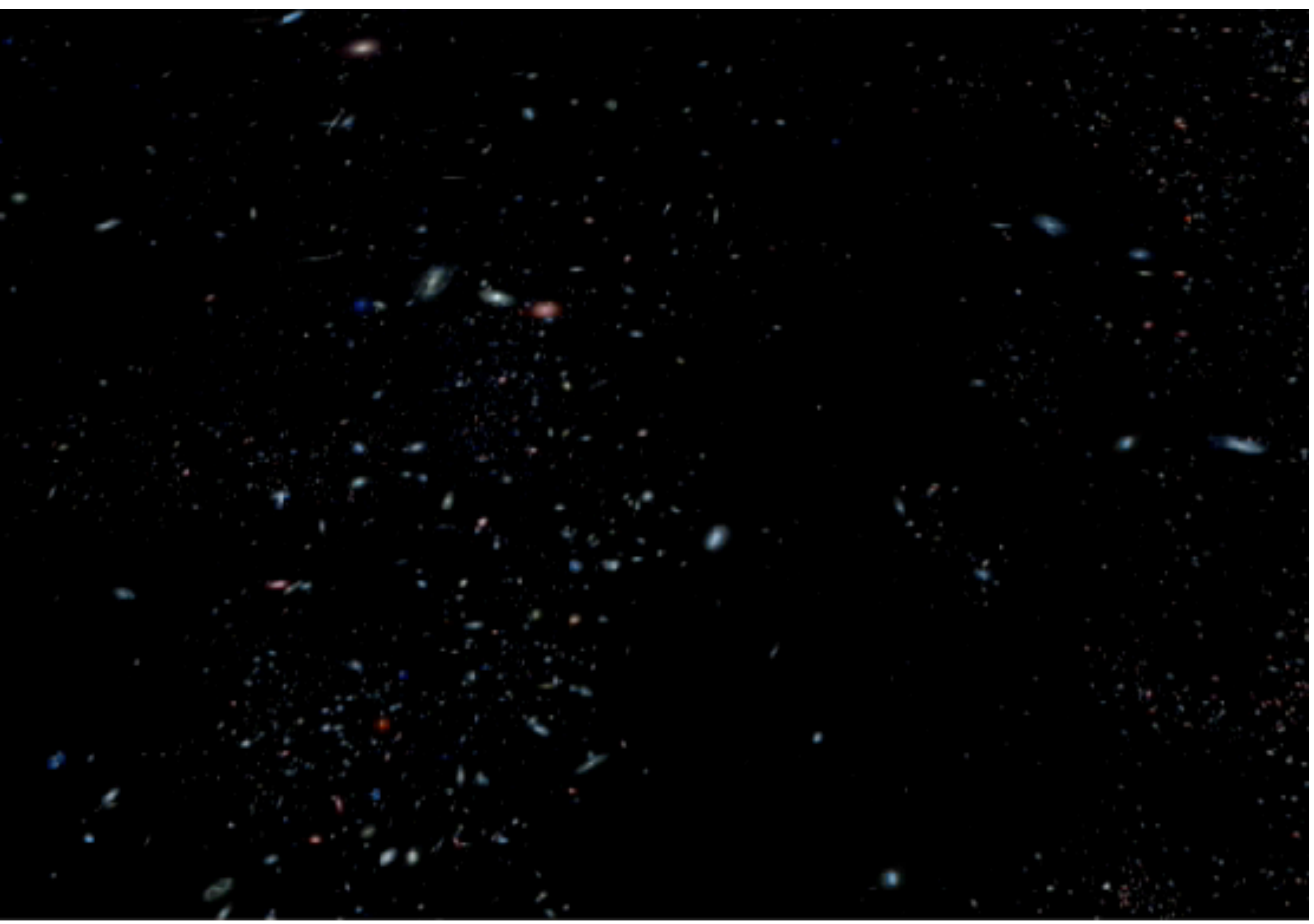
Power Spectrum



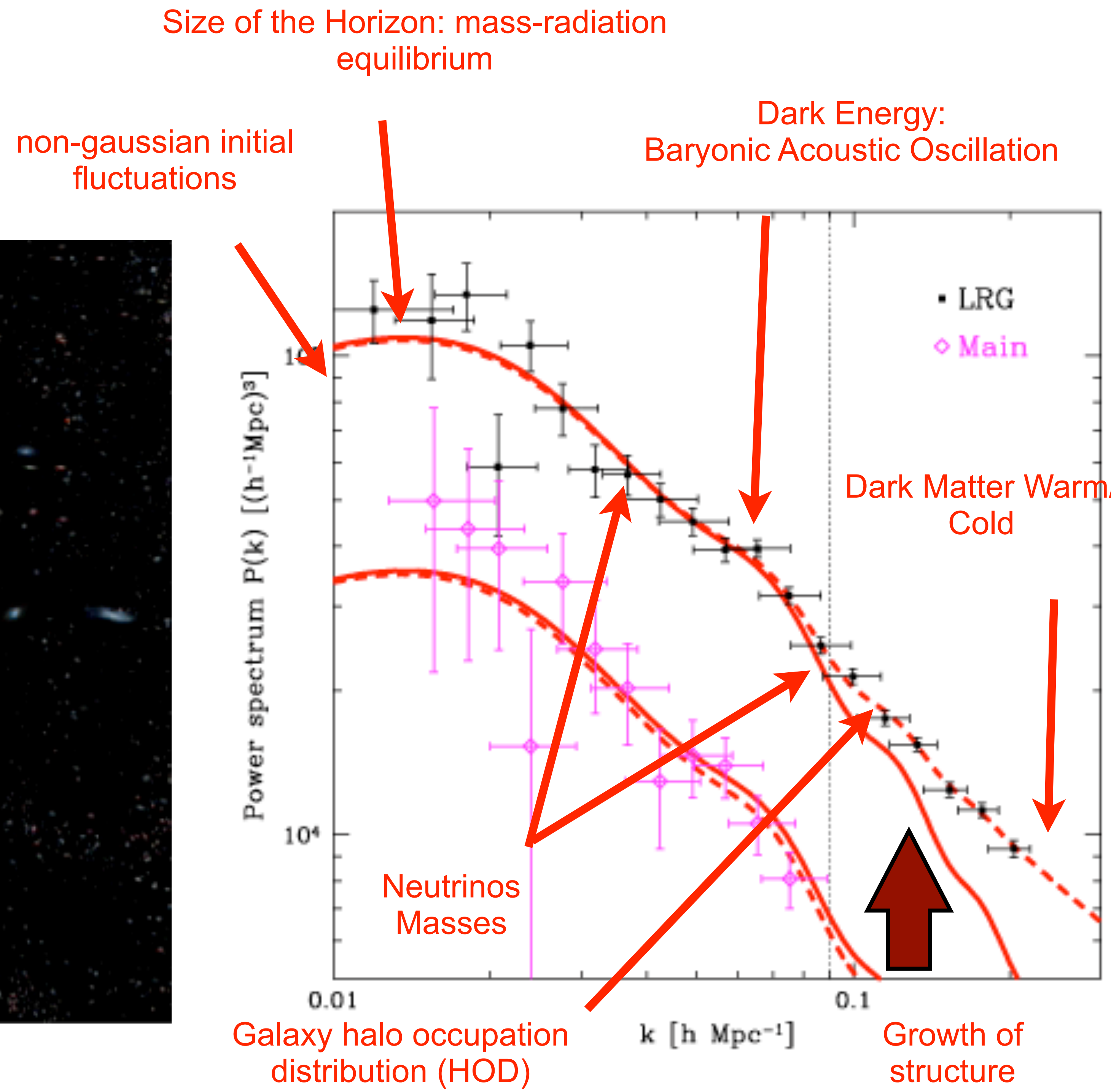
Tegmark et al 2004

Galaxy Power Spectrum

3D mapping of the position of galaxies

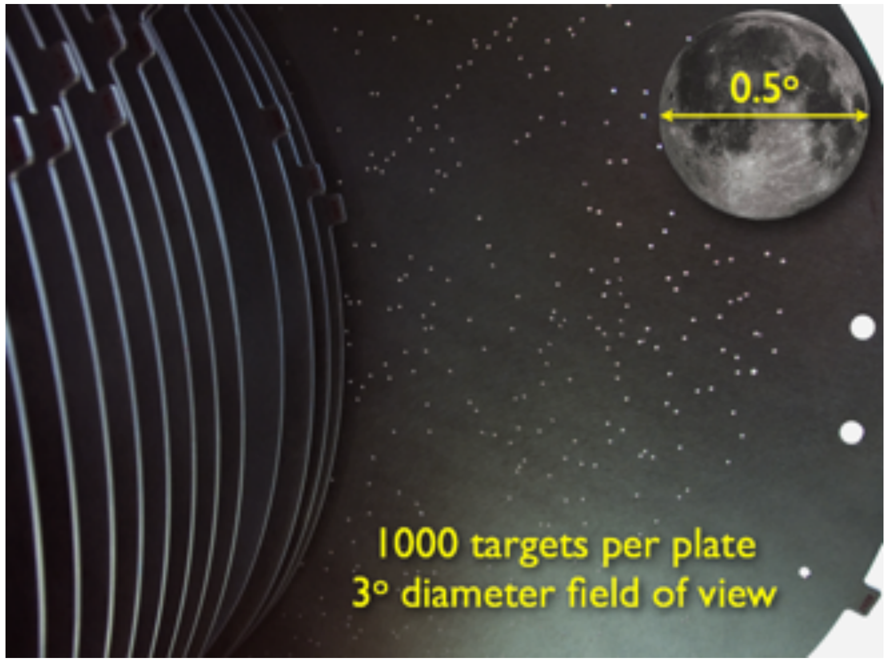


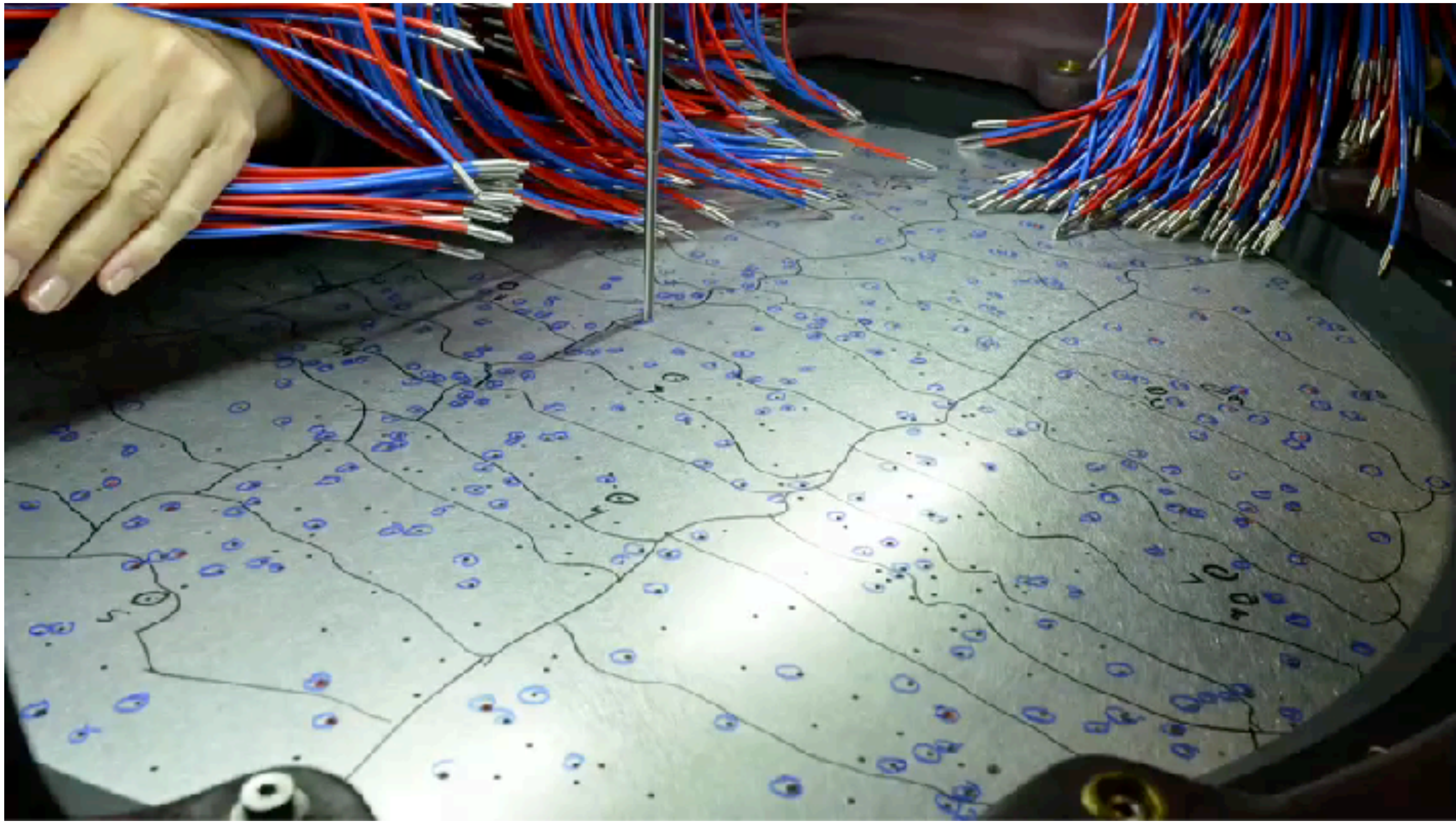
Distribution of galaxies (SDSS)



The Sloan Telescope & BOSS spectrograph

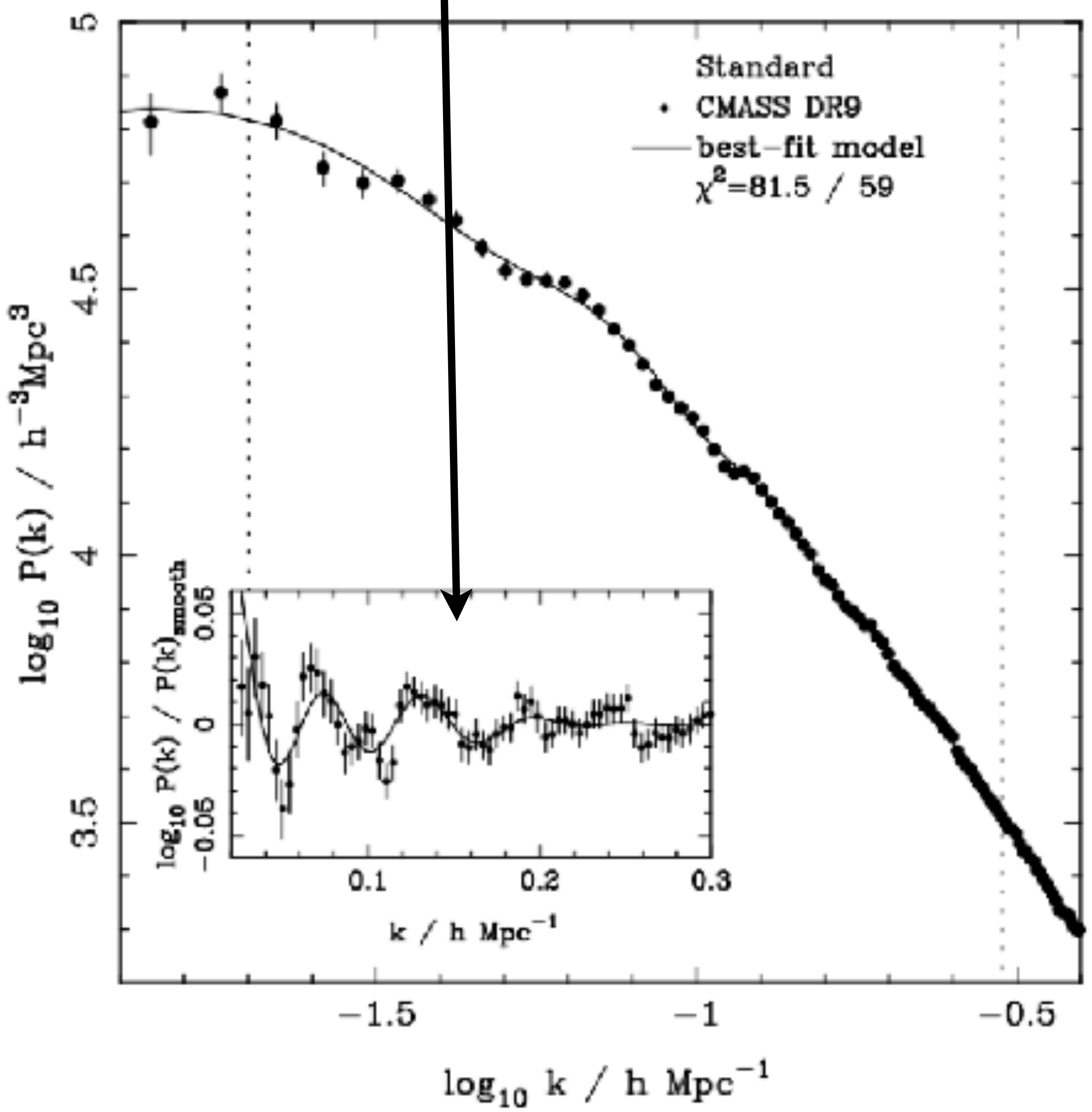
- 90 cm aluminium plate with 1000 holes for fibers,
- 45 min to plug for typical 1 hour observation on sky
- up to 9 plates observed per (good) night
- 1.5 millions redshifts in ~4 years
- The best multiplexing spectroscopic facility still



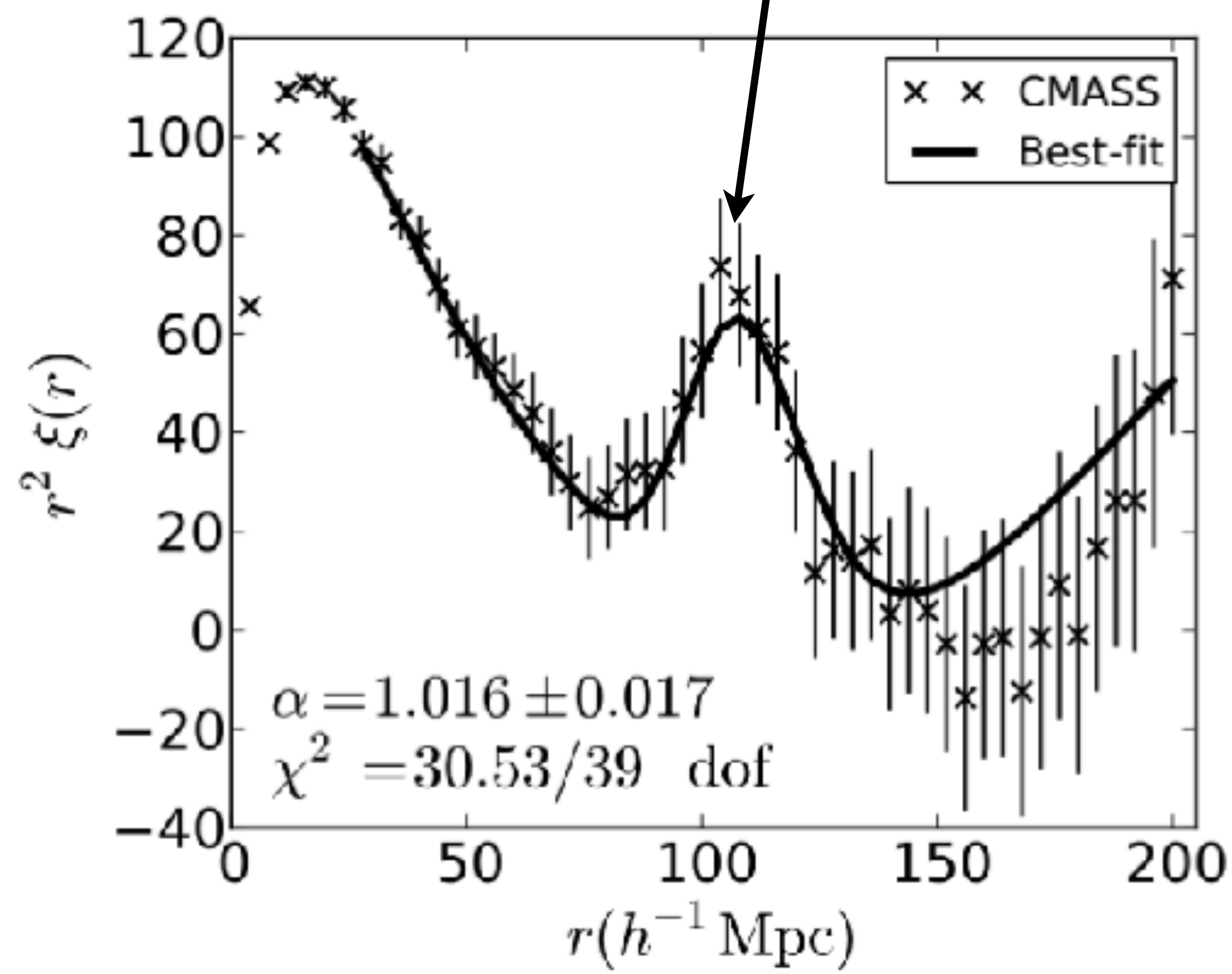


Power Spectrum & Correlation function

BAO wiggles



BAO peak



As measured by the SDSS/BOSS survey (2012)

Evolution of the density fluctuation

- Density fluctuations grow with time:

