

Gravitational Lensing - 3

Jean-Paul KNEIB

March 28, 2023: the distant Cosmic Seahorse galaxy with James Webb Space Telescope



Quiz

- What is microlensing?
- For what scientific purpose are we using microlensing?
- Have you got/checked the GravLens3 iphone App?
- What is the largest camera on Earth? In Space? (For weak lensing application)
- What is weaklensing?
- What is the reduced shear?
- What is the Bullet Cluster? What do we learn from it?

Outline

- Modelling strategy of strong lens systems (focusing on cluster of galaxies)
- Combining Lensing and other techniques
- Galaxy-galaxy lensing

Modeling Strong Lensing

Lensing Equations

Lens Mapping:

$$\vec{\theta}_S = \vec{\theta}_I - \frac{2\mathcal{D}}{c^2} \vec{\nabla} \phi_N^{2L}(\vec{\theta}_I) = \vec{\theta}_I - \vec{\nabla} \varphi(\vec{\theta}_I)$$

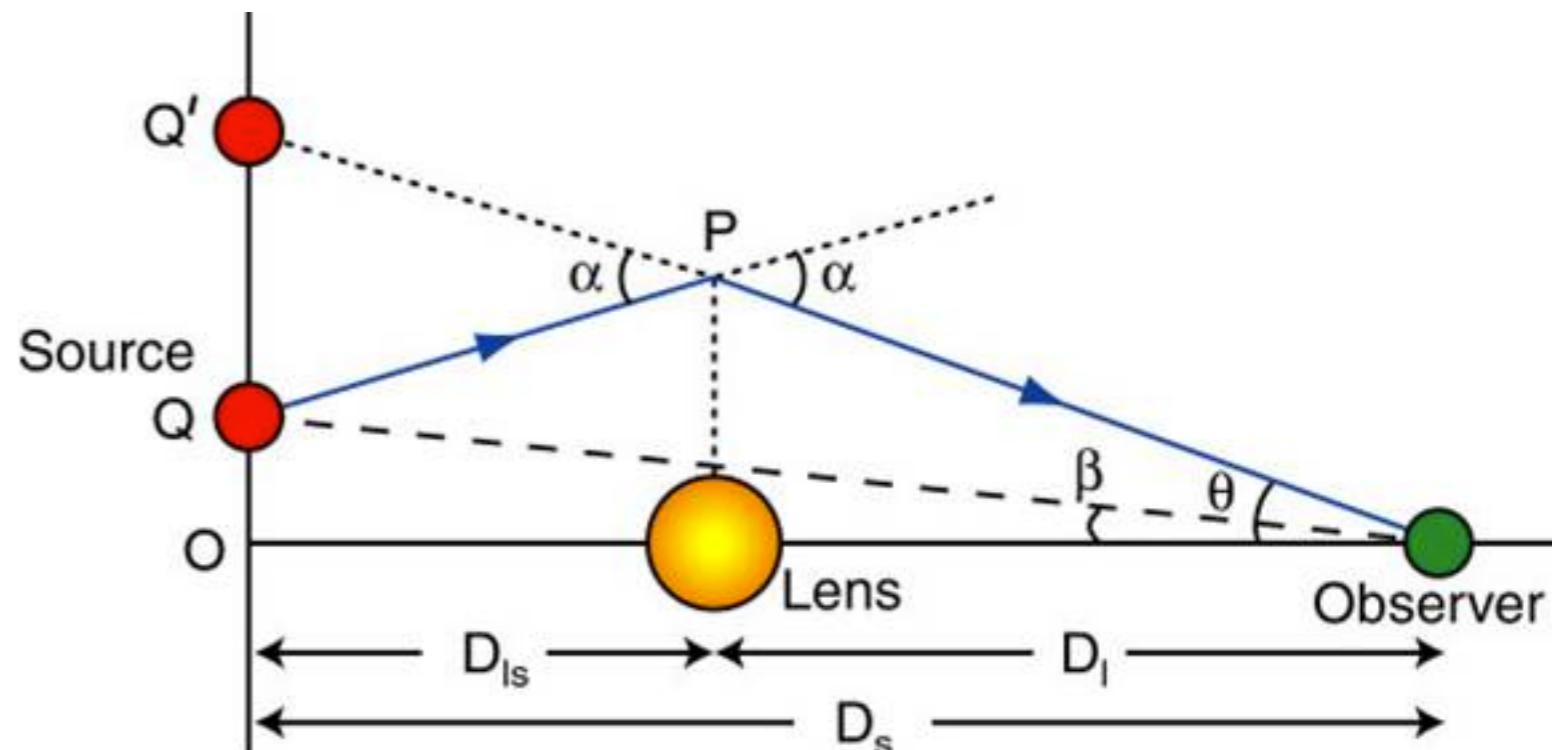
φ : lensing potential

⇒ Link with catastrophe theory

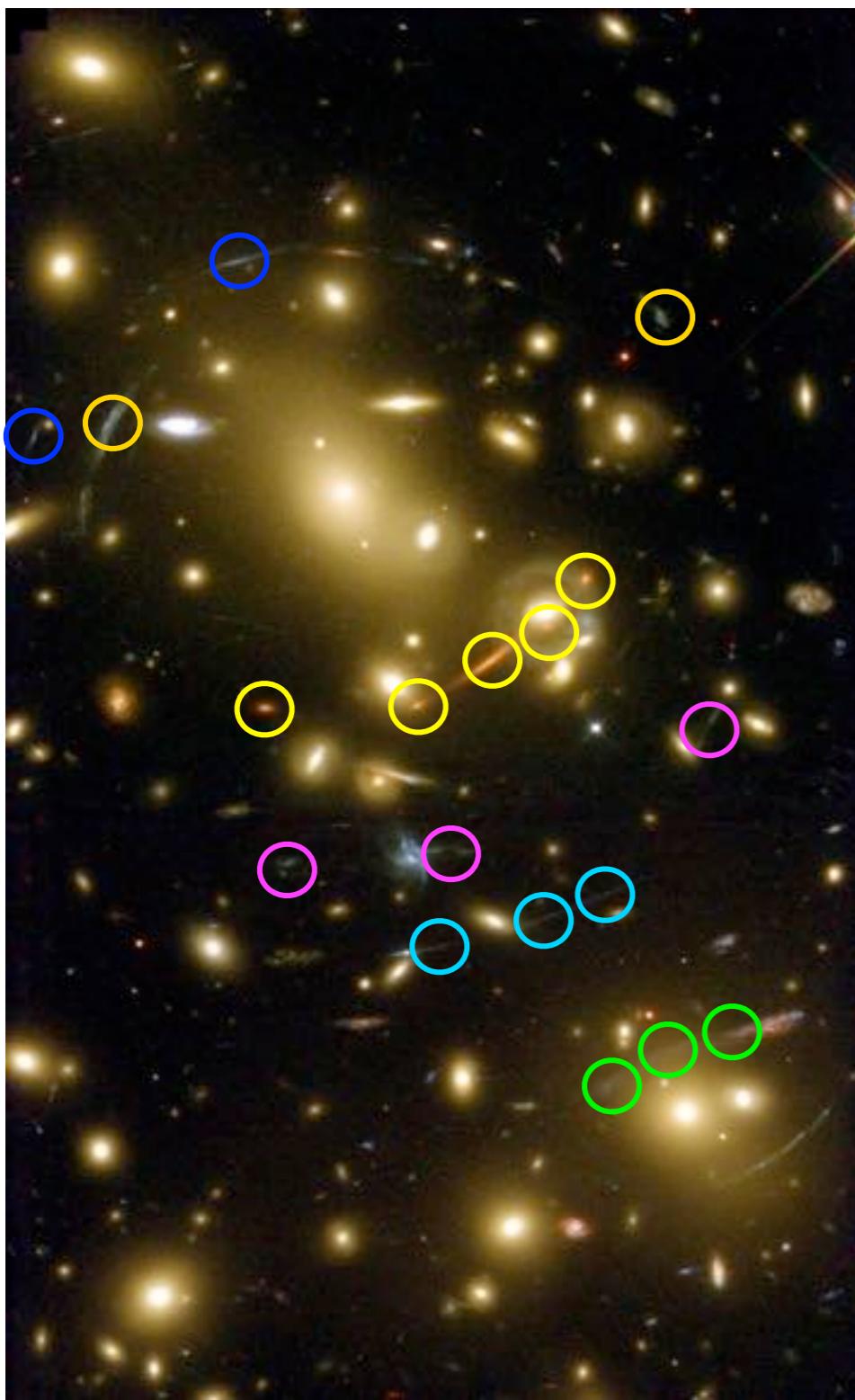
⇒ Parameters: Distances and Mass

⇒ Purely geometrical: *Achromatic*

Lensing Potential



Strong Lensing Cluster Modeling and Errors



Constraints:

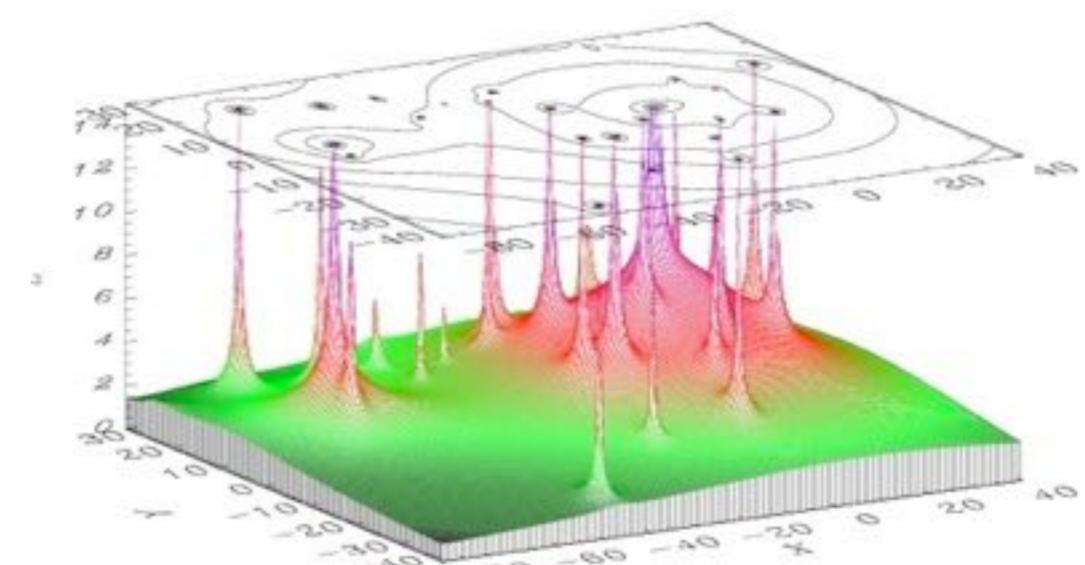
- Multiple images (position, redshift, flux, shape)
- Single images with known redshift
- Light/X-ray gas distribution

Model parameterisation

- Need to include **small scales**: galaxy halos (parametric form scaled with light)
- Large scale: DM/X-ray gas (parametric form or multi-scale grid)

Model optimisation

- e.g. Bayesian approach (robust errors)
- **Not a unique solution**: “most likely model and errors”
- *Predict amplification value and errors => cluster as telescopes*



Jullo et al 2007, Jullo & Kneib 2009

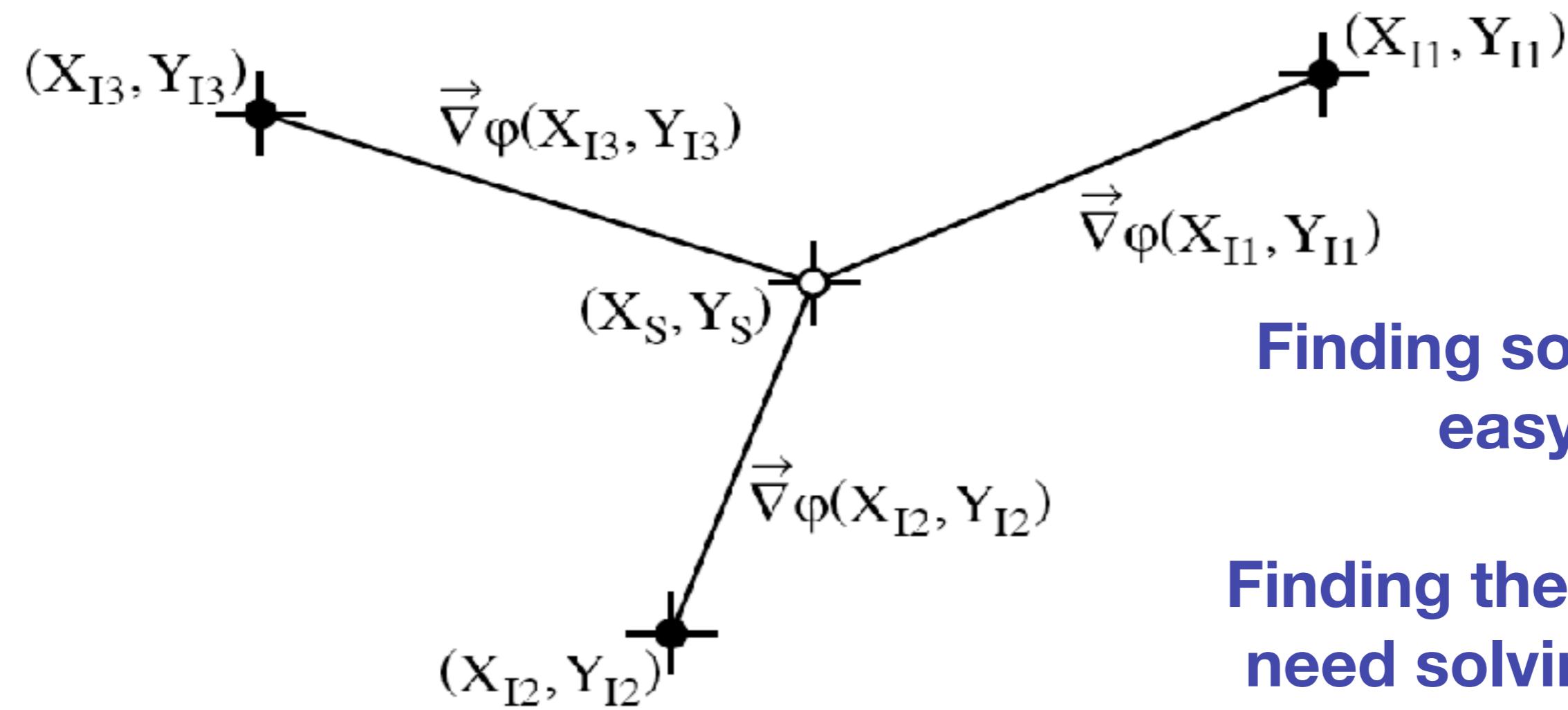
LENSTOOL public software

<https://projets.iam.fr/projects/lenstool/wiki>

Strong Lensing

Lensing equation have multiple solution (Strong lensing):

$$\theta_s = \theta_I - \nabla \varphi(\theta_I)$$



**Finding source is
easy!**

**Finding the images
need solving a 2D
equation (ray tracing)**

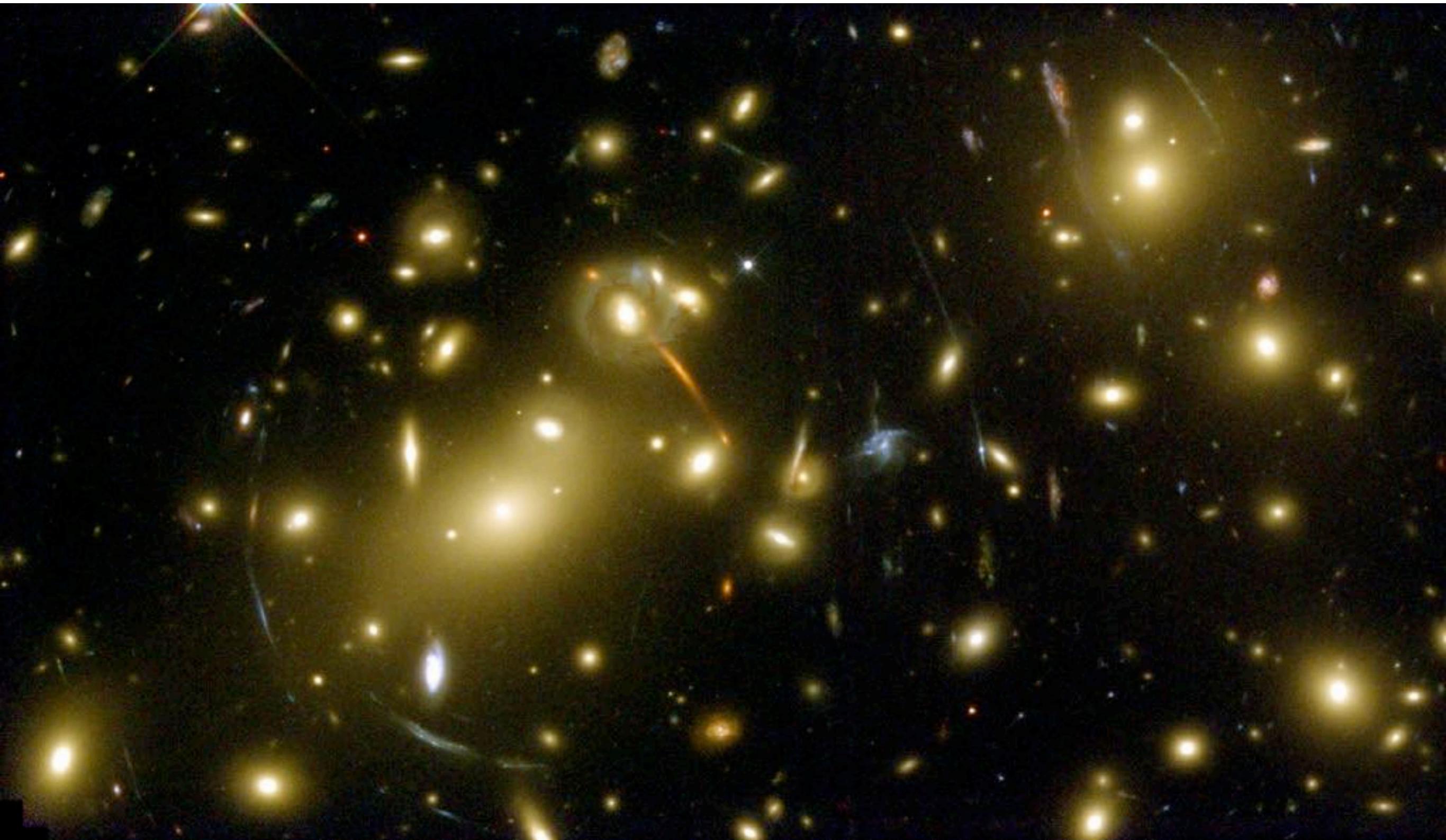
Modeling

- Finding Multiple images

Finding Multiple images

- Need GOOD (high-resolution) data
 - *Really this means HST or JWST quality data!*
- Morphology (should agree with rules of **image parity**)
- **Color** (could do from ground observation but hard)
- Spectroscopic confirmation (important for lensing strength)
- Modeling confirmation/finding
- Still missing an automatic software for multiple image identification! The human eye is still the best!

Best strong lensing data: Hubble (color) images

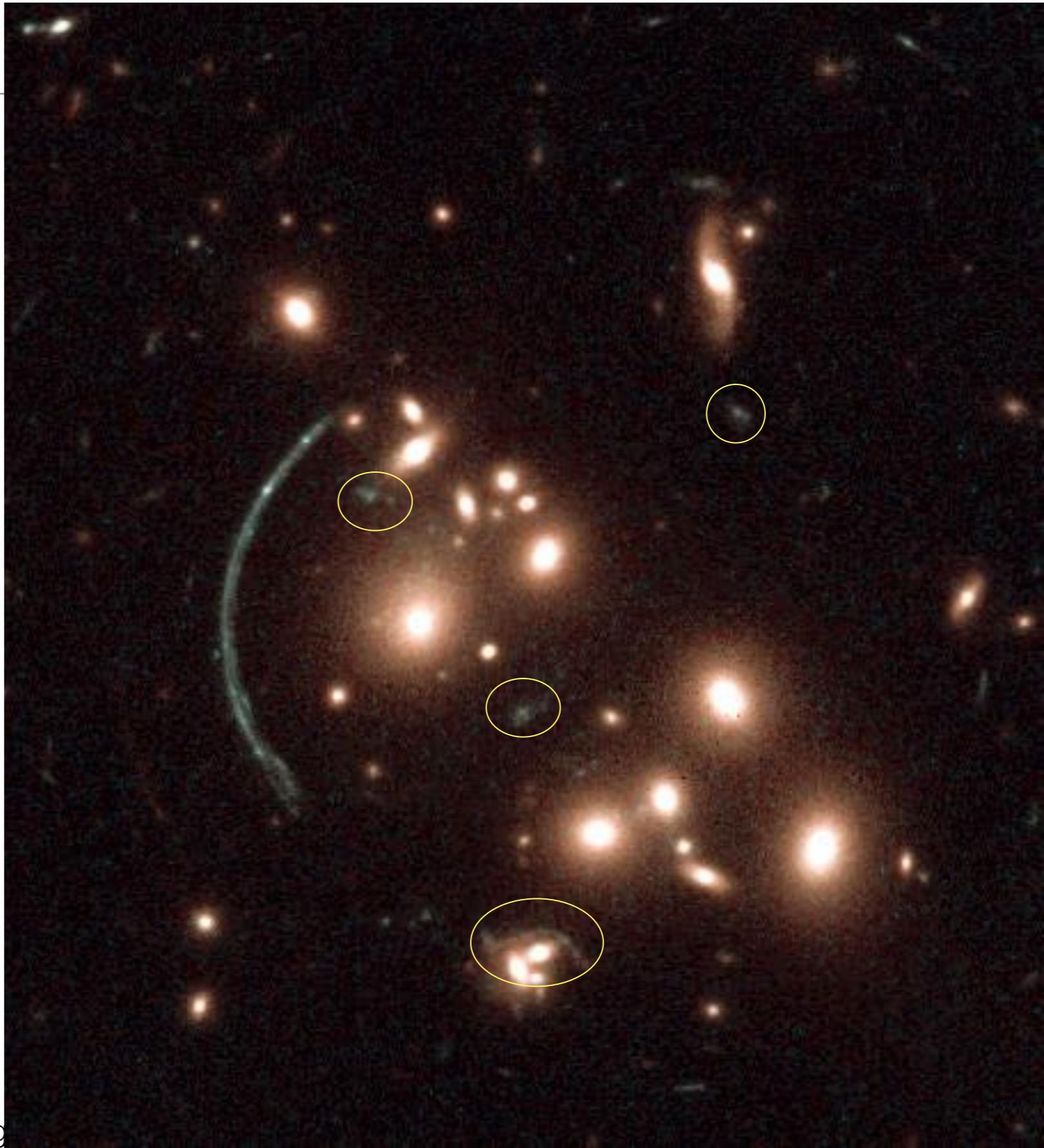


Abell 2218 at $z=0.175$

How to identify multiple images ?

Extreme distortion:
Giant arcs are the
merging of 2 or 3 (or
possibly more)
multiple images

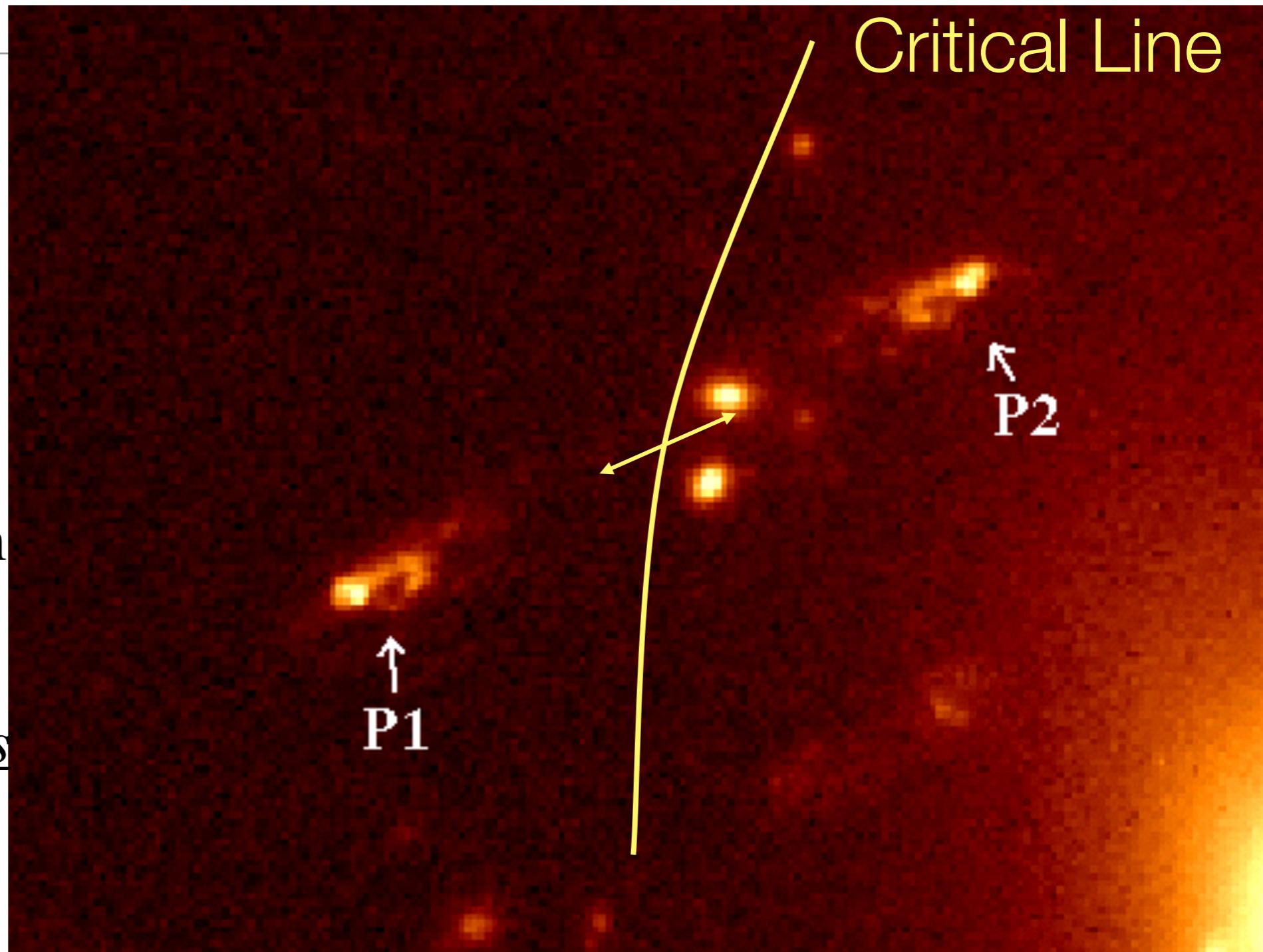
**Giant arc in
Cl2244-04,
 $z=2.24$,
Septuple image**



How to identify multiple images ?

Morphology: *Change of parity across a critical line.*

Note: lensing amplification is a gain in the angular size of the sources. Allow to *resolve distant sources* and study their size and morphologies.



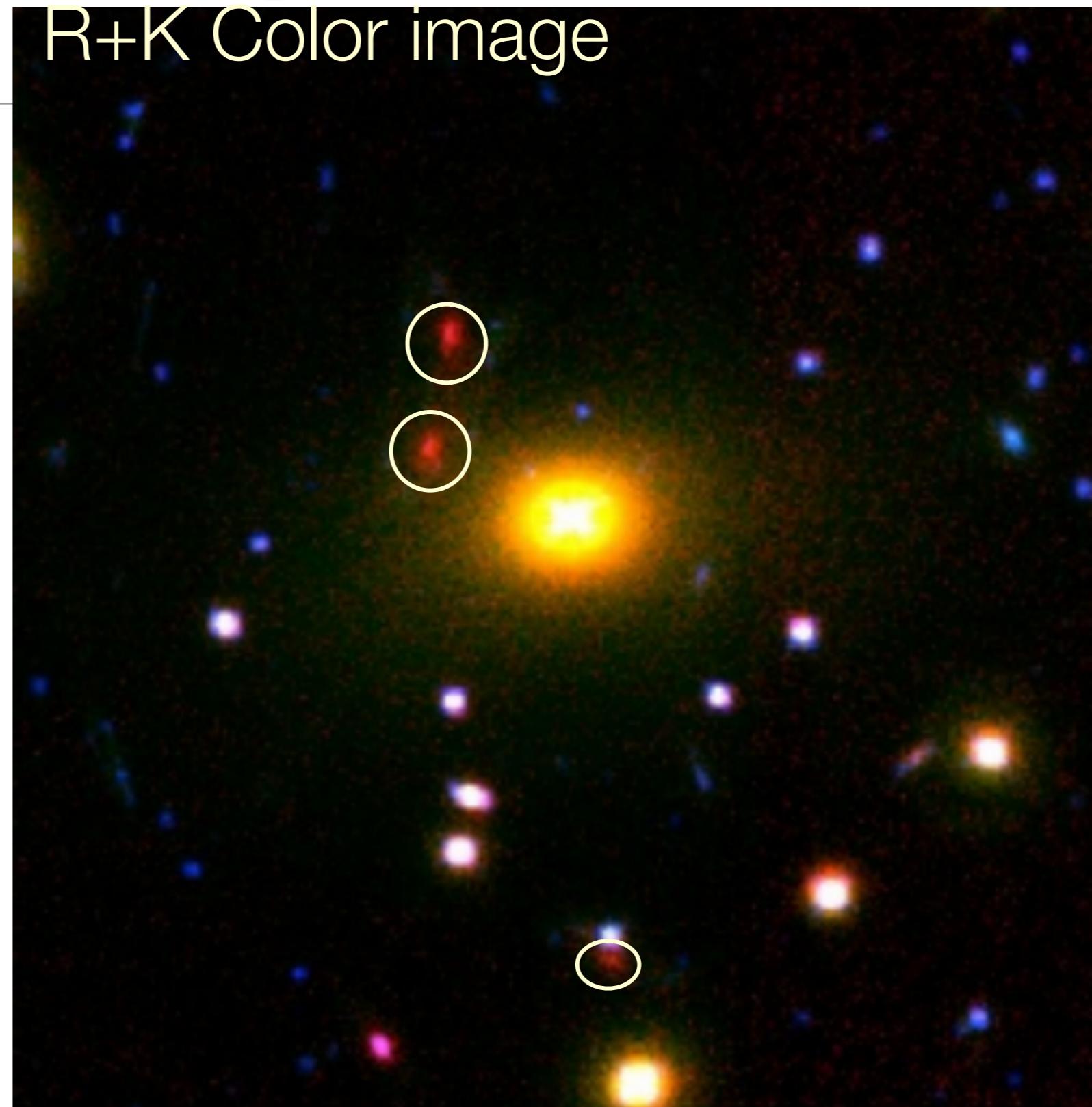
Lensed pair in AC114, $z=1.86$

How to identify multiple images ?

