

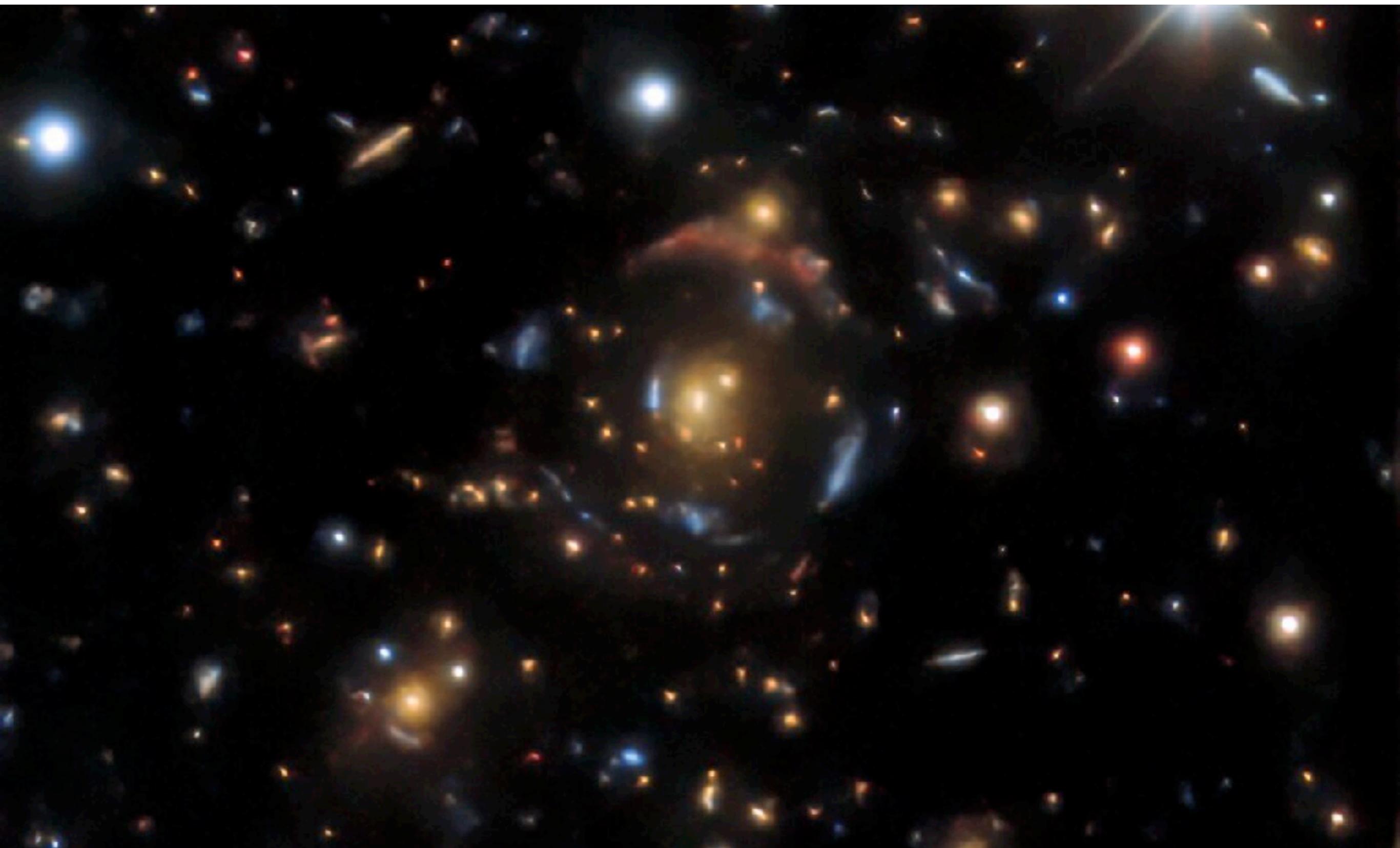
Gravitational Lensing - 2

Jean-Paul KNEIB

May 7, 2020

<https://arxiv.org/abs/2005.04730>

Artificial Intelligence finds more than 1,200 Gravitational Lensing Candidates In the DESI Legacy Imaging Survey



Quiz - 1

- How to explain the effect of gravitational lensing?
- Who had the first vision? How can it be useful?
- What is strong lensing? Is it common?
- What is weak lensing?
- What does mean “multiple images”?
- What are the main strong lensing configuration for an elliptical lens?
- What are the “critical lines”?
- What are the “caustic lines”?
- What is the Einstein radius?