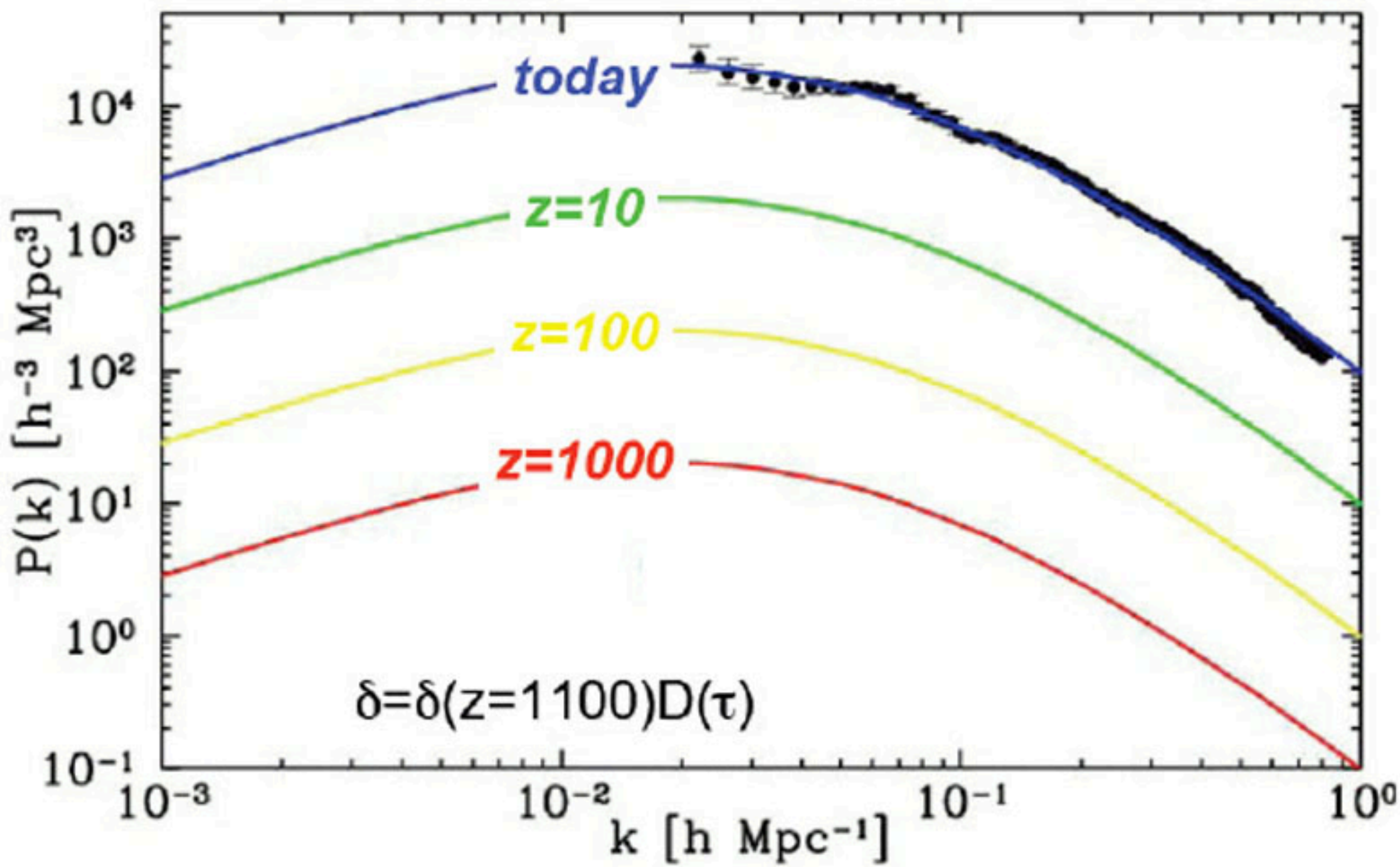
$$\delta(\mathbf{r},t)=D_+(t)\delta(\mathbf{r},t_0)$$

- Similarly both the correlation function and the power spectrum are expected to grow with time (linear perturbation theory $\delta<<1$):

$$\xi(\mathbf{r},t)=D_+^2(t)\xi(\mathbf{r},t_0)$$

$$P(k,t)=D_+^2(t)P_0(k)$$

Power Spectrum evolution



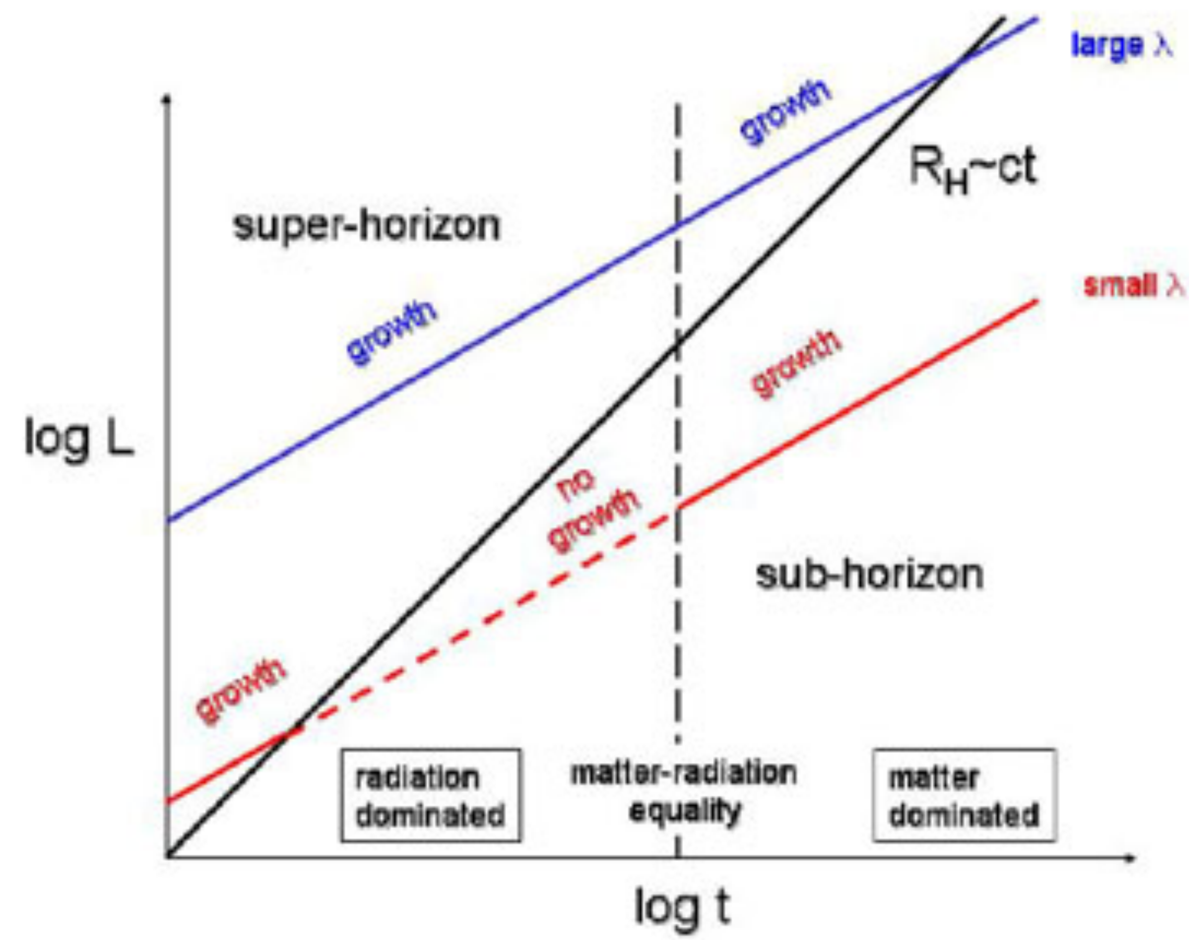
Initial Power Spectrum

- *In the radiation dominated area, there is no particular length scale in the Universe. Hence the Power Spectrum should be scale free.* Thus it should be a power law: $P(k) \sim k^n$
- The Harrison-Zeldovich spectrum corresponds to $n=1$
- This simple model needs to be corrected as in the matter dominated era the perturbation will evolved differently whether their scale is smaller or larger than the horizon scale. We need thus to correct the Power Spectrum by a **Transfer Function** $T(k)$:

$$P_0(k) = Ak^n T^2(k)$$

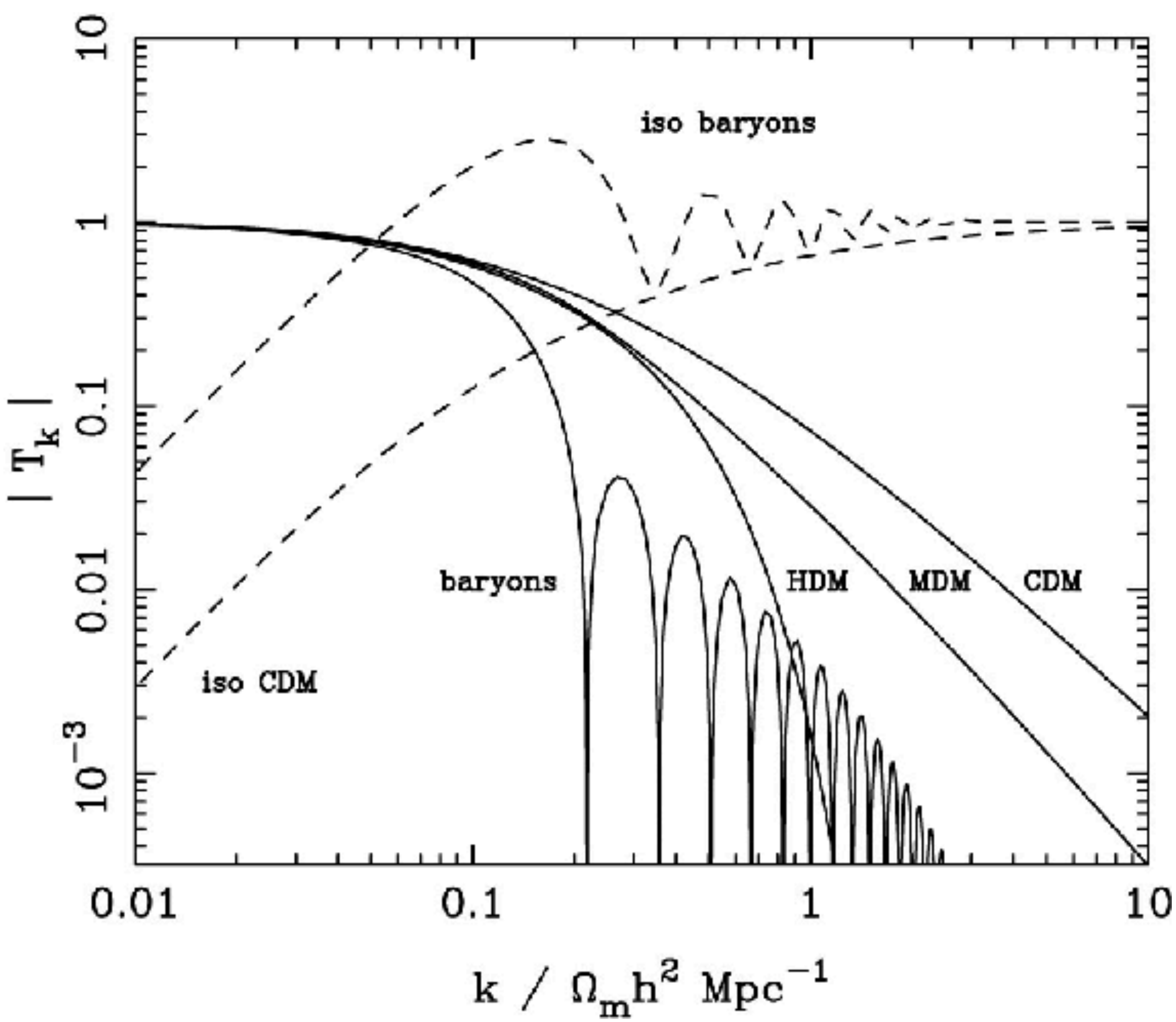
Transfer Function

- A matter density perturbation that enter the horizon during radiation-dominated epoch ceases to grow until matter start to dominate.
- Small perturbations will stop to grow ($T(k)\sim k^{-2}$), large perturbations will keep to grow ($T(k)=1$).
- The Matter-Radiation Equilibrium (MRE) epoch depends on the nature of the dark matter

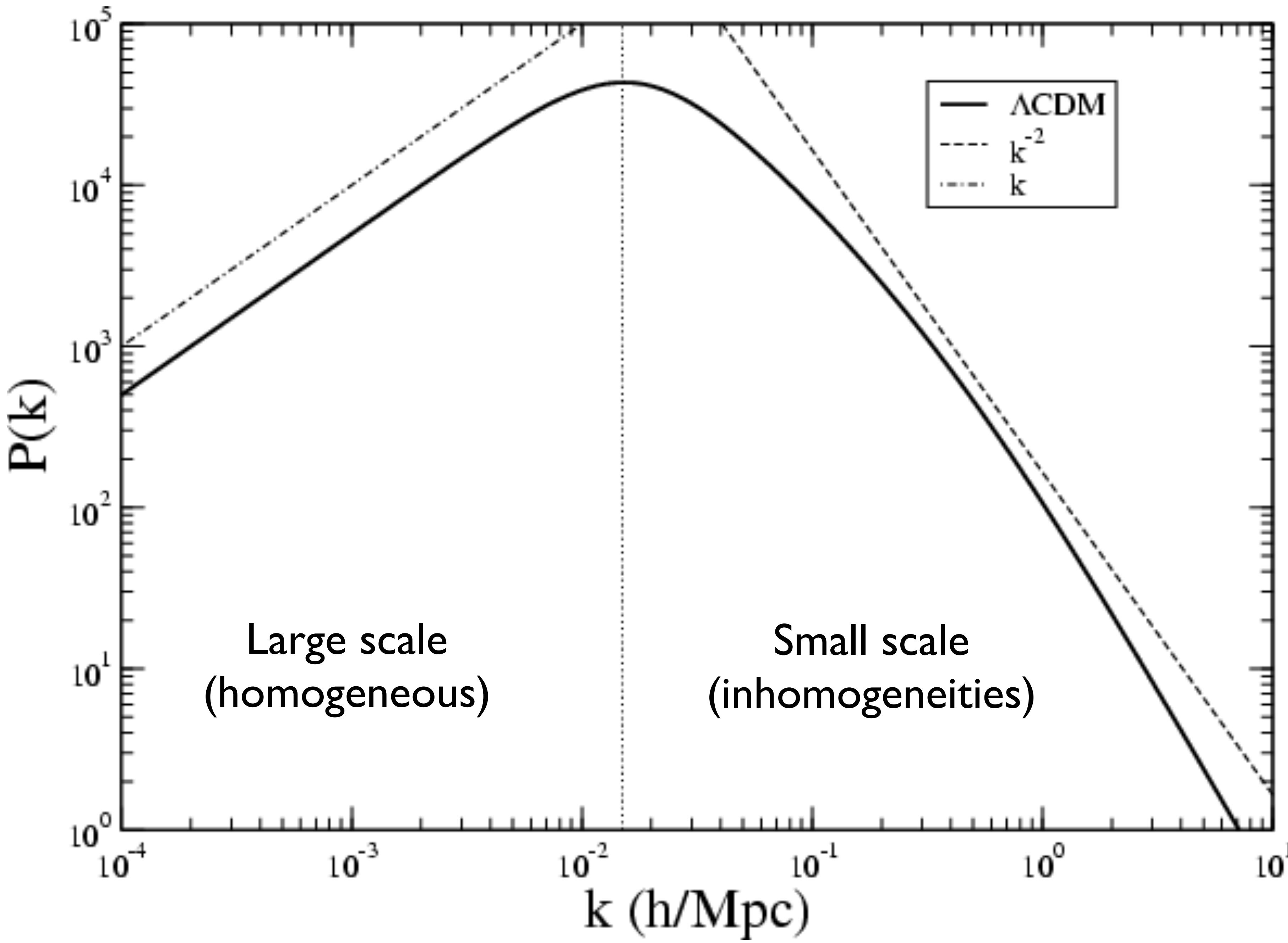


Transfer Function

- The transfer function can be computed for any cosmological model once the nature of the matter considered is specified (baryon, DM) - *suppress fluctuation on* ***small*** *scales*

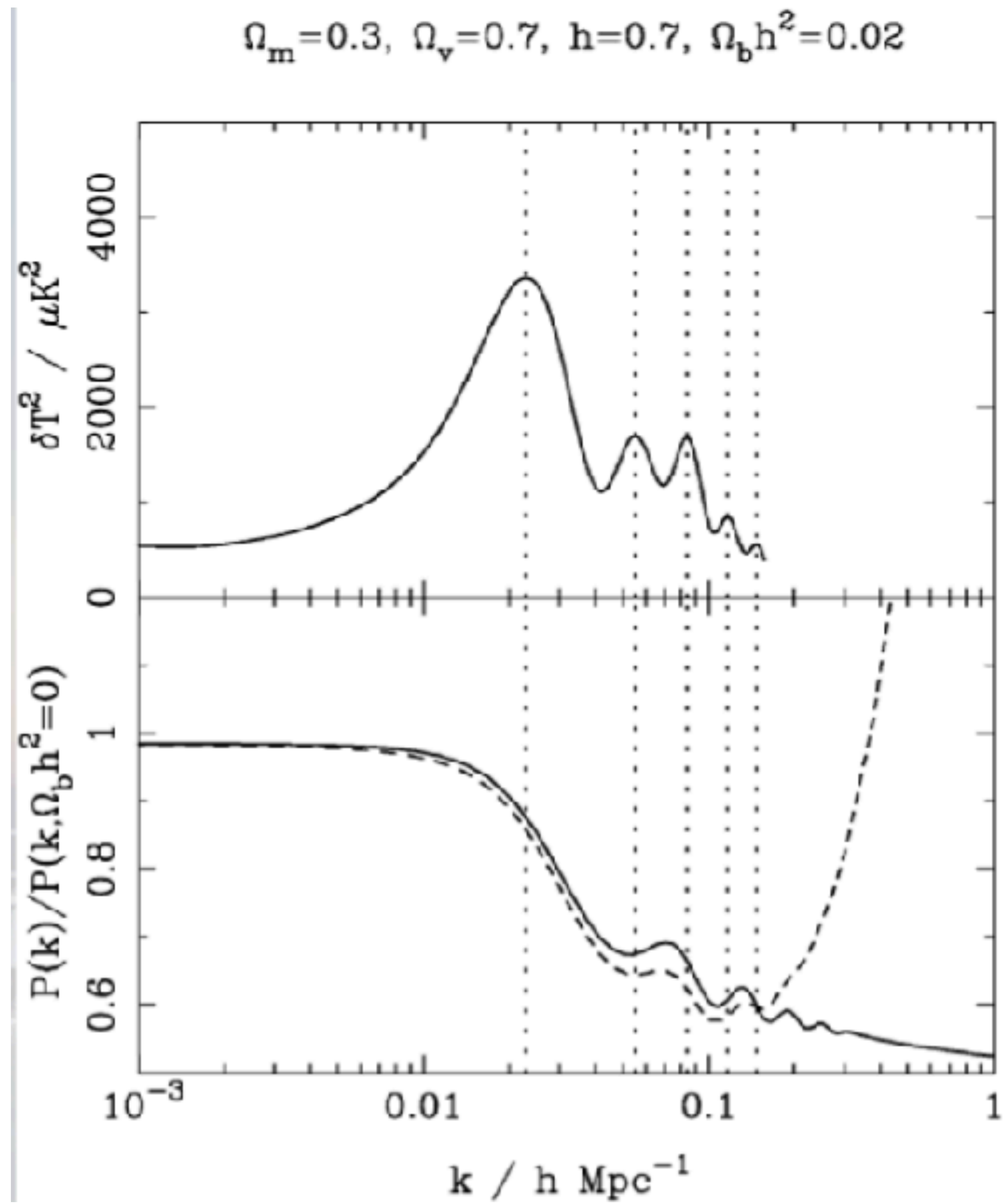


Power Spectrum



CMB and LSS power spectra

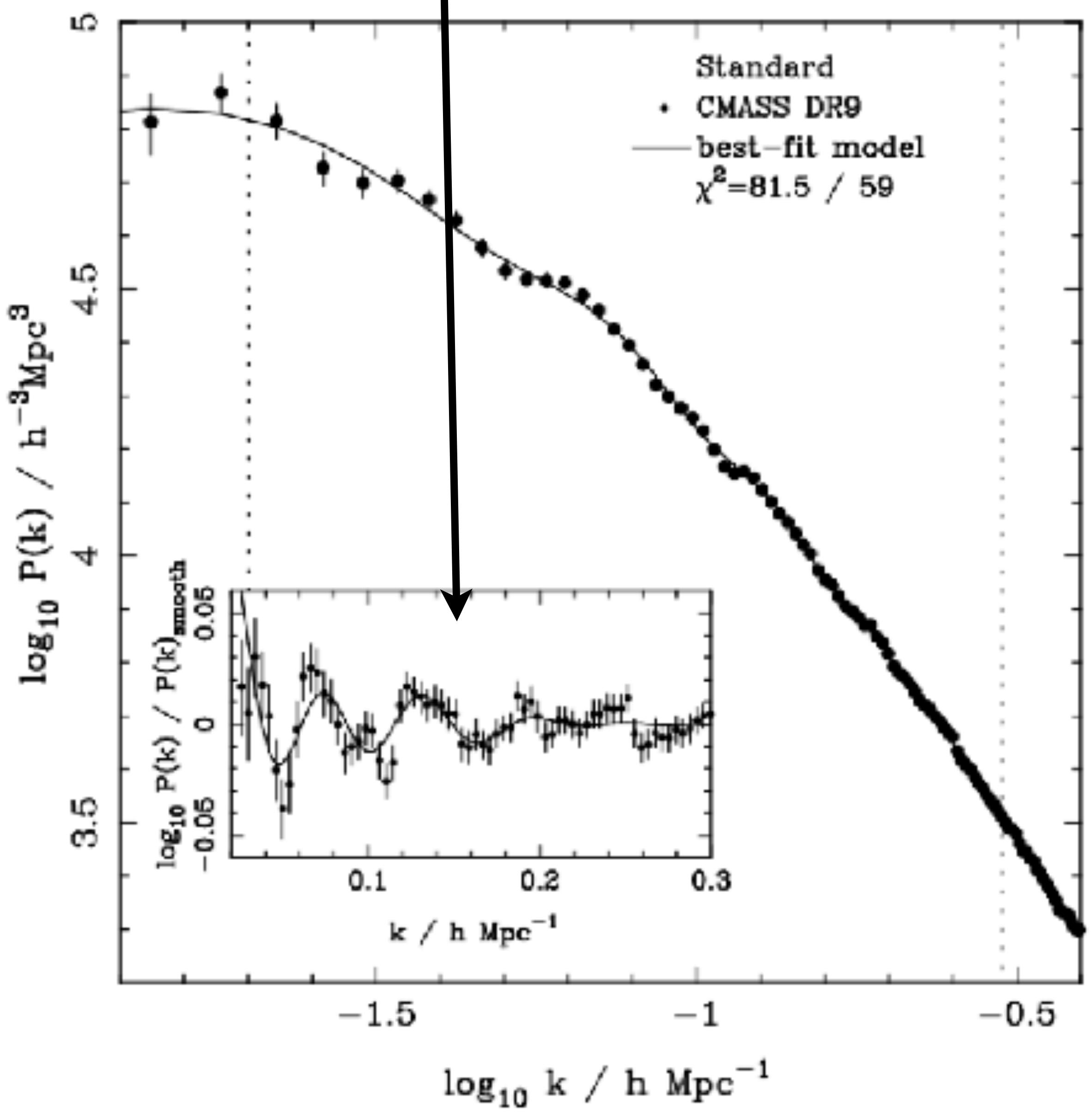
- Temperature fluctuations (acoustic oscillations) in the CMB power spectrum transforms into fluctuations in the mass/galaxy distribution
- This can be studied with large galaxy surveys.