Extreme similar colors:

Example of a triple ERO system at $z \sim 1.6$ (Smith et al 2002) lensed by Abell 68

Interest of magnification is to allow to resolved the morphology of these systems (see **Johan Richard presentation**)



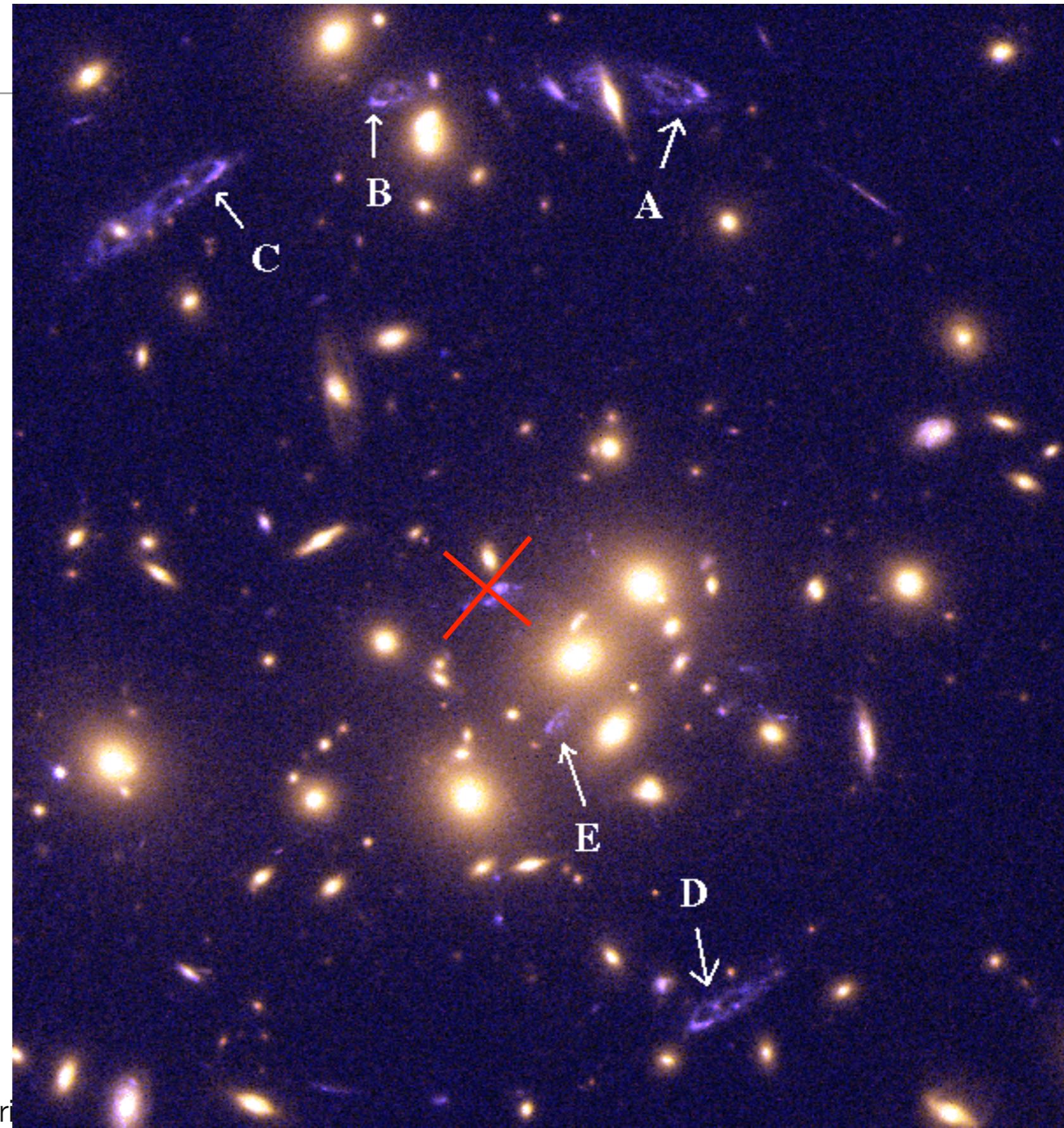
Abell 68: ERO triple image at $z \sim 1.6$

How to identify multiple images ?

Color and
Morphology:

Lens model can help
for the identification
when different
solution are possible

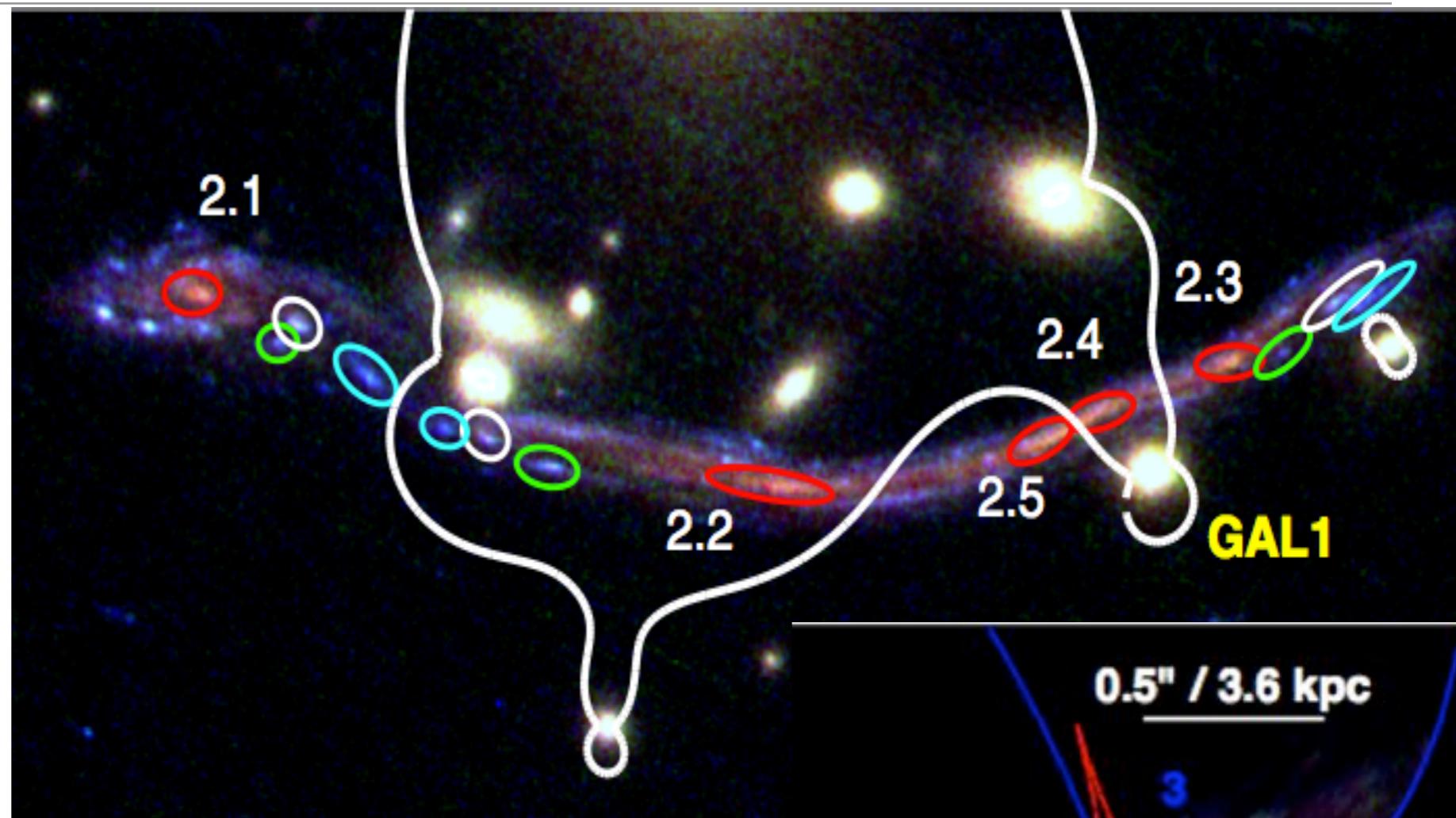
Quintuple arc ($z=1.67$)
in
Cl0024+1654 ($z=0.39$)



How to identify multiple images ?

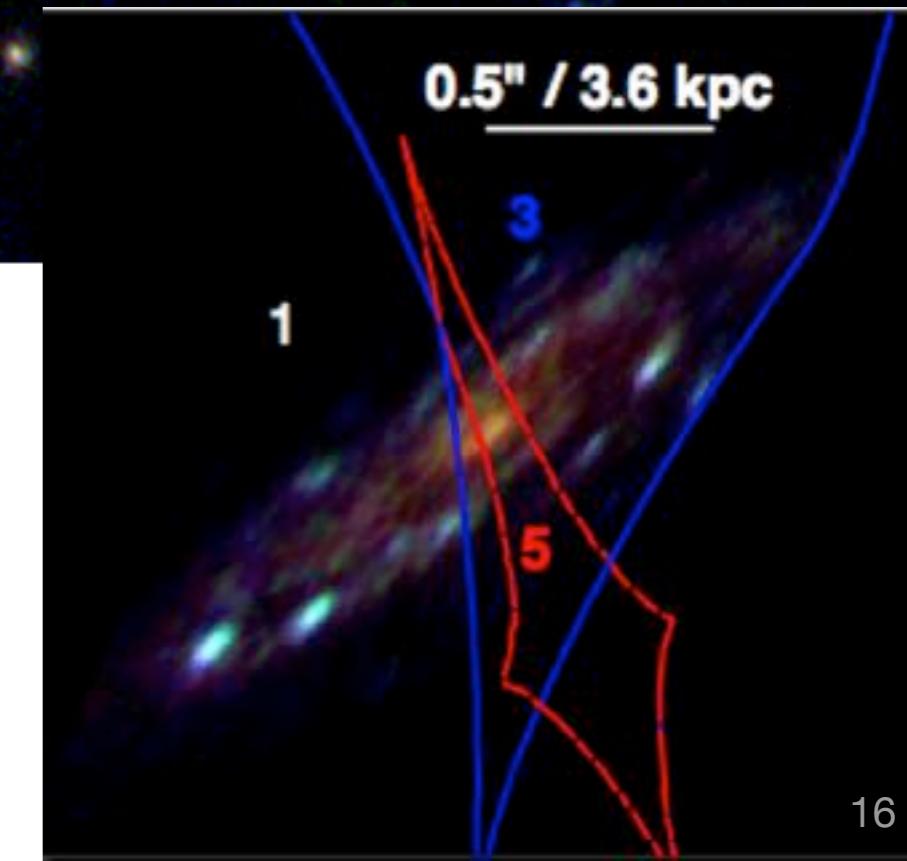
Giant arc in Abell 370 by Richard et al., 2010

HST multi-color images help understand giant arc morphologies...



... and allow unlensed source reconstruction (Richard et al 2010)

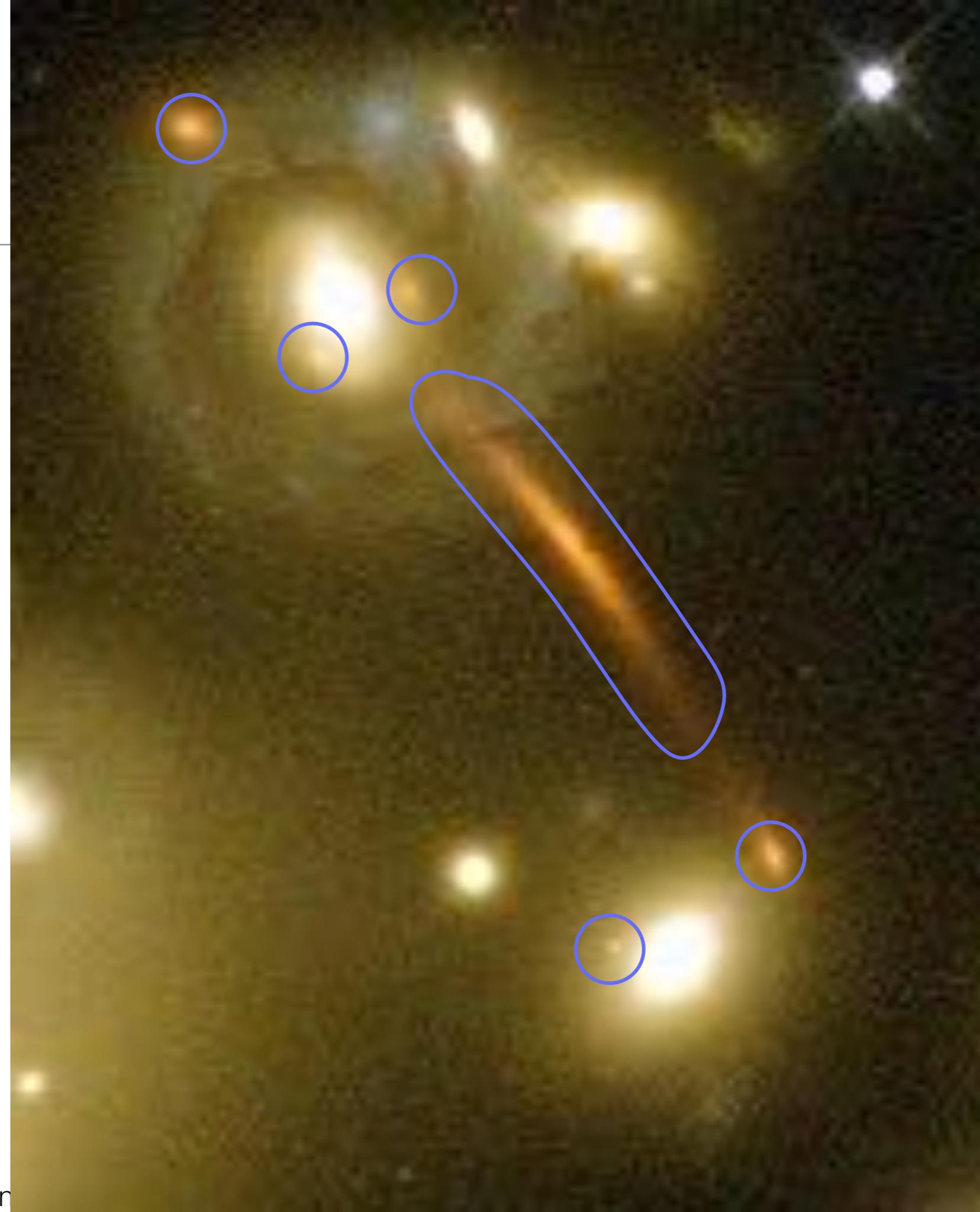
Here the source is at redshift $z=0.725$



Strong Galaxy-Galaxy Lensing in Cluster

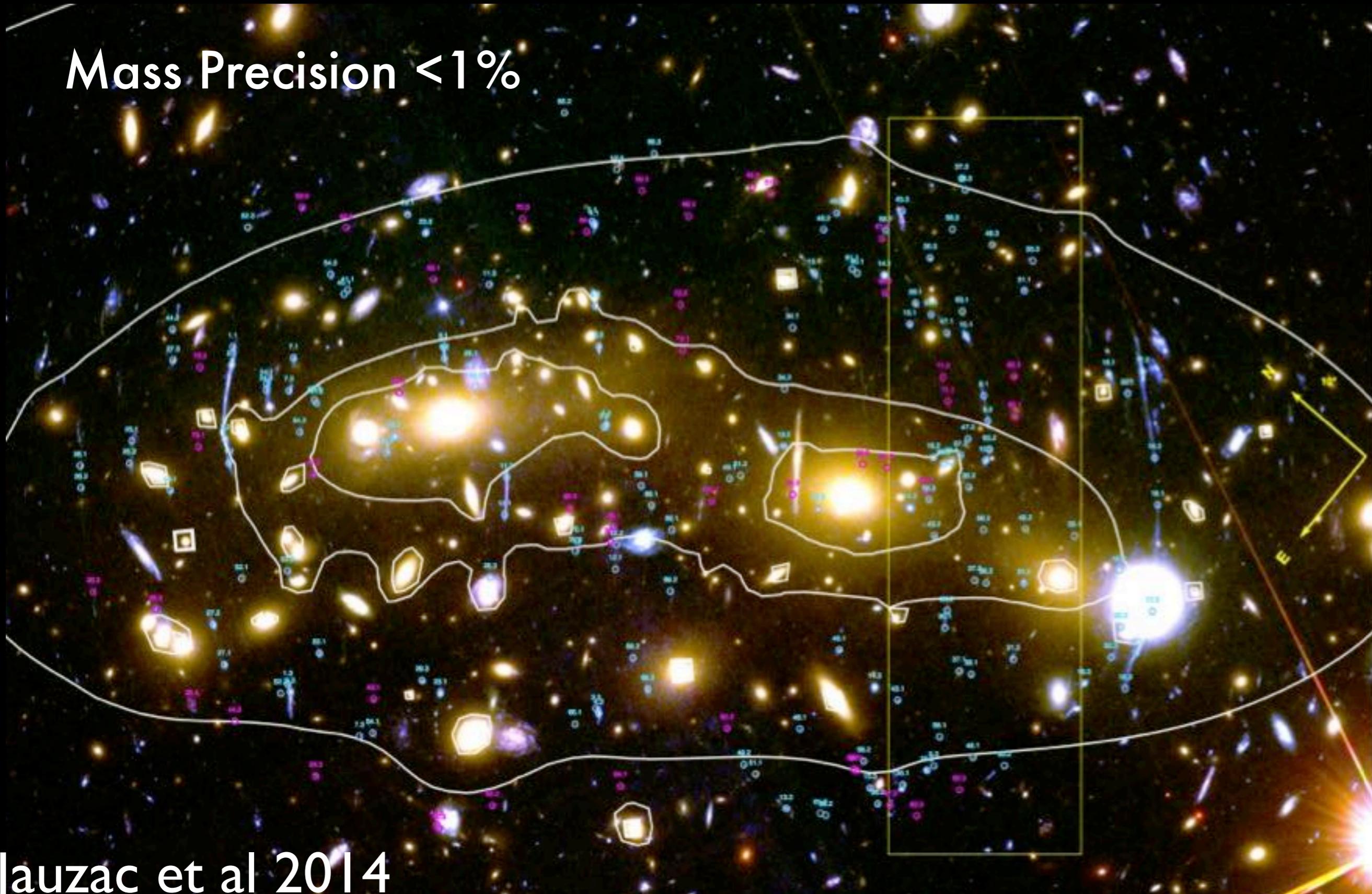
Cluster Galaxies are breaking arcs into smaller ones, adding new images of the lensed galaxy.

Abell 2218, arc at $z=0.702$, with 8 images identified (the arc is the merging of 2 images)



>200 Multiple Images in MACSJ0416

Mass Precision <1%



Modeling

- Which Mass Model?
- What Mass Model Parametrisation?

Singular Isothermal Sphere - 1

- To first approximation stars or other mass components are like particles in a gas.

- Let's consider an ideal gas:

$$p = \frac{\rho k T}{m}$$

- The temperature T is related to the 1D velocity dispersion of the particles with:

$$m \sigma_V^2 = k T$$

- Thus: $p = \rho \sigma_V^2$

Singular Isothermal Sphere - 2

- Consider a spherical shell, its mass is:

$$dM = 4\pi r^2 \rho dr$$

- The gravitational force between the inside and the shell can be expressed as:

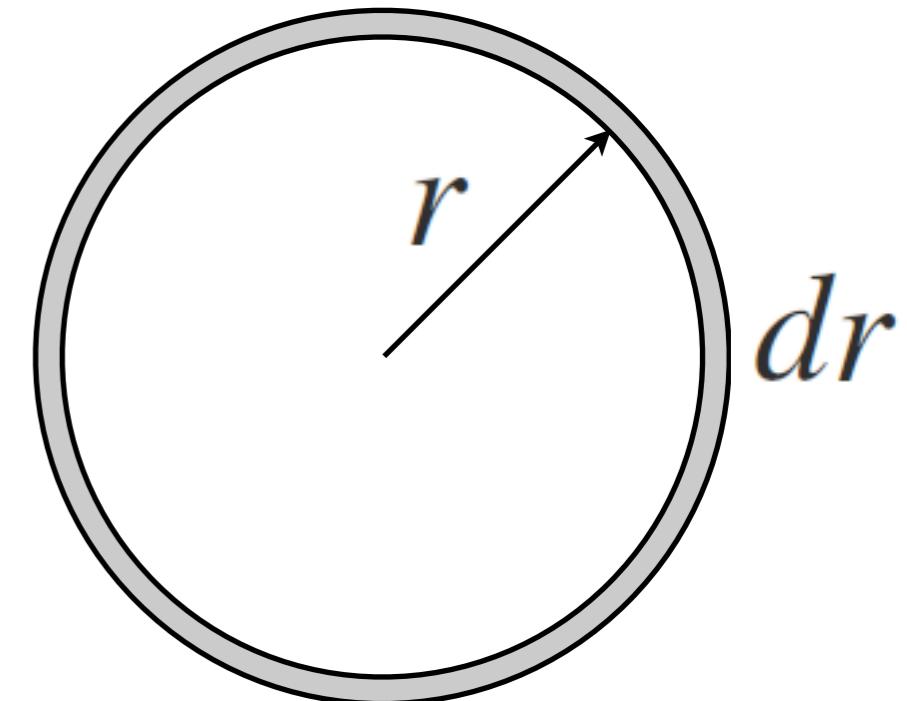
$$dF = -\frac{GM(r)dM}{r^2}$$

- The pressure variation is thus:

$$dp = \frac{dF}{4\pi r^2} = -\frac{GM(r)}{r^2} \rho dr \quad \text{and} \quad p = \rho \sigma_v^2$$

- One solution of this equation is:

$$\rho = \frac{\rho_0}{r^2} = \frac{\sigma_v^2}{2\pi G} \frac{1}{r^2}$$



Singular Isothermal Sphere - 3

- Total mass is thus:

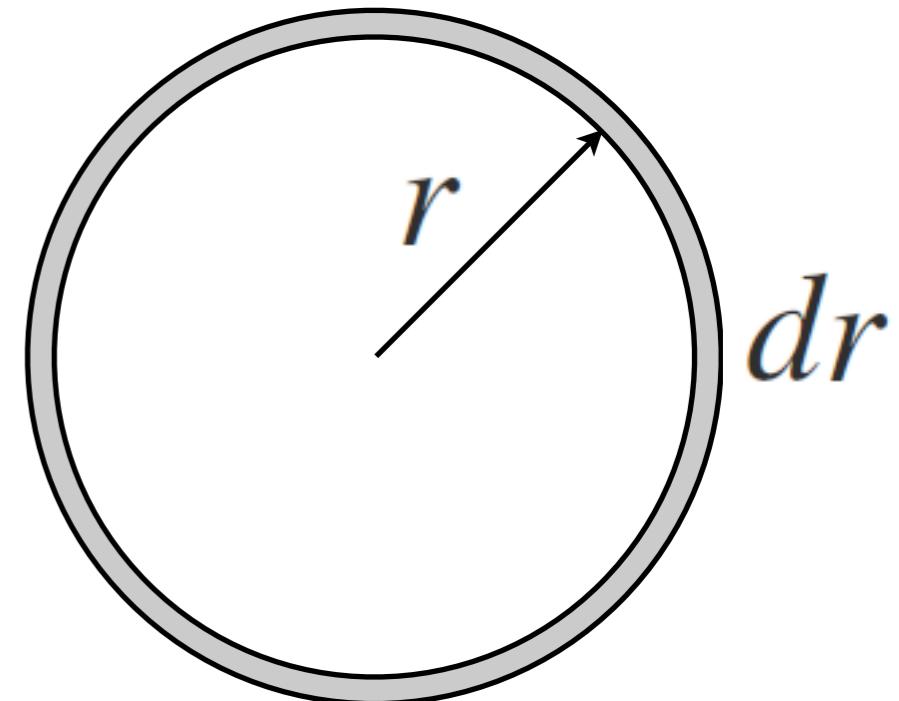
$$M(r) = 4\pi\rho_0 r = \frac{2\sigma_v^2}{G} r$$

- Projected surface density:

$$\Sigma(R) = \frac{\sigma_v^2}{2G} \frac{1}{R}$$

- Issues:

- Mass diverges at large radius
- Mass density diverge at small radius



Truncated Isothermal Sphere with a core

- Adding a core (softening the central density spike):

$$\rho(r) = \frac{\sigma_v^2}{2\pi G} \frac{1}{r_c^2 + r^2}$$

- Truncating the profile at large radius:

$$\rho(r) = \frac{\sigma_v^2}{2\pi G} \left(\frac{1}{r_c^2 + r^2} - \frac{1}{r_{cut}^2 + r^2} \right)$$

- Pseudo isothermal profile:

- Mass converge at large radius $\rho \sim r^{-4}$ $M_{tot} \sim \sigma_v^2 r_{cut}$

- Mass density diverge at small radius $\rho \sim cste$

Navarro-Frenk-White (NFW) mass profile

- The NFW profile correspond to the 3D mass distribution of dark matter fitted to dark matter haloes identified in N-body simulations by Julio Navarro, Carlos Frenk and Simon White (1996, 1997).

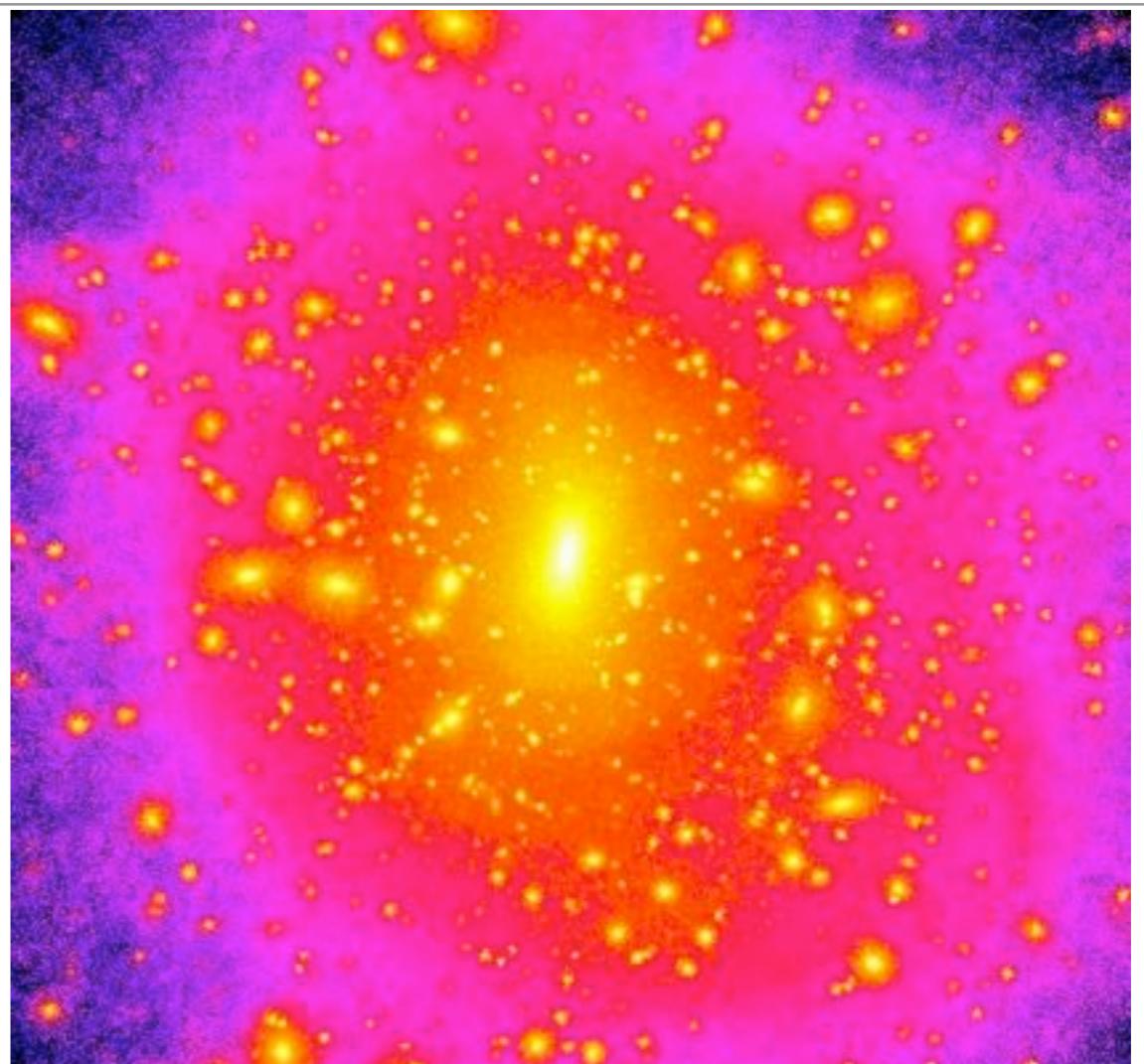
$$\rho = \frac{\rho_0}{r/r_s (1+r/r_s)^2}$$

- The NFW profile is one of the most commonly used model profiles for dark matter halos.
- **However:**
 - Mass diverges at large radius
 - Mass density diverge at small radius

More recent CDM-only Simulations

(e.g. Navarro et al. 2004; Diemand et al. 2004; Tasitsiomi et al. 2004 + others)

- Convergence achieved down to $\sim 0.003r_{\text{vir}}$...roughly the size of massive galaxies. Baryons are important for progress!!
- Density profiles obtained using different codes and initial conditions agree.
- The generalised NFW density profile is a good fit to simulations with β between 1.0 & 1.5. There is significant scatter.
- Sersic profiles seems to be an alternative to generalised NFW expression (Merrit et al 2005)



Generalized NFW

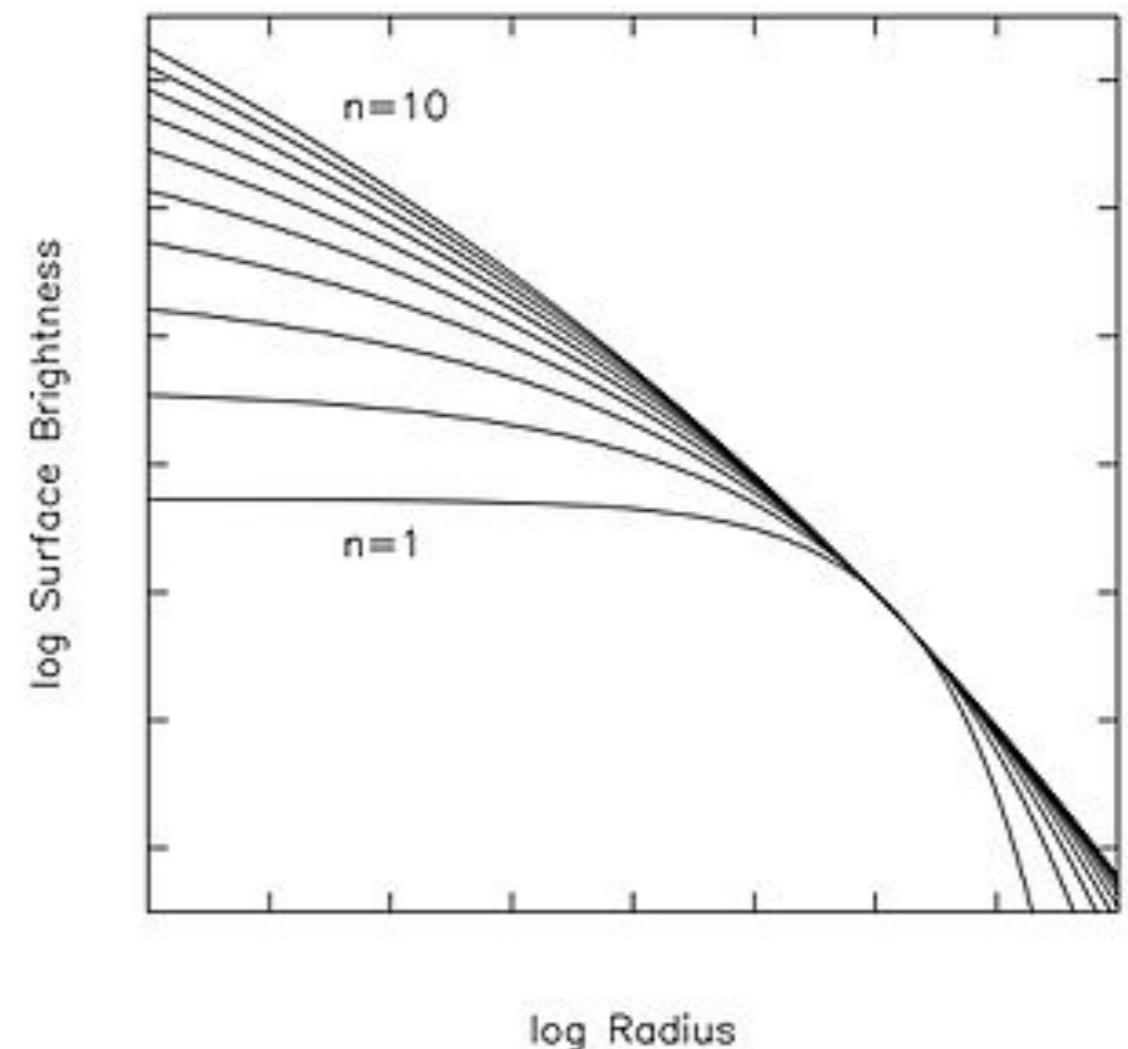
$$\rho = \frac{\rho_0}{(r/r_s)^\beta (1+r/r_s)^{3-\beta}}$$