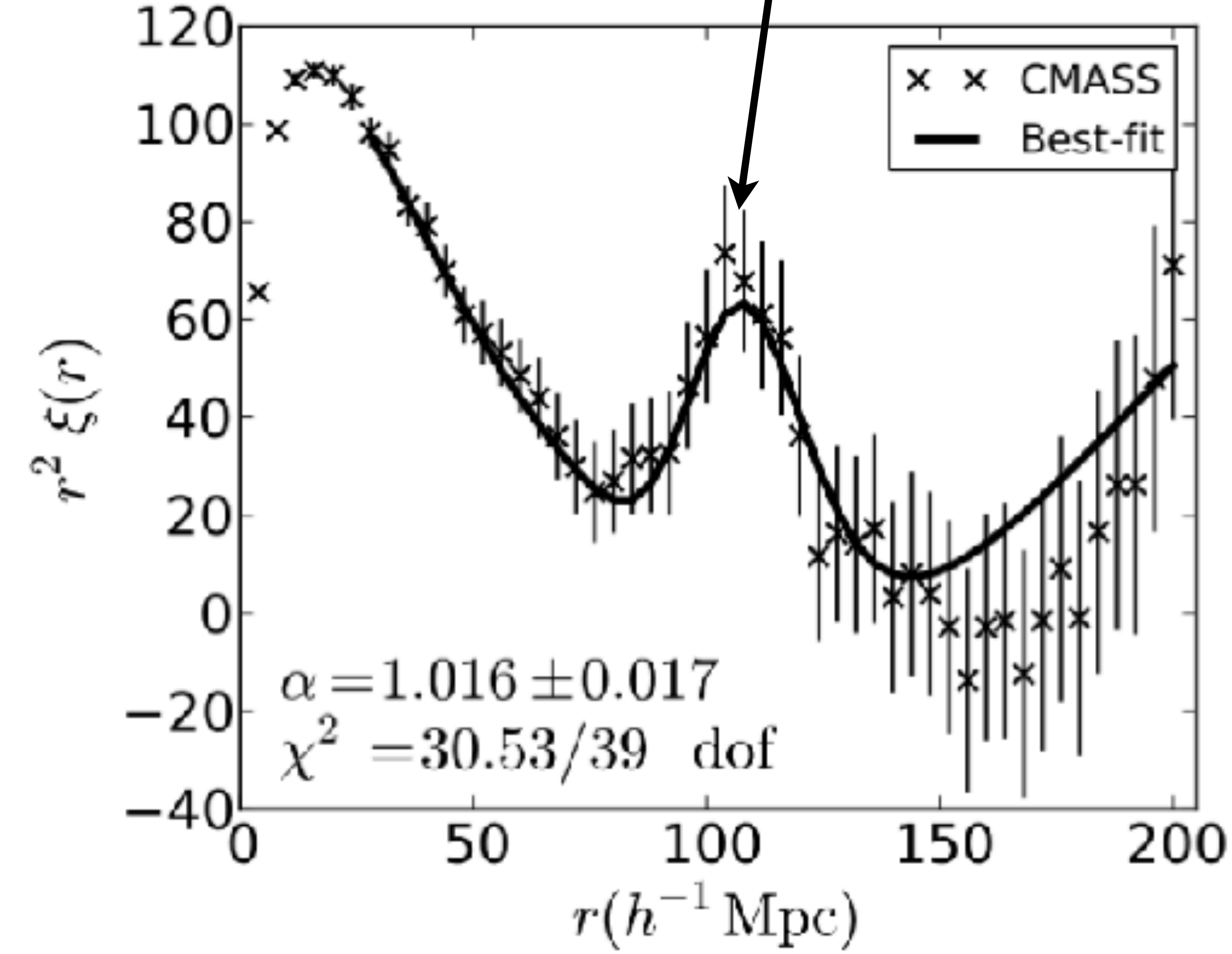


Power Spectrum & Correlation function

BAO wiggles

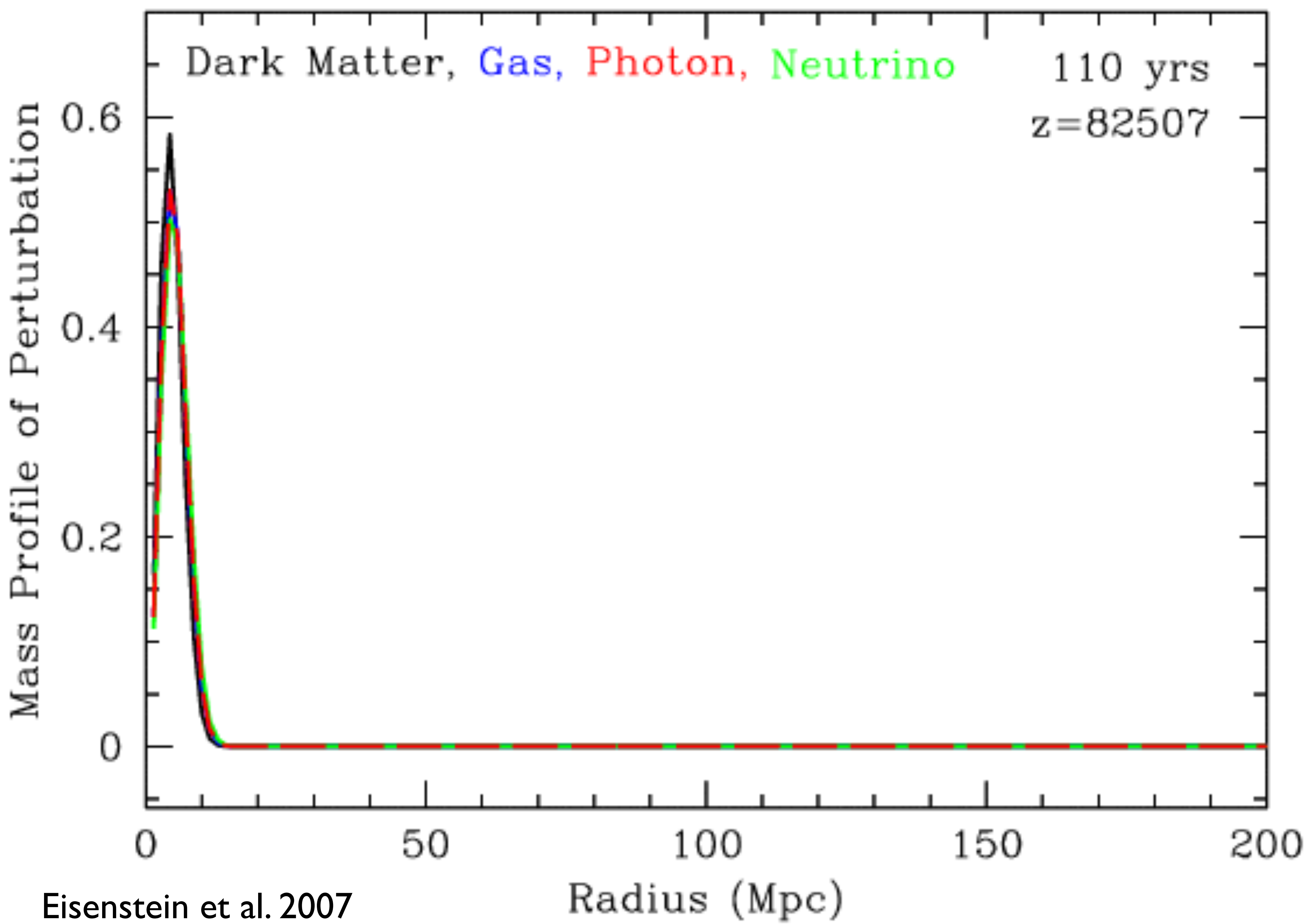
BAO peak





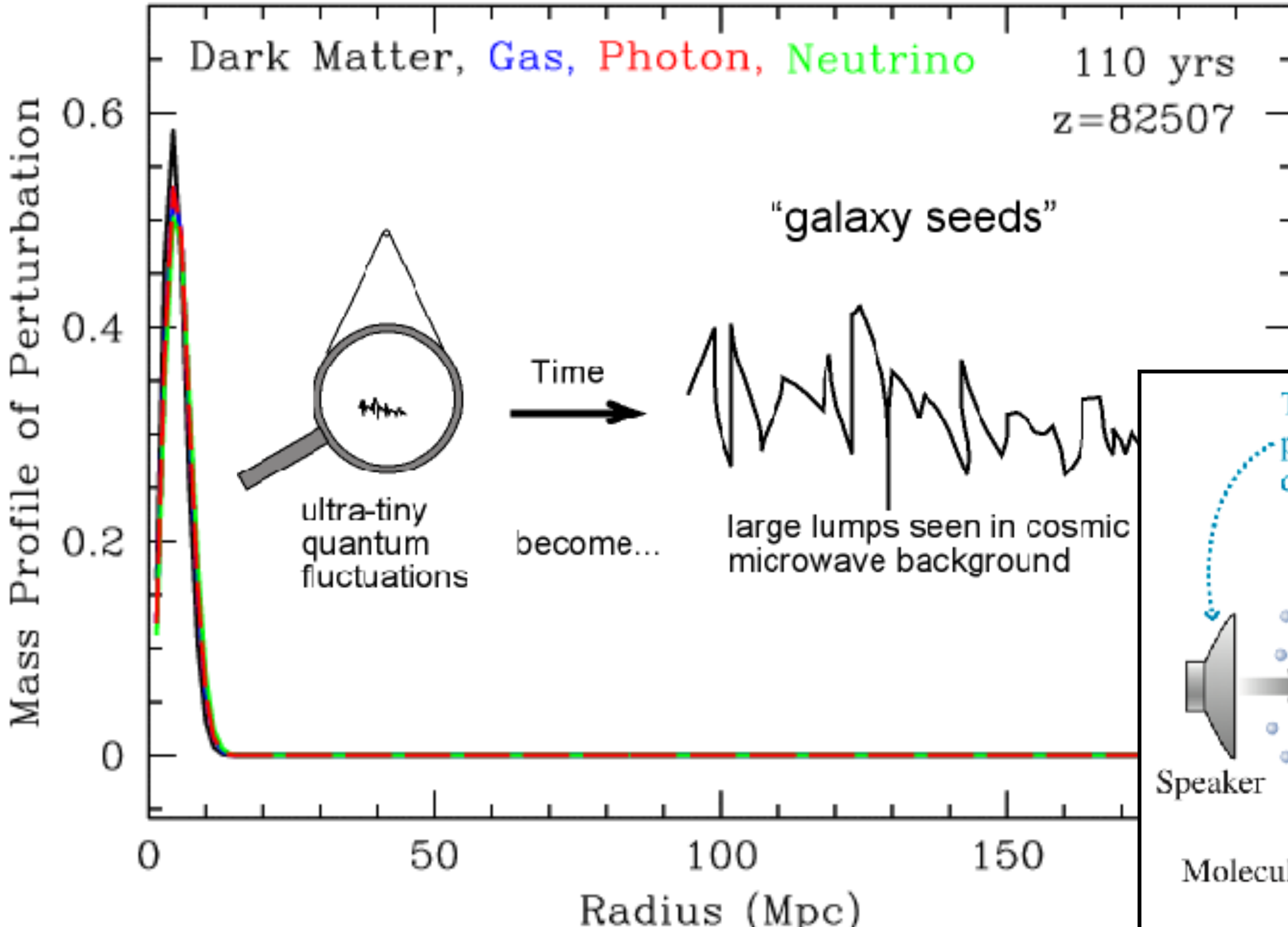
As measured by the SDSS/BOSS survey (2012)

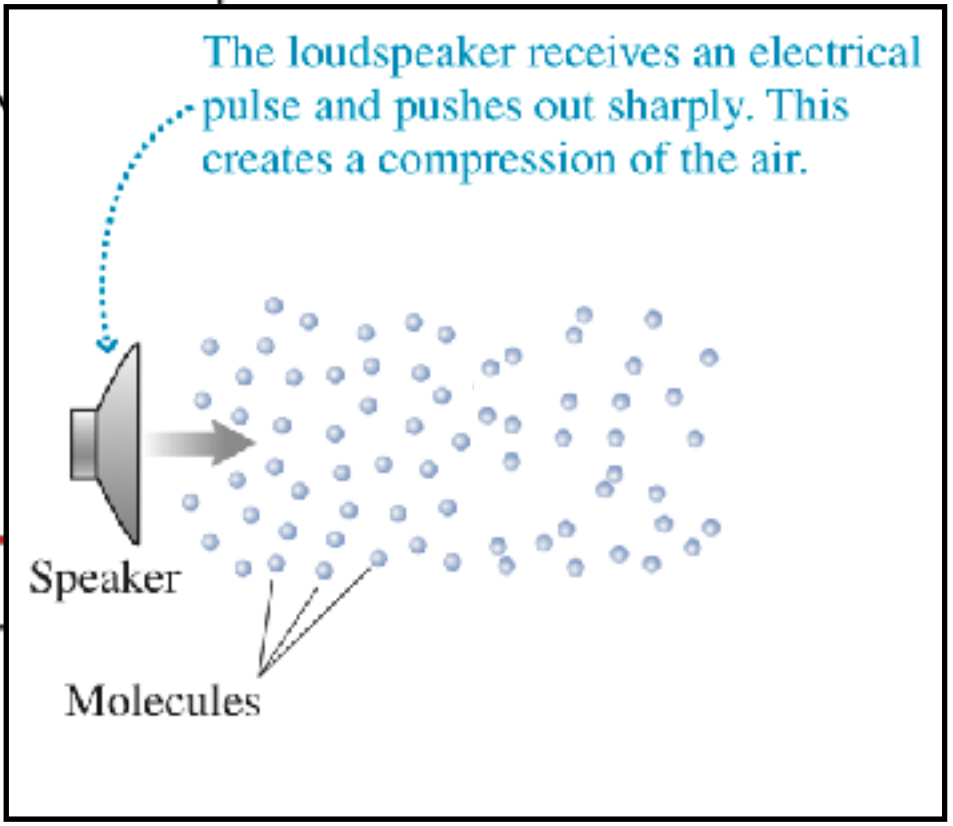
The sound horizon



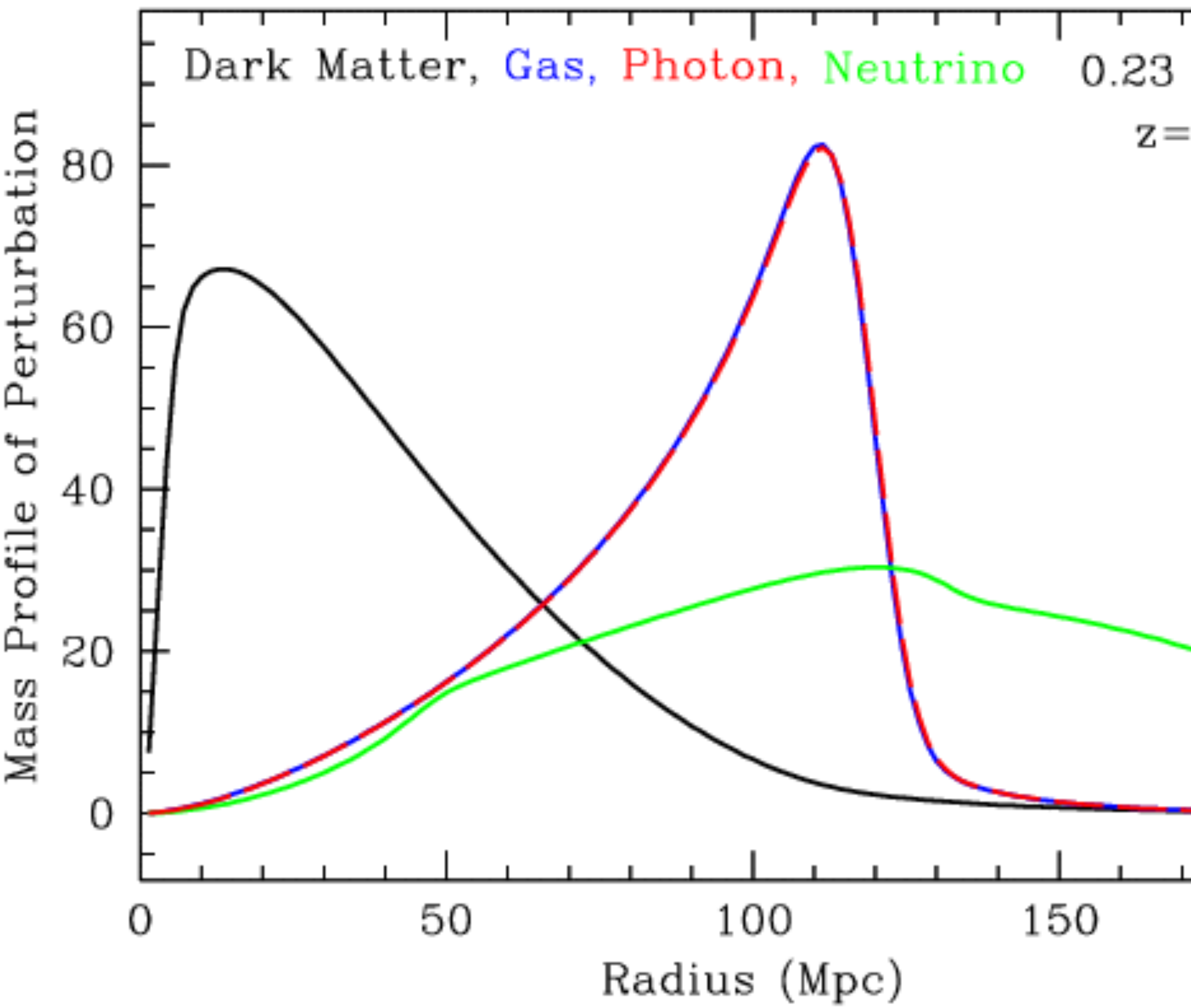
Eisenstein et al. 2007

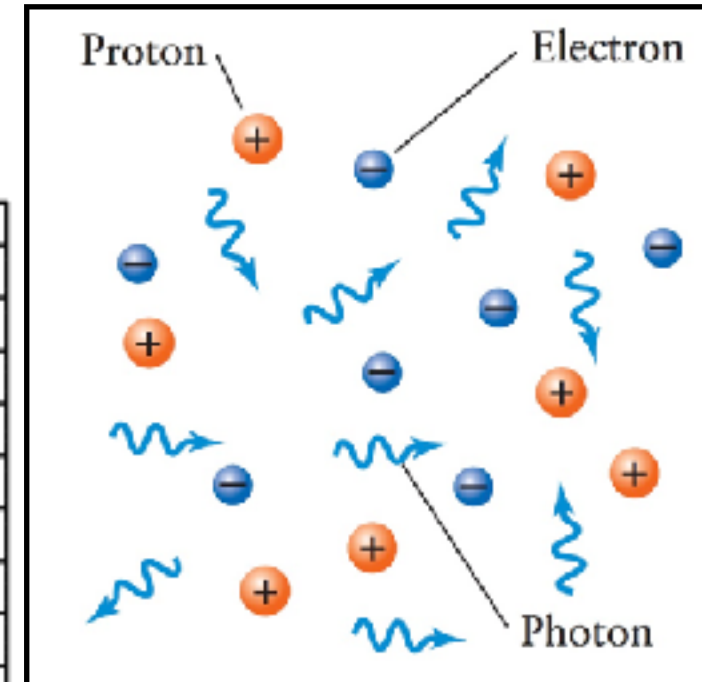
Baryon Acoustic Oscillations

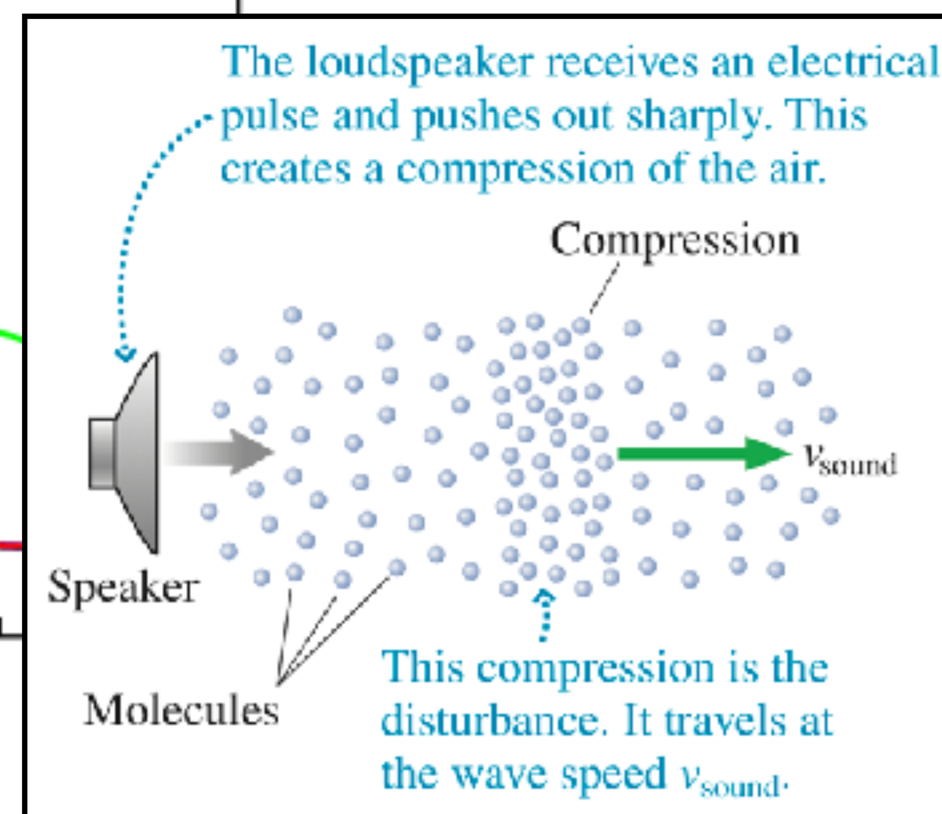




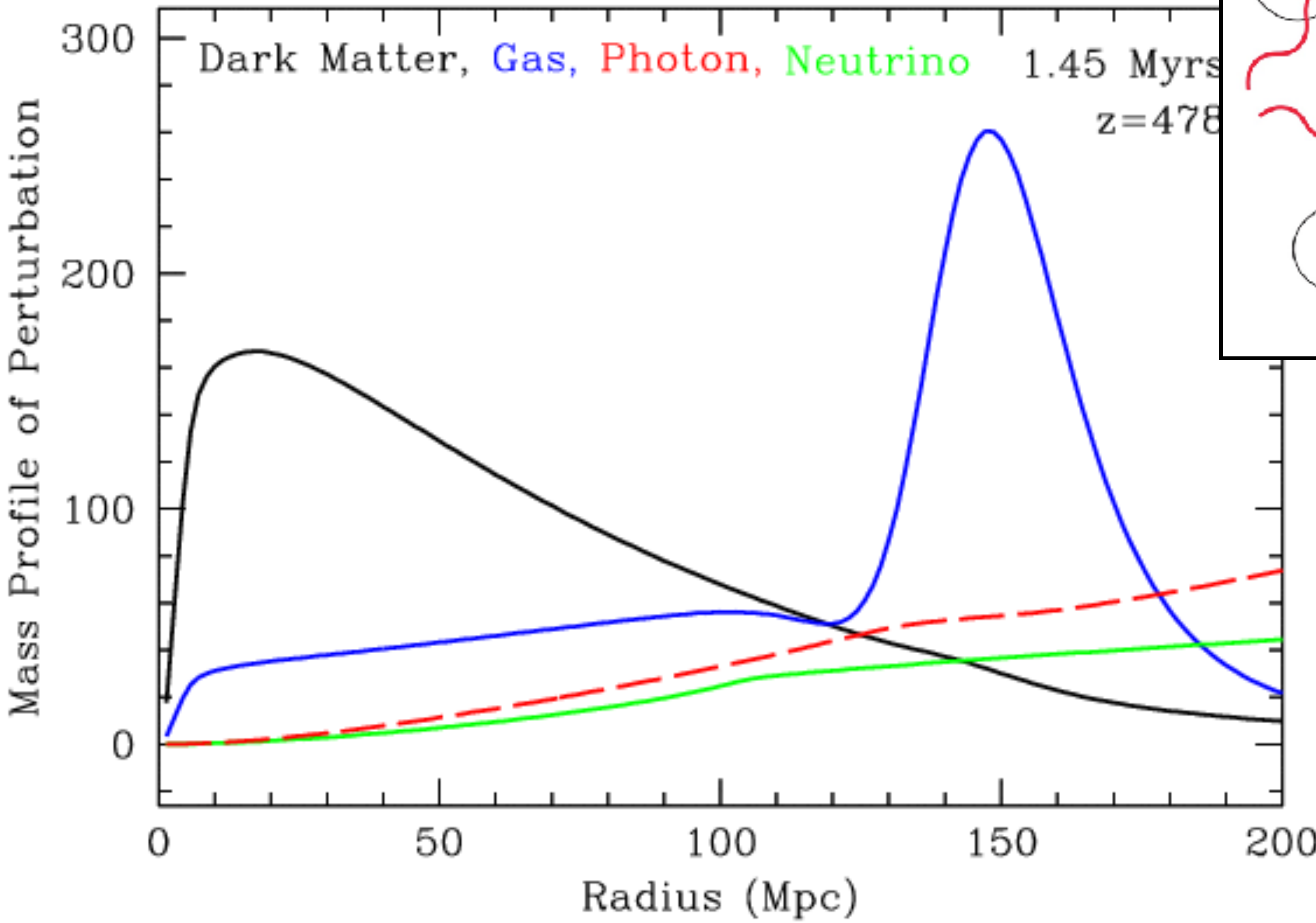
Baryon Acoustic Oscillations

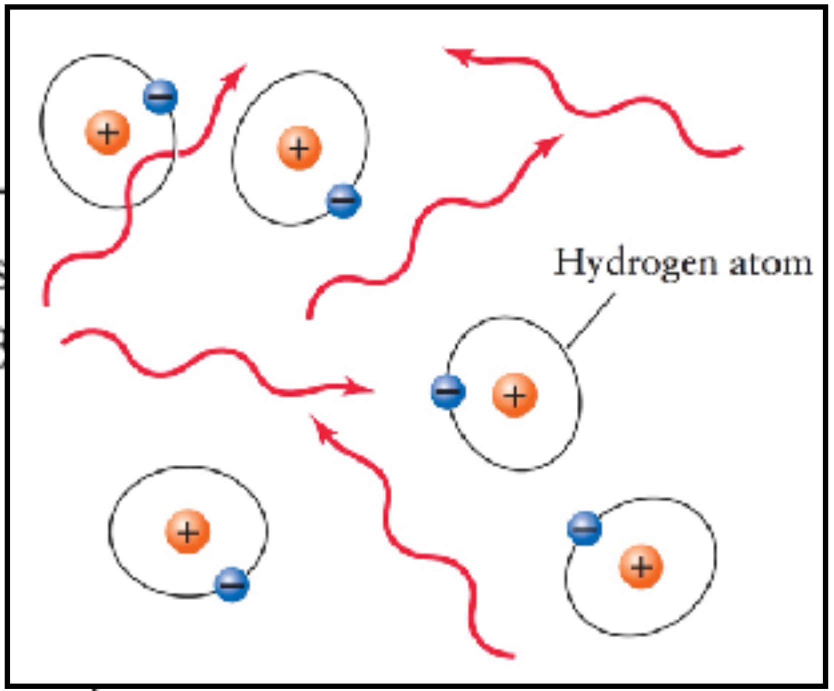




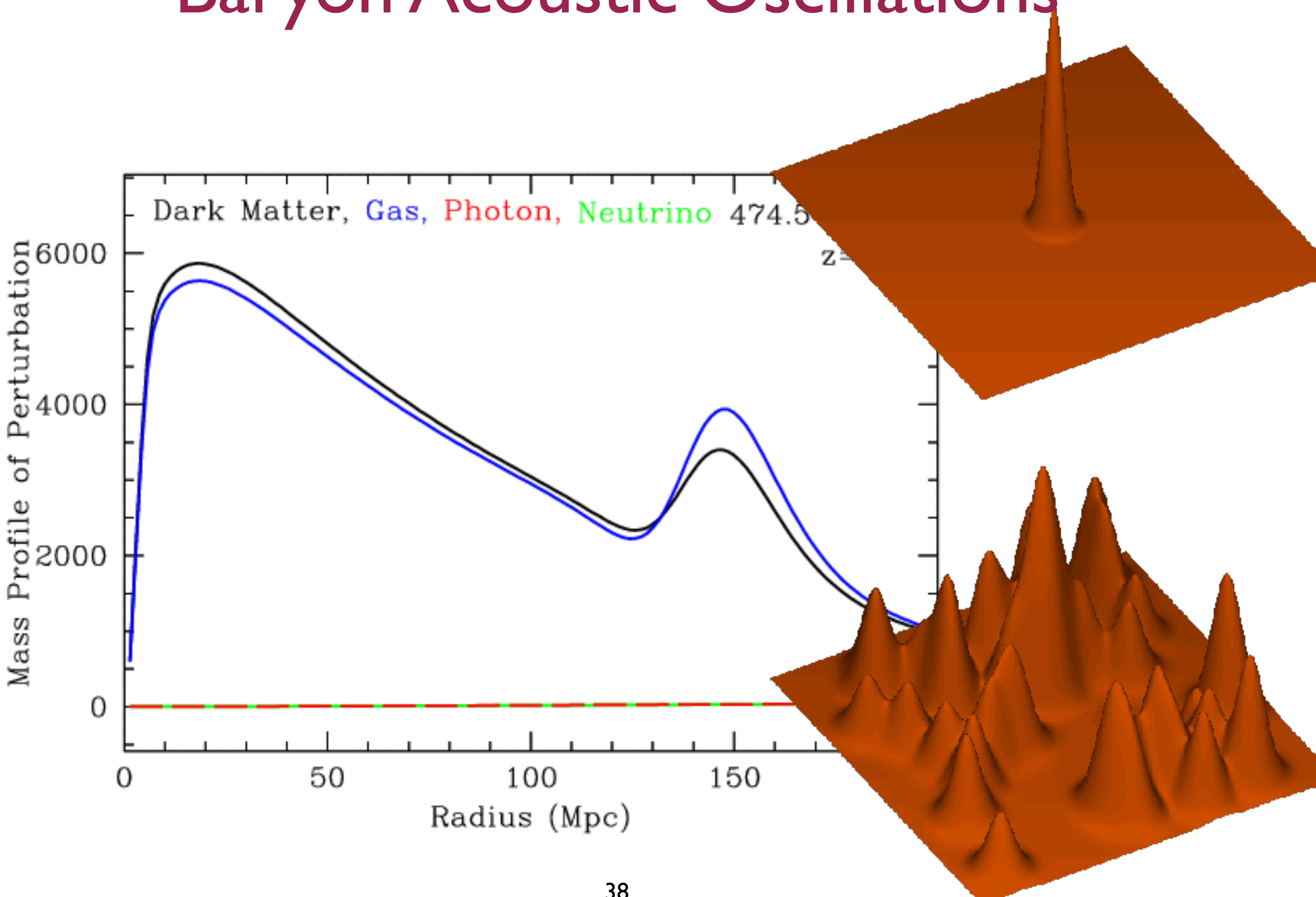


Baryon Acoustic Oscillations



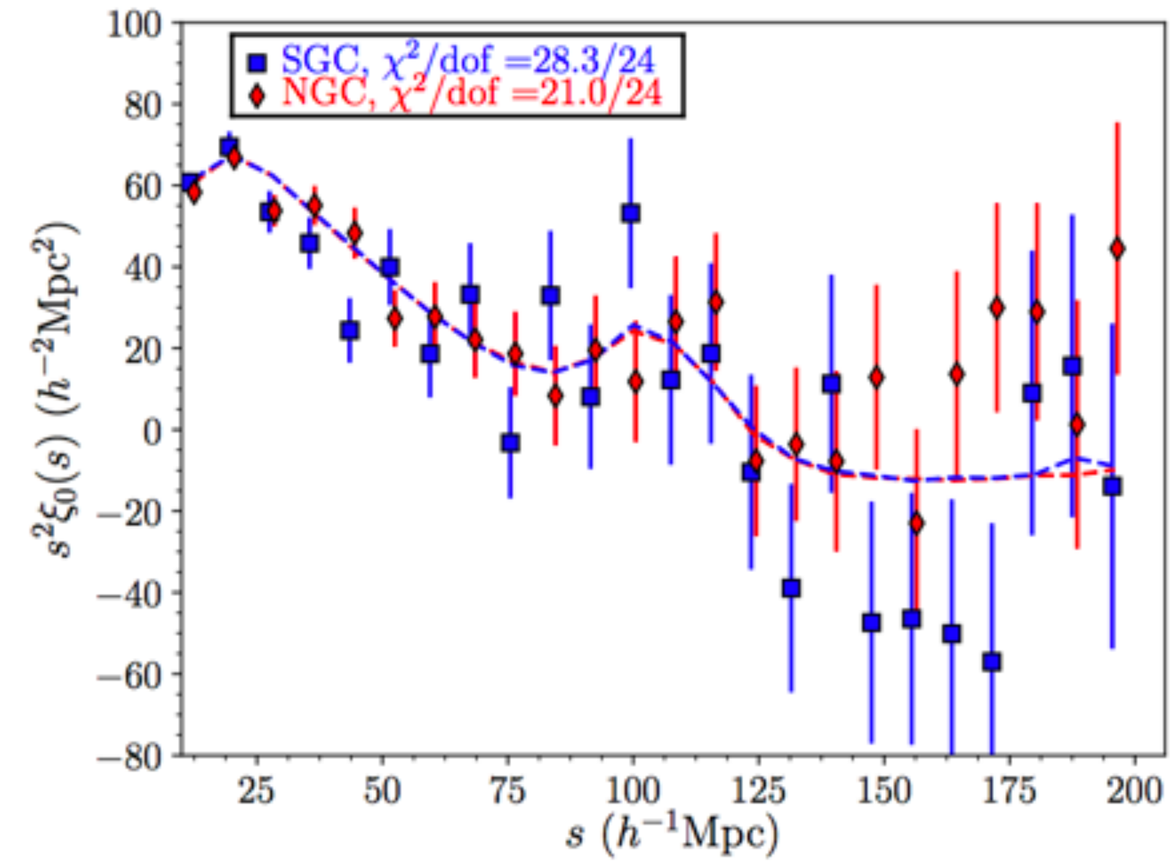


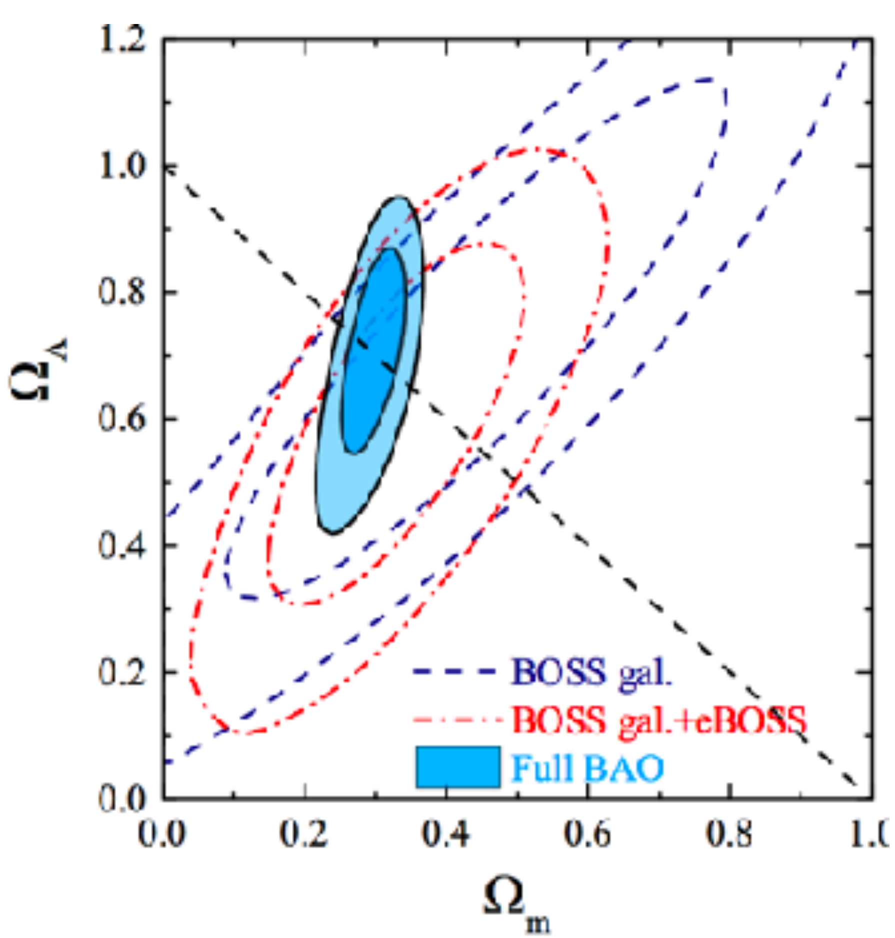
Baryon Acoustic Oscillations

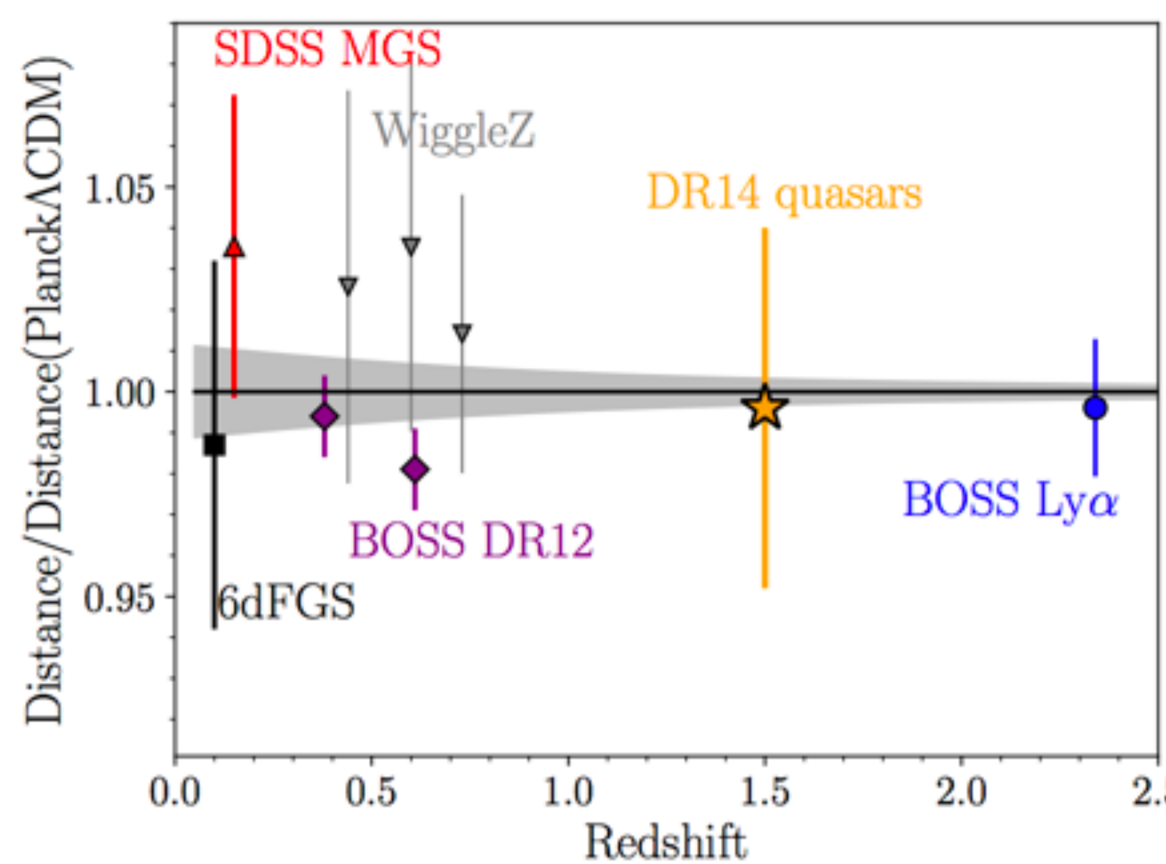


BAO peak in the quasar distribution

eBOSS measured the redshift of ~150'000 new quasars (Ata et al 2017) and obtained the first measurement of the BAO peak at z~1.5.