Sersic mass profile

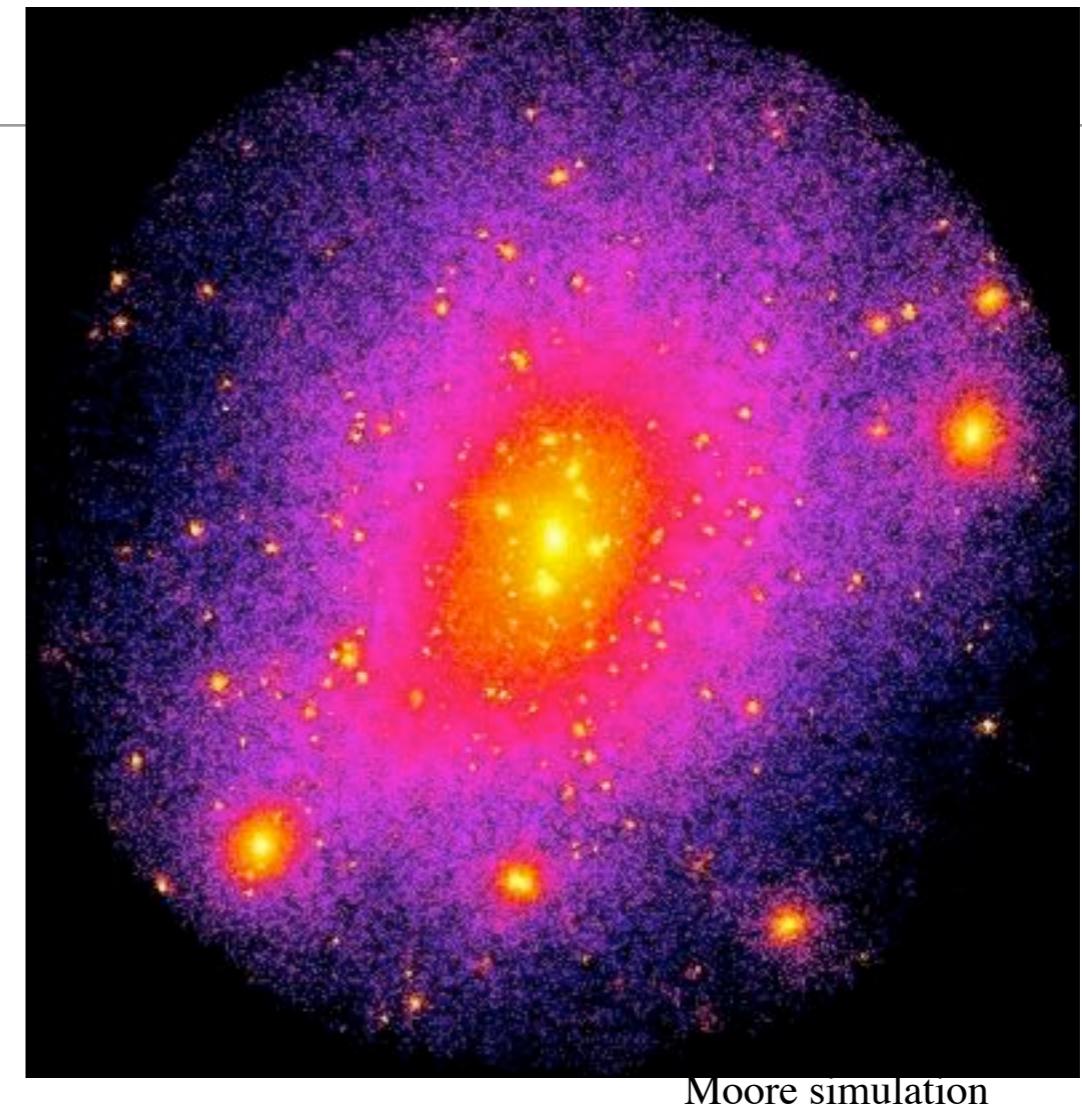
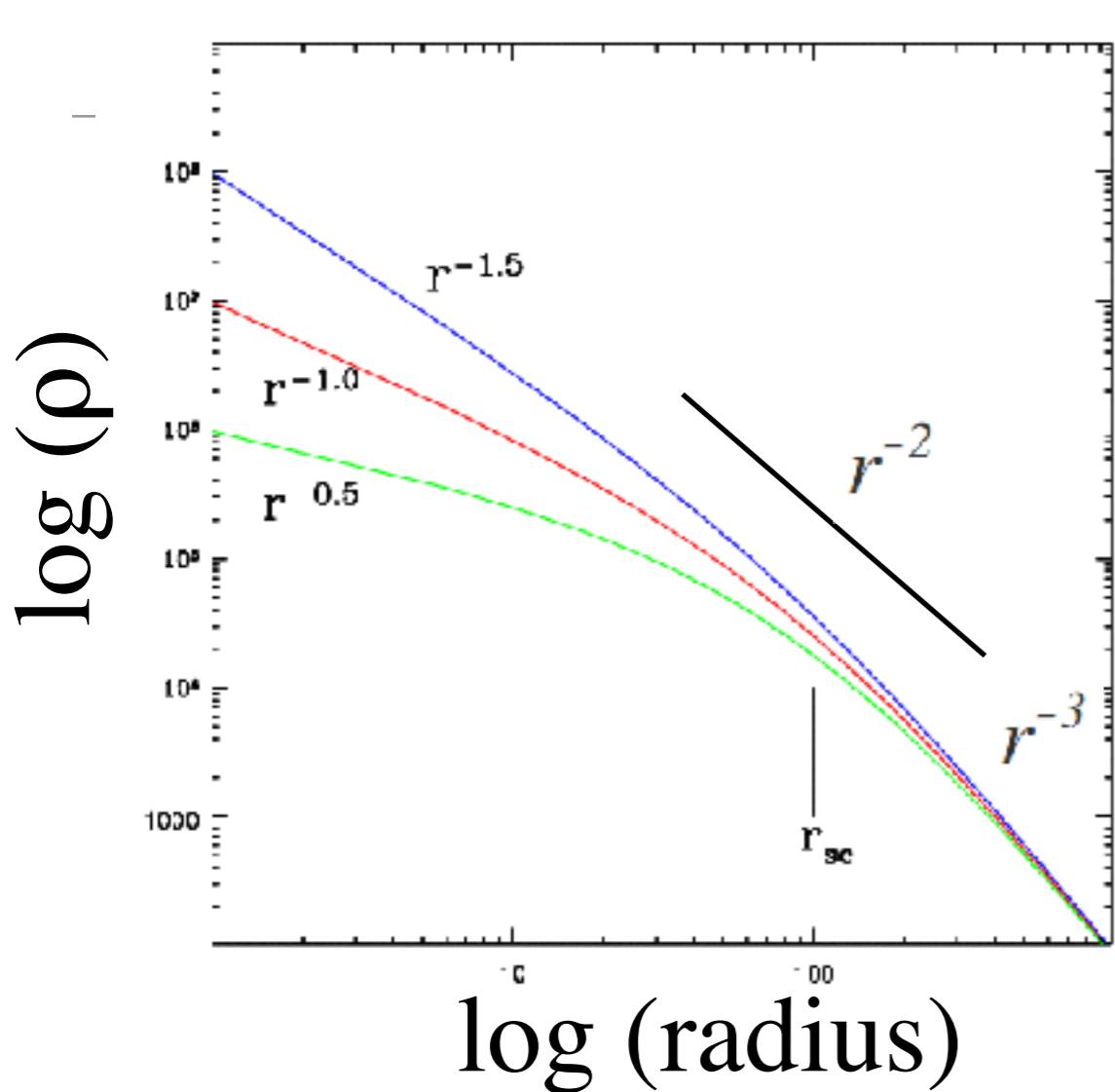
- General profile used to fit galaxy surface brightness profile:

$$I(R) = I_e \exp \left\{ -b_n \left[\left(\frac{R}{R_e} \right)^{1/n} - 1 \right] \right\}$$

- n : Service Index
- $n=1$ Exponential profile (spiral like galaxy)
- $n=4$ de Vaucouleurs profile (elliptical like galaxy)
- Also used to describe Dark Matter halo profile



Generalised NFW profile



Inner profile: $\rho \propto r^{-\beta}$ NFW: $\beta \rightarrow 1.0$ Moore: $\beta \rightarrow 1.5$

What is the inner slope of cluster DM profiles?

What is the TOTAL density profile?

Mass model representation of complex system

- large scale cluster component+sum of galaxy halo components ([DM+gaz]+galaxy halos):

$$\phi_{tot} = \phi_{cluster} - \sum_i \phi_{halos}^i$$

Kneib et al 1996

- need to scale the galaxy halo components; for example for a pseudo isothermal mass distribution:

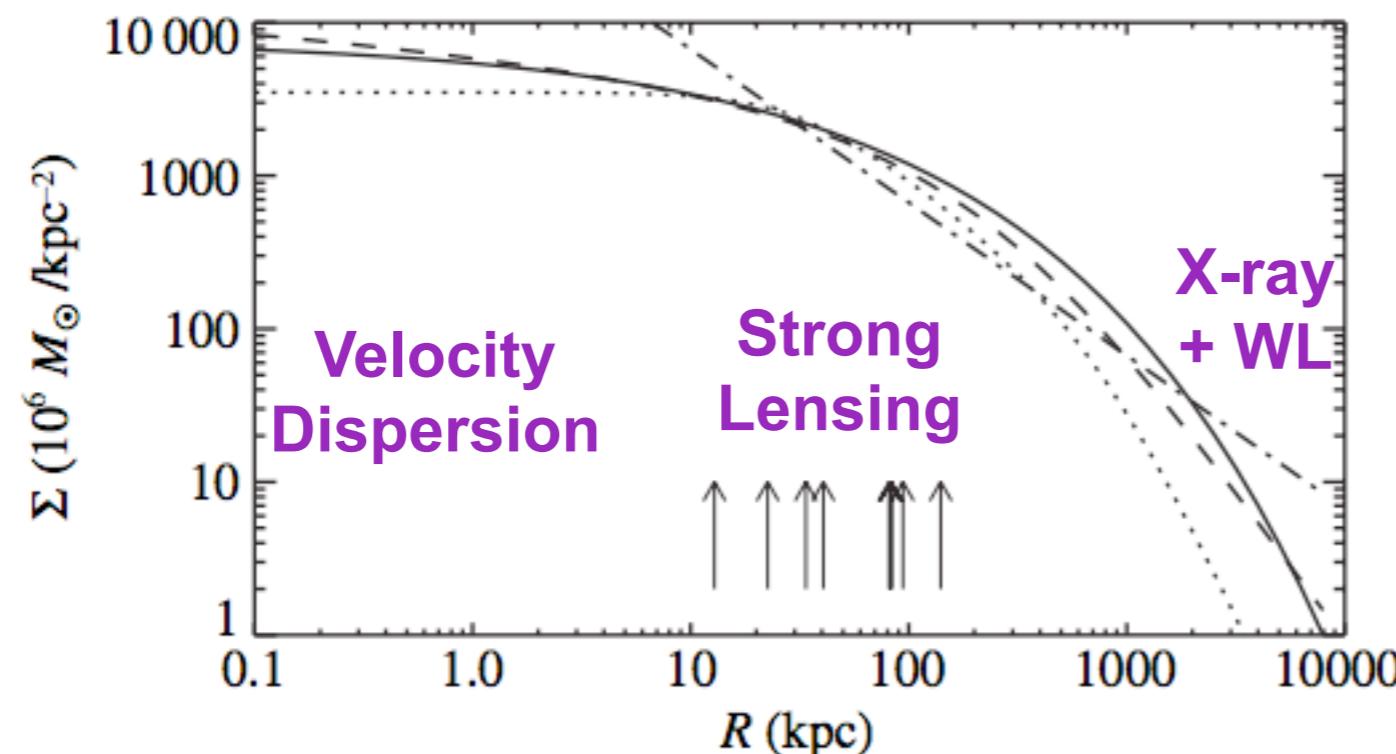
$$\sigma = \sigma_* \left(\frac{L}{L_*} \right)^{1/4} \quad r_{cut} = r_{cut}^* \left(\frac{L}{L_*} \right)^\eta$$

- Hence: $\frac{M}{L} \propto L^{\eta-1/2}$

$$\eta = 1/2 \text{ Constant M/L}$$
$$\eta = 0.8 \text{ Fundamental Plane Scaling}$$

Mass profile for a mass clump?

- Mass profile should match theoretical or numerical simulations in order to be close to reality (avoid Gaussian function for example):
 - isothermal model (singular => cored & truncated, circular => elliptical): PIEMD
 - NFW model => gNFW, Sersic, Einasto (beware at infinite values, truncate?)

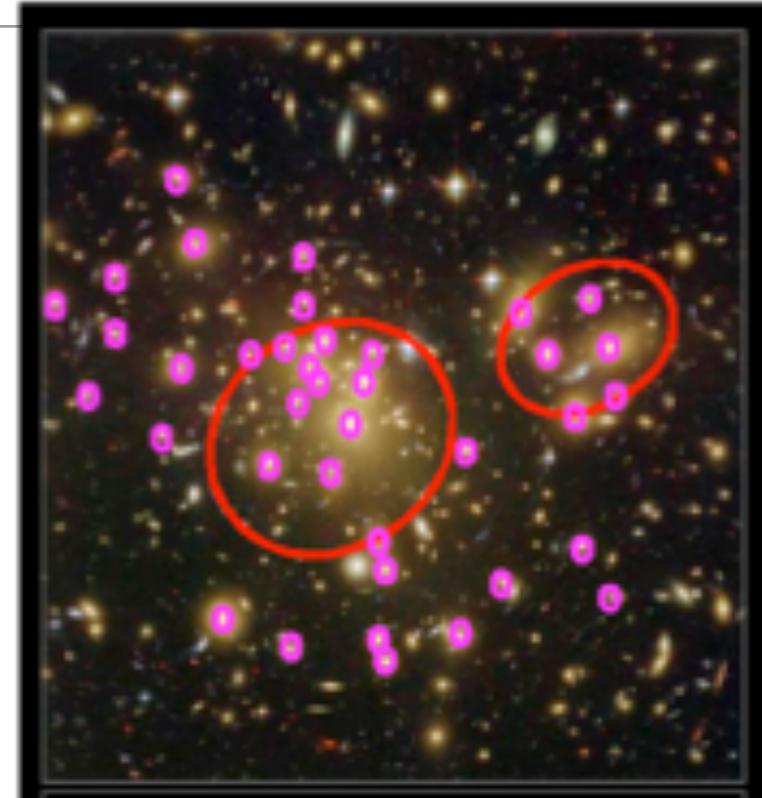


Additional data
useful to constraints
further the mass
profile

Strong lensing modeling strategies

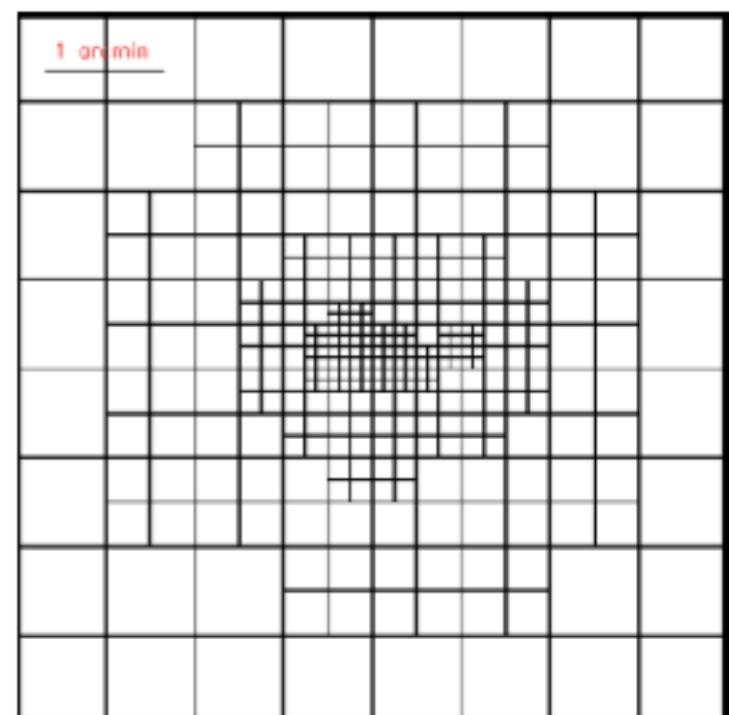
Observationally motivated models

- Decomposition into halos
- Simple clusters
- Few constraints
- Good fit with few constraints



Grid-based models

- Decomposition into pixels/clumps
- Complex clusters
- Lots of constraints
- Better fit with lots of constraints



Modeling

Likelihood estimator, optimisation

Lens Modeling with Multiple Images

- **One system with N images:**

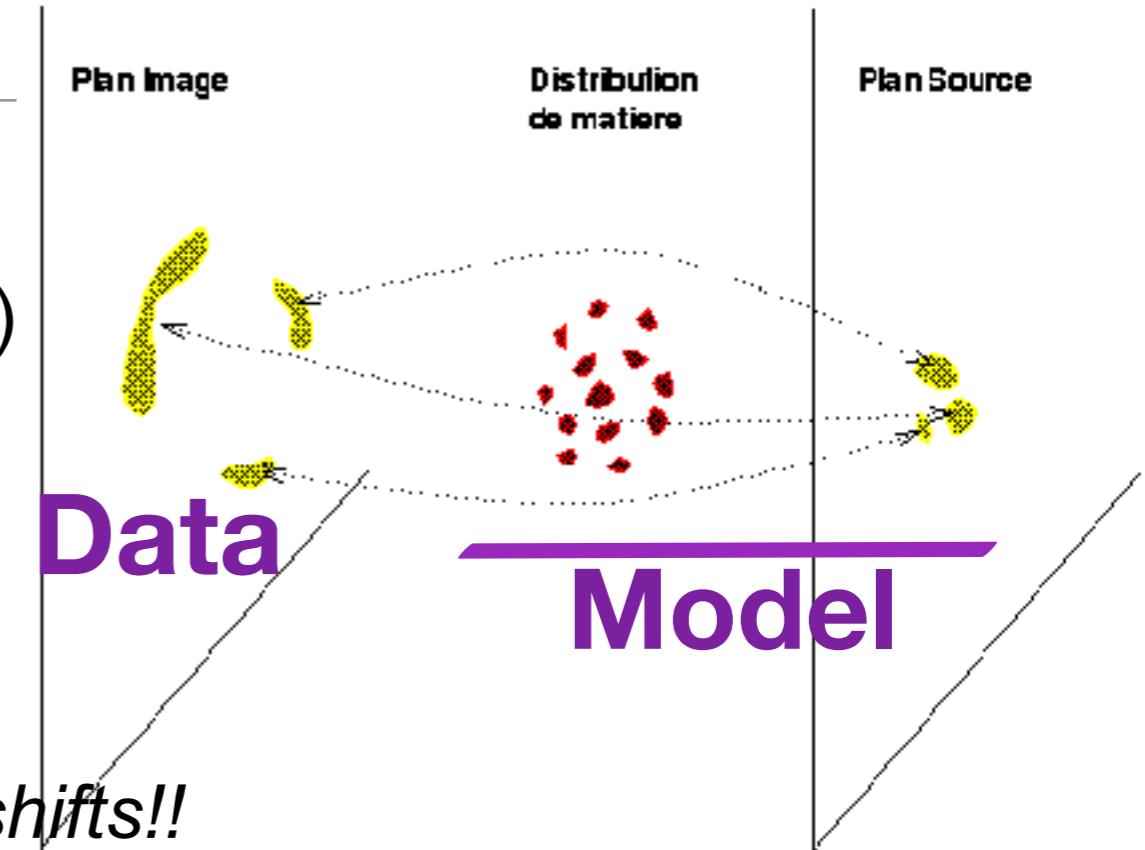
- # of constraints: $2N$, $3N$ (image position+flux)
- # of unknown: 2, 3 (source position+flux)
- # of free parameter: $2(N-1)$, $3(N-1)$
Double: 2, 3 Triple: 4, 6 Quad: 6, 9

- η systems of N images:

- # of free parameters: $2(N-1)\eta$, $3(N-1)\eta$
- *need to subtract the number of unknown redshifts!!*

A1689 with ACS ~30 triples: <120, <180

[=> deep JWST observations may reach ~1000]



⇒ parametric models have been favoured as there was generally a small number of constraints

⇒ Introduce other constraints:

critical line location and/or external constraints from X-ray observations or velocities (of stars in central galaxy)

Maximum Likelihood expressions

Likelihood of the *image positions* can be computed:

- in the **source plane** [*FAST no lens inversion needed, bad error estimate!*]
- or in the **image plane** [*better! real error estimate possible, SLOW*]

Source plane:

$$\chi_{S_i}^2 = \sum_{j=1}^{n_i} \frac{[x_S^j(\boldsymbol{\theta}) - \langle x_S^j(\boldsymbol{\theta}) \rangle]^2}{\mu_j^{-2} \sigma_{ij}^2}$$

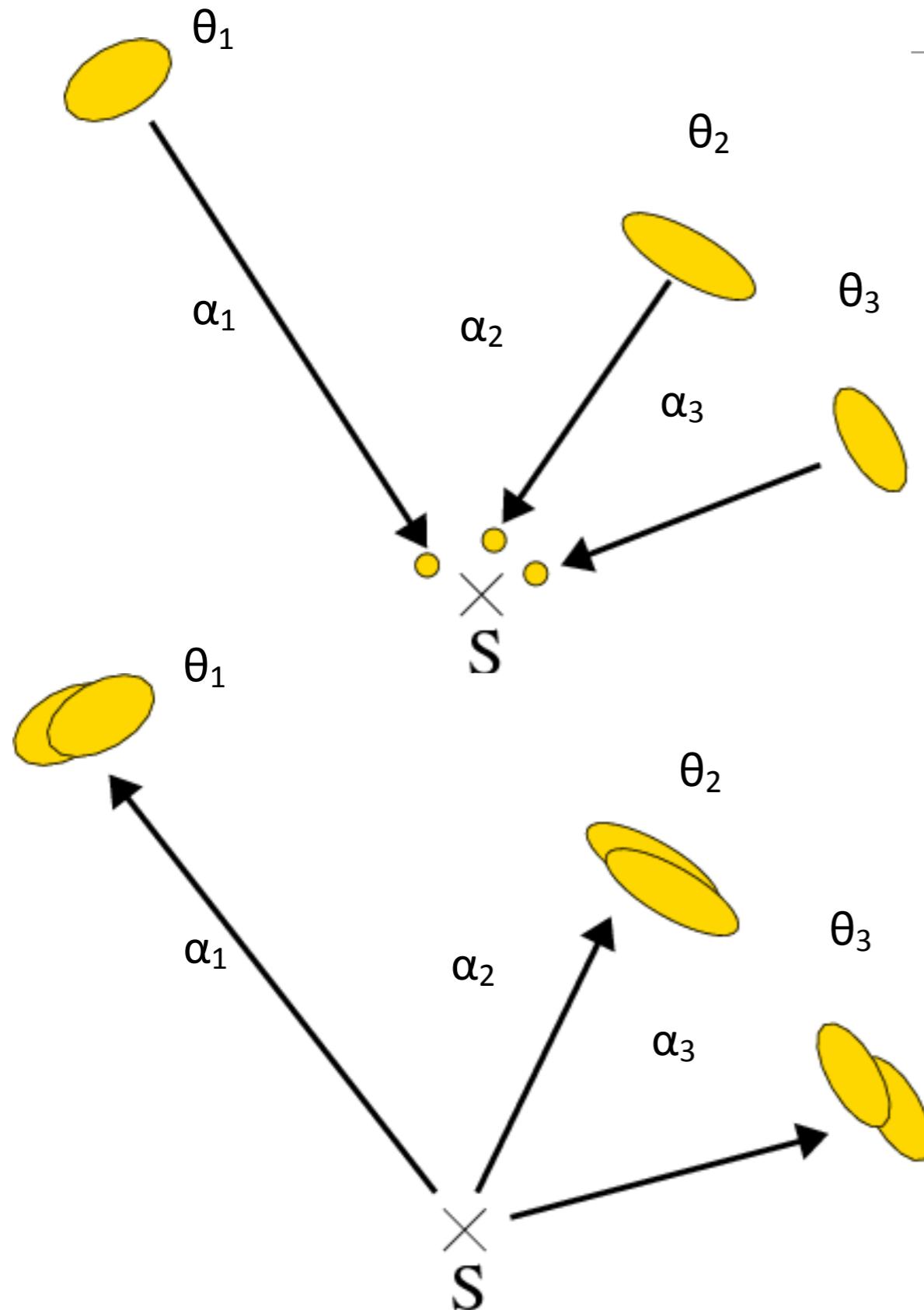
Image plane:

$$\chi_i^2 = \sum_{j=1}^{n_i} \frac{[x_{\text{obs}}^j - x^j(\boldsymbol{\theta})]^2}{\sigma_{ij}^2}$$

Jullo et al 2007

$\boldsymbol{\theta}$: source, and model parameters

Lens modeling strategy



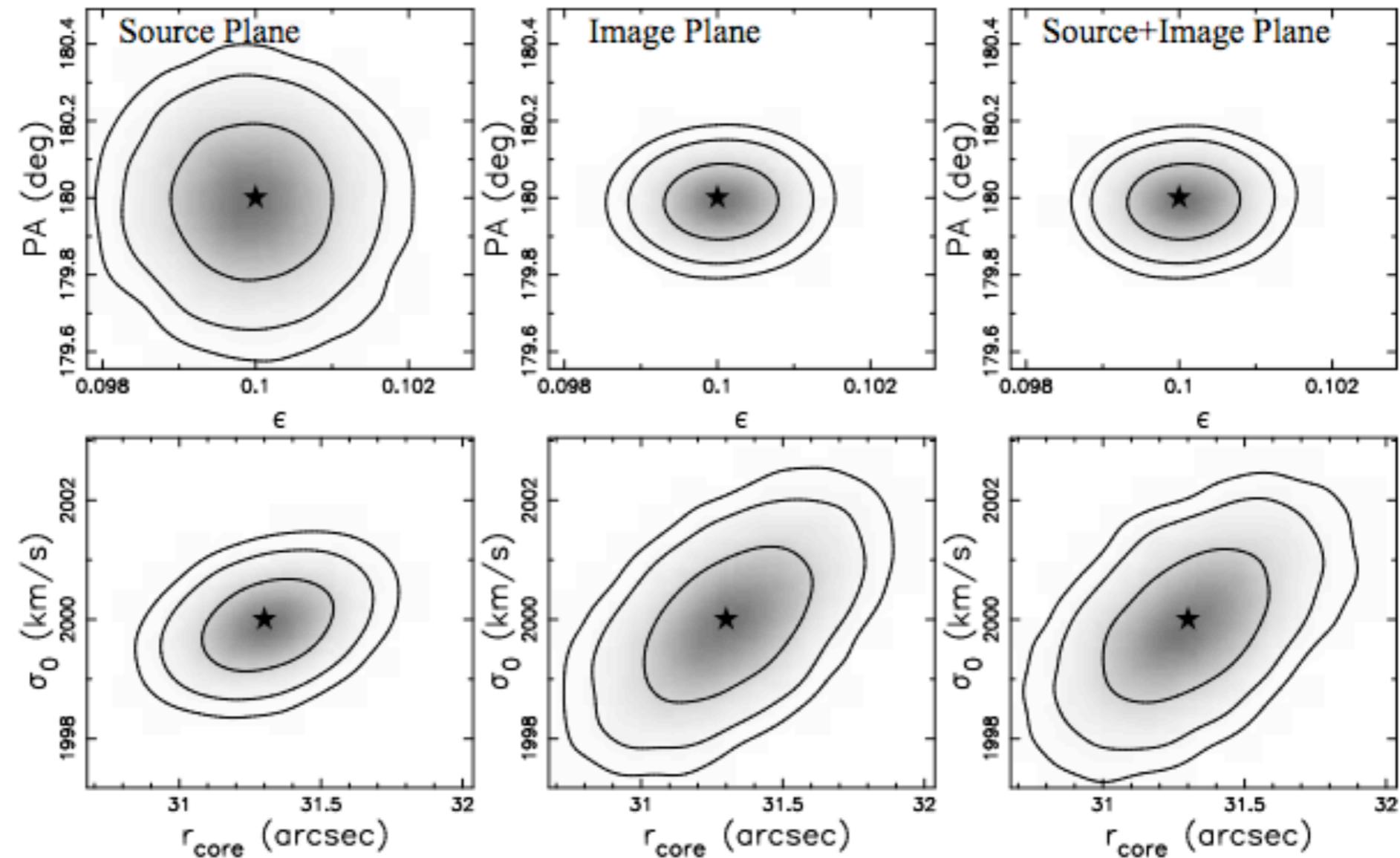
Identification of multiple image systems

Modeling strategy:
Source Plane
vs
Image Plane

The fit is good when the **predicted** and **observed** images overlap each others (within the positioning accuracy!)

Source vs. Image plane modeling

Jullo et al 2007



- Source plane only => bad error estimate, **possible bias in some parameters** (depending on the constraints)
- Source+image => ~10 times faster than image plane optimisation

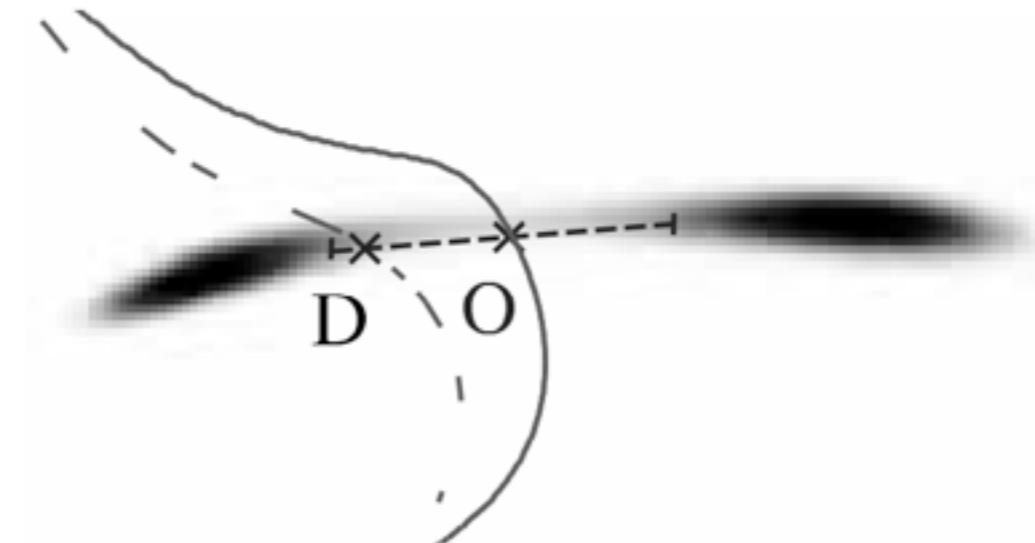
Other (lensing) constraints

- Constrain from critical line position

$$\chi^2_{cl} = \frac{\|\mathbf{O} - \mathbf{D}\|^2}{\sigma_{cl}^2}$$

- Constrain from flux ratios

$$\chi^2_{flux} = \sum_{i=1}^N \frac{(f_i - f_i^{model})^2}{\sigma_i^2}$$



Although ideally, we would like to do the fitting at data pixel level !

This yet to be fully implemented (massive computation)

Jullo et al 2007

MCMC optimisation (LENSTOOL)

Jullo et al 2007

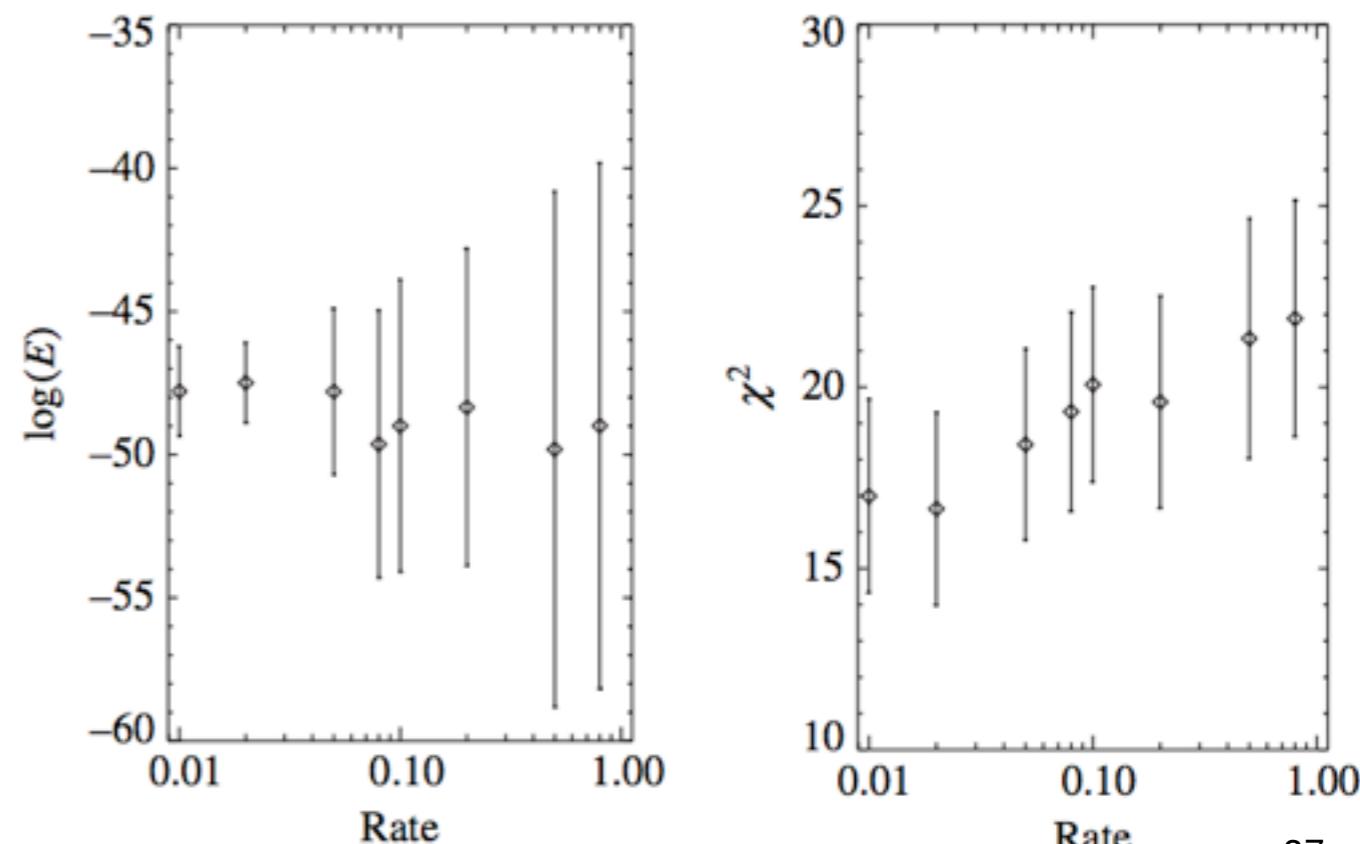
- Bayesian approach (problem with many parameters, need for proper error estimates)
- Bayes Theorem

$$\Pr(\boldsymbol{\theta}|D, M) = \frac{\text{Likelihood} \quad \text{Prior}}{\text{Posterior} \quad \text{evidence}}$$
$$\Pr(\boldsymbol{\theta}|D, M) = \frac{\Pr(D|\boldsymbol{\theta}, M)\Pr(\boldsymbol{\theta}|M)}{\Pr(D|M)}$$

- Use of selective annealing (rewrite of Bayes theorem) - control convergence speed

$$\Pr(\boldsymbol{\theta}|D, M) = \frac{\Pr(D|\boldsymbol{\theta}, M)^\lambda \Pr(\boldsymbol{\theta}|M)}{\Pr(D|M)}$$

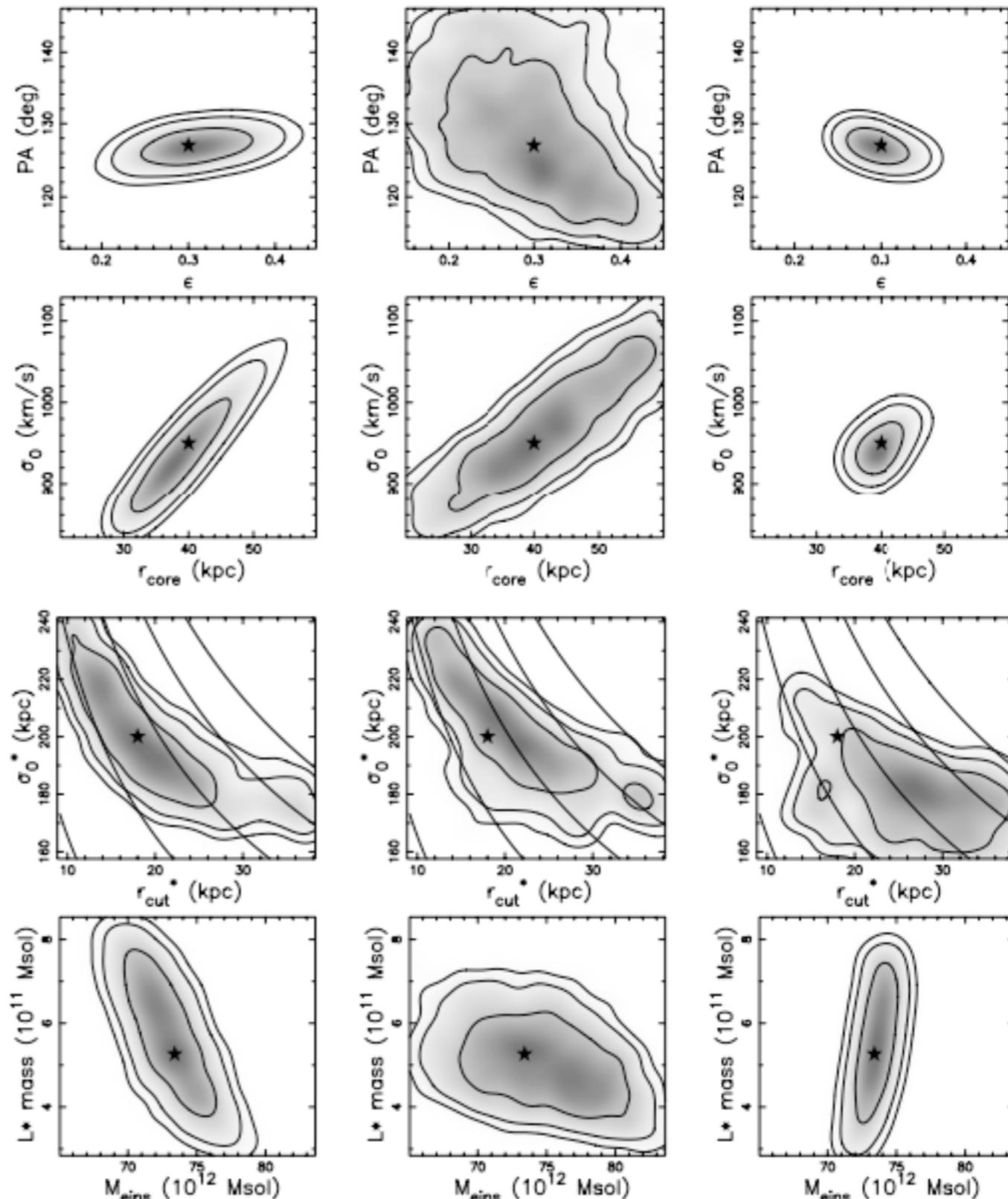
- Burning phase goes 0=>1 depending on the 'Rate' value
- 'Rate' does impact on the convergence
- Sampling phase to derive errors



MCMC optimisation (LENSTOOL)

Jullo et al 2007

- Example of degeneracies for 3 different simulated clusters
- Easy parameter estimation
- Can include cosmological parameters in the optimisation (Jullo et al 2010)



Modeling: Mass Distribution Measurement

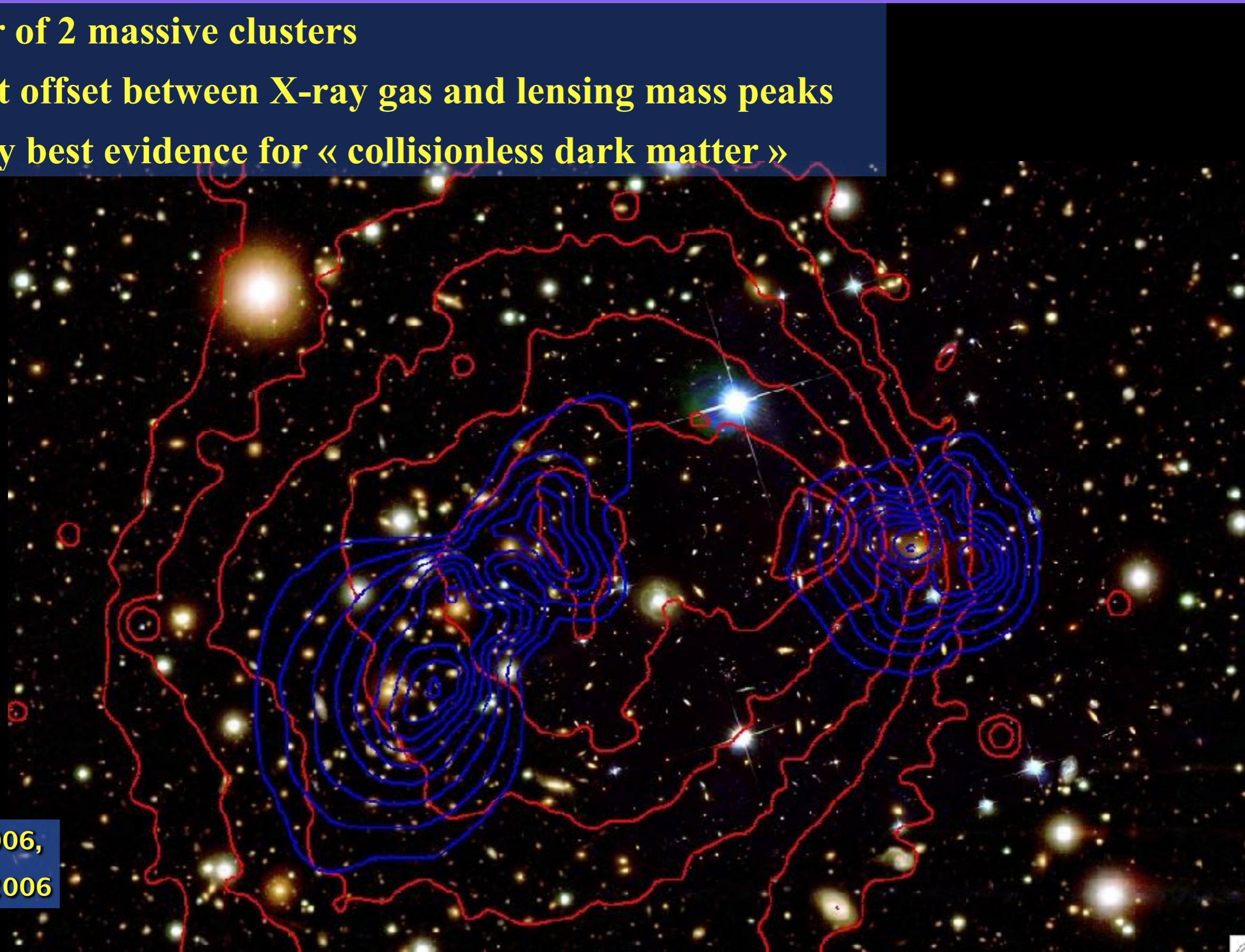
Mass Distribution Measurement

- Why do we measure cluster mass ?
- Central mass profile ? => learn about DM and baryon interactions
- Large scale mass profile and substructures ? => structure formation paradigm, halo models, filaments (use of with weak lensing)
- Case of mergers => probe nature of DM
- Comparison of the distribution of the different components => scaling relations, cluster thermodynamics
- Use cluster as telescopes

The “Bullet Cluster”: Direct Proof of existence of DM

- Encounter of 2 massive clusters
- Significant offset between X-ray gas and lensing mass peaks

⇒ probably best evidence for « collisionless dark matter »



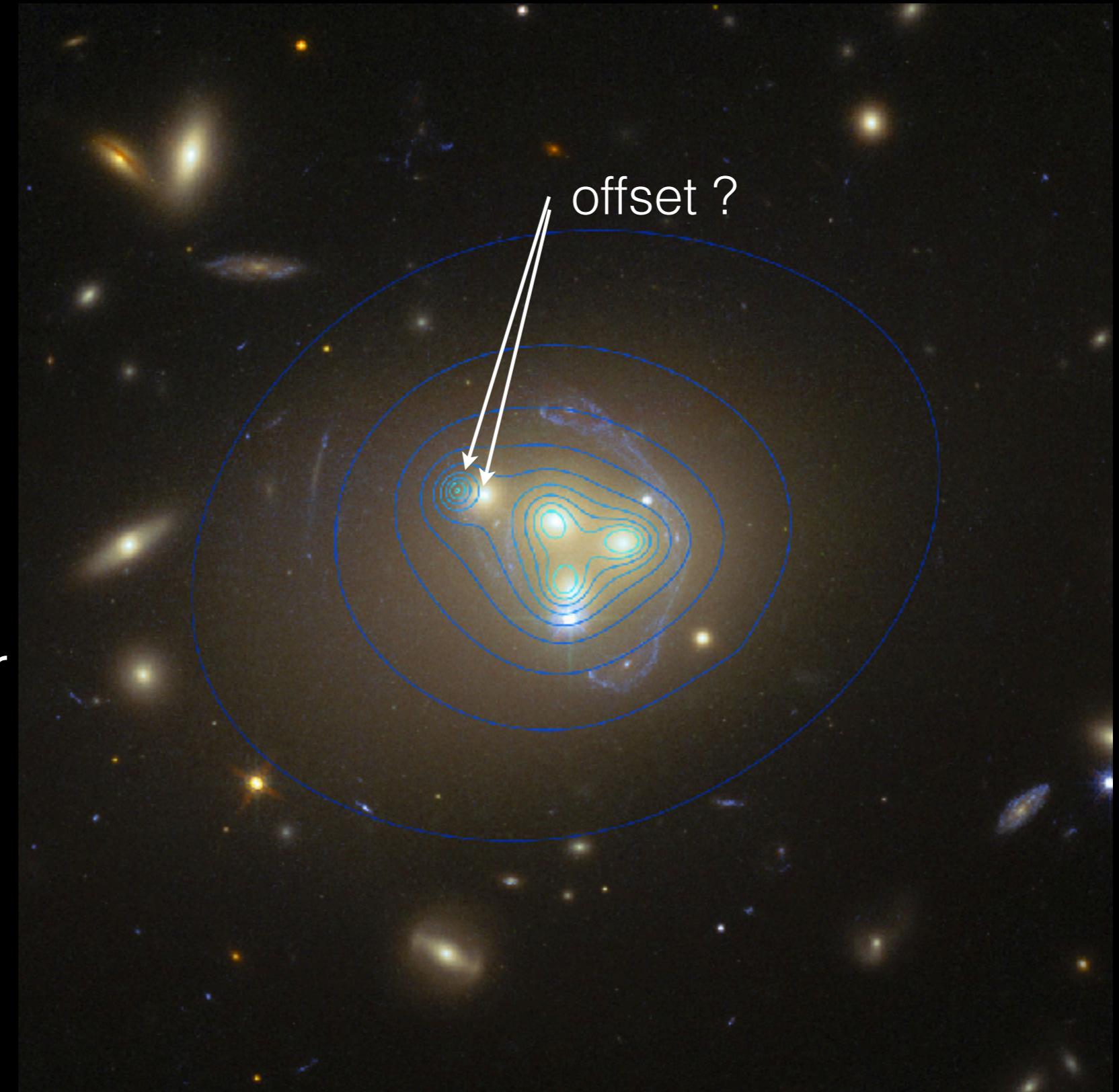
Clowe et al 2006,
Bradac et al 2006

Is Dark Matter interacting with baryonic matter?

Measurement of the offset between Dark and Baryonic Matter can inform us on their possible interaction.

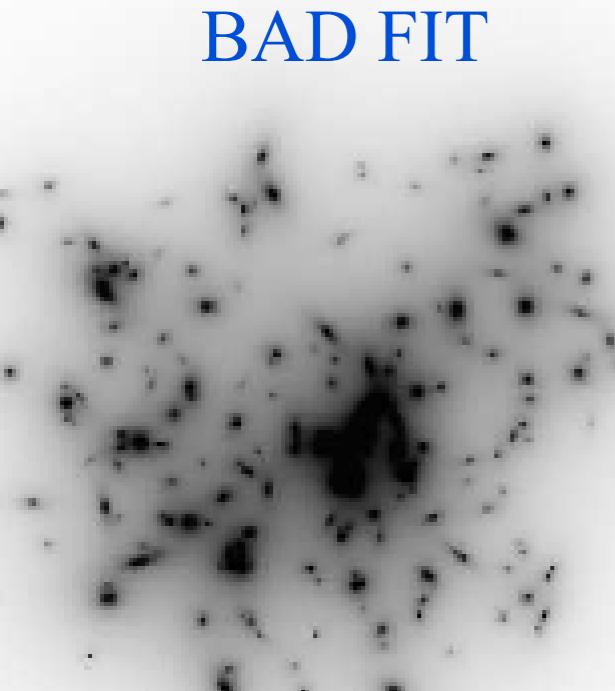
Example of a galaxy infalling in the cluster A3827.

Offset of DM and light => sign of DM self interaction?



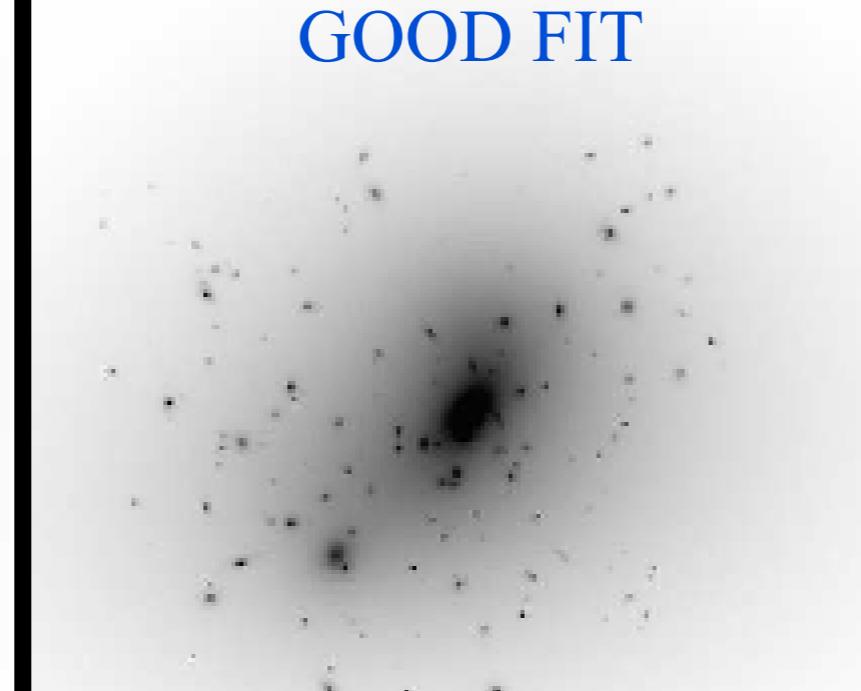
Where is the Matter in A2218?

BAD FIT

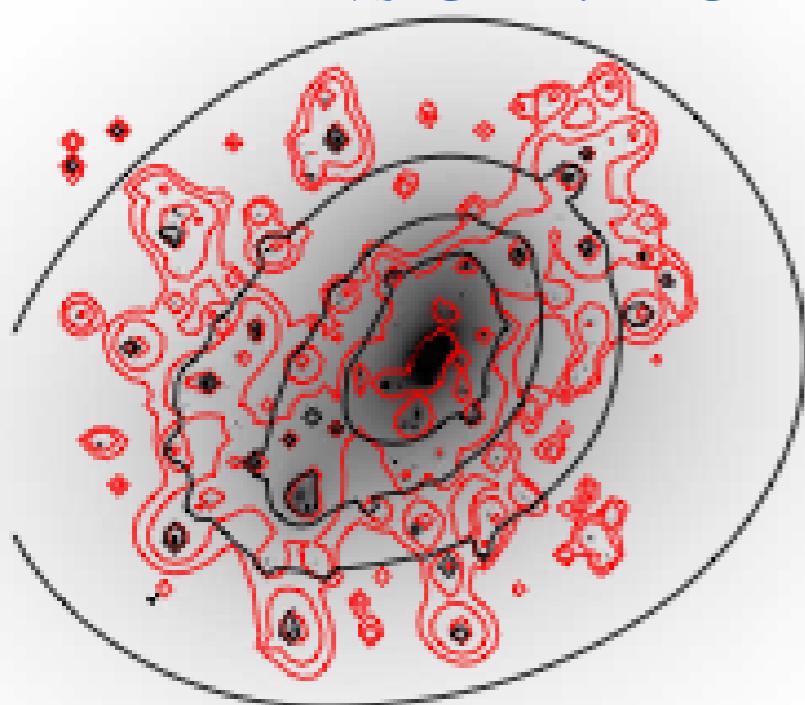


Mass scales with stellar mass

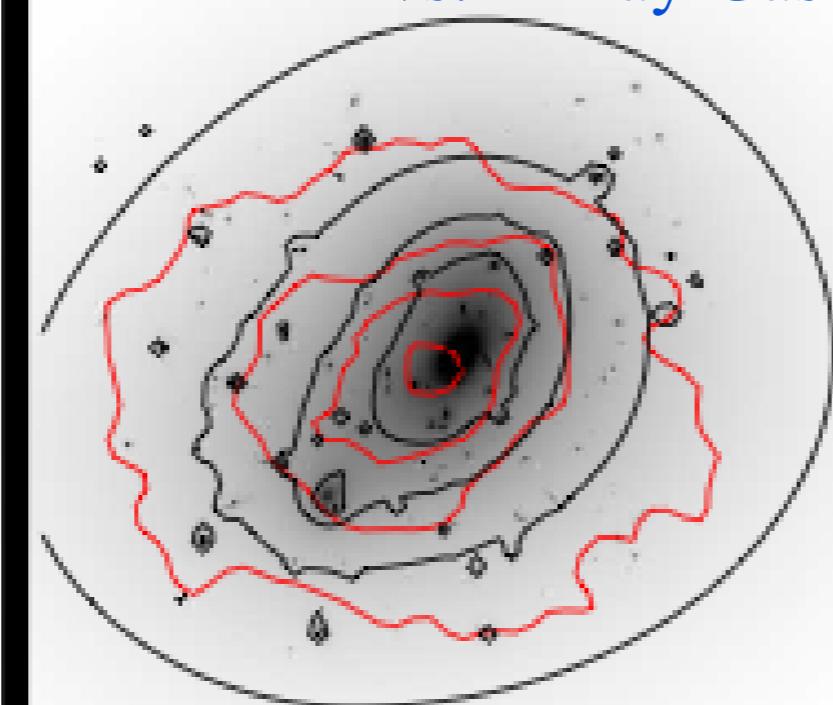
GOOD FIT



MATTER vs GAL. LIGHT



MATTER vs. X-Ray Gas

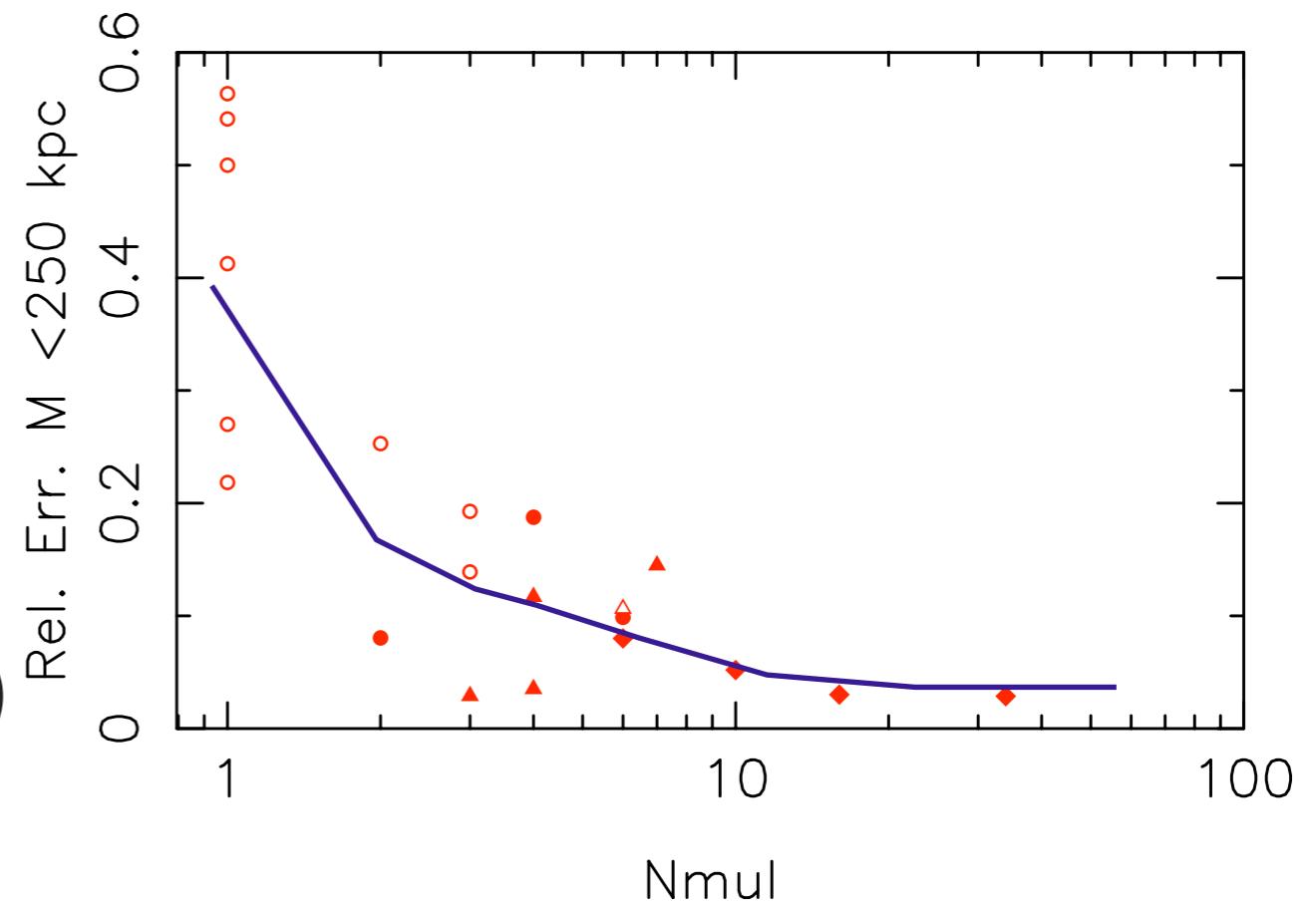


Strong Lensing constraints in Abell 2218:

- Mass distribution proportional to the stellar mass produce a BAD FIT to the lensing data
- Require large scale mass distribution (cluster DM)
- Important difference between DM, Galaxy distribution and X-ray gas (different physics)
- But, scaling relation should exists

Cluster lens model accuracy ?

- Mass model accuracy depends on the number of multiple images.
- Mass accuracy limitations:
 - multiple images identification
 - redshift of multiple images
 - priors on the mass models (hence on the method used)
 - galaxy halos scaling relations
 - line of sight structures (can be modelled)



Deep = Many

Deep HST/ACS multi-band imaging of massive clusters provides **MANY** multiple images:

A1689 ~40 systems

A1703 ~20 systems

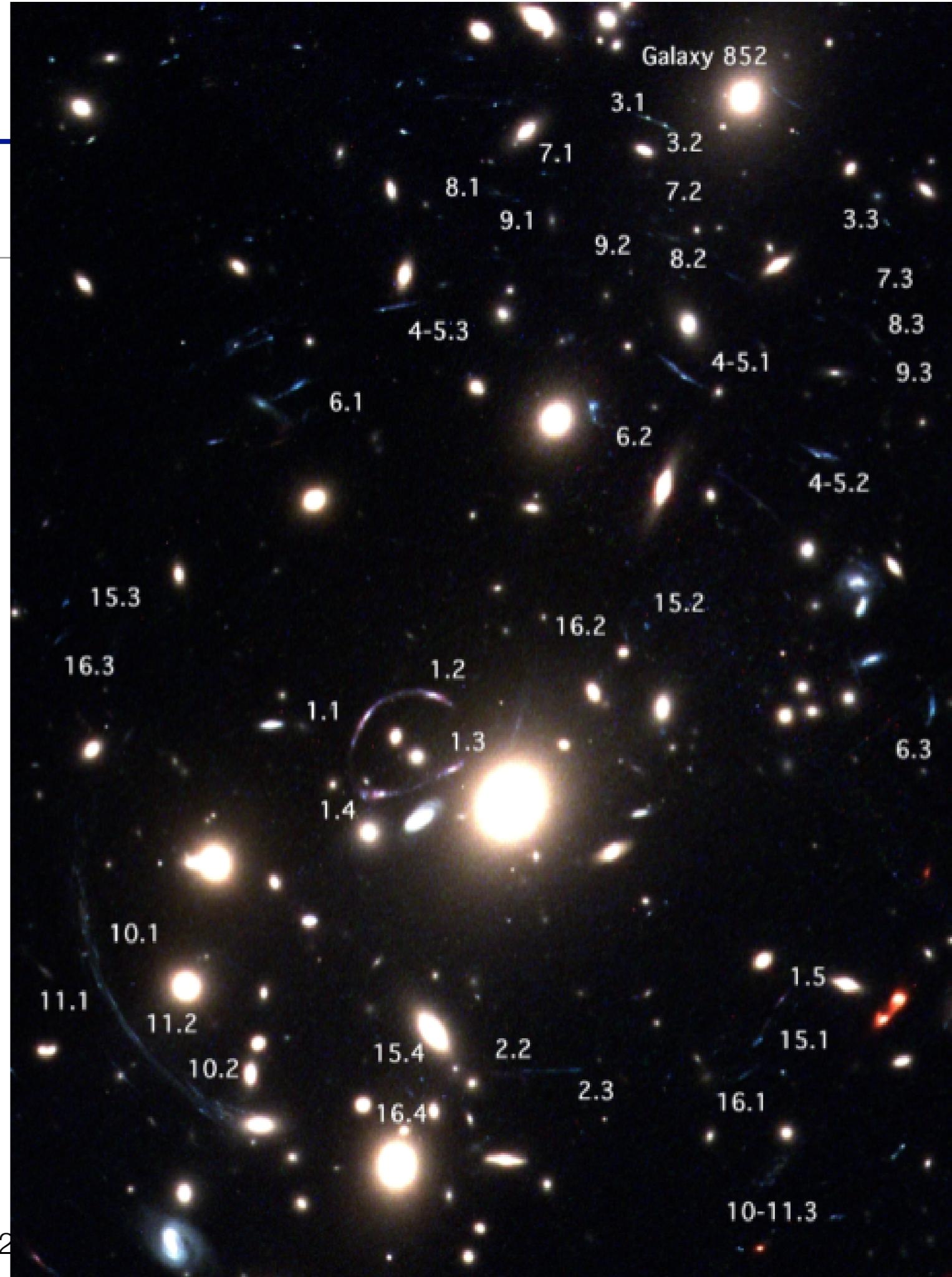
Standard parametric modelling have the RMS image position fit proportional to number of constraints = model too rigid!

Need a change of paradigm in strong lensing mass modelling

⇒ Grid approach: Jullo & Kneib

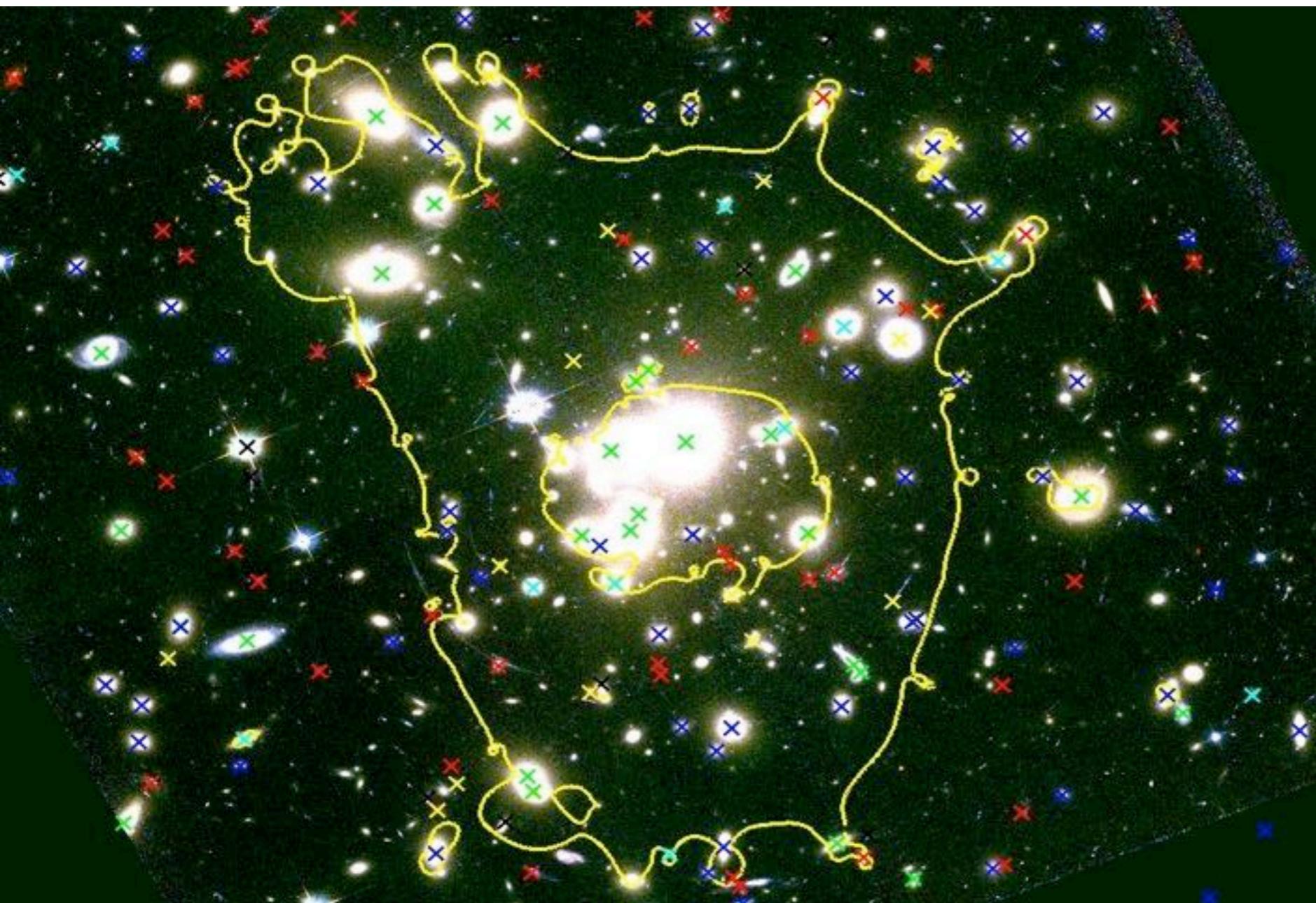
2009

Limousin et al 2008. Richard et al 2009



The most massive cluster: Abell 1689

- Mass models form different groups w. or w/o weak lensing
- Massive spectroscopic surveys (2003-2006)
- 41 multiple image systems, 24 with spectro-z with $1.1 < z < 4.9$



Broadhurst et al 2005
Halkola et al 2007
Limousin, et al. 2007
Richard et al. 2007
Frye et al 2007
Leonard et al 2007
Jullo & Kneib 2009 ...