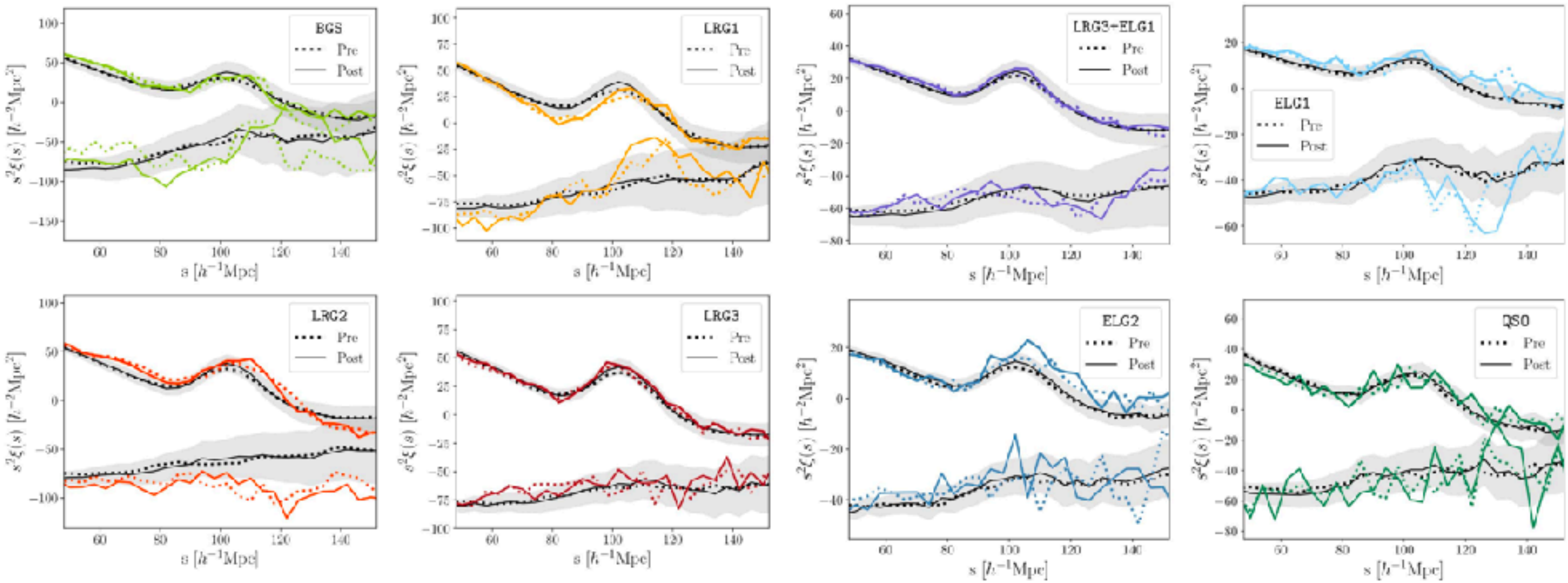




DARK ENERGY SPECTROSCOPIC INSTRUMENT

U.S. Department of Energy Office of Science

2-point correlation functions of DESI tracers



DESI Y1 BAO

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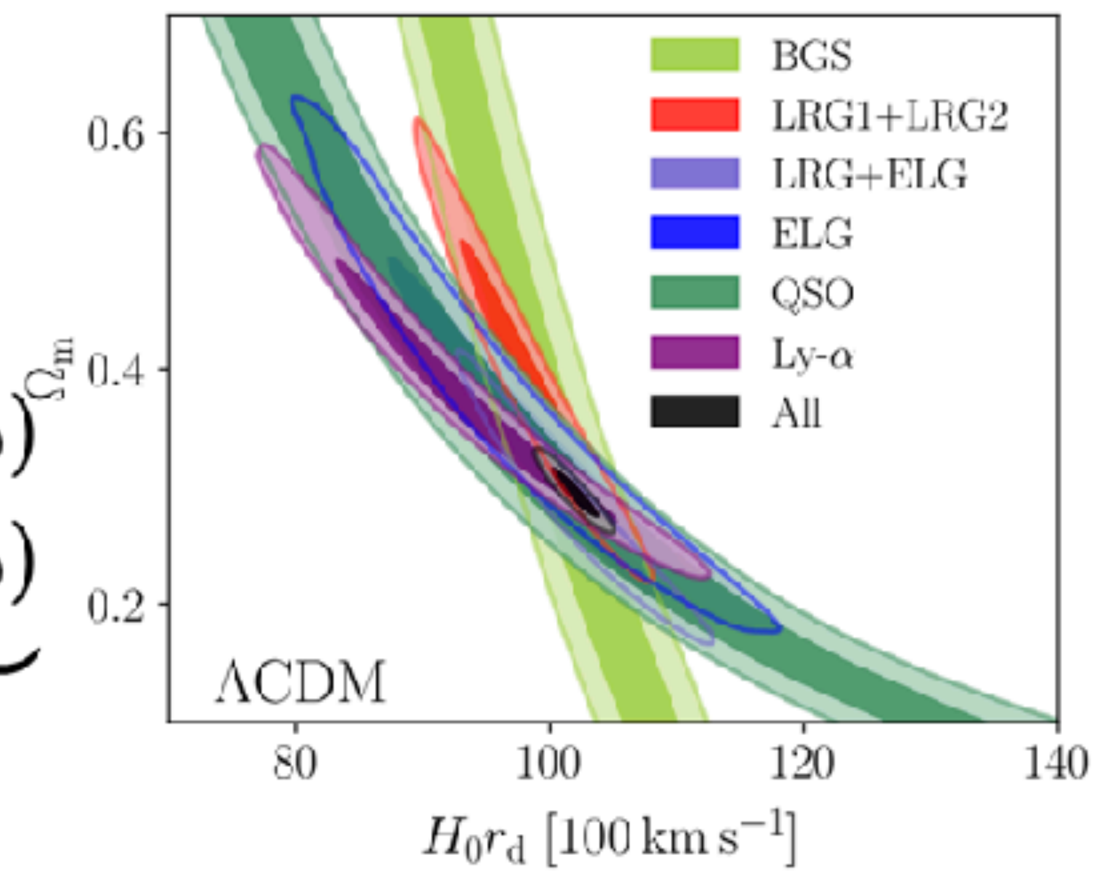
DESI BAO measurements

Consistent with each other, and complementary

$$\underbrace{\begin{aligned} \Omega_m &= 0.295 \pm 0.015 & (5.1\%) \\ H_0 r_d &= (101.8 \pm 1.3)\,[100\,\mathrm{km\,s^{-1}}] & (1.3\%) \end{aligned}}_{\text{DESI}}$$



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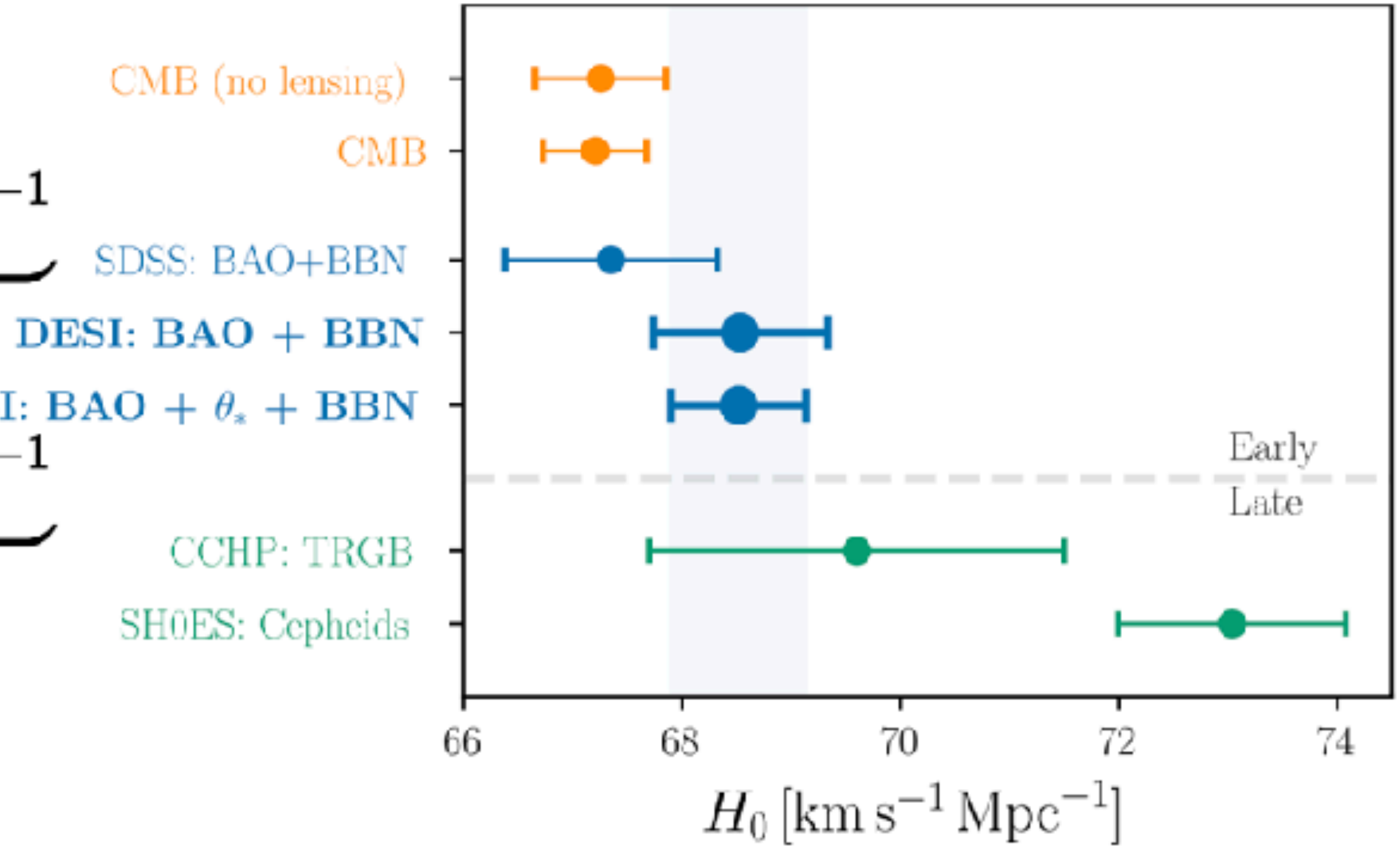
Hubble constant

$$\underbrace{H_0 = (68.53 \pm 0.80)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}}_{\textbf{DESI + BBN}}$$

$$\underbrace{H_0 = (68.52 \pm 0.62)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}}_{\textbf{DESI} + \theta_* + \textbf{BBN}}$$

- Consistency with SDSS
- In agreement with CMB
- In 3.7σ tension with SH0ES



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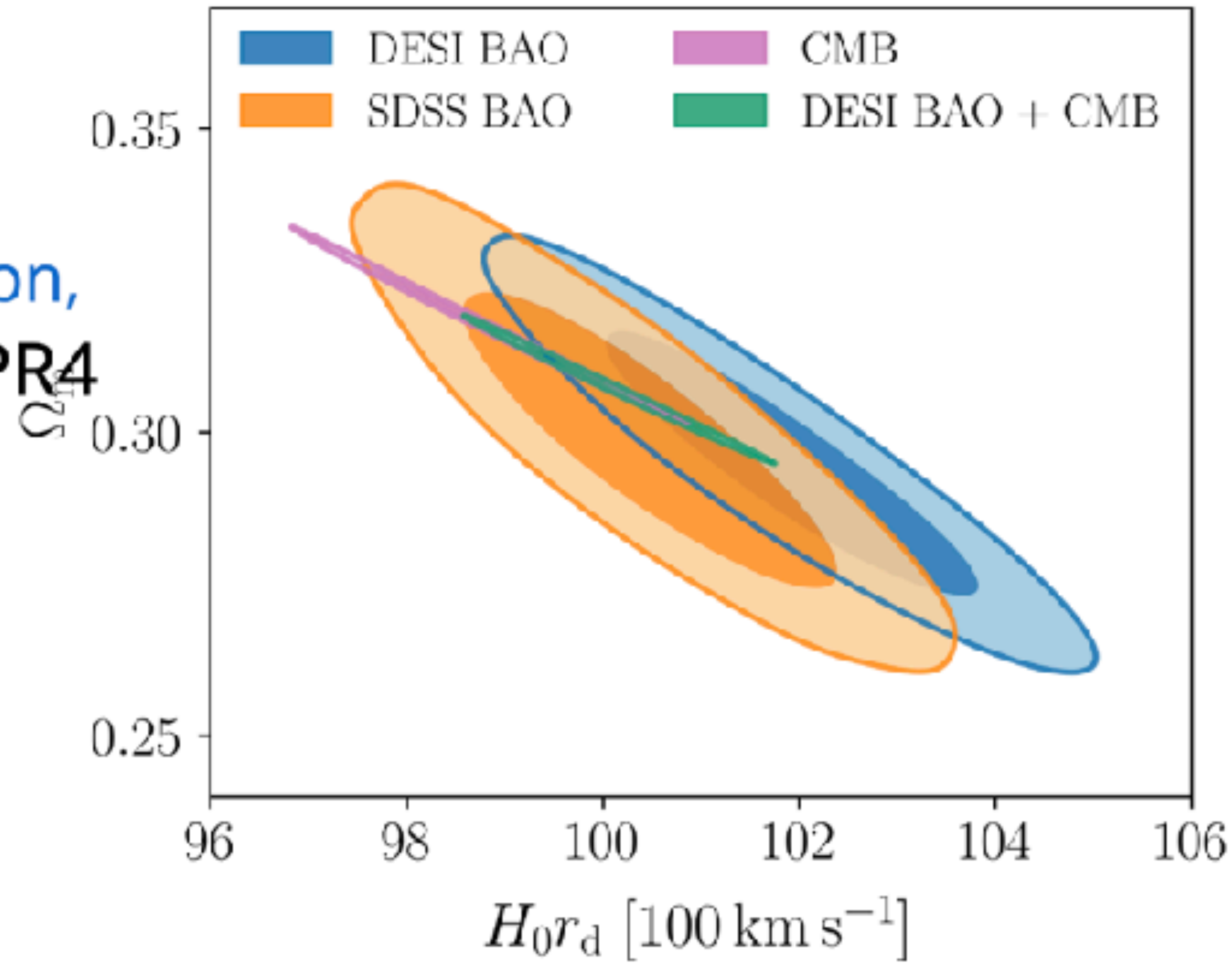
Consistency with other probes

DESI Y1 BAO consistent with:

- SDSS (eBOSS Collaboration, 2020)
- primary CMB: Planck Collaboration, 2018 and CMB lensing: Planck PR4 + ACT DR6 lensing ACT Collaboration, 2023, Carron, Mirmelstein, Lewis, 2022

https://arxiv.org/abs/1807.06209

$$\underbrace{\Omega_m = \mathbf{0.3069 \pm 0.0050\ (1.6\%)}}_{\text{DESI + CMB}}$$



Dark Energy Equation of State

DARK ENERGY SPECTROSCOPIC INSTRUMENT
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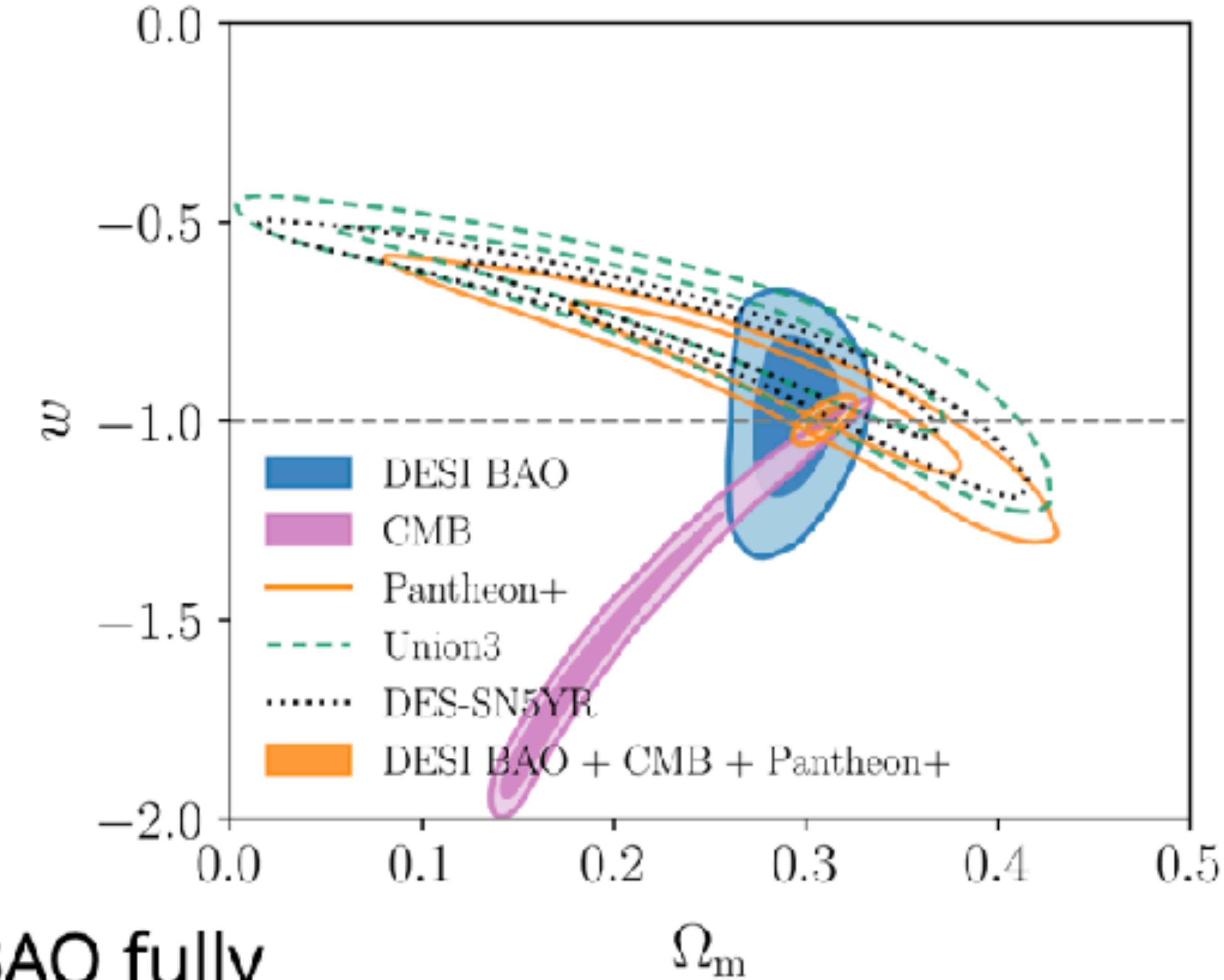
Constant EoS parameter w

$$\underbrace{\begin{aligned} \Omega_\mathrm{m} &= 0.293 \pm 0.015 \quad (\mathbf{5.1\%}) \\ w &= -0.99^{+0.15}_{-0.13} \quad (\mathbf{15\%}) \end{aligned}}_{\text{DESI}}$$

$$\underbrace{\begin{aligned} \Omega_\mathrm{m} &= 0.3095 \pm 0.0065 \quad (\mathbf{2.1\%}) \\ w &= -0.997 \pm 0.025 \quad (\mathbf{2.5\%}) \end{aligned}}_{\text{DESI + CMB + Pantheon+}}$$



Assuming a **constant** EoS, DESI BAO fully compatible with a cosmological constant…

DARK ENERGY SPECTROSCOPIC INSTRUMENT
U.S. Department of Energy Office of Science

Sum of neutrino masses

Internal CMB degeneracies limiting precision on the sum of neutrino masses

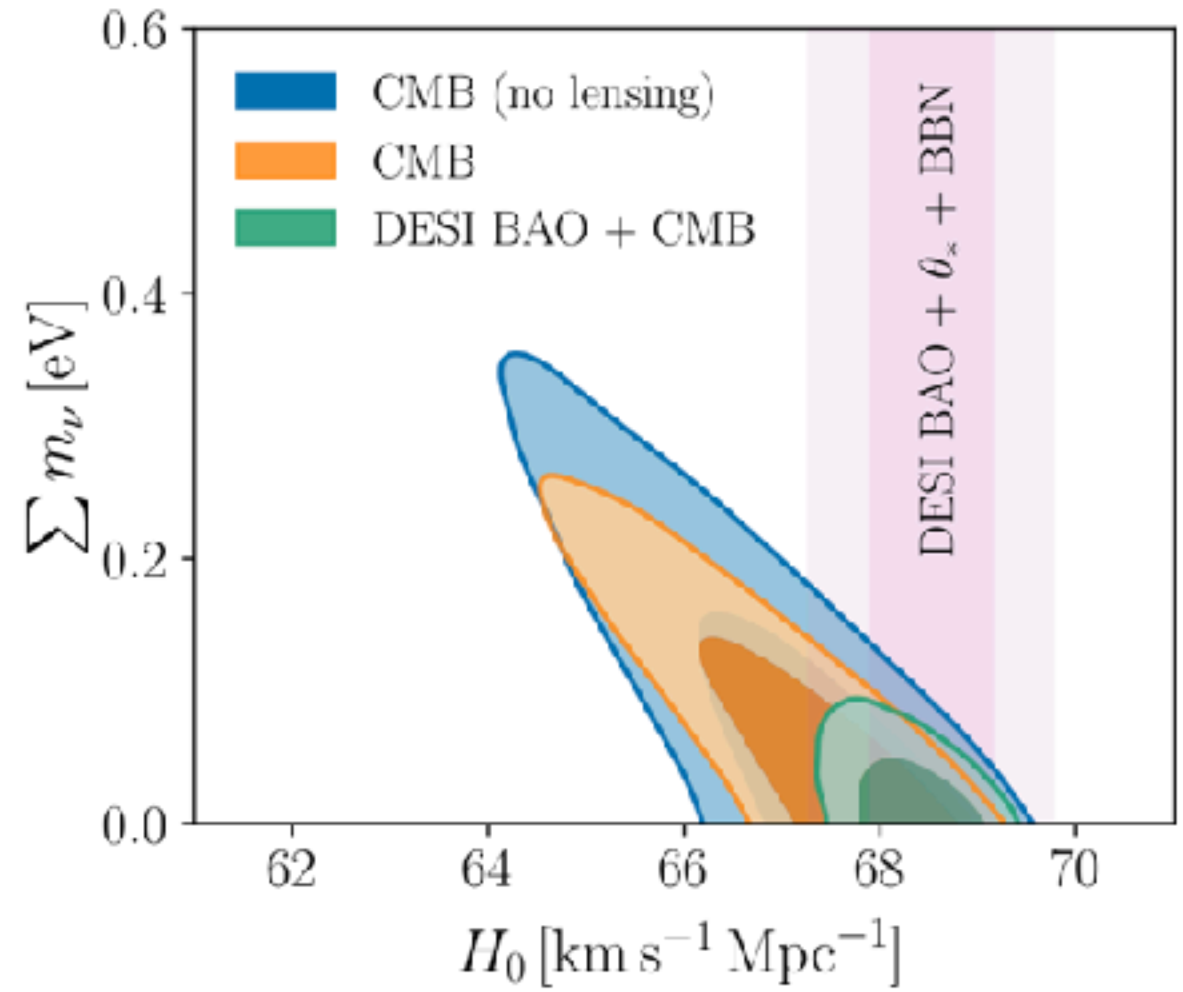
Broken by BAO, especially through H_0

Low preferred value of H_0 yields

$\sum m_\nu < 0.072\,\mathrm{eV}$ (95%, DESI + CMB)