X KECK/LRIS
X VLT/FORS
X CFHT/MOS
X MAGELLAN
/LDSS2
X Littérature

2014-2016: Hubble Frontier Fields
One of the most ambitious HST project
More than 1000 orbits projects!



MACSJ0416 : Before HFF ...

Previous GL Analysis :

Zitrin et al. 2013, *ApJ*, 762, 30

- 34 SL multiple images
- no WL data

PreHFF GL analysis :

Johnson et al. 2014, *arXiv 1405.0222*

Coe et al. 2014, *arXiv 1405.0011*

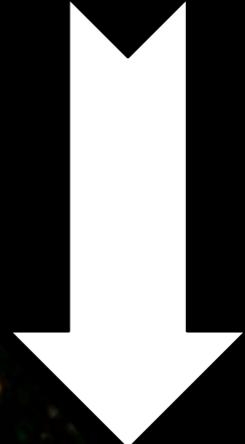
Richard, Jauzac et al. 2014, *MNRAS*, 444, 268

- 47 SL multiple images
- ~ 50 WL gal.arcmin $^{-2}$

MACSJ0416 : ... After HFF !!!

Jauzac et al. 2014a, *MNRAS*, 443, 1549
Jauzac et al. 2014b, *arXiv*, 1406.3011

194 multiple images
~100 WL gal.arcmin⁻²



MACSJ0416 :
the MOST constrained
lensing cluster to date !!!

Multiple Images in MACSJ0416

SL-only analysis

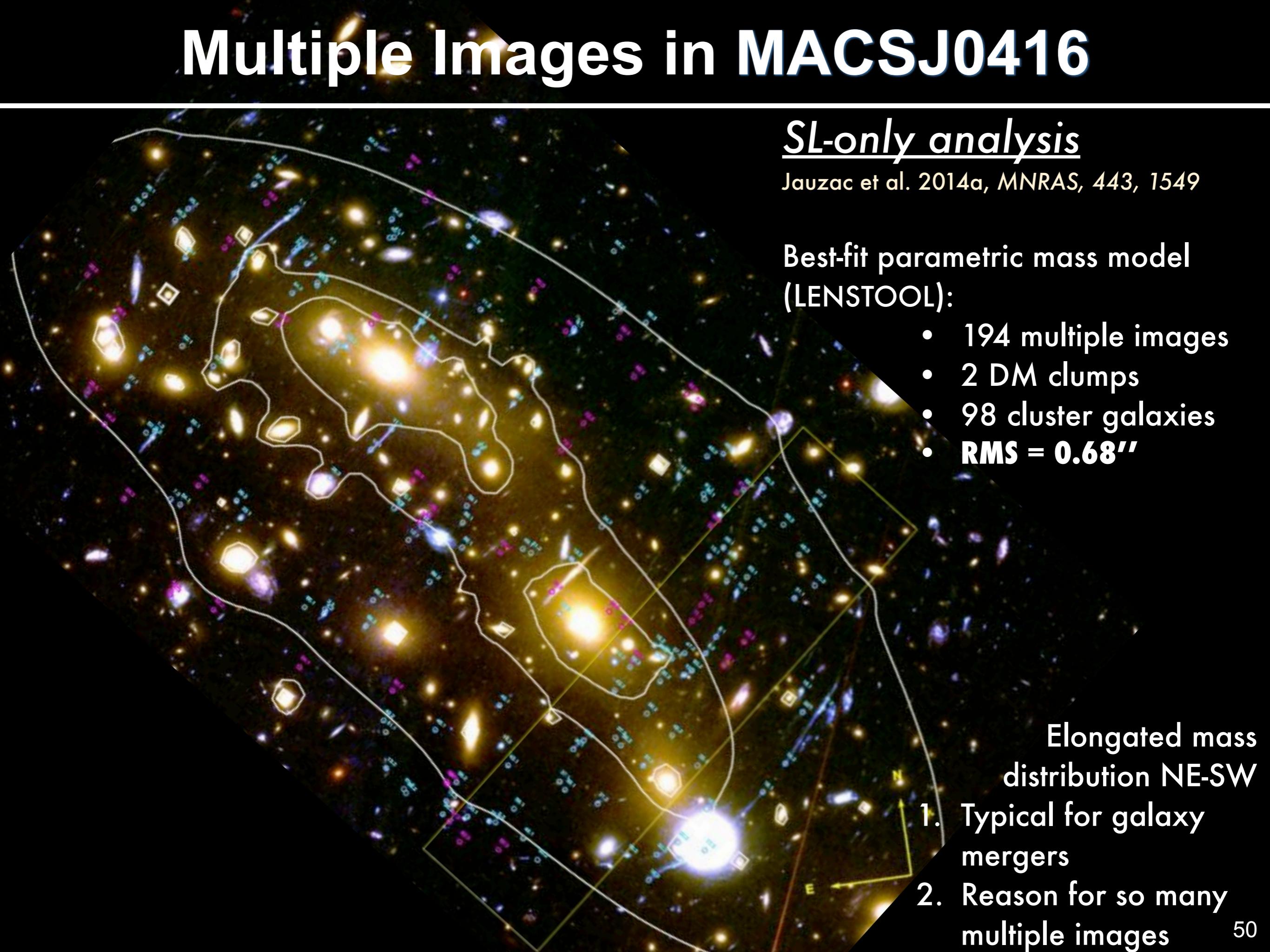
Jauzac et al. 2014a, MNRAS, 443, 1549

Best-fit parametric mass model
(LENSTOOL):

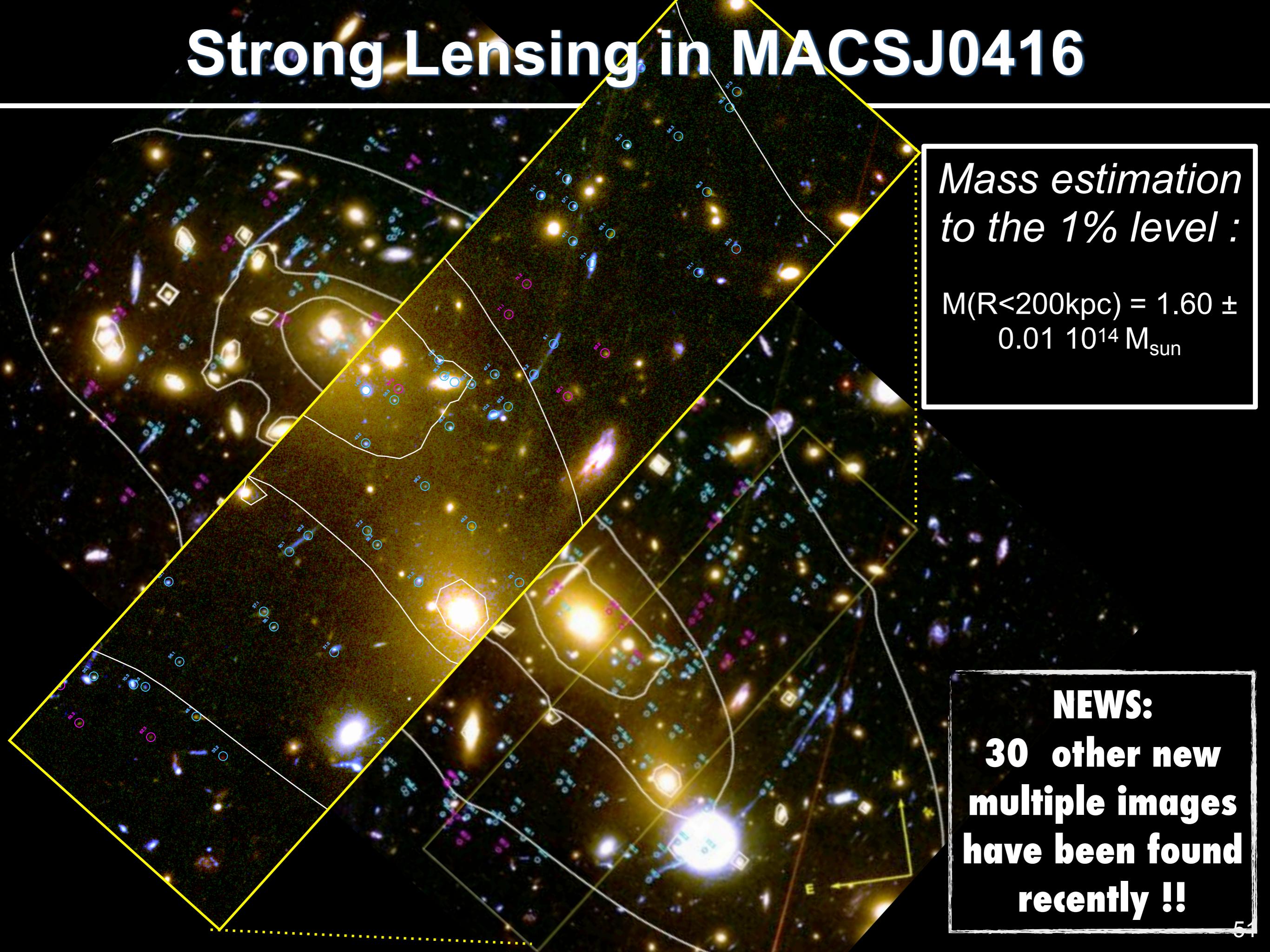
- 194 multiple images
- 2 DM clumps
- 98 cluster galaxies
- **RMS = 0.68''**

Elongated mass
distribution NE-SW

1. Typical for galaxy
mergers
2. Reason for so many
multiple images



Strong Lensing in MACSJ0416

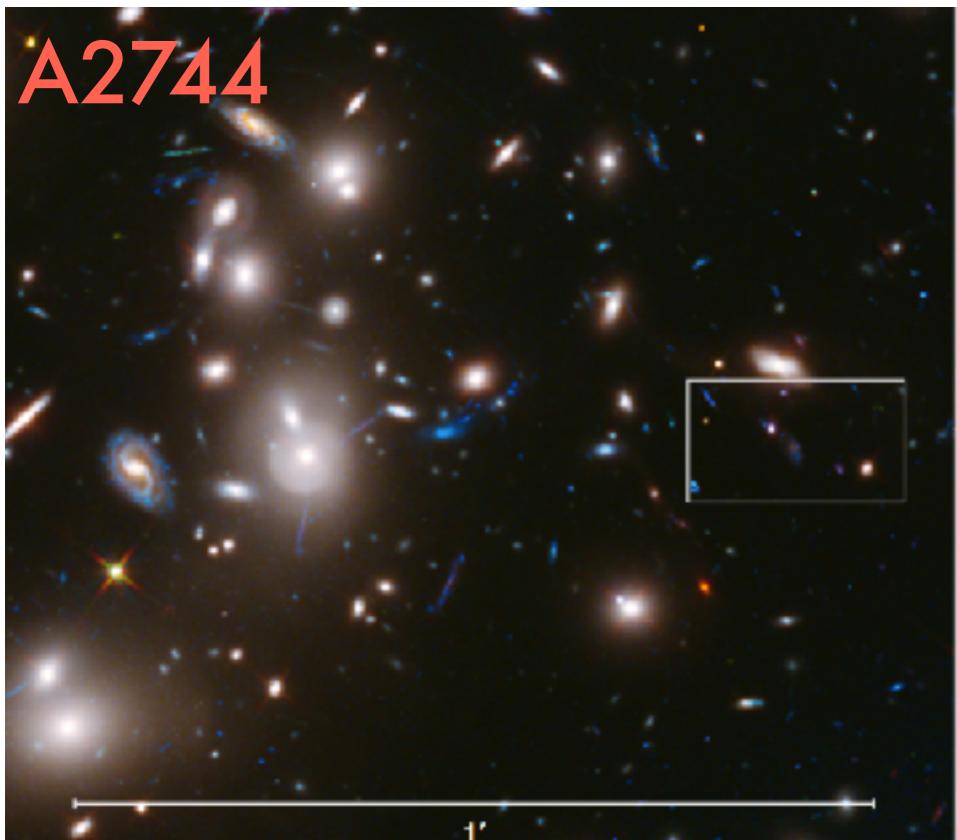


*Mass estimation
to the 1% level :*

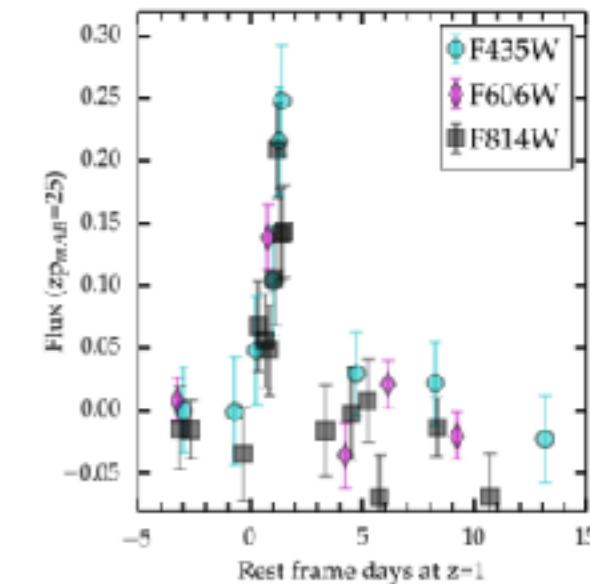
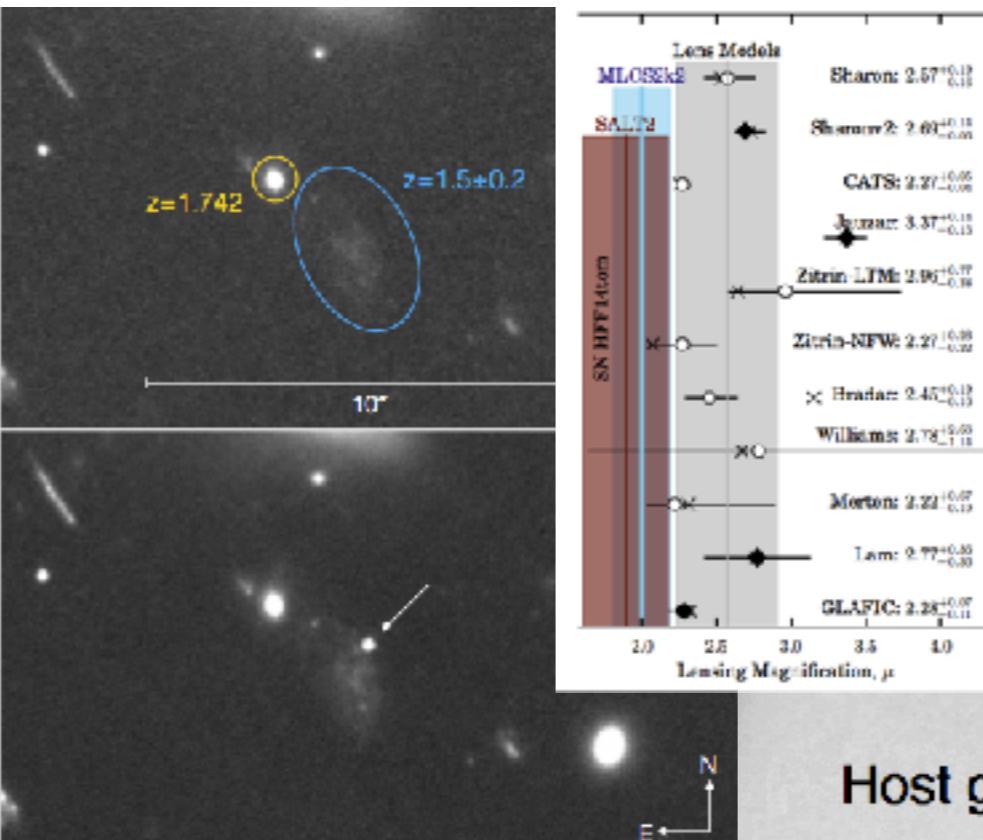
$$M(R < 200 \text{ kpc}) = 1.60 \pm 0.01 \cdot 10^{14} M_{\text{sun}}$$

NEWS:
**30 other new
multiple images
have been found
recently !!**

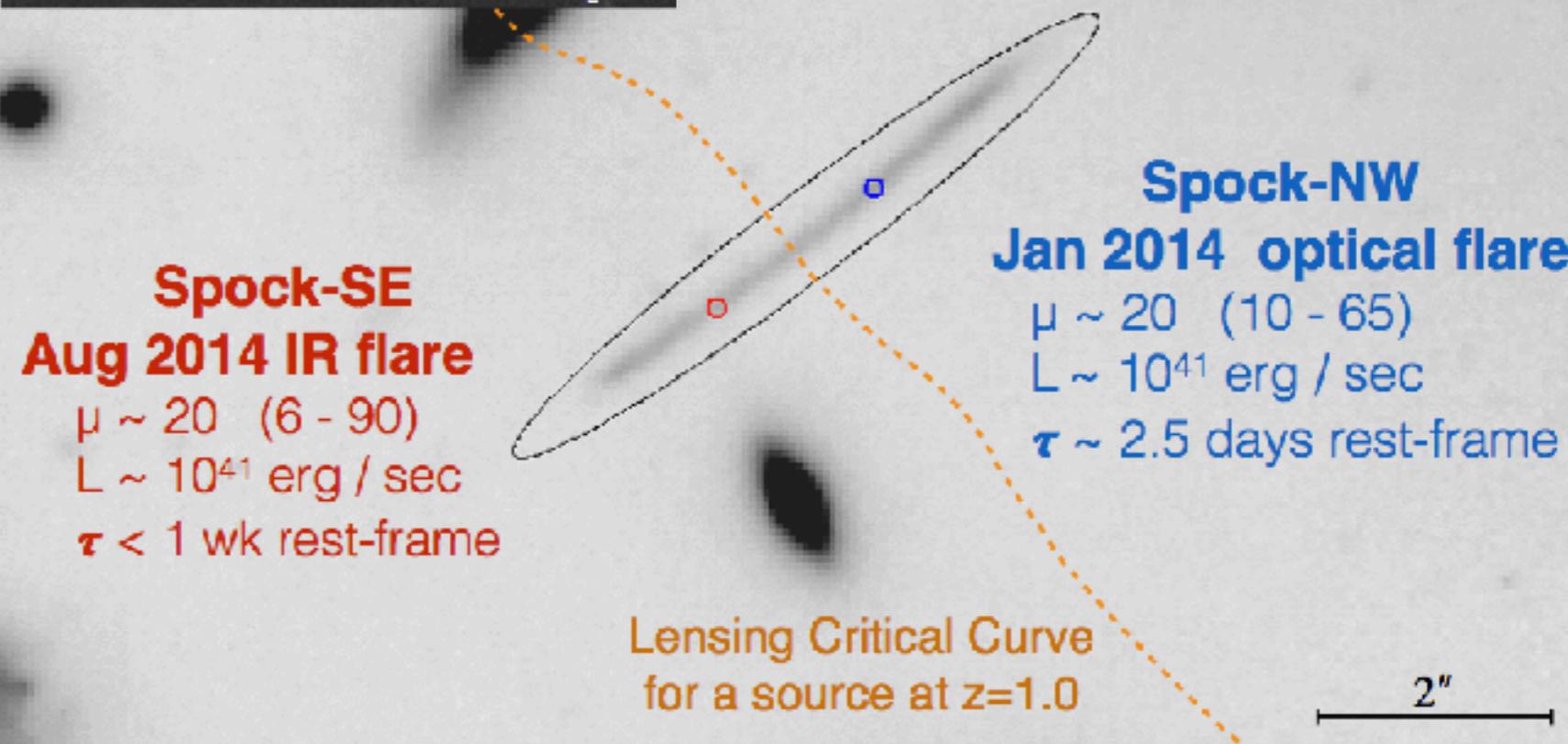
Supernovae Discoveries



Tom: Single image Type Ia
Spock: Double image Peculiar SN



MACS0416
 Host galaxy at $z = 1.00 \pm 0.02$



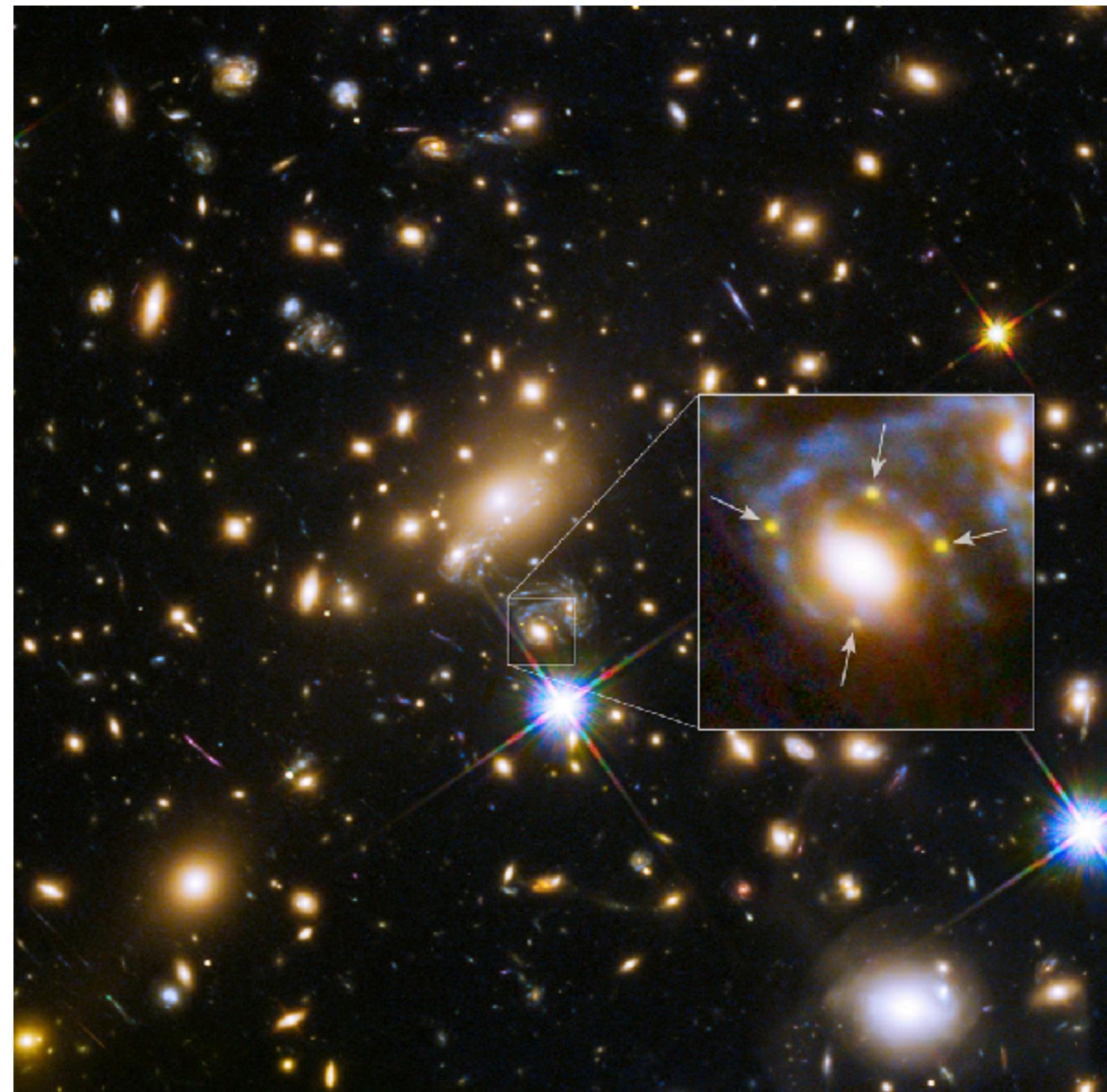
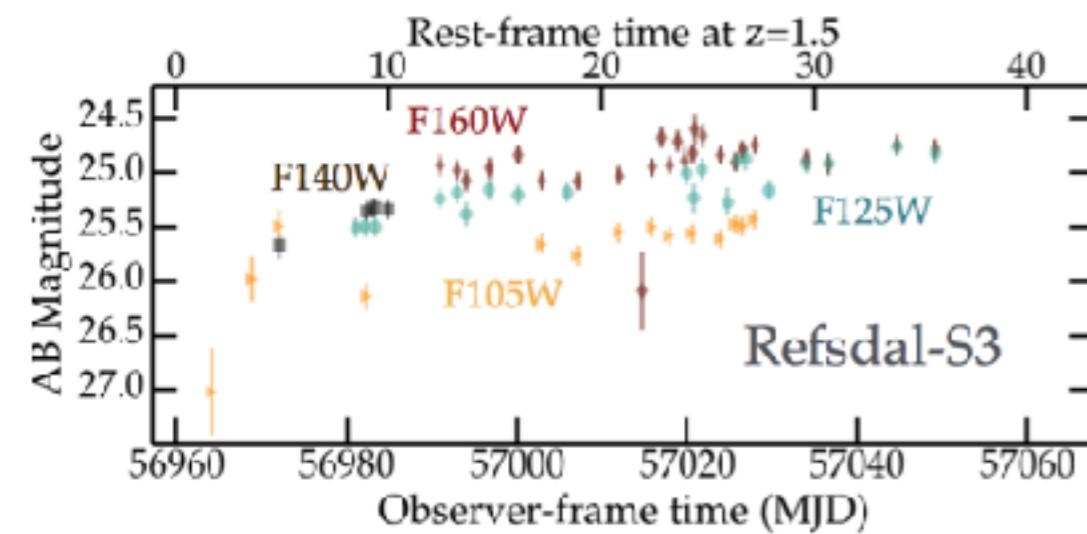
Supernovae Discoveries

Multiple images: “**Refsdal**”

Supernova - not a SNIa

Both cluster and galaxy scale
lensing.

Prediction of a new image in
couple of months. Important for
 H_0 measurement.

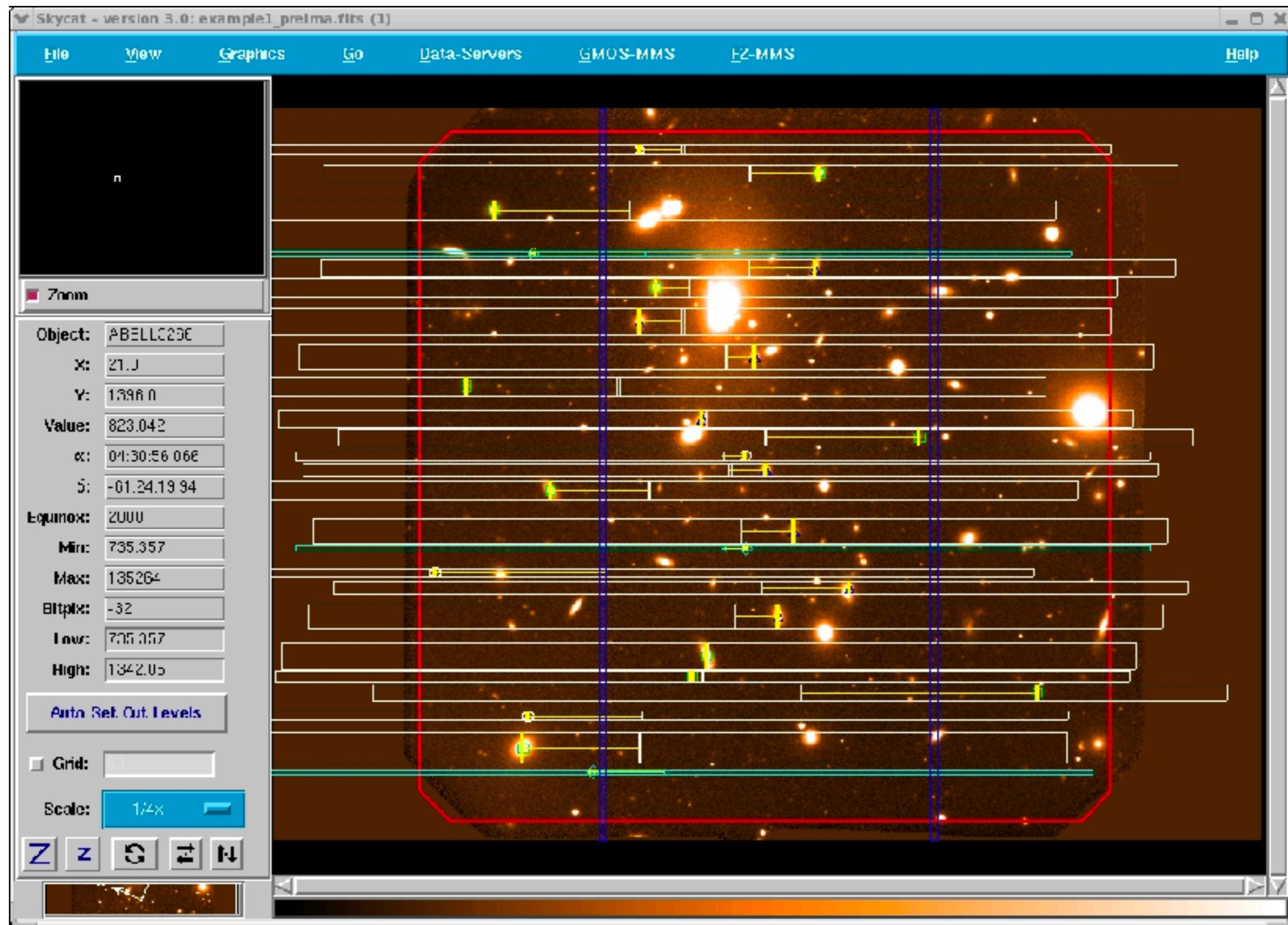


How to measure redshift: spectrograph !

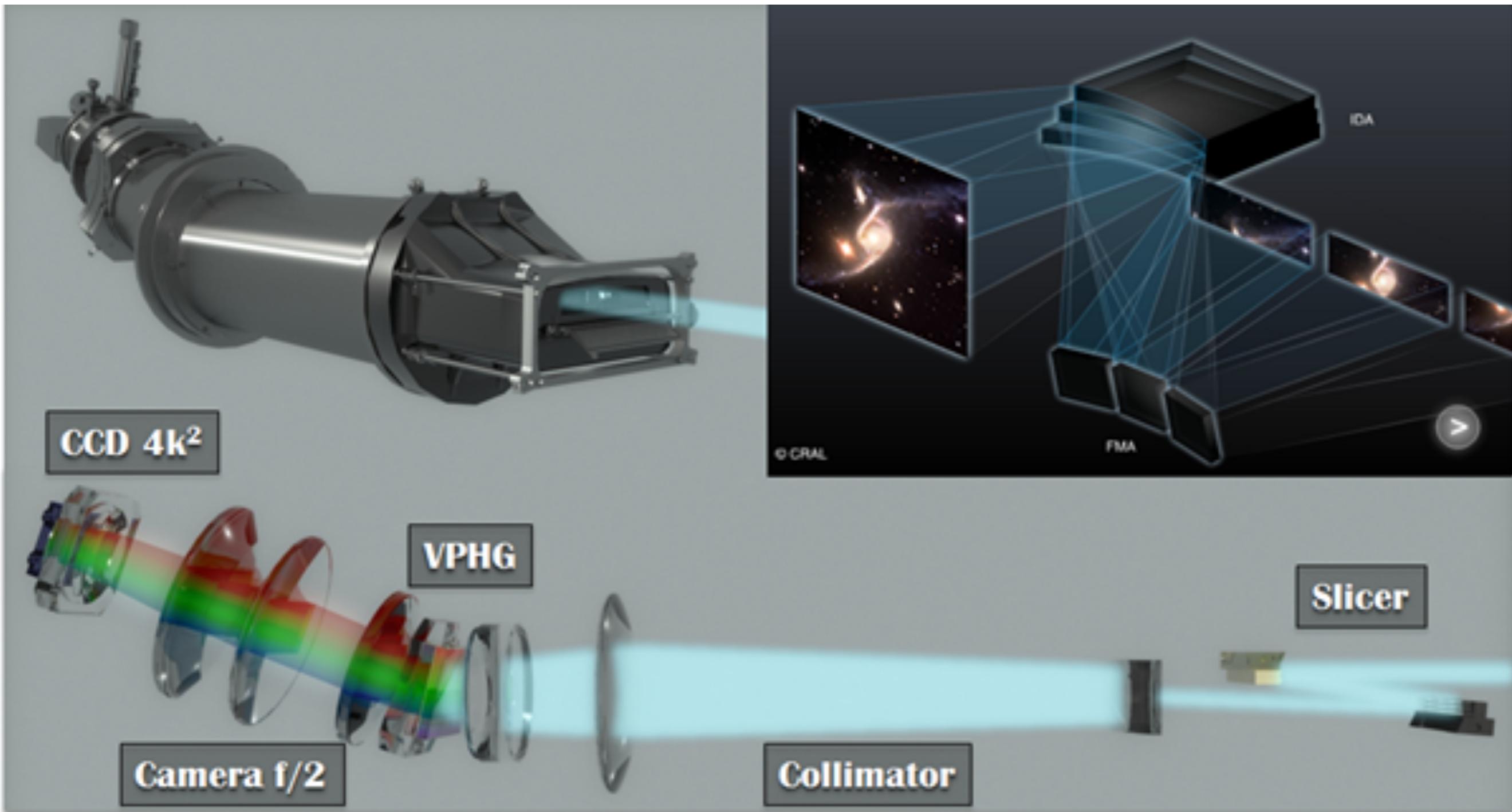
- **Classical spectroscopy is using slit-masks:**
 - this allow to measure at best 20-30 redshift in a 5x5 square.arcmin region
 - VLT/FORS or Keck/LRIS are the most powerful spectrograph of this class
- **Advanced spectroscopy is using integral-field unit spectrograph:**
 - this allow spectroscopy measurement of any object in the field of view
 - the VLT/MUSE spectrograph is the most powerful instrument



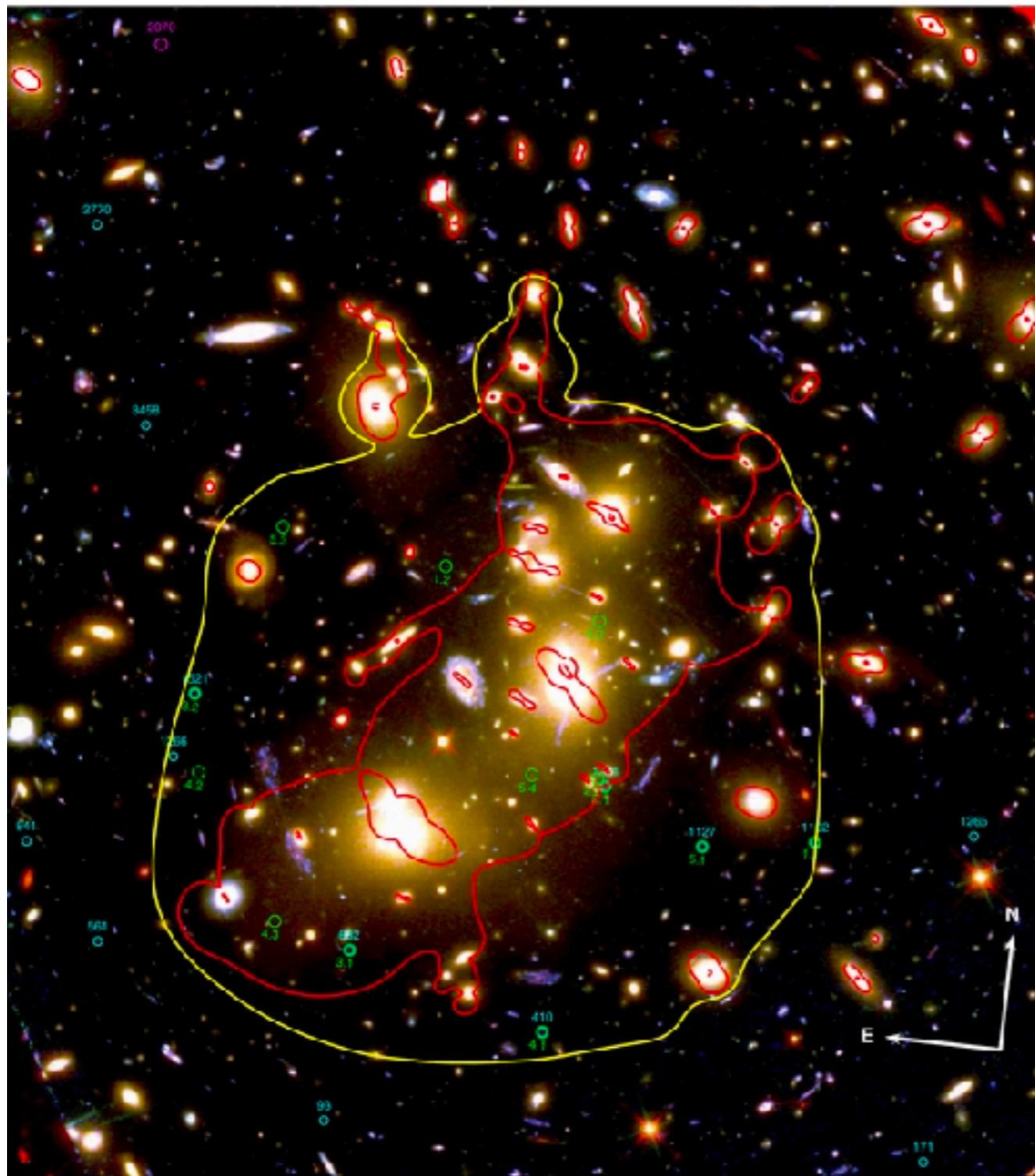
Slit Mask Spectroscopy



VLT/MUSE spectrograph



A2744 Frontier Fields Observation



Red: critical lines

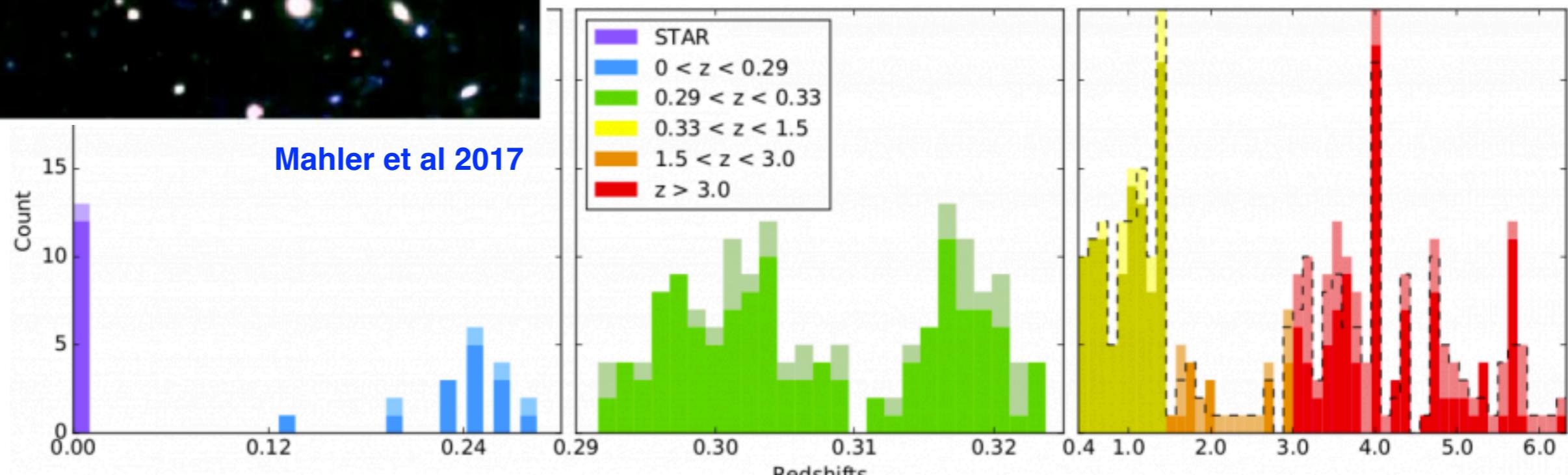
Yellow: enclose region
of multiple images

Green: $z > 6$ galaxy
candidates

MUSE Spectroscopy of the A2744 Cluster



Mahler et al 2017



Integral Field Spectroscopy is very powerful to identify both foreground, cluster and background galaxies.

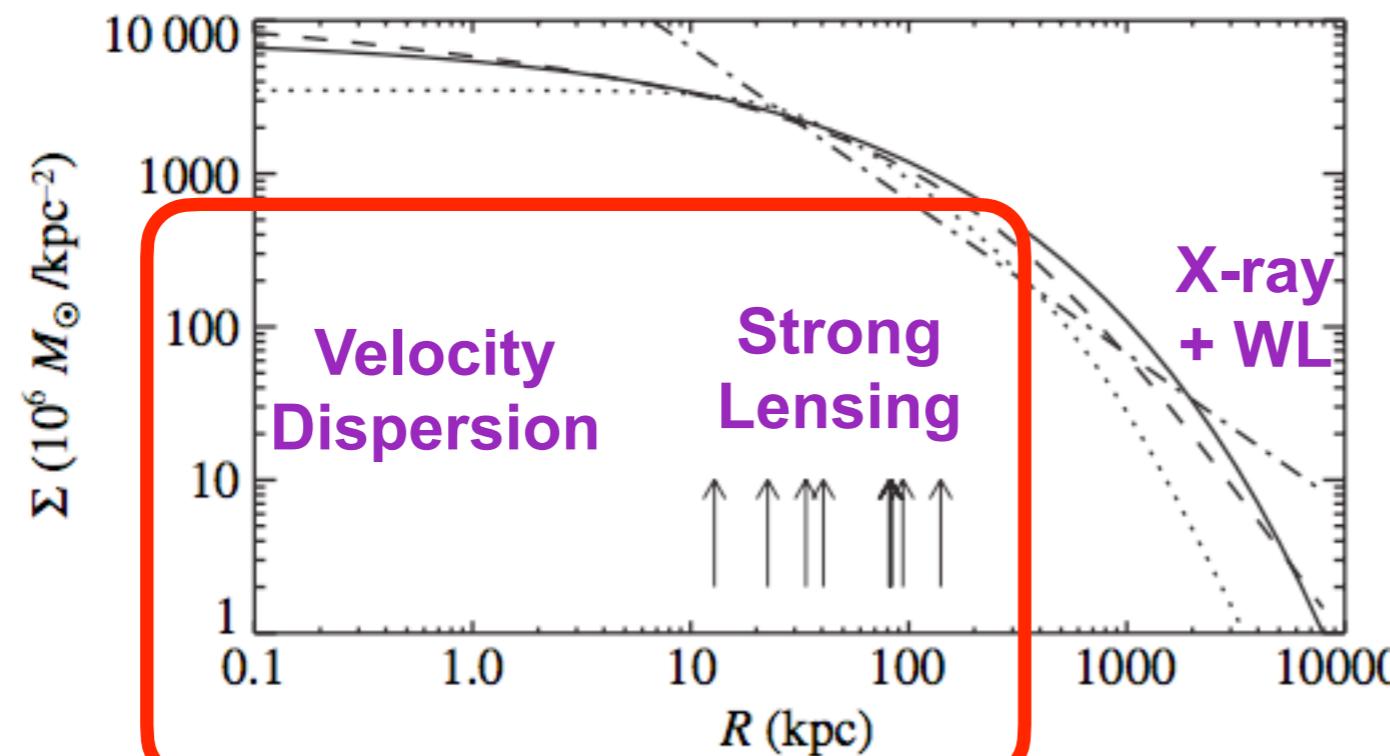
MUSE wavelength range from 0.55 to 0.95 micron

This limits the detection of galaxy redshift in the $1.5 < z < 3.0$ window.

Combining Lensing and other techniques

Mass profile for a mass clump?

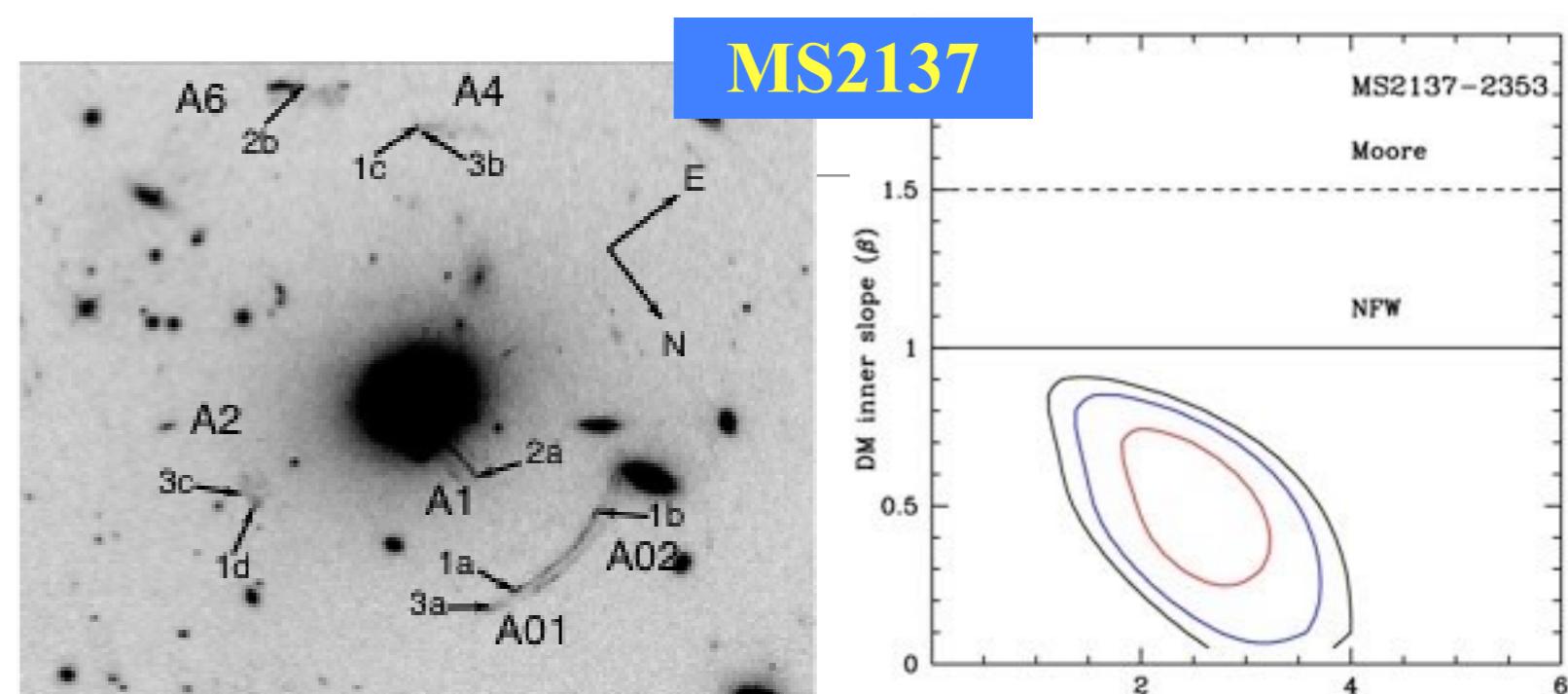
- Mass profile should match theoretical or numerical simulations in order to be close to reality:
 - isothermal model (singular => cored & truncated, circular => elliptical): PIEMD: Pseudo Isothermal Elliptical Mass Distribution
 - NFW model => gNFW, Sersic, Einasto (beware at infinite values, truncate?)



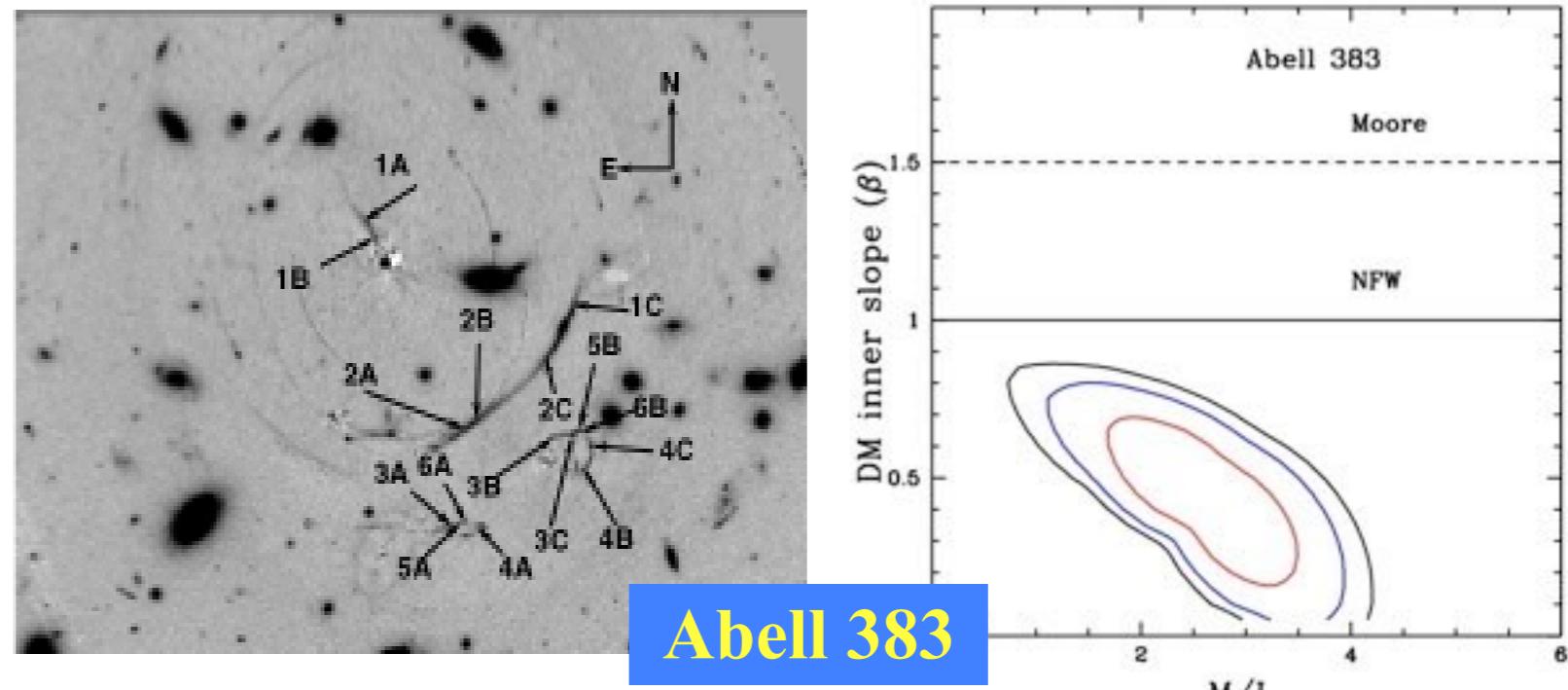
Additional data useful to constraints further the mass profile

Mass Profile of Clusters (SL+Dynamics)

- DM simulation predicts a universal profile; what is observed in the inner core?
- Combination of strong lensing (radial and tangential arcs) + dynamical estimates from the cD galaxies
- Some degeneracies, but indication of a flatter profile than canonical NFW: $-0.5 < \beta < -1$
- “Flat” core found in other clusters (RCS0224, Cl0024)
- Possibly probe DM & Baryon coupling?



New detailed modeling



Abell 383

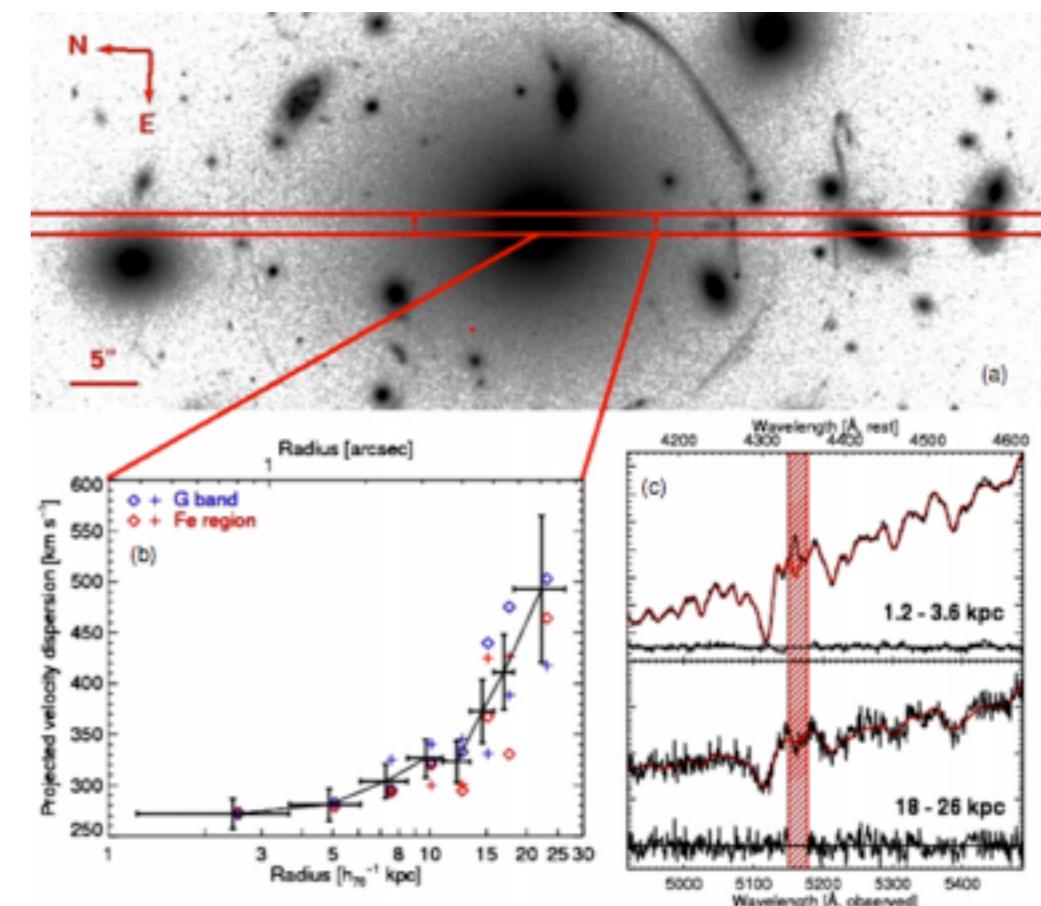
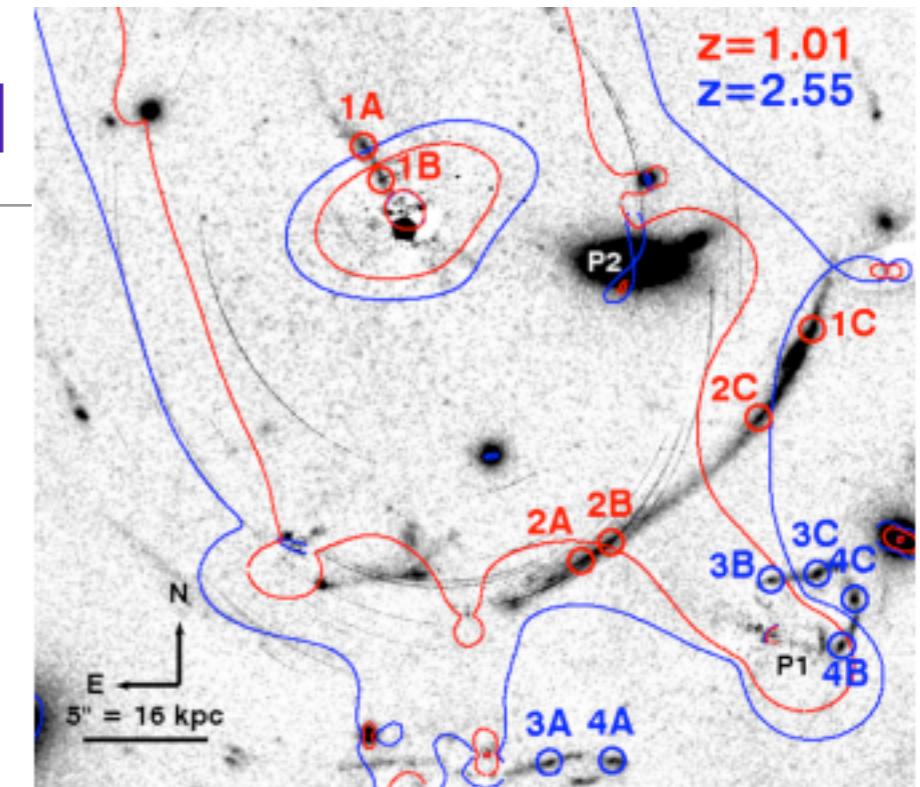
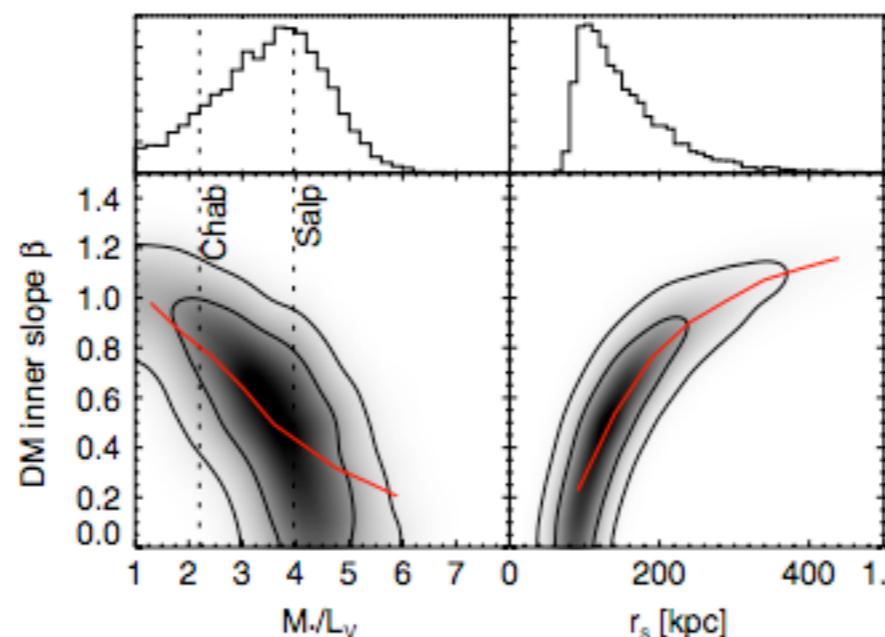
Sand, et al. 2007

Mass Slope Measurement

for Abell 383

Newman et al 2011

- Best measurement yet of velocity dispersion of stars in a cD galaxy!
- Best constraints on the inner slope profile of Dark Matter (slope shallower than -1)
=> limited by the knowledge of mass/luminosity of stars
- in conflict with DM only simulation, but baryon/DM interaction are likely to be important.



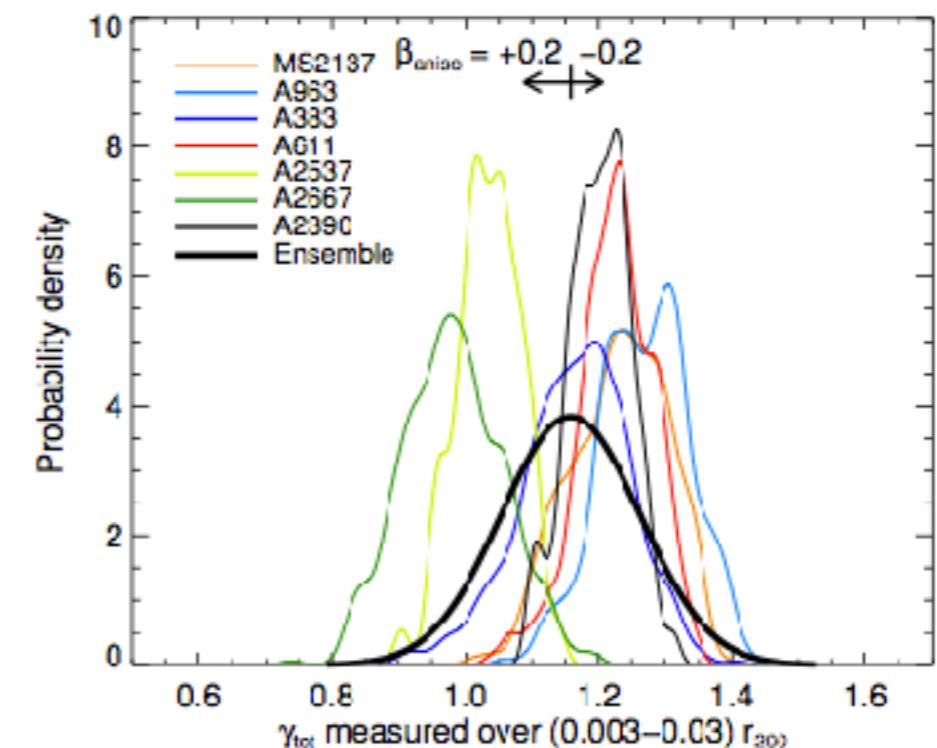
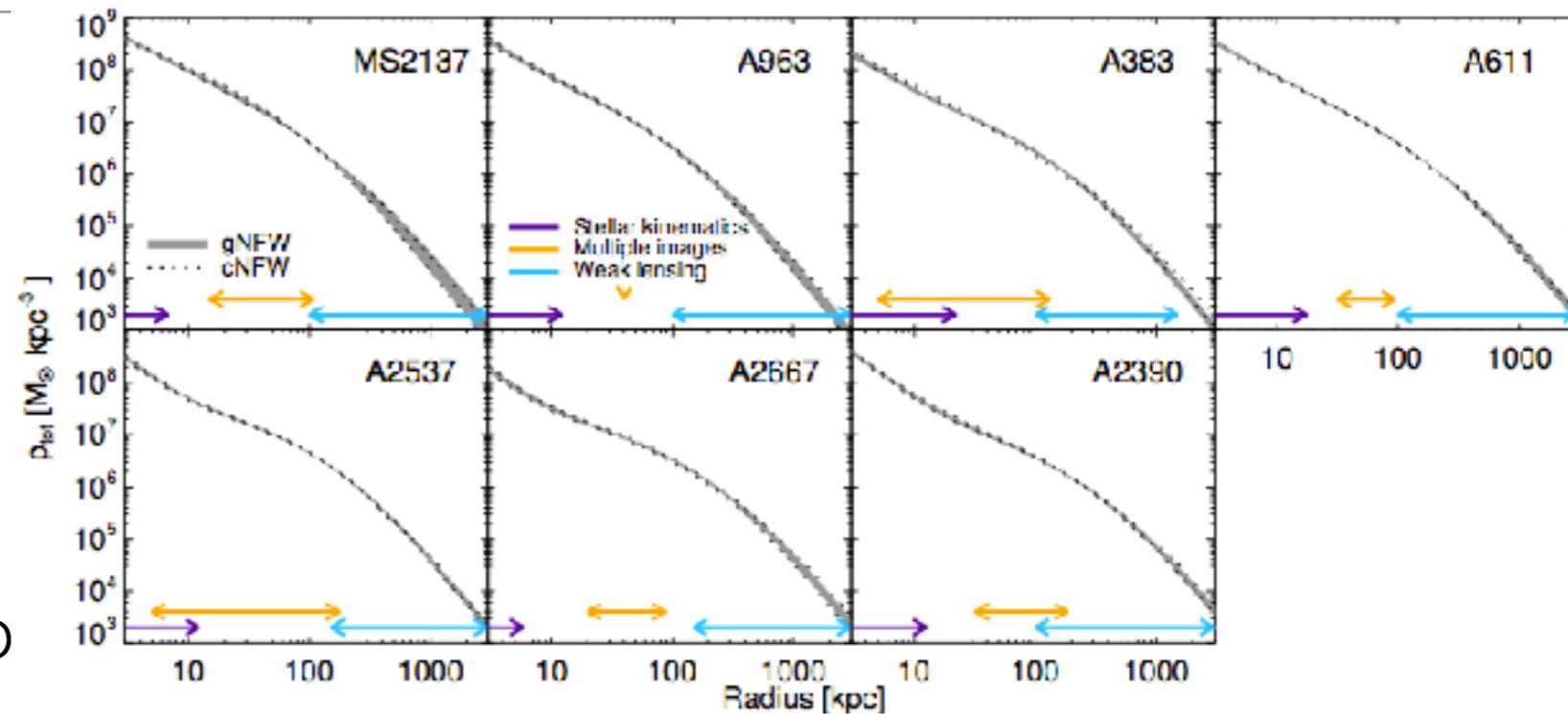
Mass Profile of Clusters (SL+Dynamics)

Newman et al. 2012

DM simulation predicts a universal profile; what is observed profile?