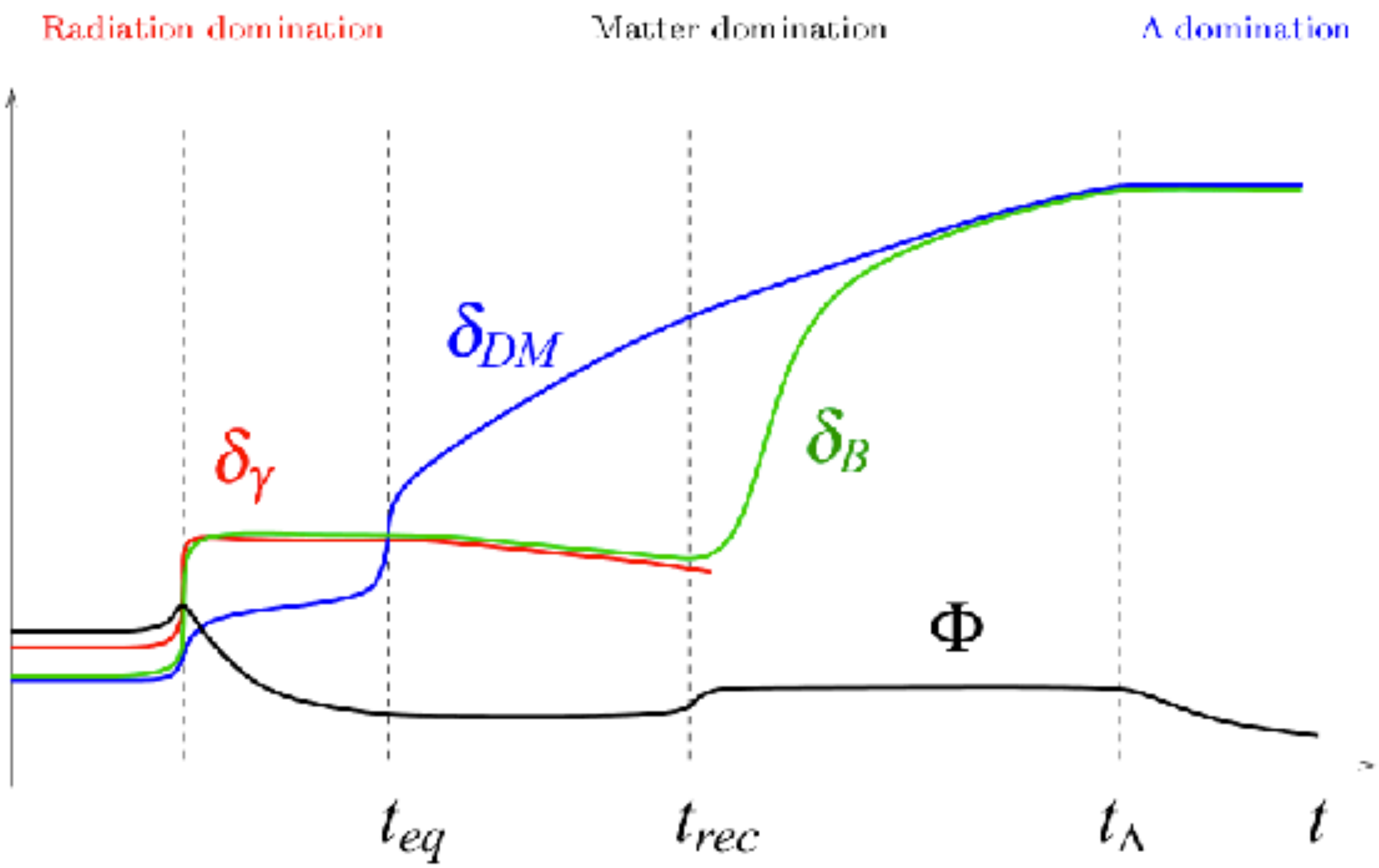
Limit relaxed for extensions to ΛCDM

$\sum m_\nu < 0.195\,\mathrm{eV}$ for $w_0 w_a$CDM



Evolution of the density fluctuations

- The growth of density fluctuations will depends on the nature of the particle, and the epoch in the universe.
- Photons and baryons are coupled until the time of recombination.
- Dark Matter fluctuations were larger than the baryon fluctuations at recombination thus explaining the small temperature fluctuations in the CMB



Non-Linear structure evolution

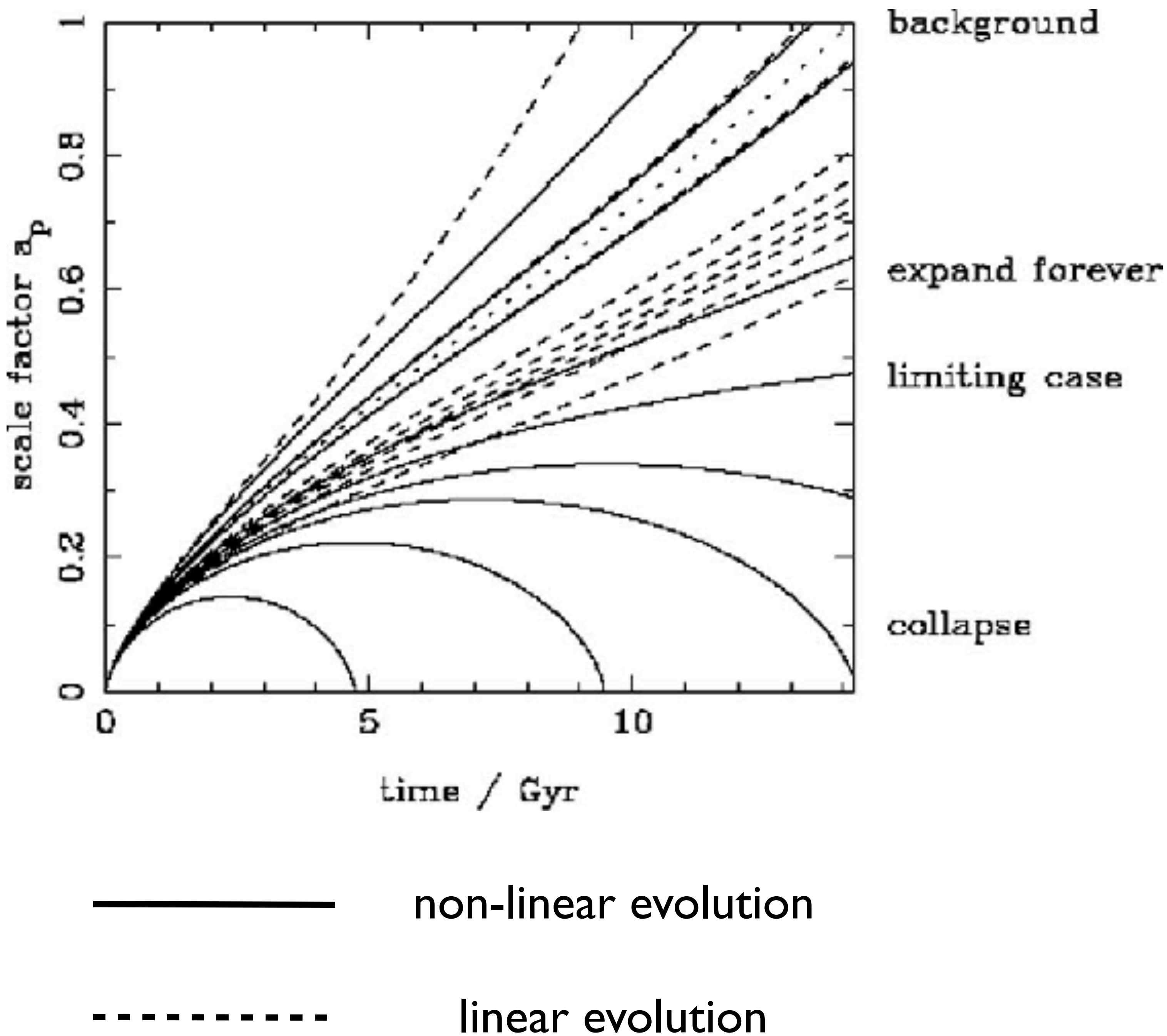
- When density fluctuation are large $\delta >> 1$, the previous formalism does not hold anymore.

- Either analytical solutions is possible in very limited cases (spherical collapse model), or one needs to conduct numerical simulations.

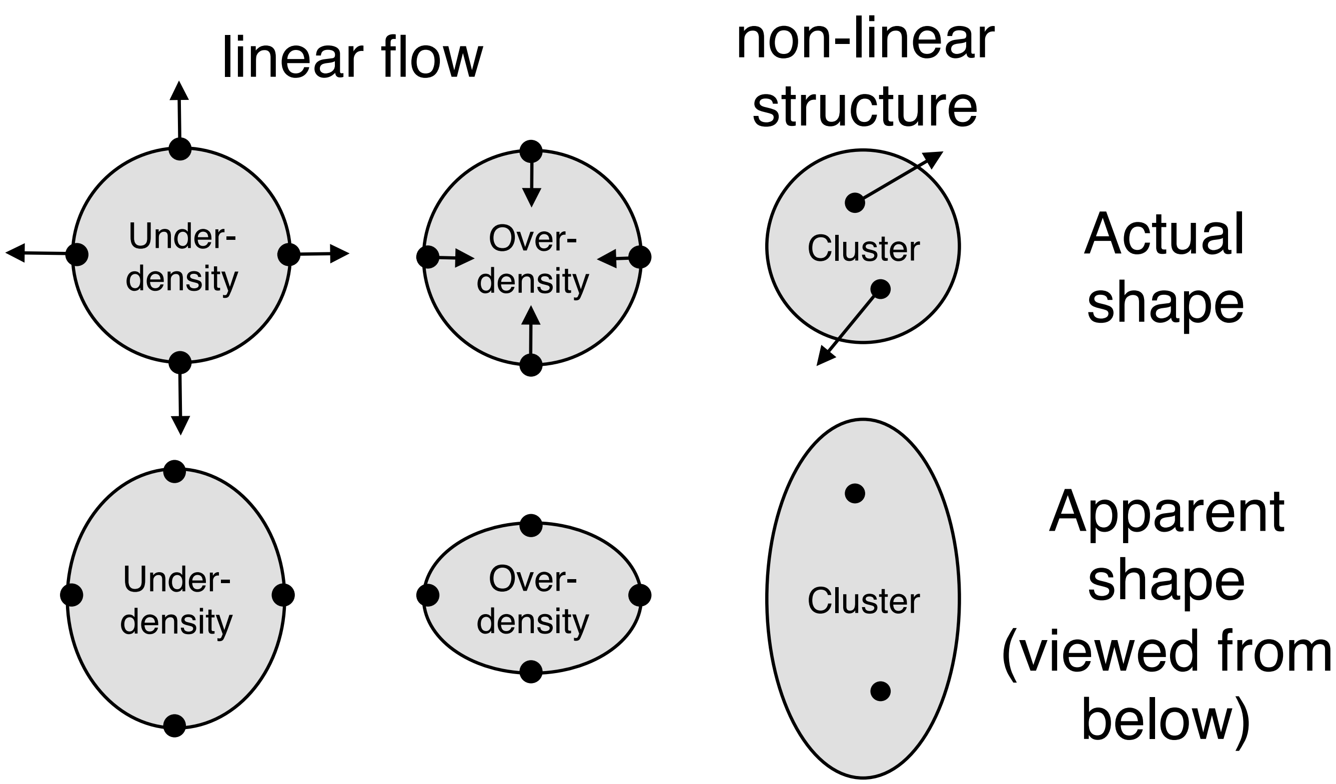
Model of Spherical collapse

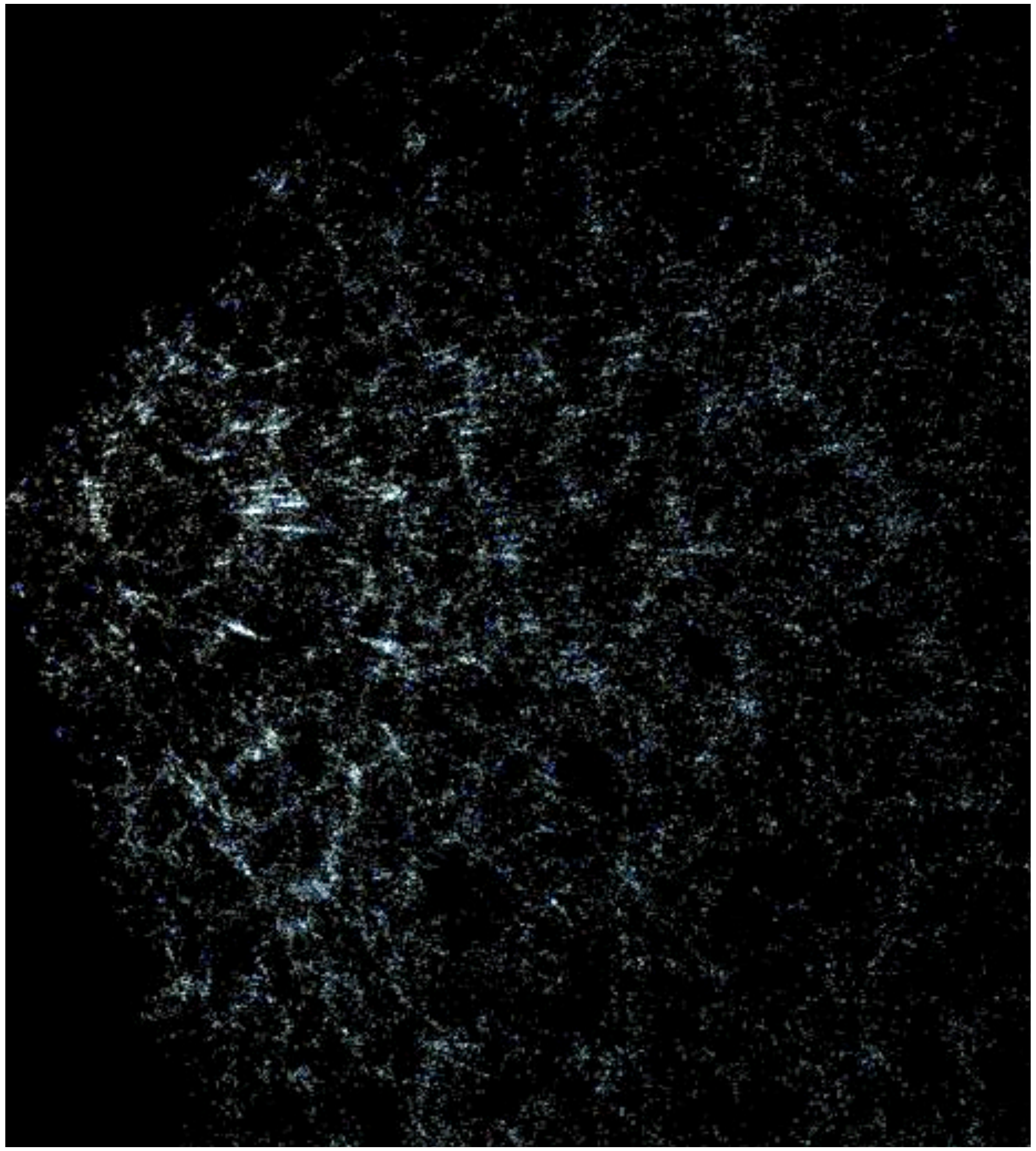
- If the density fluctuation is high, we can consider that it evolves like a closed universe and eventually it will collapse.
- In the Einstein de Sitter model the collapse take place when:

$$\delta_o > \delta_c = \frac{3}{20}(12\pi)^{2/3} \sim 1.69$$



Redshift-Space Distortions



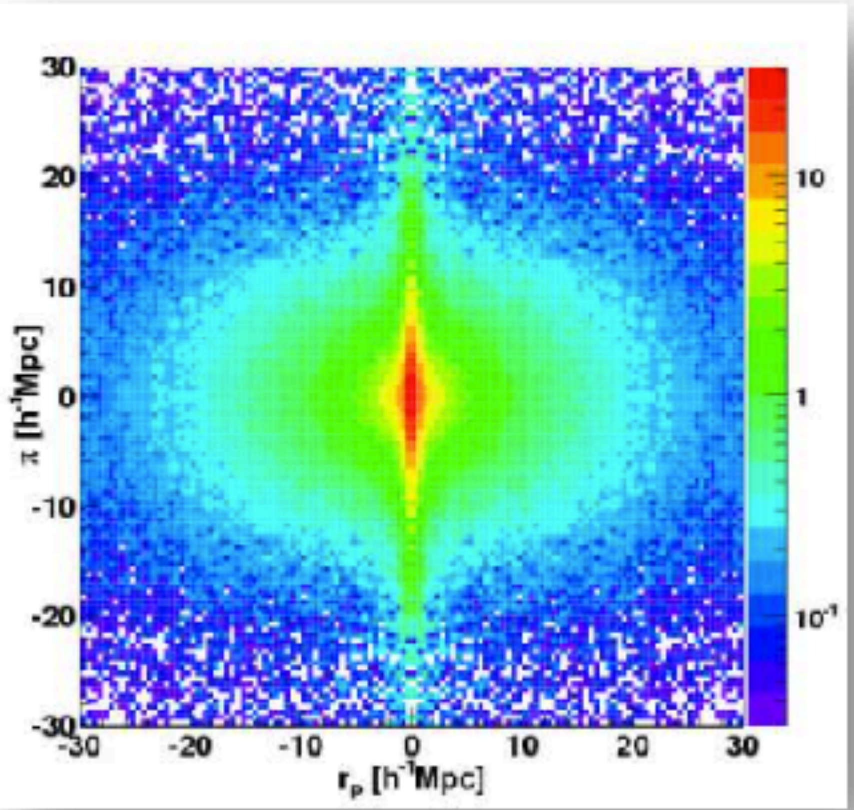


- Observed redshift depends on both Hubble expansion and additional "peculiar velocity"
- Galaxies move with structure growth
- extra clustering depends on amplitude of peculiar velocities

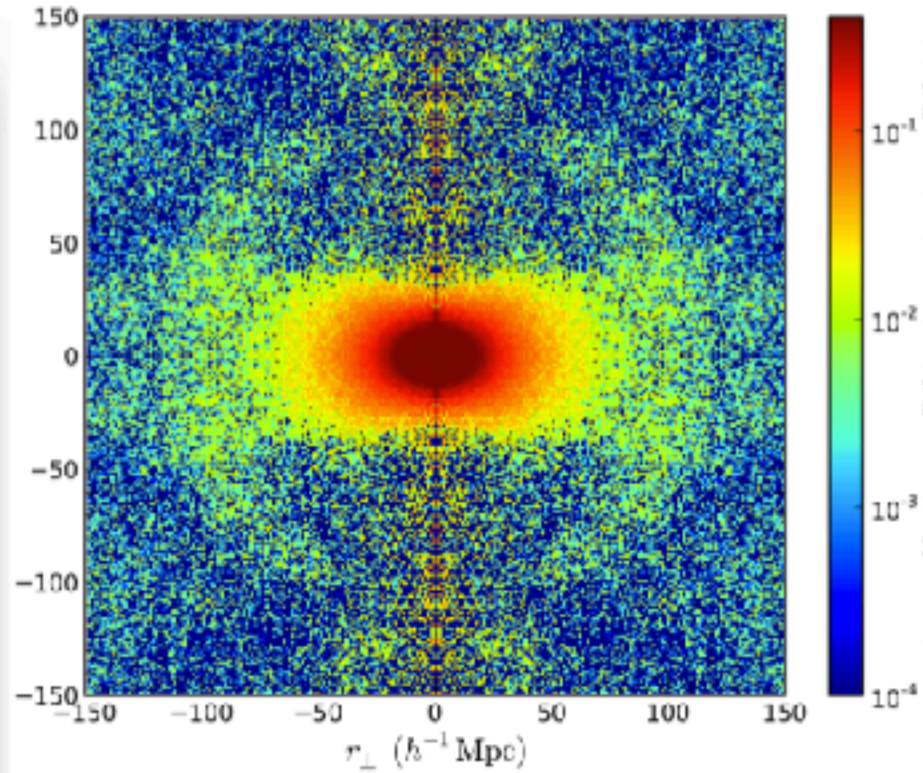
$$f(z)\sigma_8(z) \propto \frac{dG}{d\log a}$$