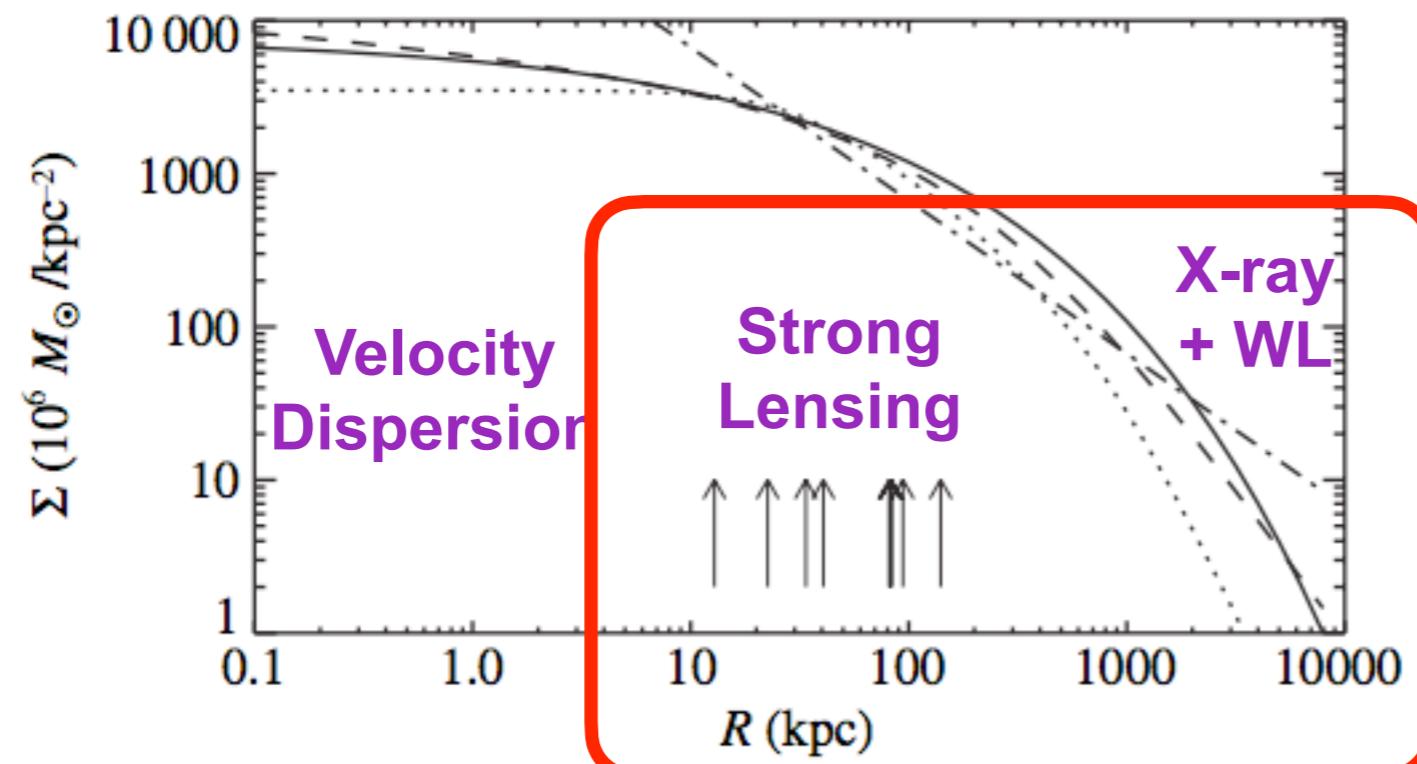
- Combination of strong lensing, weak lensing and dynamical estimates from the cD galaxies to measure ρ_{tot} between 3-3000 kpc.
- Sample of 7 galaxy clusters at redshift $z \sim 0.25$

The slope of the density profile $\gamma_{\text{tot}} \sim 1.16$ is in agreement with DM simulations



Mass profile for a mass clump?

- Mass profile should match theoretical or numerical simulations in order to be close to reality:
 - isothermal model (singular => cored & truncated, circular => elliptical): PIEMD: Pseudo Isothermal Elliptical Mass Distribution
 - NFW model => gNFW, Sersic, Einasto (beware at infinite values, truncate?)



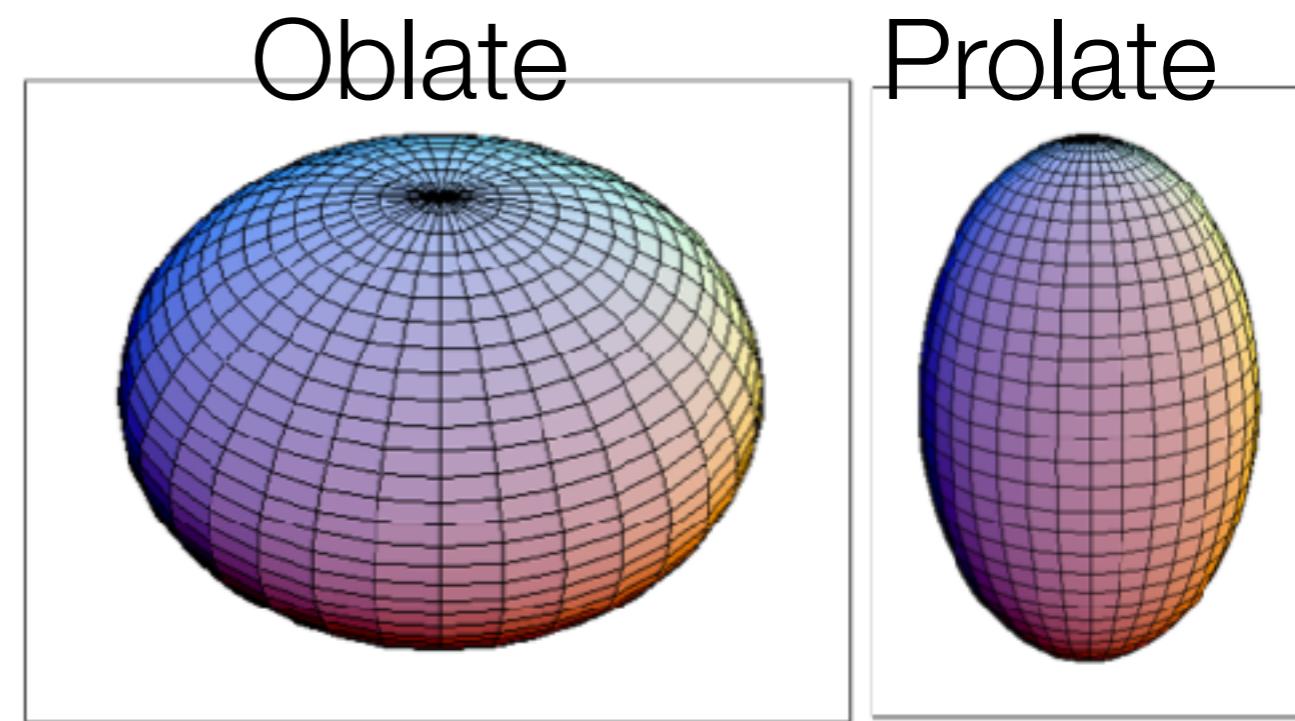
Additional data useful to constraints further the mass profile

Cluster triaxiality

Dark matter halos are triaxial

Mass discrepancies when Lensing
and X-ray estimates when a
Spherical shape is assumed

Strong lensing clusters constitute a
highly biased population of **prolate**
halos with major axis aligned along
the LOS (Oguri & Blandford 2009,
Meneghetti et al. 2010)



Cluster triaxiality

Morandi, Pedersen & Limousin 2010a,b

Joint X-ray, SZ & Lensing analysis

X-ray flux

$$S_X = \frac{1}{4\pi(1+z)^4} \Lambda(T_{\text{proj}}^*, Z) \int n_e n_p dz'$$

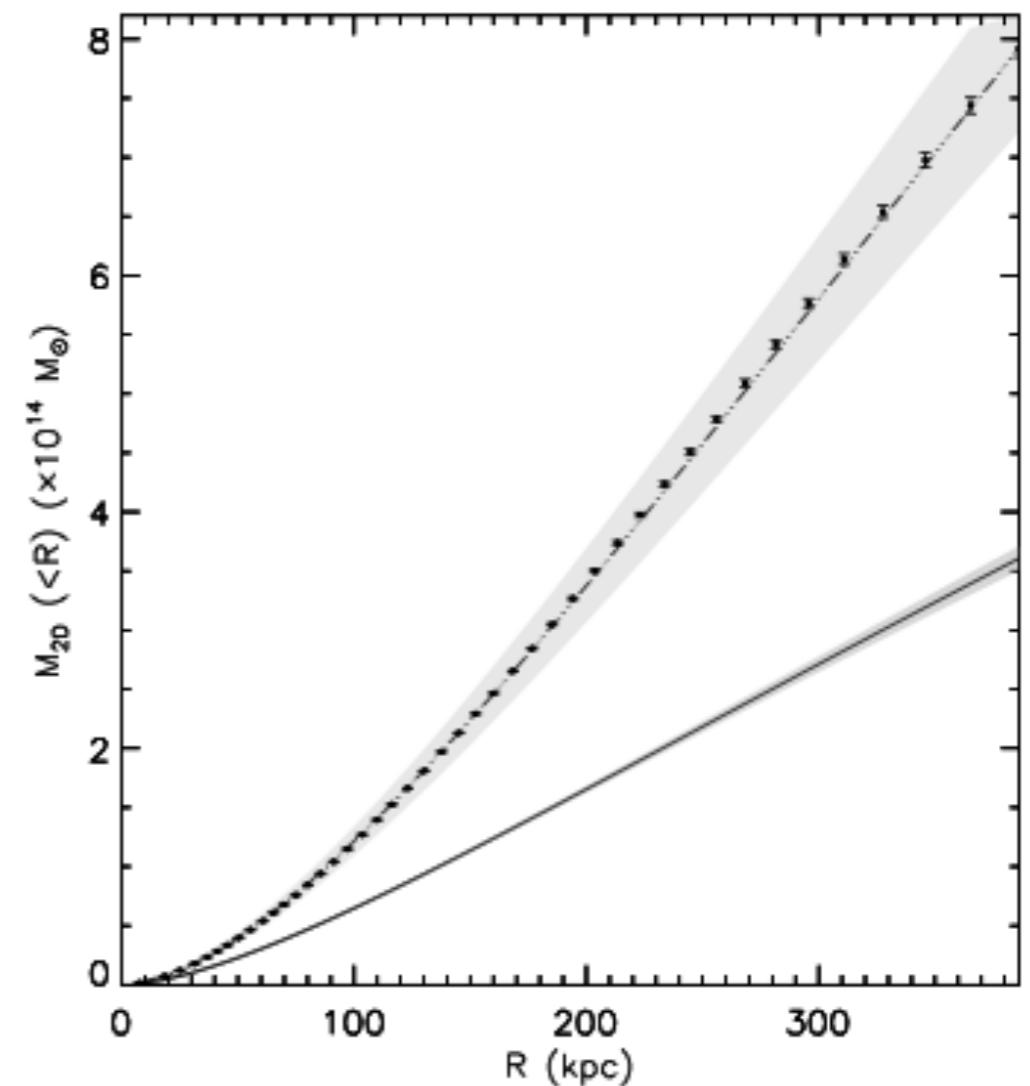
SZ effect

$$\frac{\Delta T(\nu)}{T_{\text{cmb}}} = \frac{\sigma_T}{m_e c^2} \int P(\mathbf{r}) f(\nu; T(\mathbf{r})) dz'$$

Lensing mass

$$\Sigma = \int_{-\infty}^{\infty} \rho(R) dz'$$

→ Estimation of the 3D shape of the gravitational potential (DM)



Solving the X-ray-Lensing mass discrepancy in MACS1423
Morandi et al. 2010

COSMOS Survey

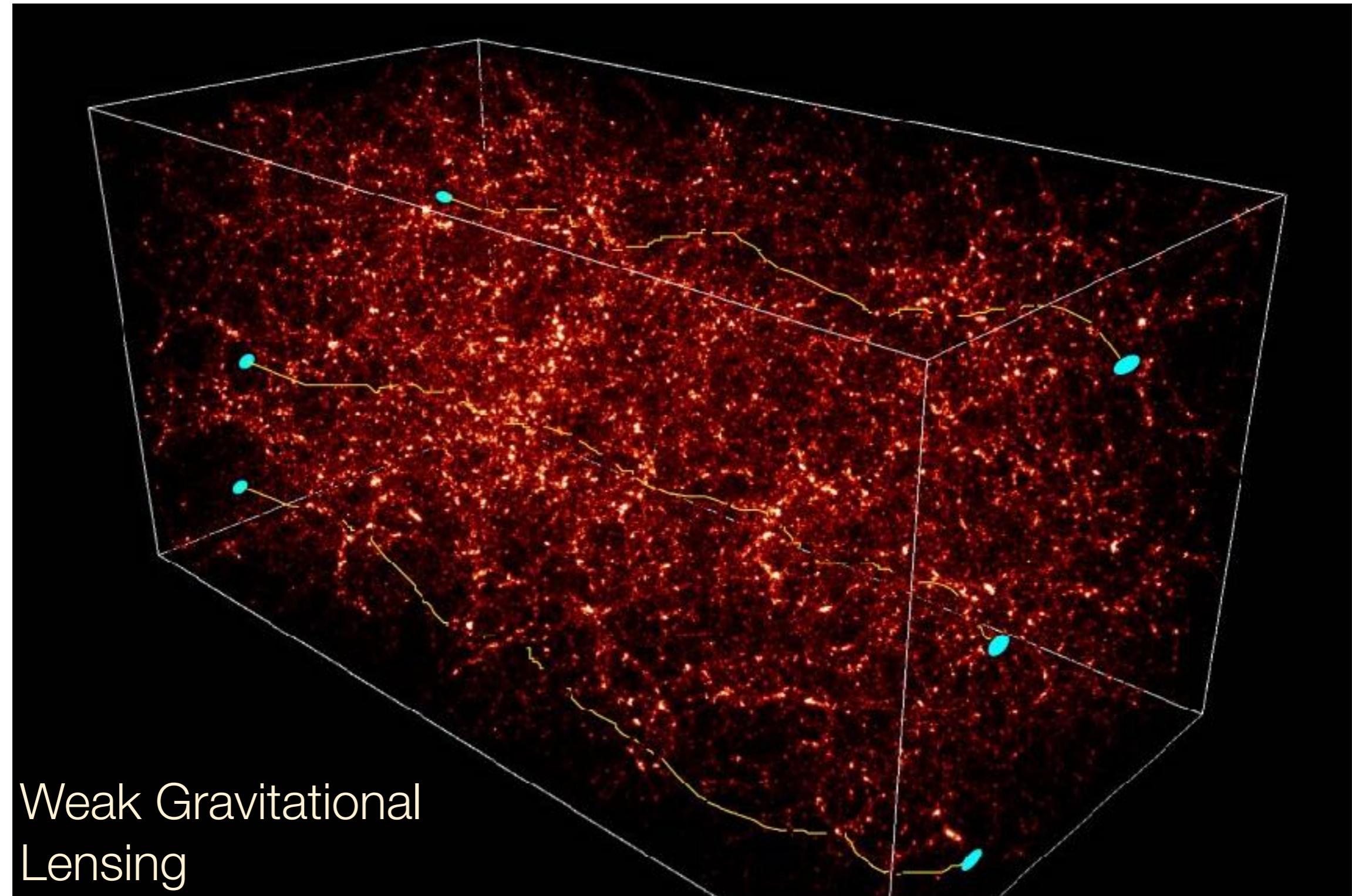
COSMOS Survey: 3D mapping of Dark Matter



One of the largest Hubble project:

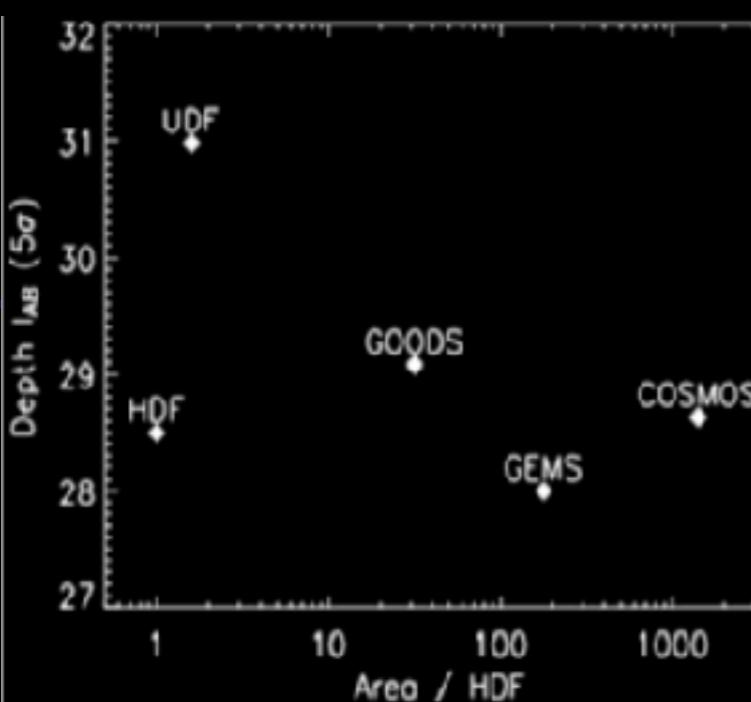
- 10% of the observing time during 2 years
- 575 images
- 9 times the moon size
- 20 Giga pixel image (0.03"/pixel)
- 1.2 millions of galaxies
- 400 000 galaxies used for the weak lensing analysis

Mapping the mass in the Universe

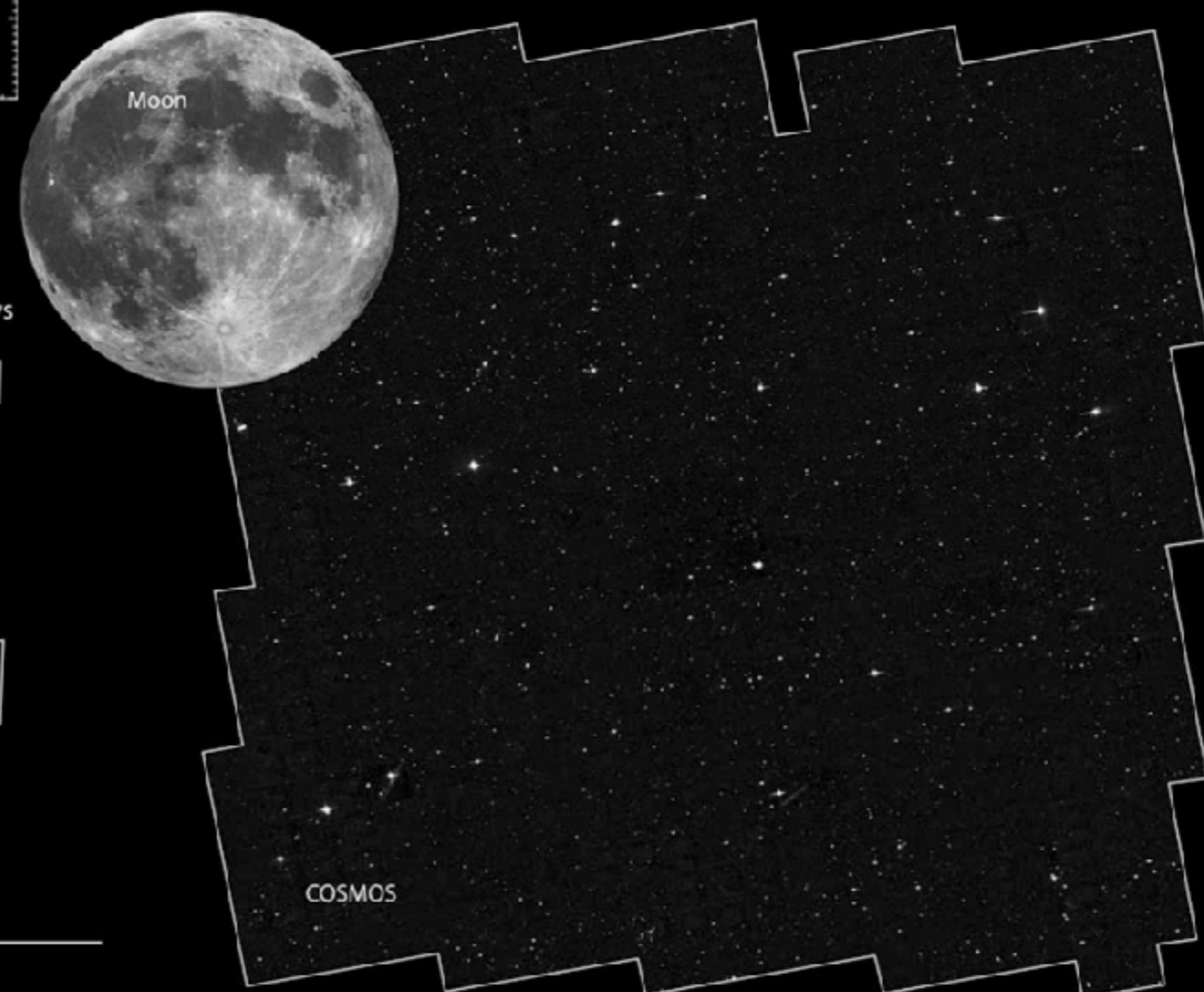
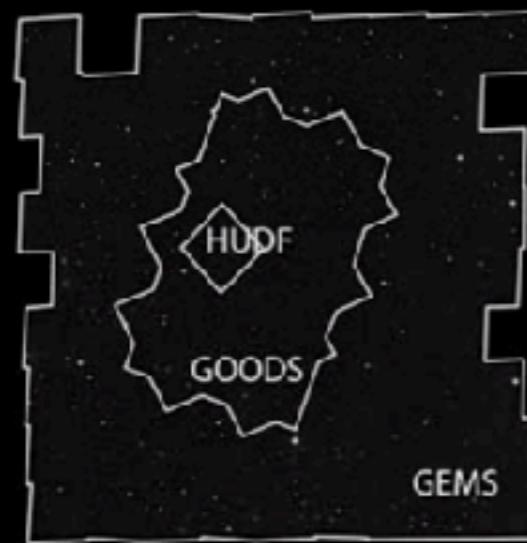


Galaxies shapes are sensitive to the mass distribution along the line of sight : a powerful tool to constrain the mass-energy content!

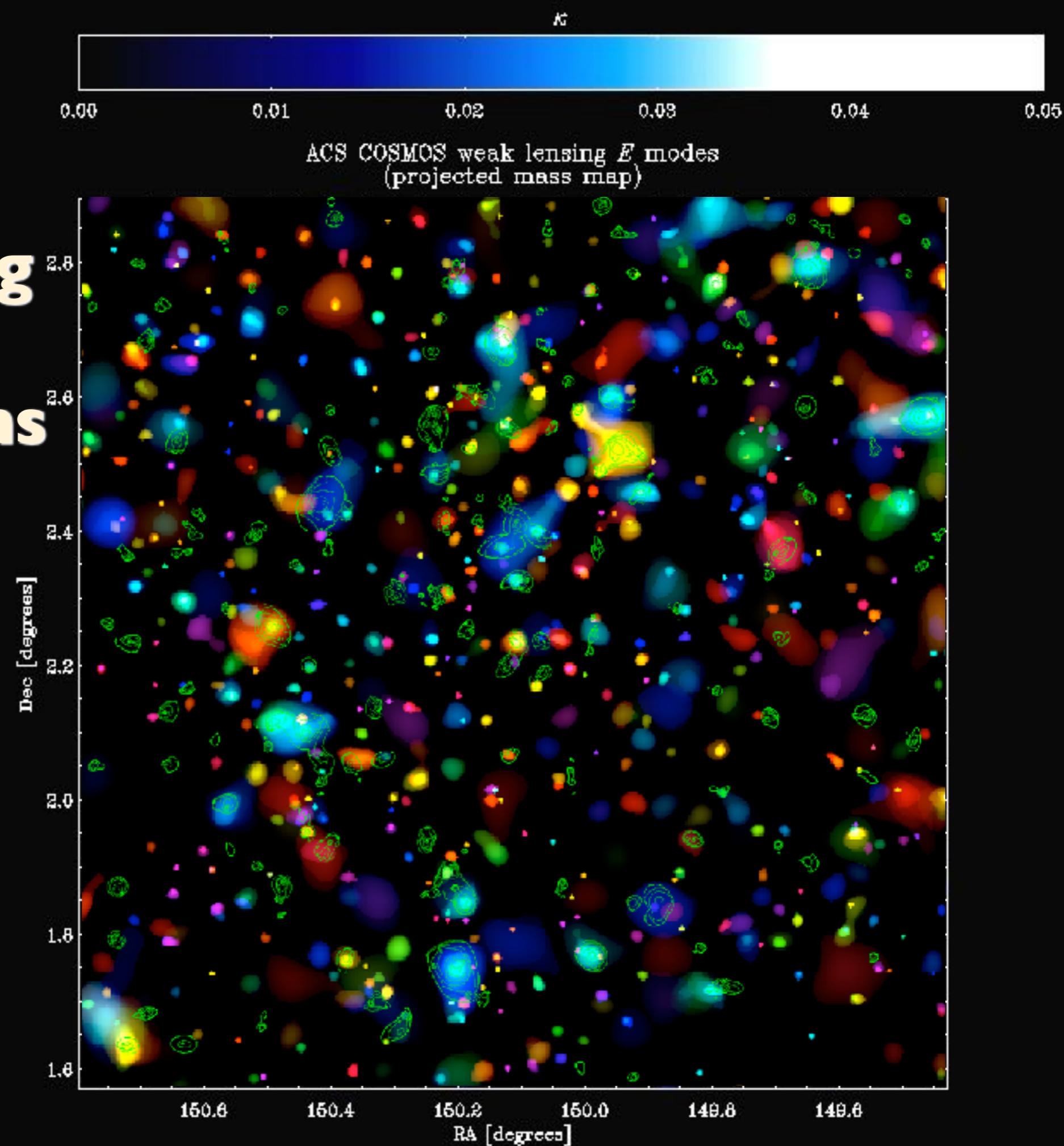
Comparison to other *Hubble* Surveys



Relative Sizes of *HST* ACS Surveys

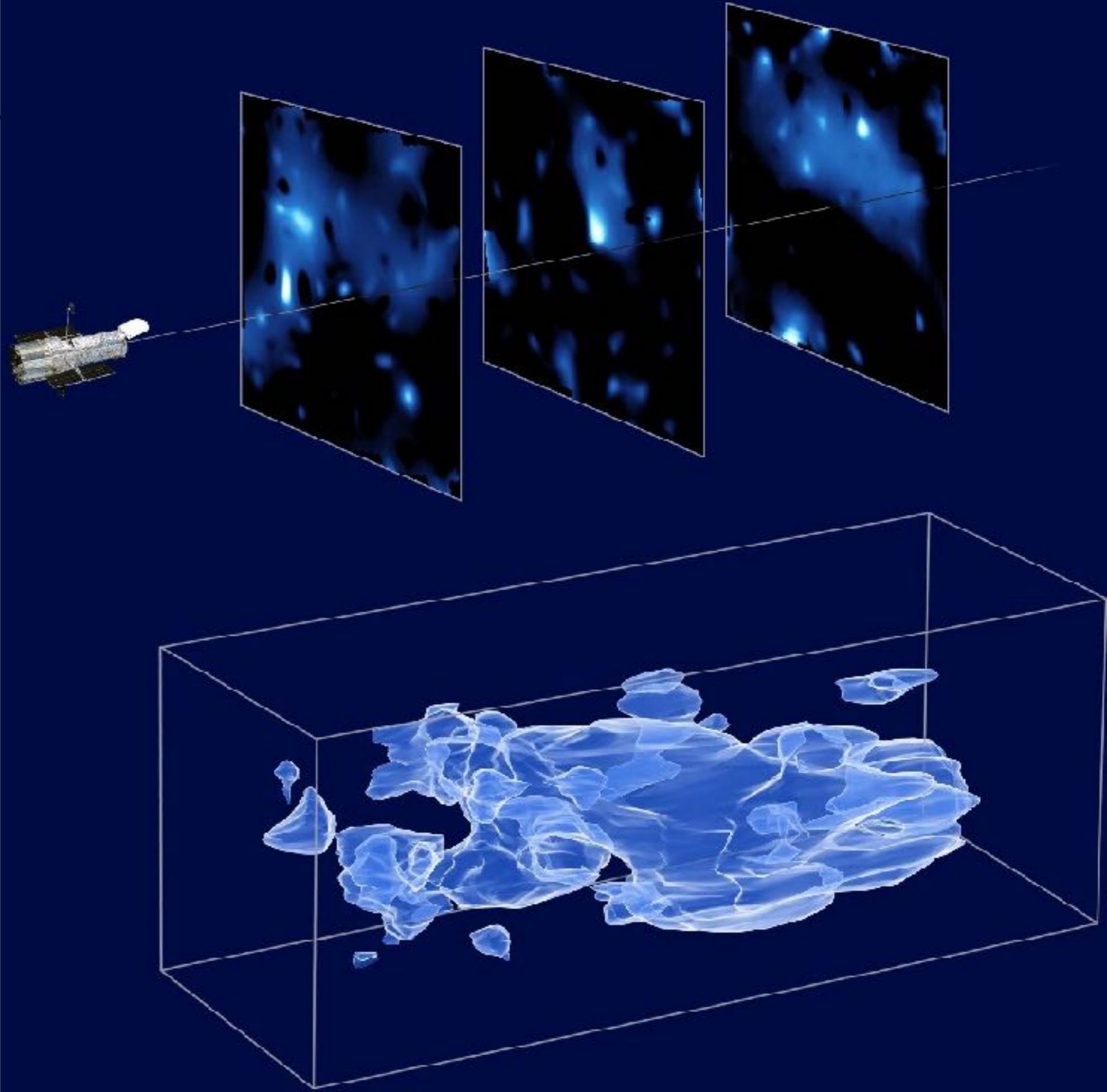


Mass mapping and concentrations of galaxies



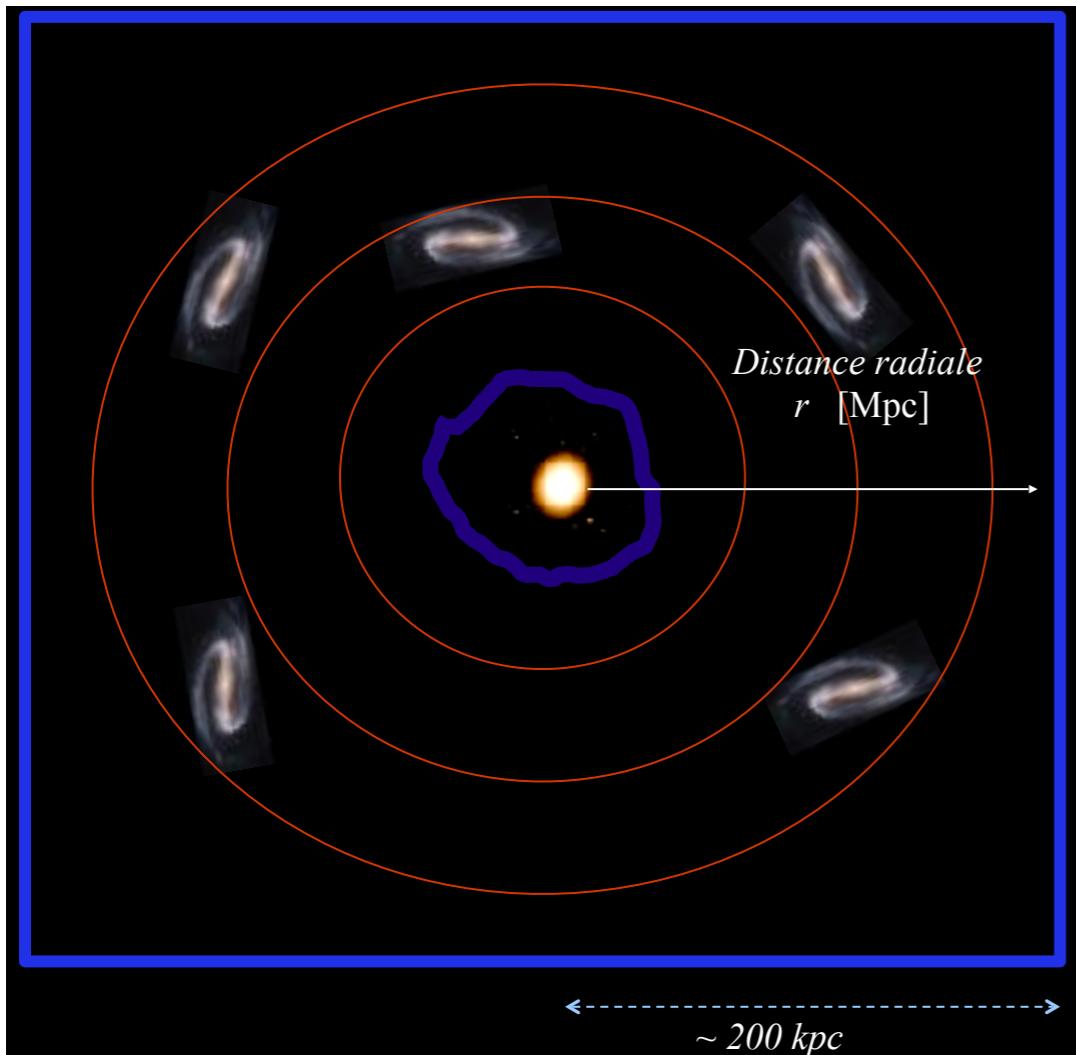
Tomography of mass distribution

combining the
lensing
information and
distances
(redshift),
=>
Mapping in 3D
of the (dark)
matter.