- where G is the linear growth rate

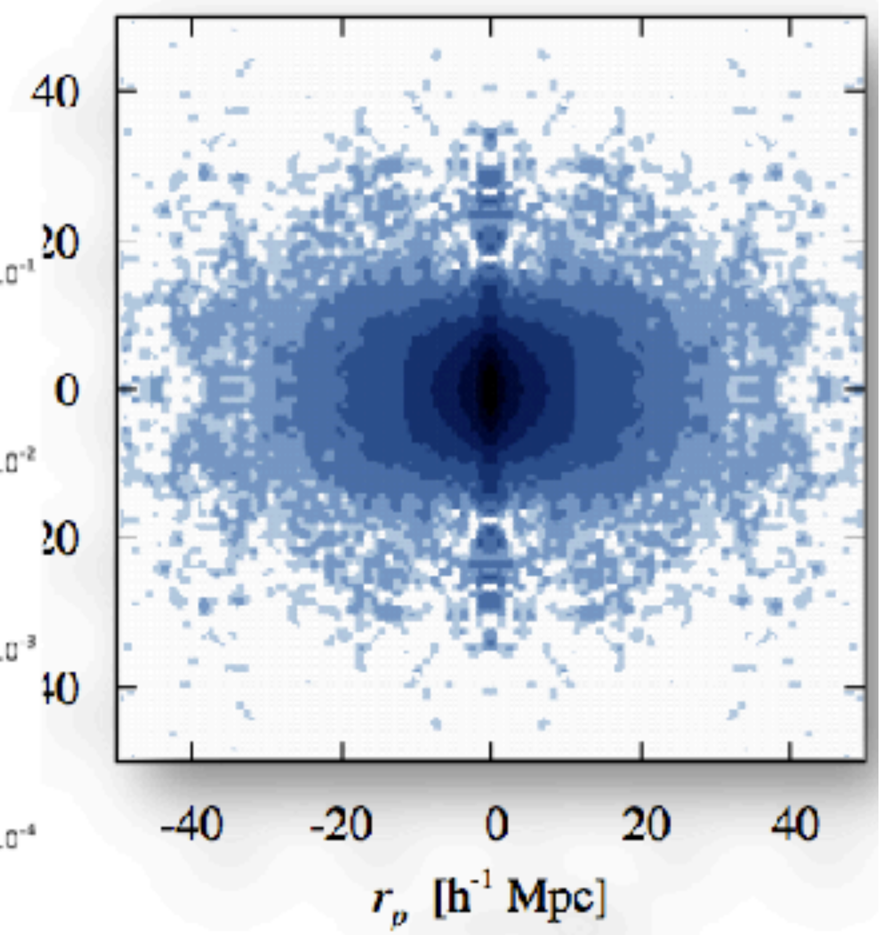
Redshift Space Distorsion



6dFGS
Beutler et al. 2012
z=0.06



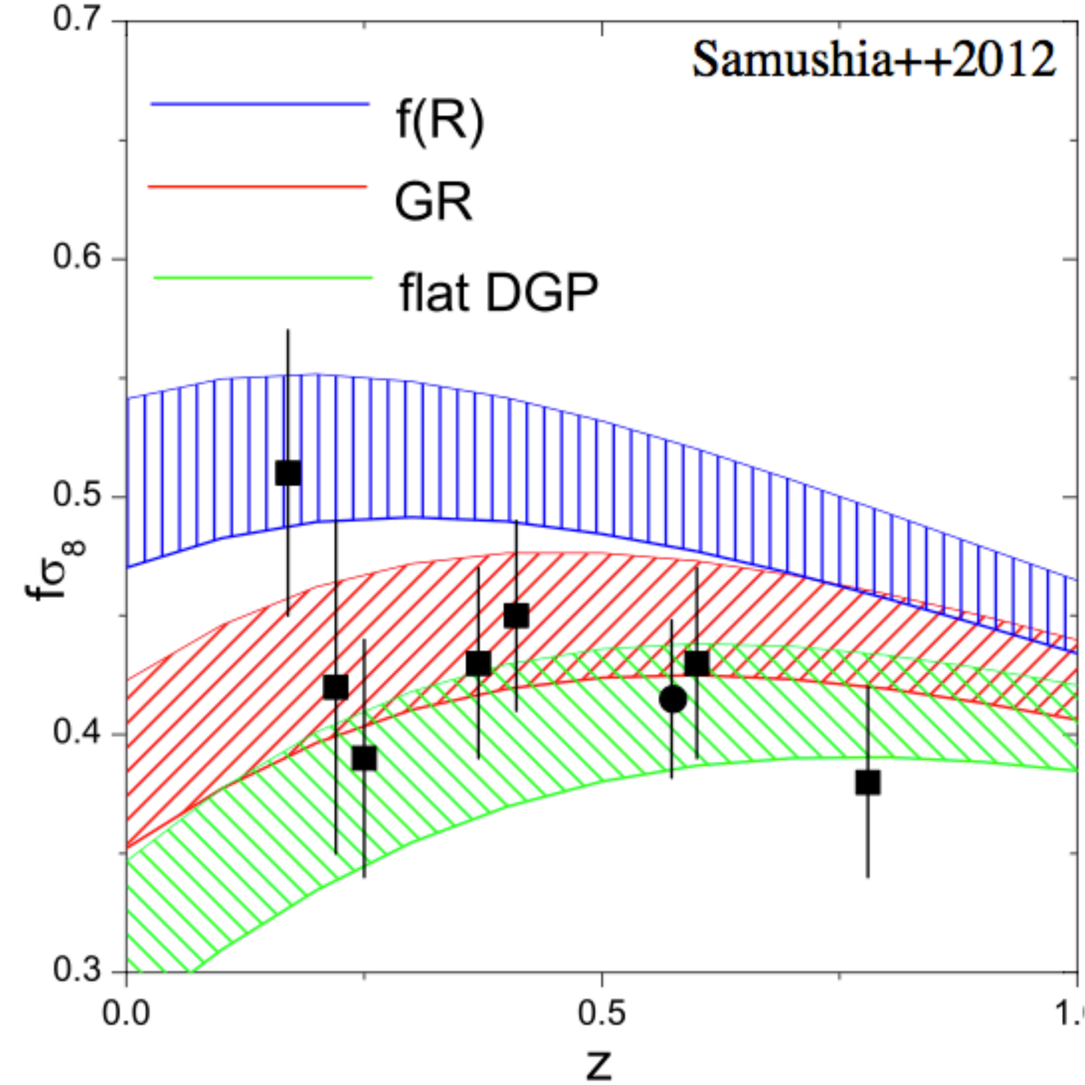
SDSS-III/BOSS
Samushia et al. 2014
z=0.57



VIPERS
de la Torre et al. 2013
z=0.8

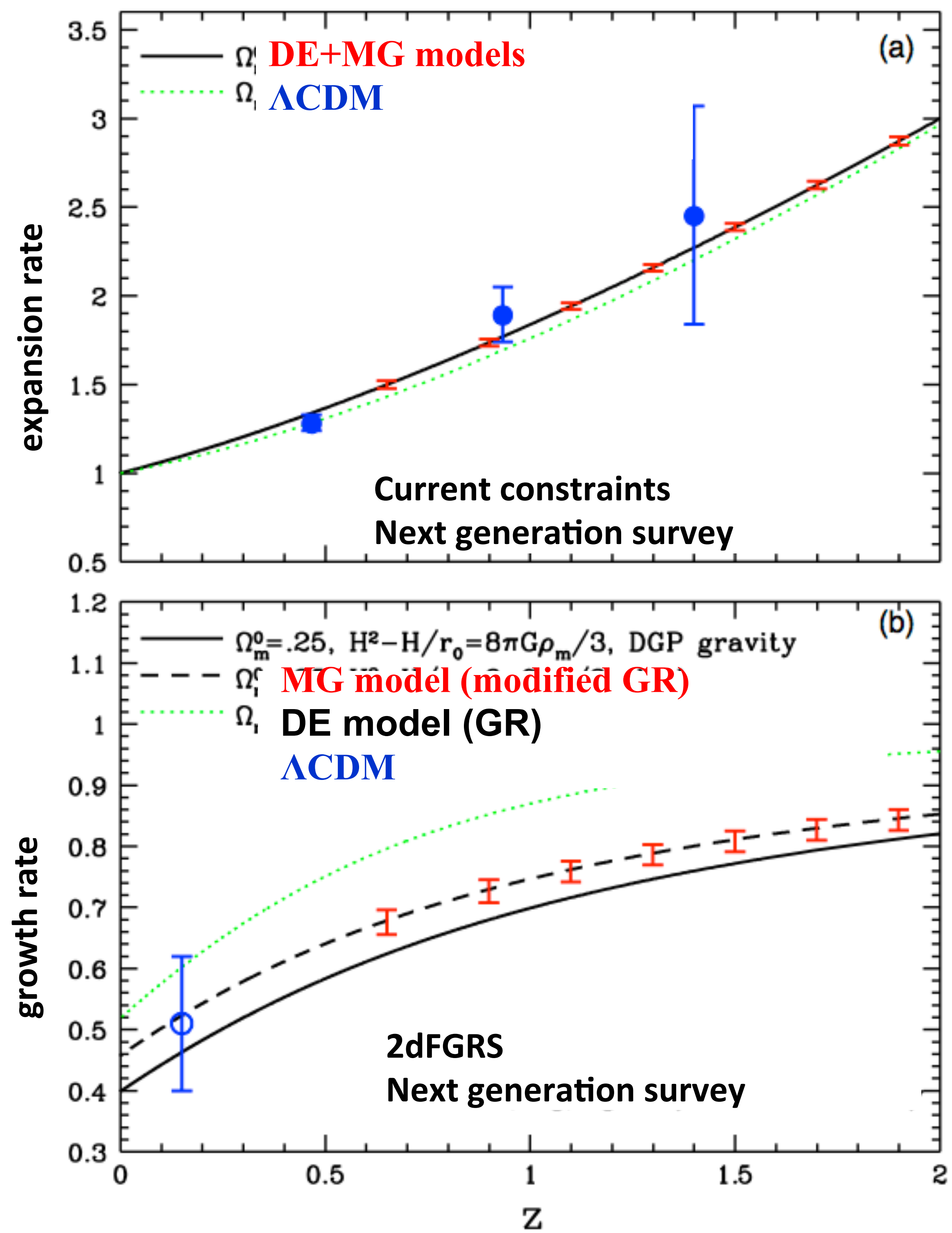
Redshift-Space Distortions

- Now measured with 8.2% accuracy simultaneously with geometry (assuming ΛCDM, Reid et al. 2012; MNRAS, 426, 2719)
- Anisotropic clustering allows huge improvement on w!
- w = -0.95 ± 0.25 (WMAP + DV(0.57)/rs)
- w = -0.88 ± 0.055 (WMAP + anisotropic)
- Provides a number of GR tests



Samushia et al. 2013; MNRAS 429, 1514

Complementarity of growth & expansion



Wang 2008: JCAP, 05, 21

Press-Schechter theory

- It builds on idea of spherical collapse and the overdensity field to create statistical theory for structure formation
  - Base on the critical density for collapse. Assume any perturbations with greater density (at an earlier time) have collapsed
  - Filter the density field to find the largest size of perturbations that has collapsed
- Can be used to give
  - mass function of collapsed objects (halos)
  - creation time distribution of halos
  - information about the build-up of structure (extended PS theory)

Press-Schechter theory

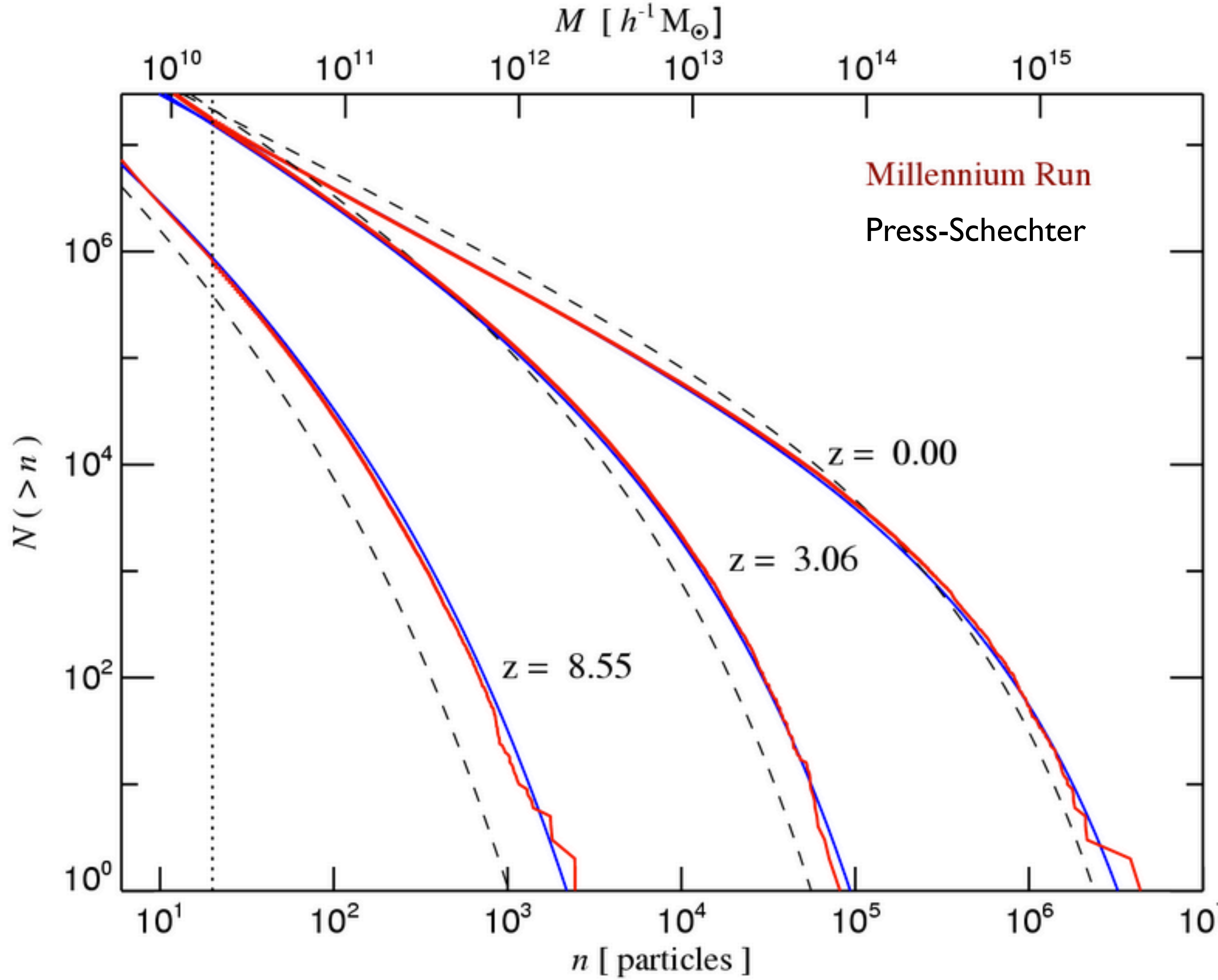
- The Press–Schechter formalism predicts that the number of objects with mass between M and M+dM:

$$n(M,z)=\frac{\rho_c r\Omega_m}{\sqrt{\pi}}\,\frac{\gamma}{M^2}\left(\frac{M}{M^*(z)}\right)^{\gamma/2} e^{\left[-\left(\frac{M}{M^*(z)}\right)^{\gamma}\right]}$$

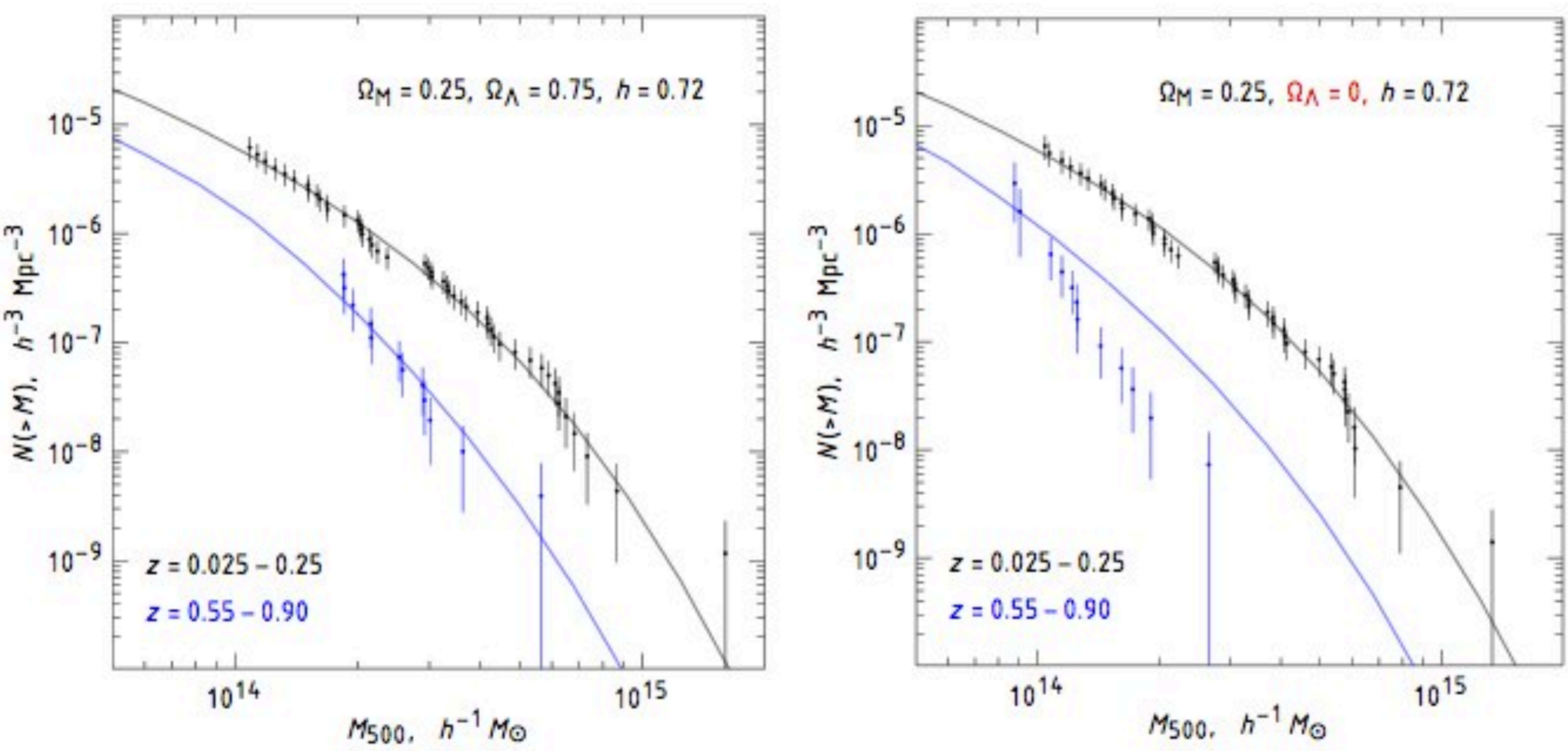
- M*(z) is the redshift dependent mass-scale above which the mass spectrum is exponentially cut off.