Galaxy-galaxy lensing techniques

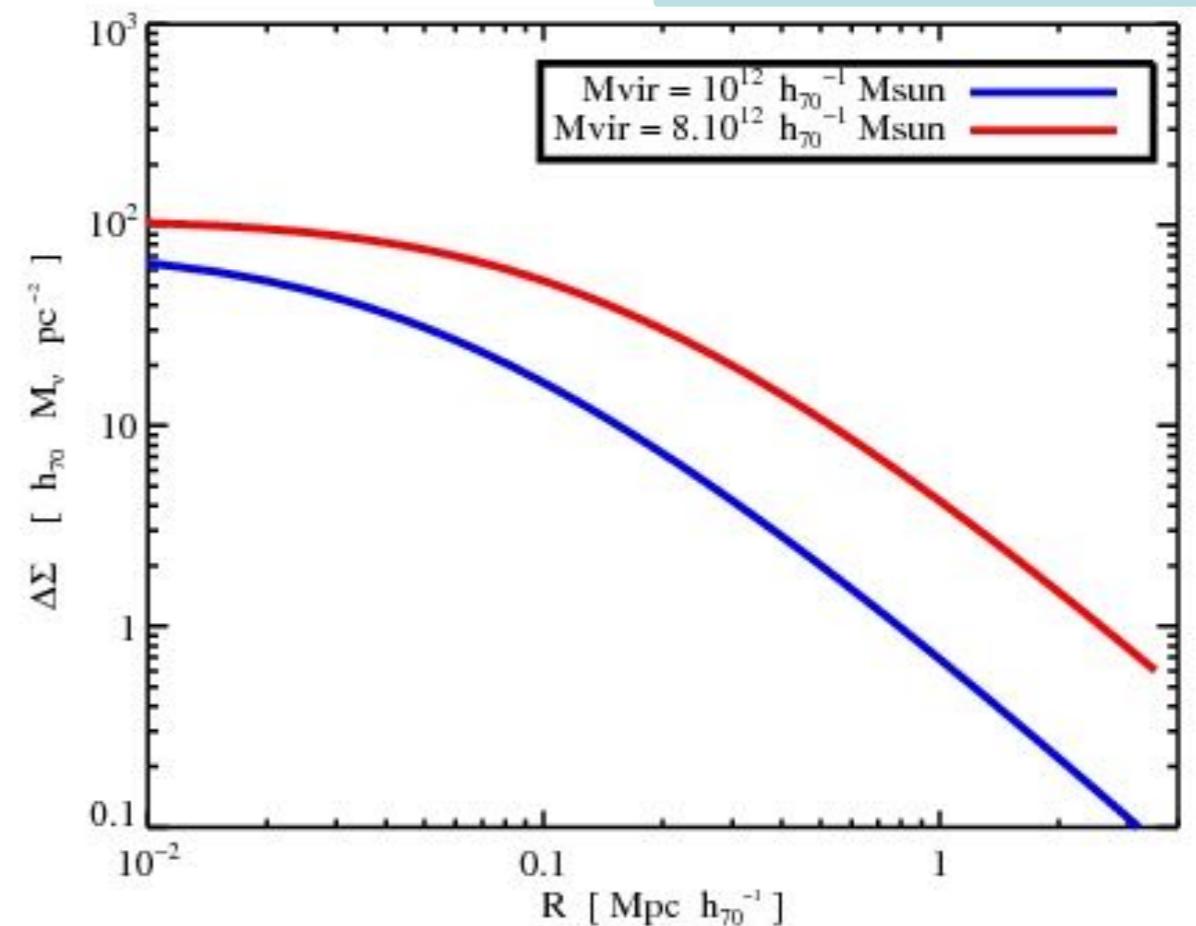
Galaxy-Galaxy Lensing



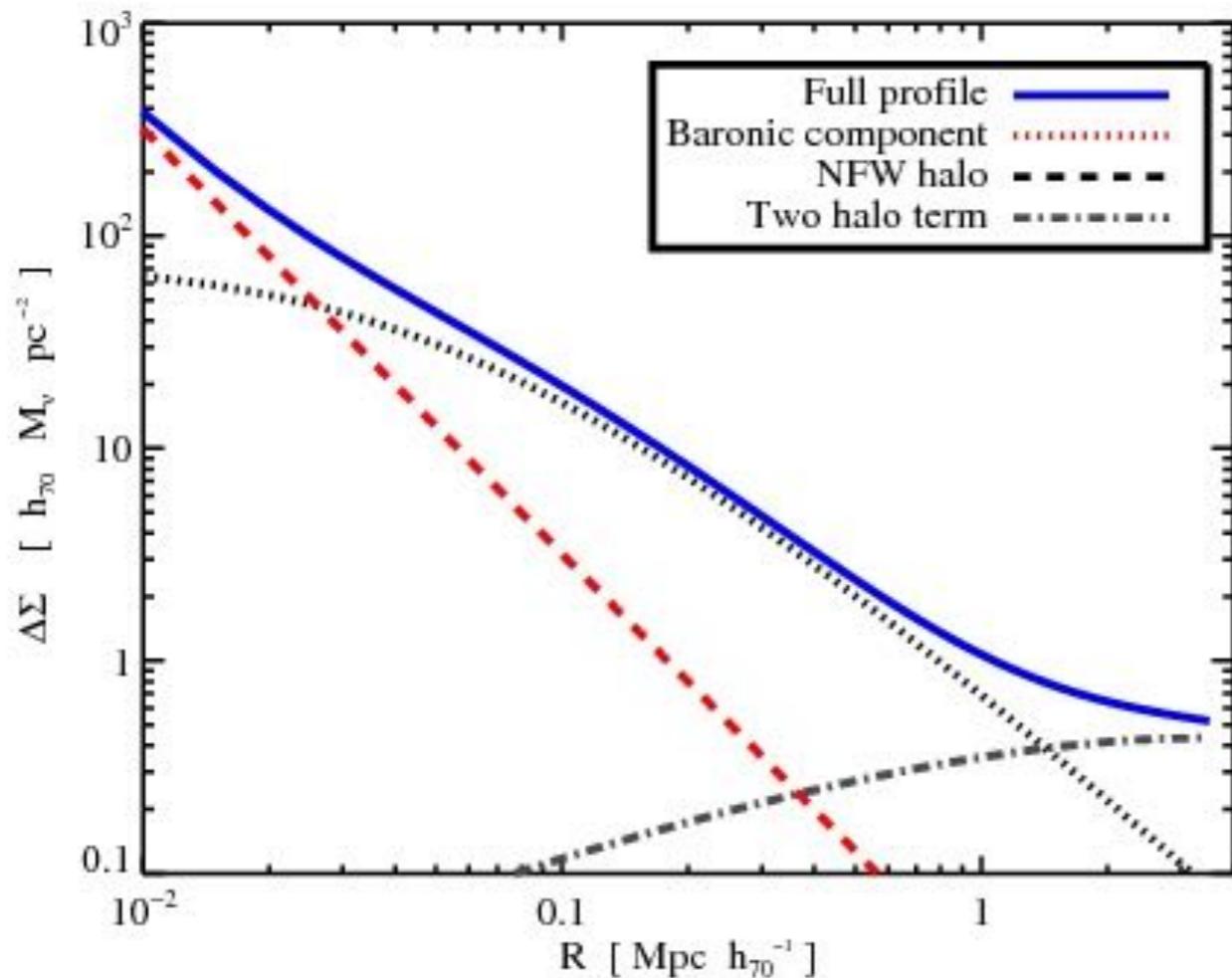
- Average tangential weak lensing signal in radial bins

$$\Delta\Sigma(r) \equiv \bar{\Sigma}(< r) - \bar{\Sigma}(r) = \Sigma_{crit} \times \gamma_t(r)$$

$$\Sigma_{crit} = \frac{c^2}{4\pi G} \frac{D_{OS}}{D_{OL} D_{LS}}$$

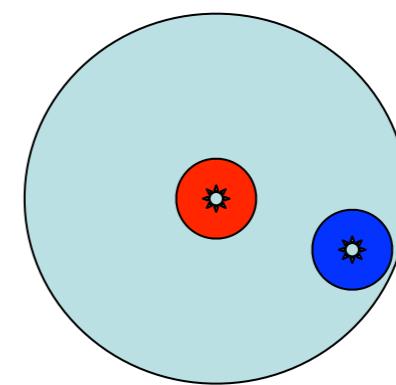


Interpreting galaxy-galaxy lensing signal

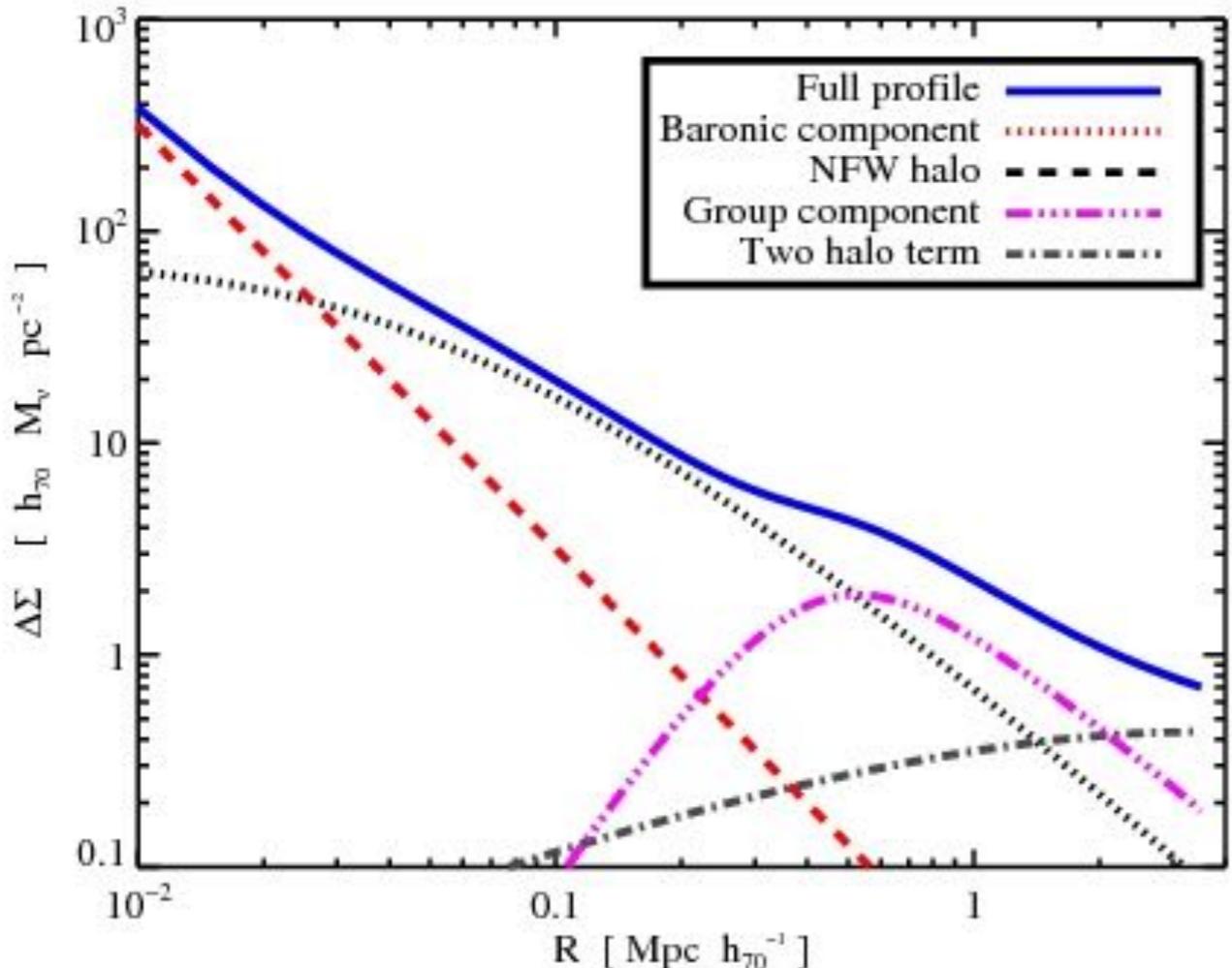


- 3 main contributions to the lensing signal:
 - Stellar mass
 - Dark Matter halo
 - Sub-halo contribution (satellite galaxies)

$$\Delta\Sigma_{\text{cent}} = \Delta\Sigma_{\text{cent}}^{\text{b}} + \Delta\Sigma_{\text{cent}}^{\text{nfw}} + \Delta\Sigma_{\text{cent}}^{\text{hh}}$$

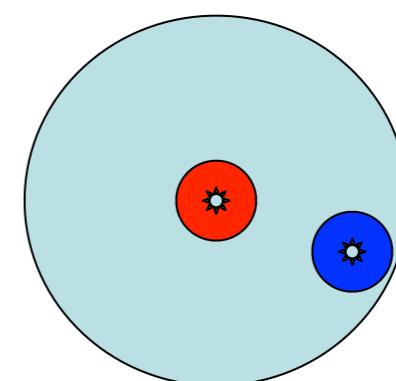


Interpreting galaxy-galaxy lensing signal

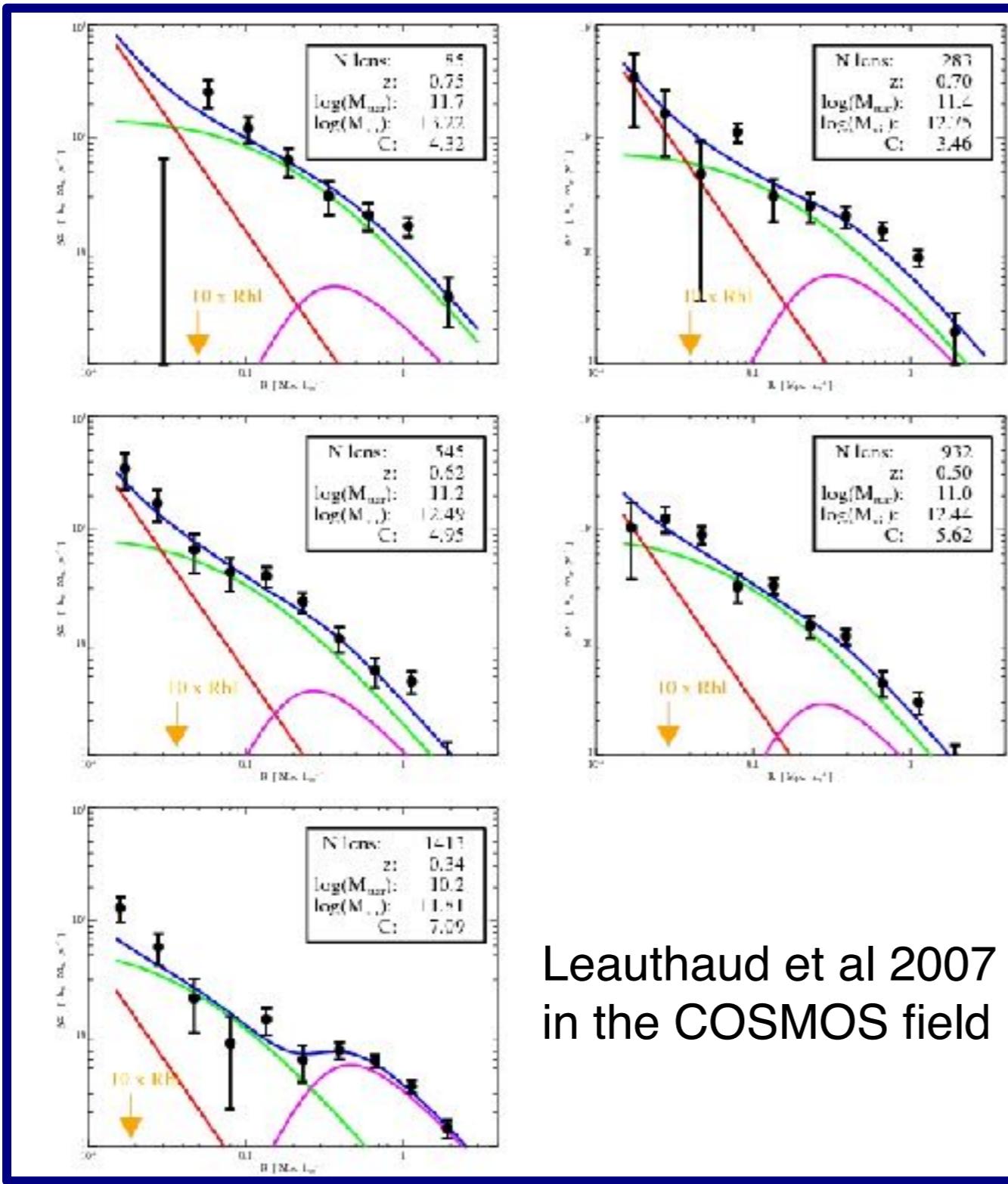


- 3 main contributions to the lensing signal:
 - Stellar mass
 - Dark Matter halo
 - Sub-halo contribution (satellite galaxies)
- Group/cluster contribution for non central galaxies

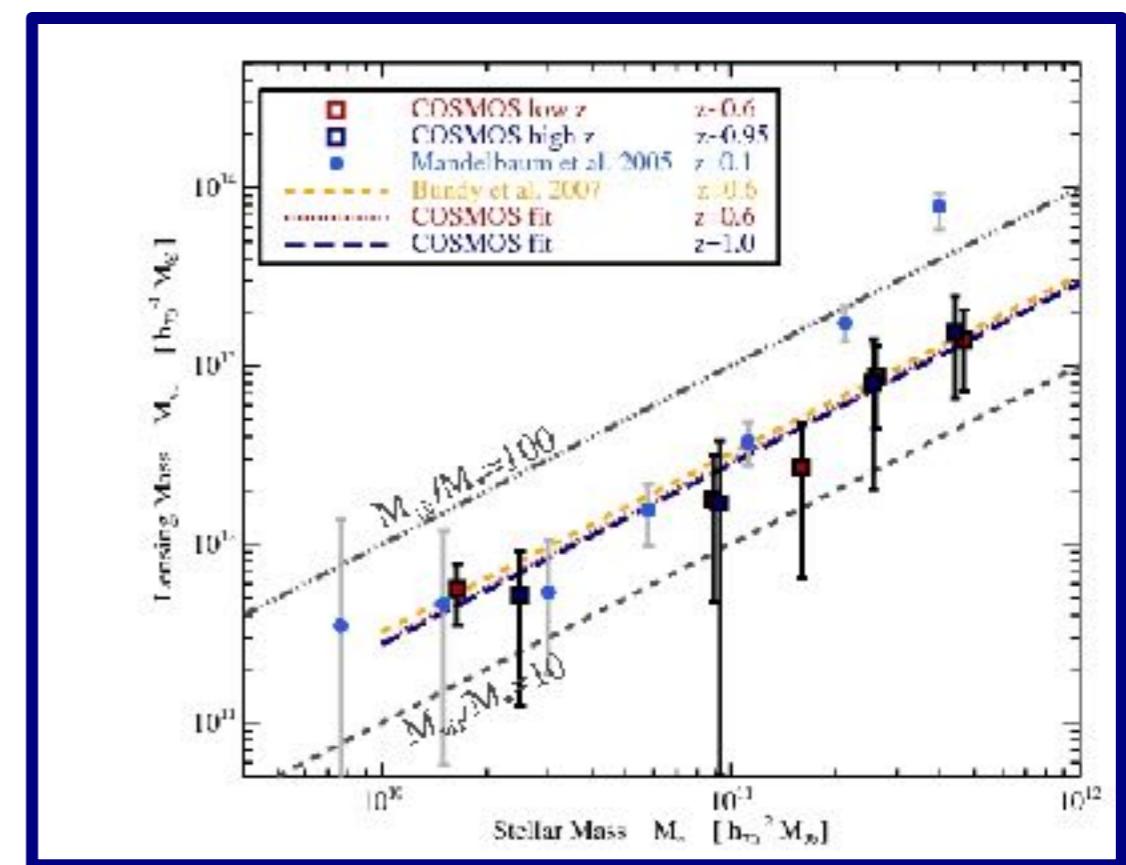
$$\Delta\Sigma_{sat} = \Delta\Sigma_{sat}^b + \Delta\Sigma_{sat}^{nfw} + \Delta\Sigma_{sat}^{host} + \Delta\Sigma_{sat}^{hh}$$



Example of measurements

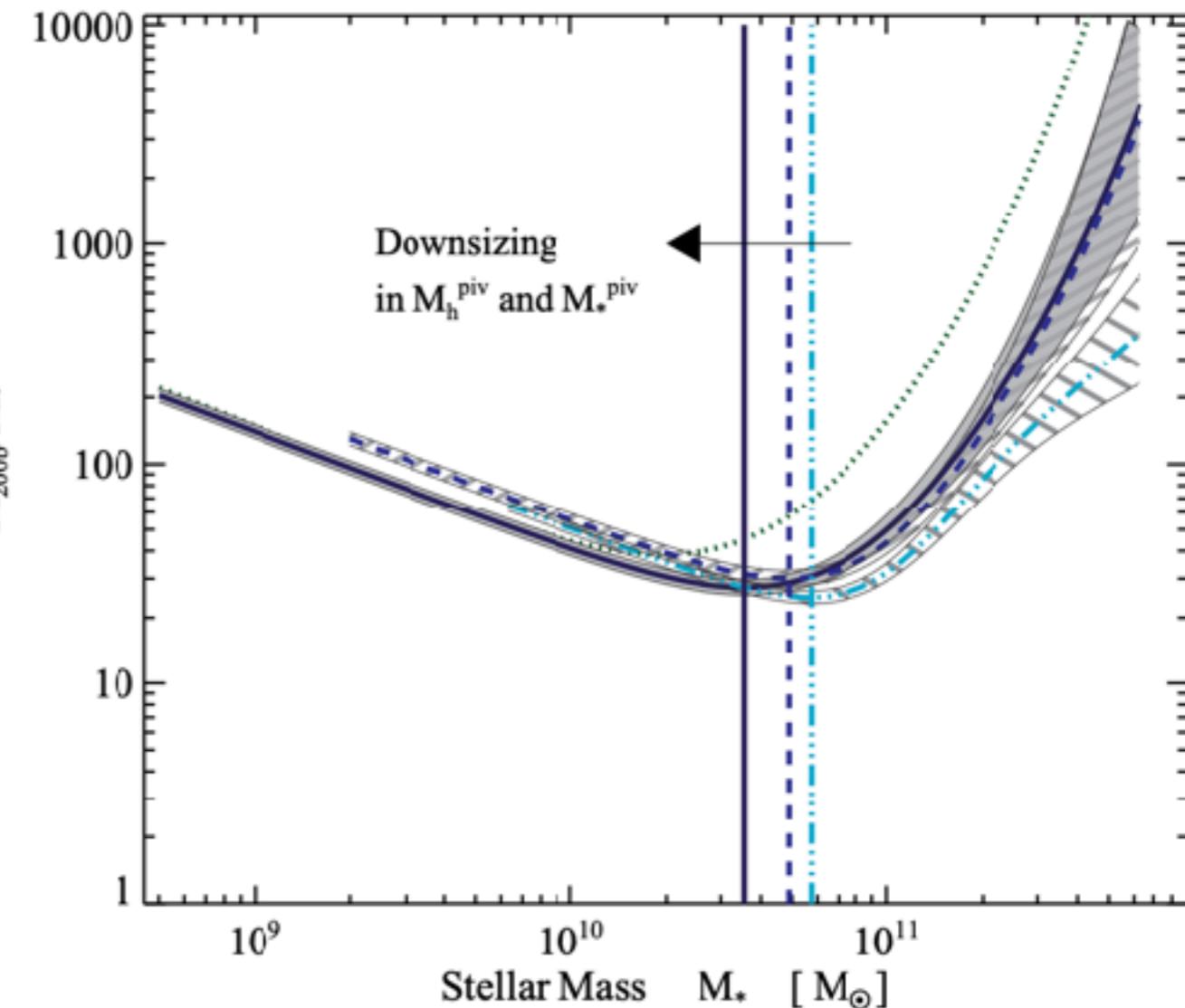
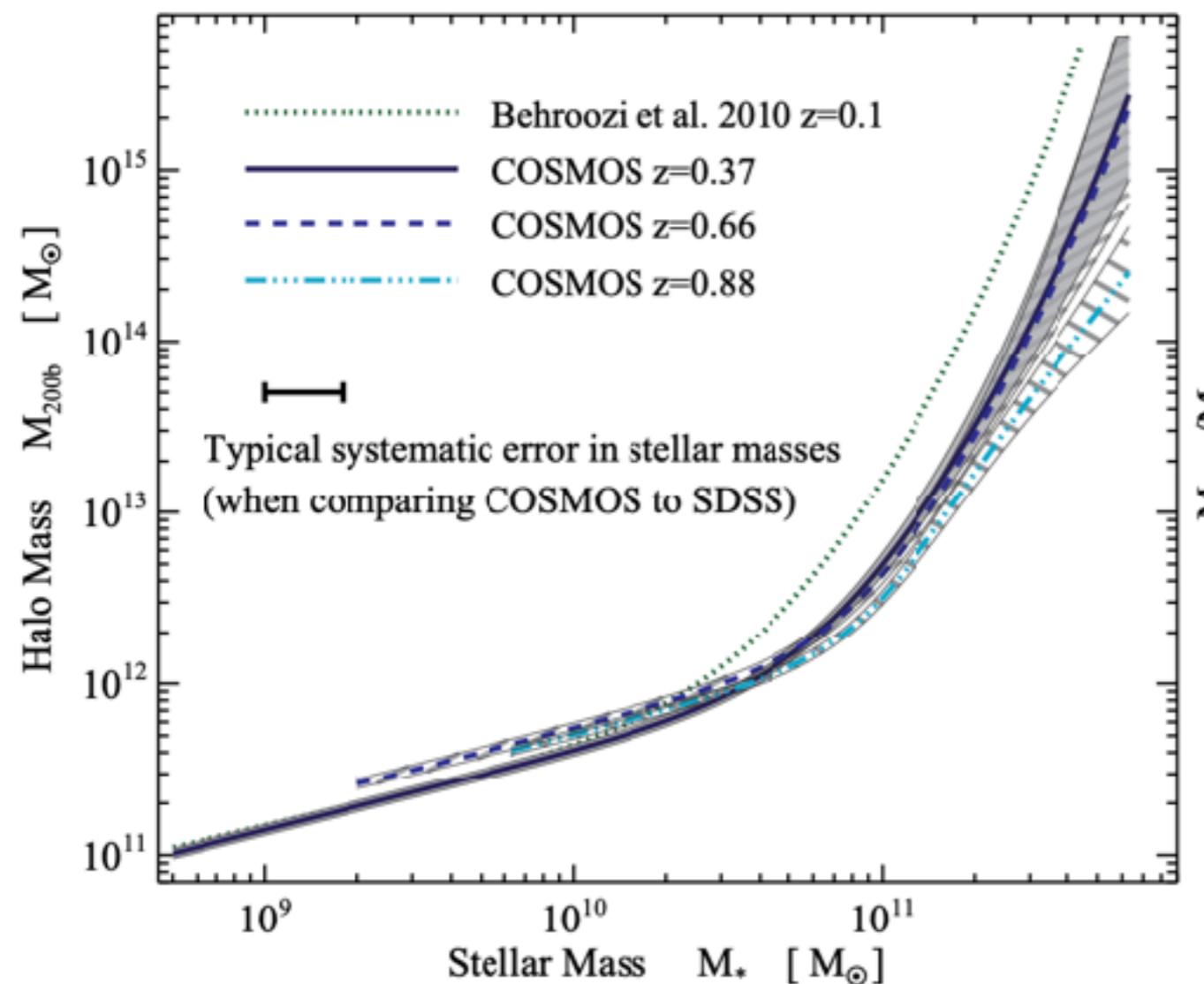


Leauthaud et al 2007
in the COSMOS field



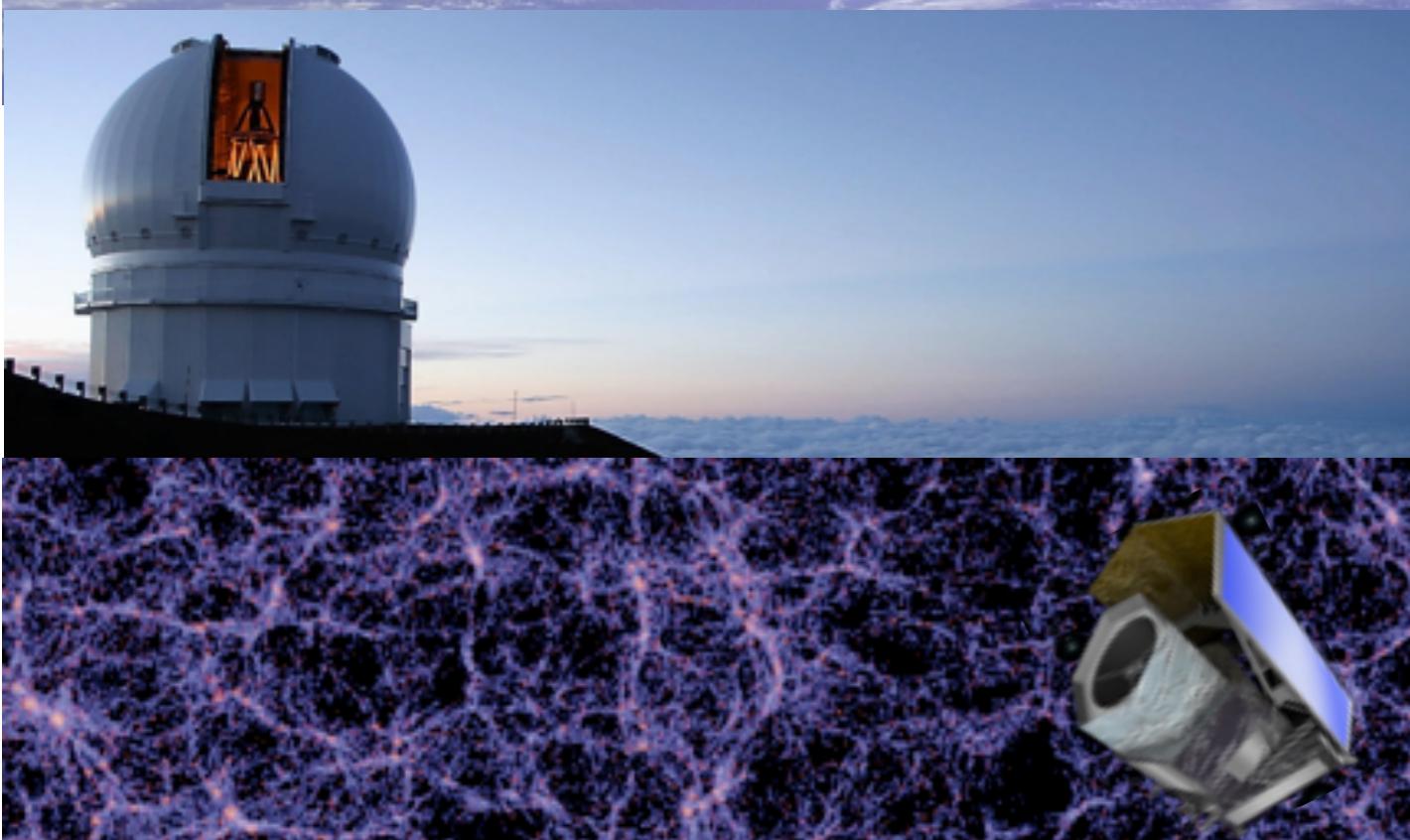
Results M^* versus Halo Mass

- Results for the COSMOS field observation (1.5 sqdeg with HST data)



Leauthaud et al 2012

Projects mapping the mass of the universe



- **Hubble** precise imaging ($0.1''$), but small field of view (clusters, COSMOS: $\sim 1.5 \text{ deg}^2$).
- **CFHT** first wide field imager $\sim 300 \text{ deg}^2$.

- **Ground projects:** KIDS(2014-22), DES(2015-22), HSC(2016-23), LSST(2025-35) from **1'500 to 20'000 deg²**

- **Euclid**: ESA space mission (2024-2029) $\sim 15'000 \text{ deg}^2$
- **SKA**: radio-telescope ? (2030-2040) $\sim 20'000 \text{ deg}^2$ (?)