$$\gamma \sim 0.5 \qquad M^*(z)=M_0^*(1+z)^{-2/\gamma}$$

Mass function from Millennium simulation

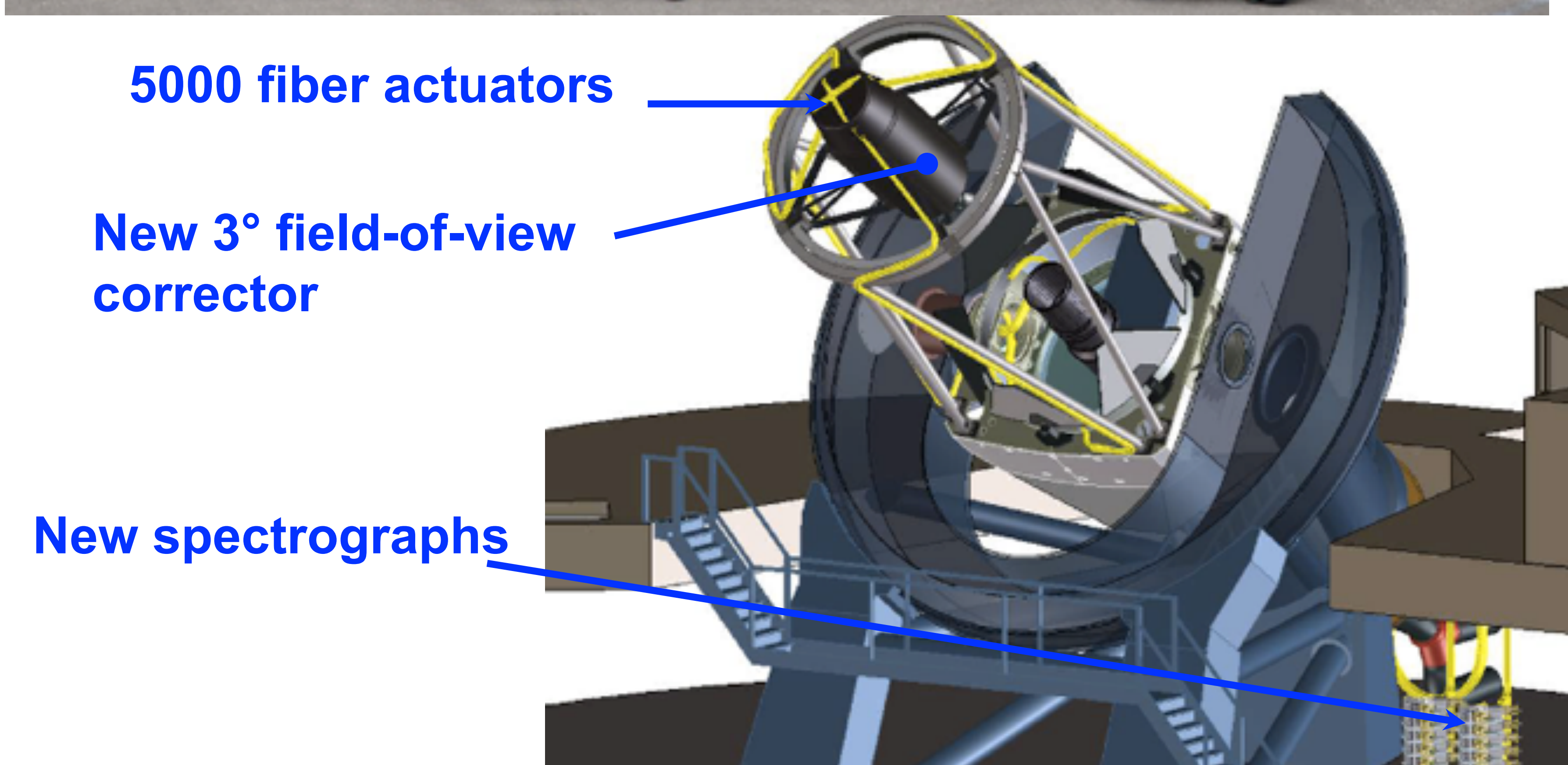


Mass function from X-ray observation

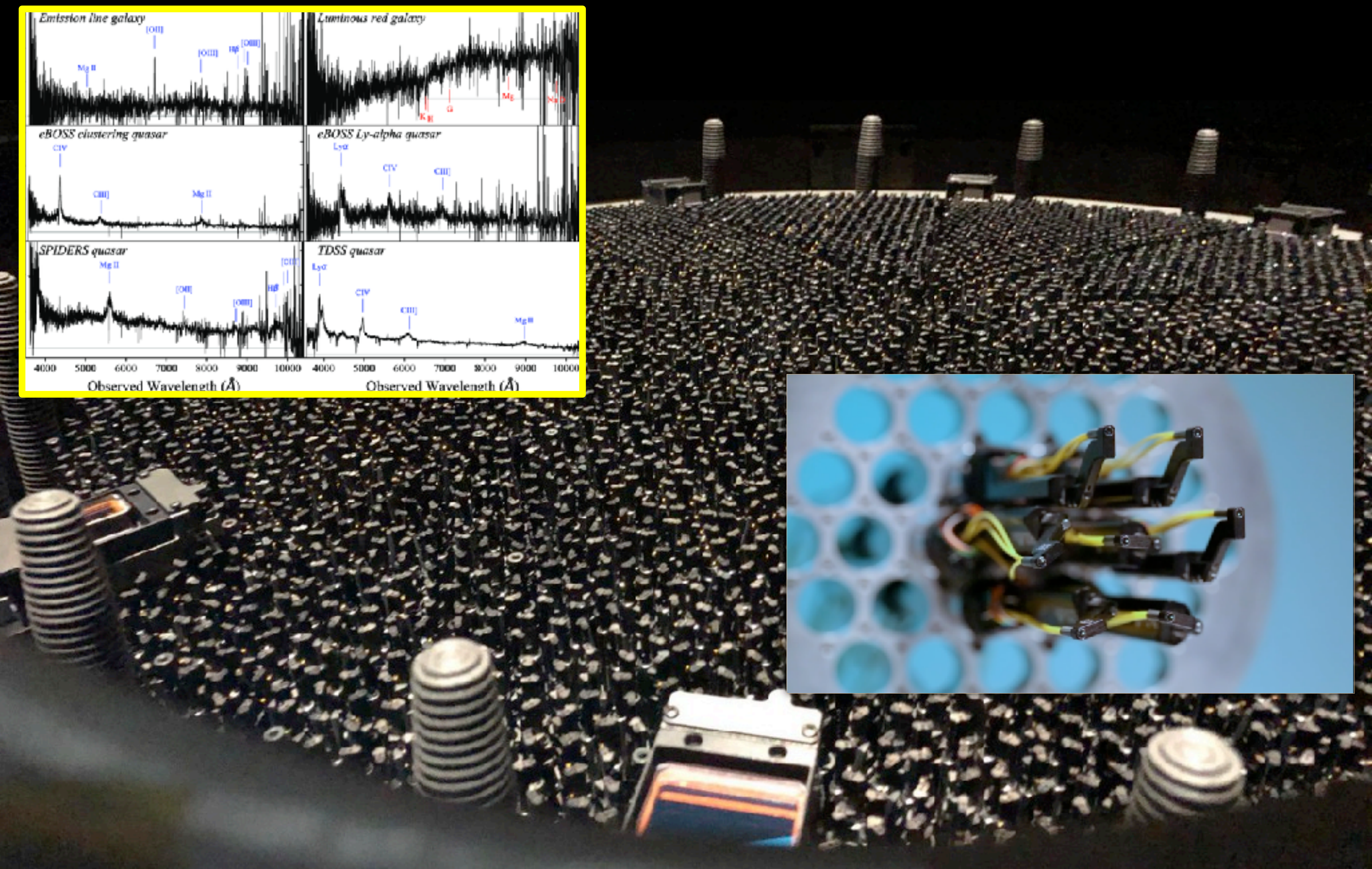


Dark Energy Spectroscopic Instrument

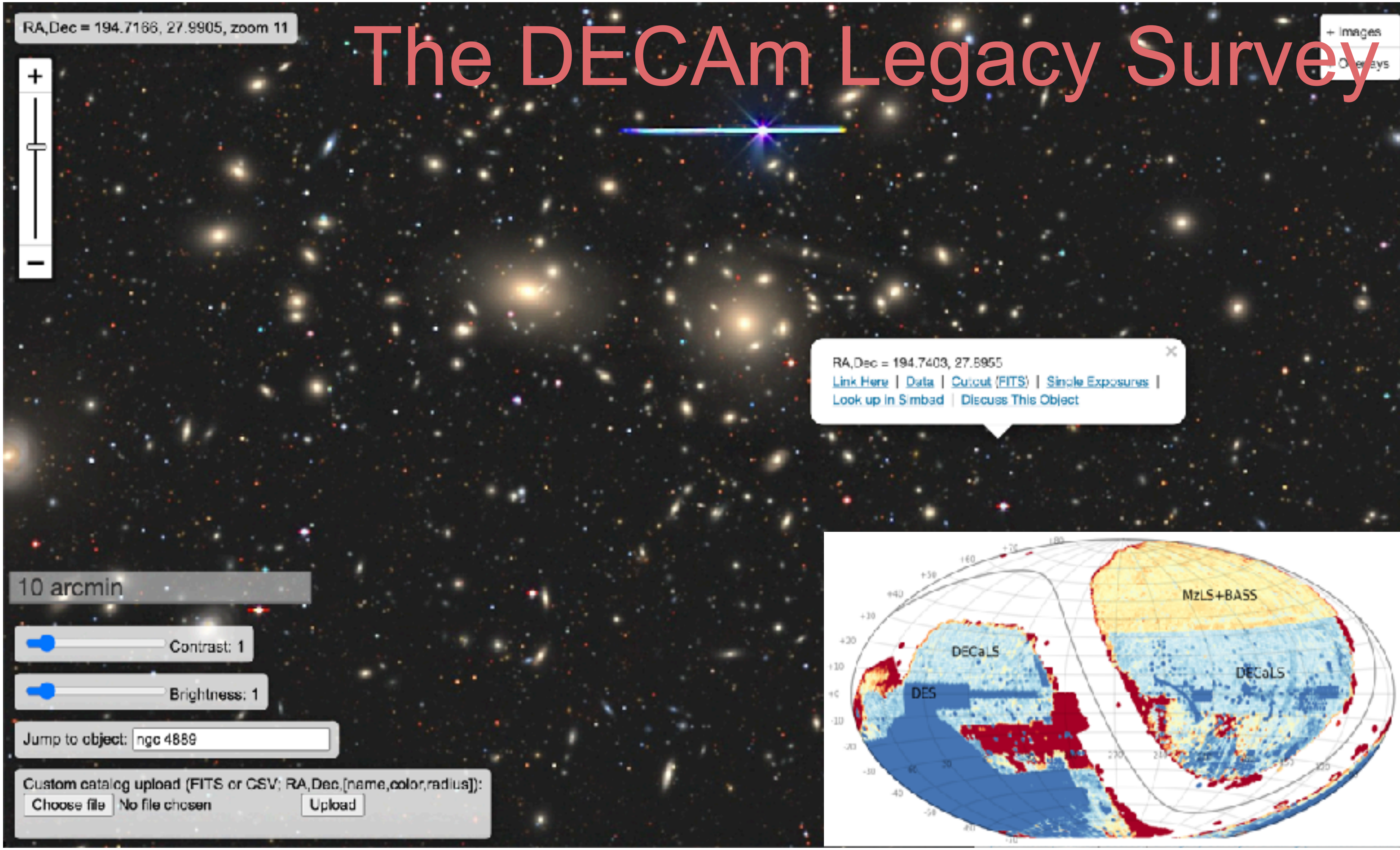
- **Map 40 million galaxies**
- Experiment to be installed on the 4m Mayall telescope at Kitt Peak in Arizona
- **International project** selected by the Department of Energy in the USA with many European countries involved (35+ institutions) **~600+ participants**
- **EPFL involved in developing the fiber-positioner robotic system.**



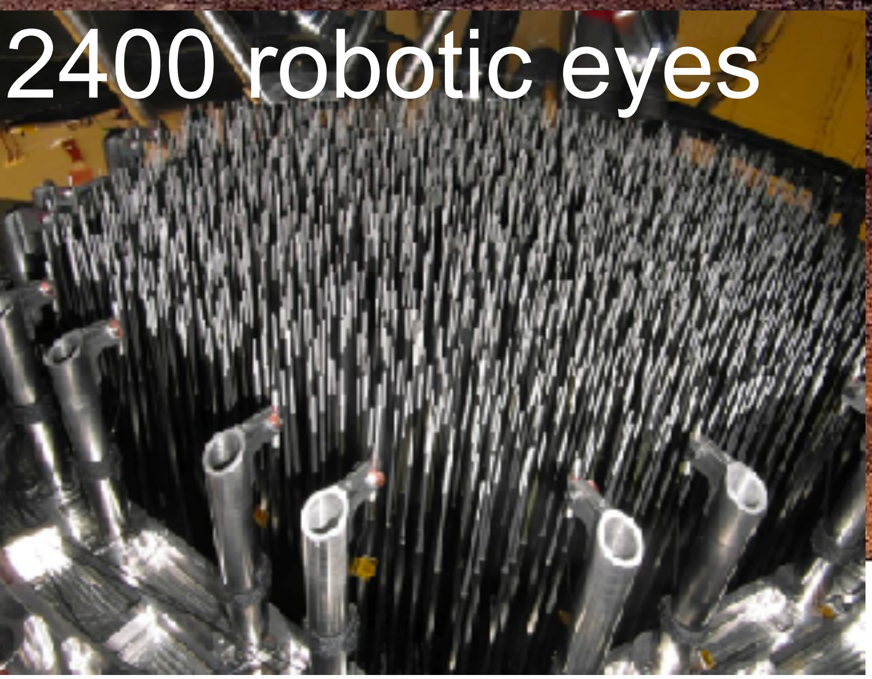
5’000 robotic eyes … 40 million galaxies by 2025

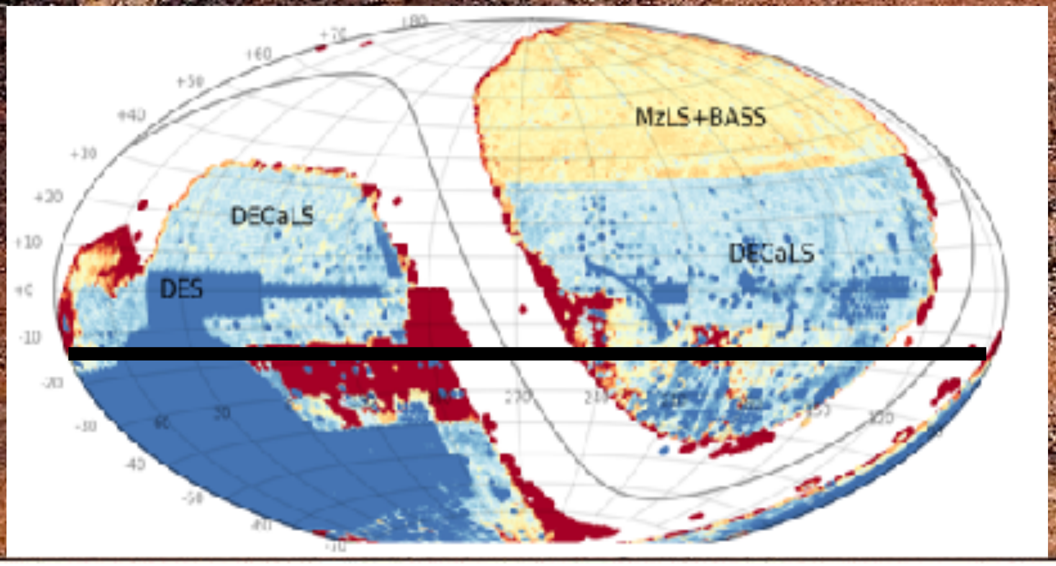


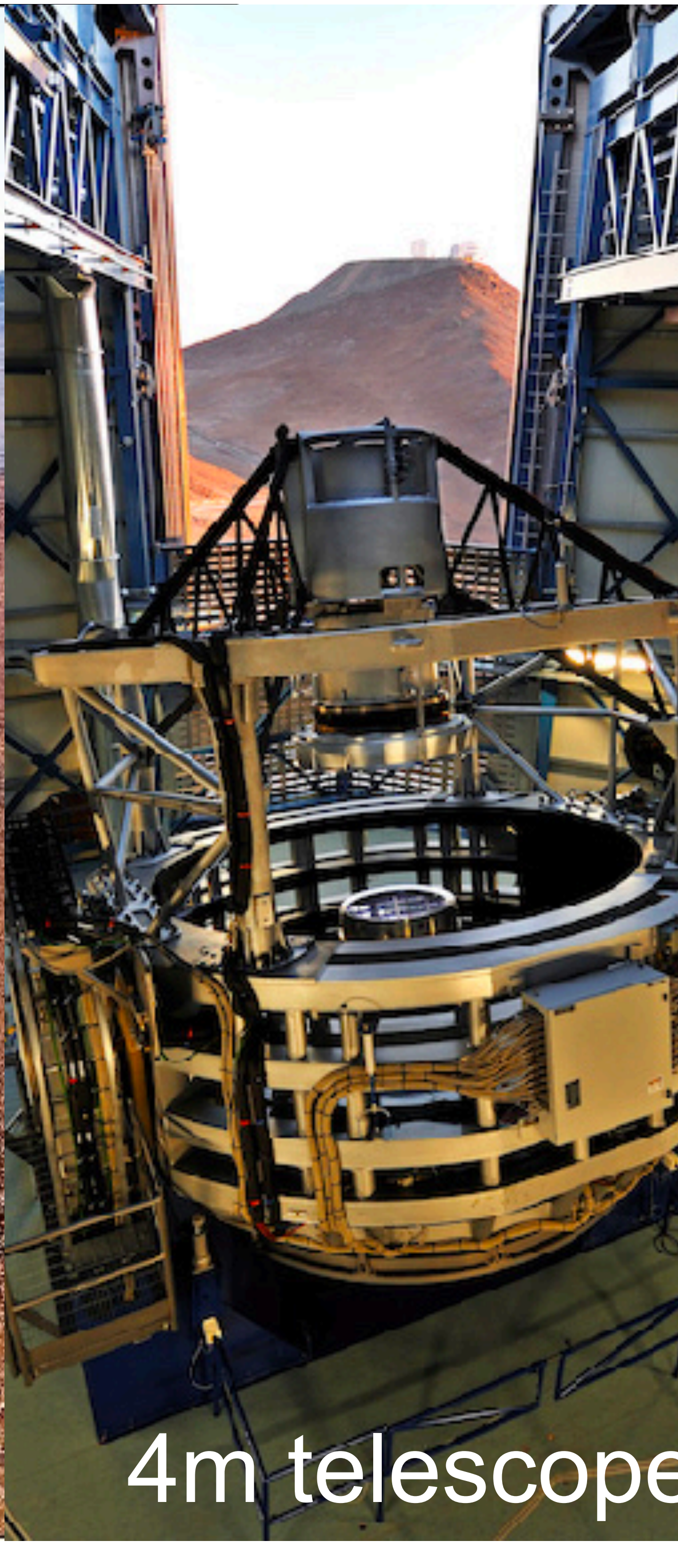
The DECAm Legacy Survey



4MOST - a southern hemisphere survey
Start of the Survey 2026

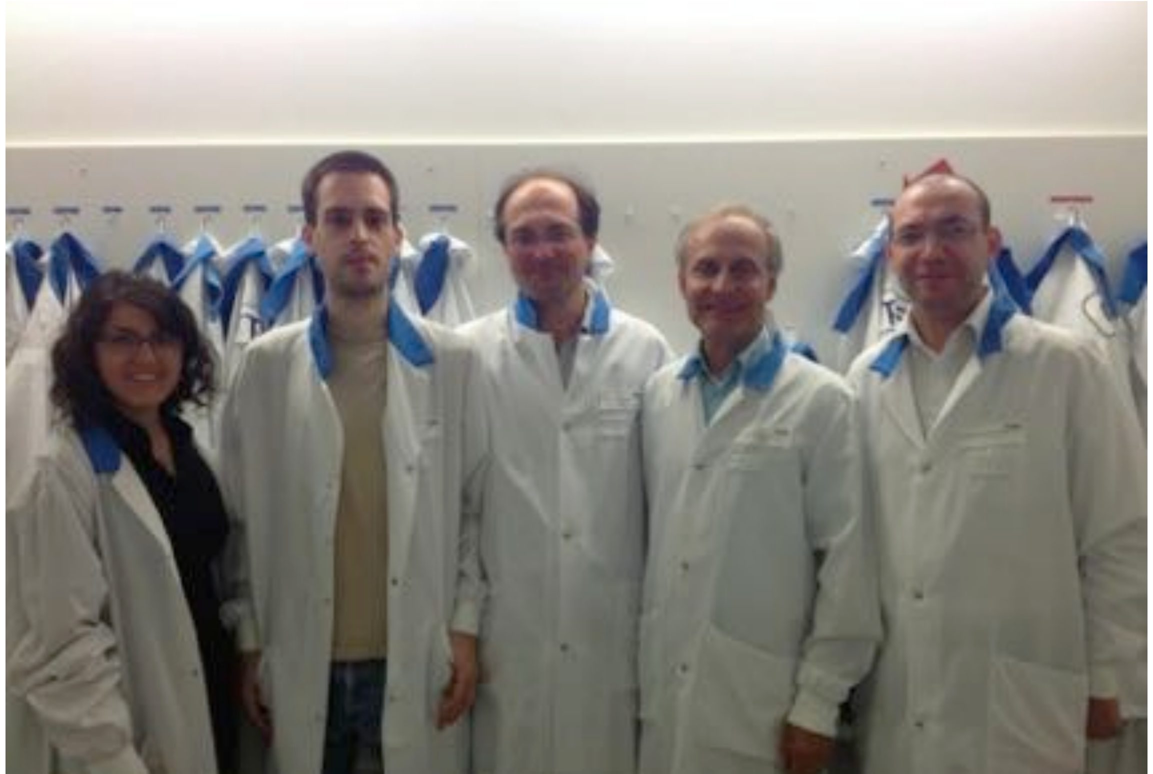






Astrobots group at EPFL

- Interdisciplinary group at EPFL to focus on robotic fiber positioner system, started in February 2013.
- New 10mm prototype constructed at MPS a Swiss company, April 2014
- **Prototype presented at the DESI collaboration meeting, May 2014 but not selected. Berkeley design was selected.**
- **Massive production (5000 units) started end of 2015, completed this spring.**
- **Instrument ready on telescope in September 2019**
- Now, involved in a new similar project: SDSS-V a project with 1000 actuators for the Sloan+Dupond telescopes. All positions have been delivered. On the telescope since end of 2021.



EPFL interdisciplinary team:

http://astrobots.epfl.ch/

Projects after DESI/4MOST

- China led: **MUST**
- USA led: **Spec-S5**
- European led: **WST**
- EPFL part of the 3 projects and developing the critical fiber positioner robots (funded by InnoSuisse)
- Cosmology, fundamental physics, mapping the universe in 3D