Lensing Equations

Lens Mapping:

Lensing Potential

$$\vec{\theta}_S = \vec{\theta}_I - \frac{2\mathcal{D}}{c^2} \vec{\nabla} \phi_N^{2L}(\vec{\theta}_I) = \vec{\theta}_I - \vec{\nabla} \varphi(\vec{\theta}_I)$$

φ : lensing potential

- ⇒ Link with catastrophe theory
- ⇒ Parameters: **Distances and Mass**
- ⇒ Purely geometrical: *Achromatic effect*

Lensing Theory

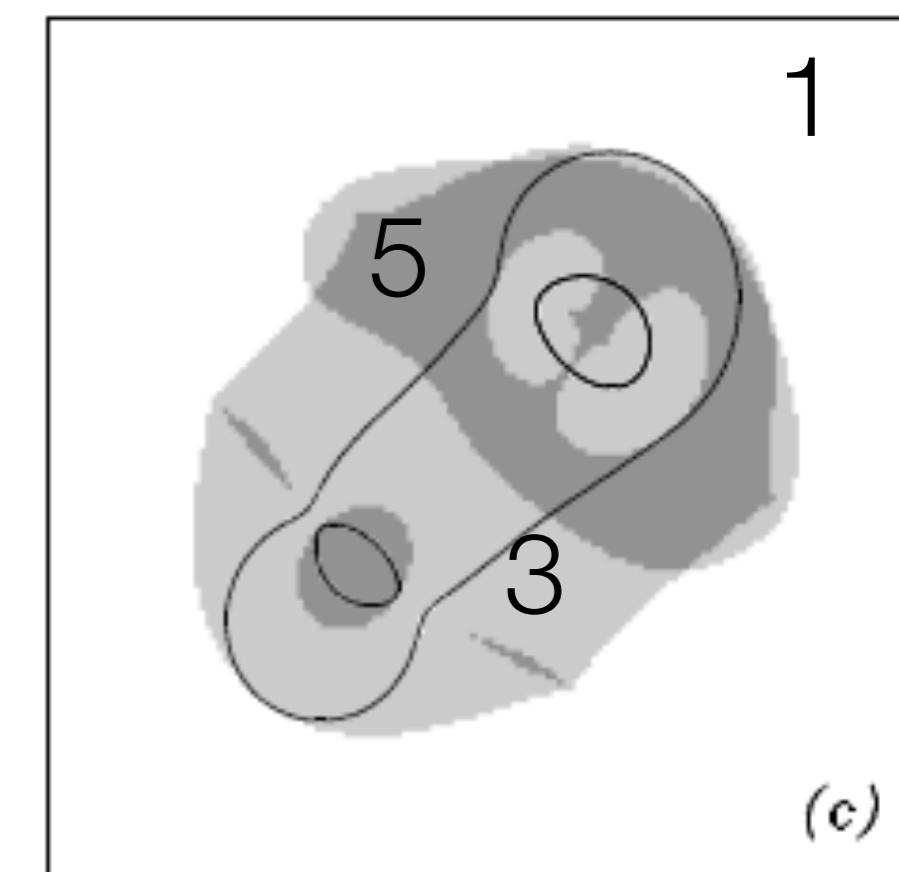
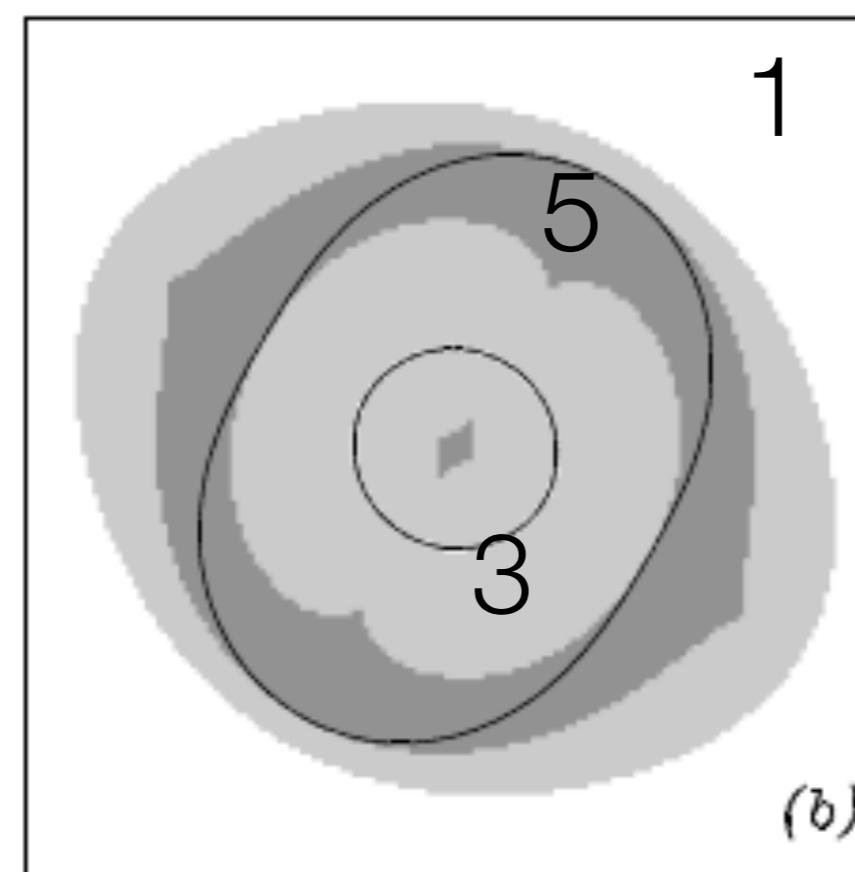
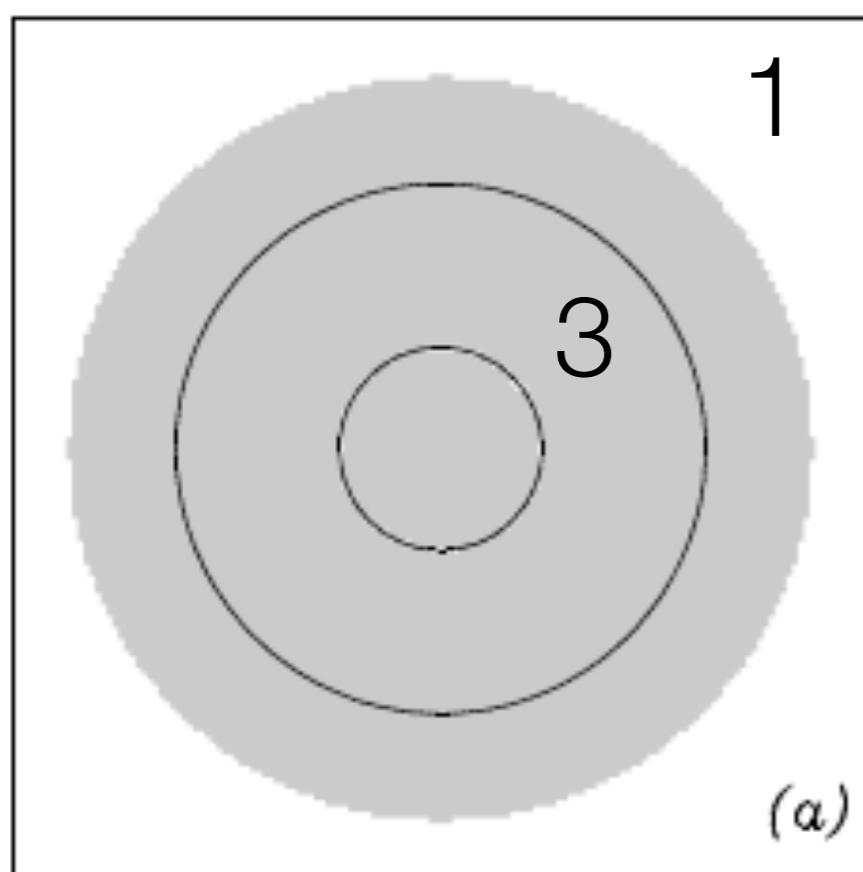
Multiple image region for:

Circular

Elliptical

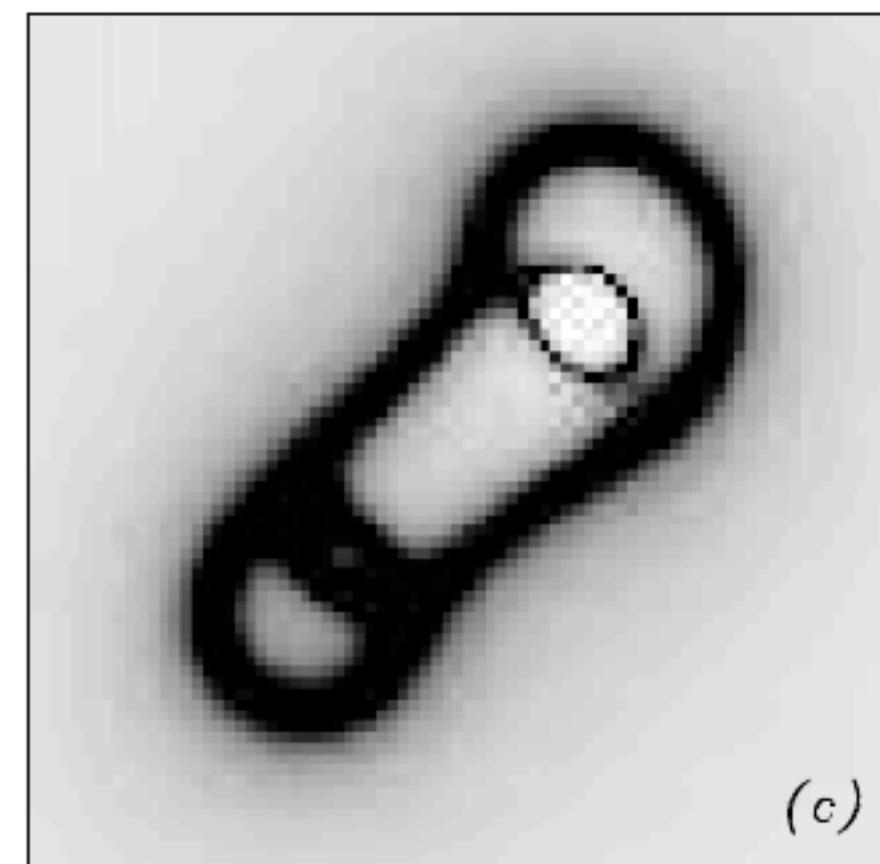
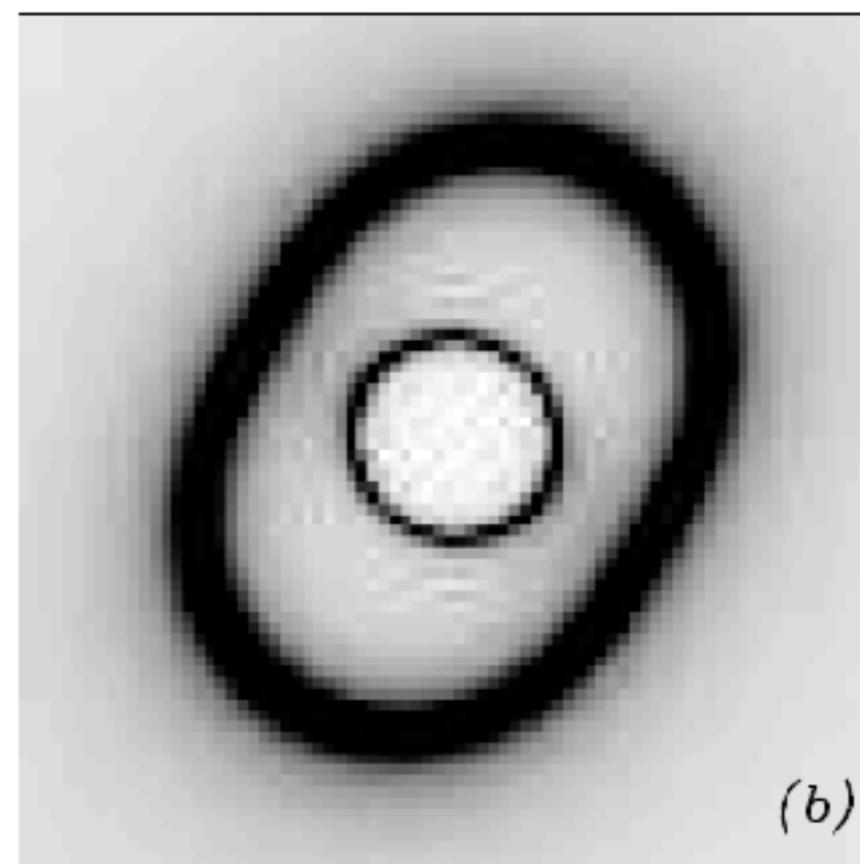
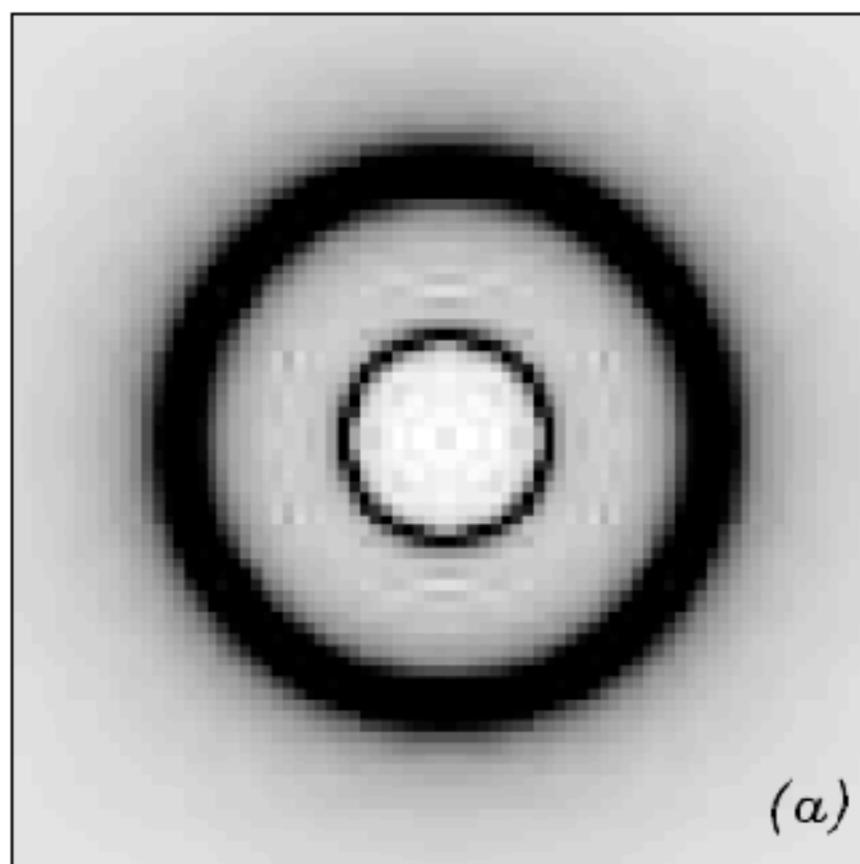
Bimodal

mass distribution



Lensing Theory

Amplification map for:
Circular Elliptical
mass distribution



Lensing Theory

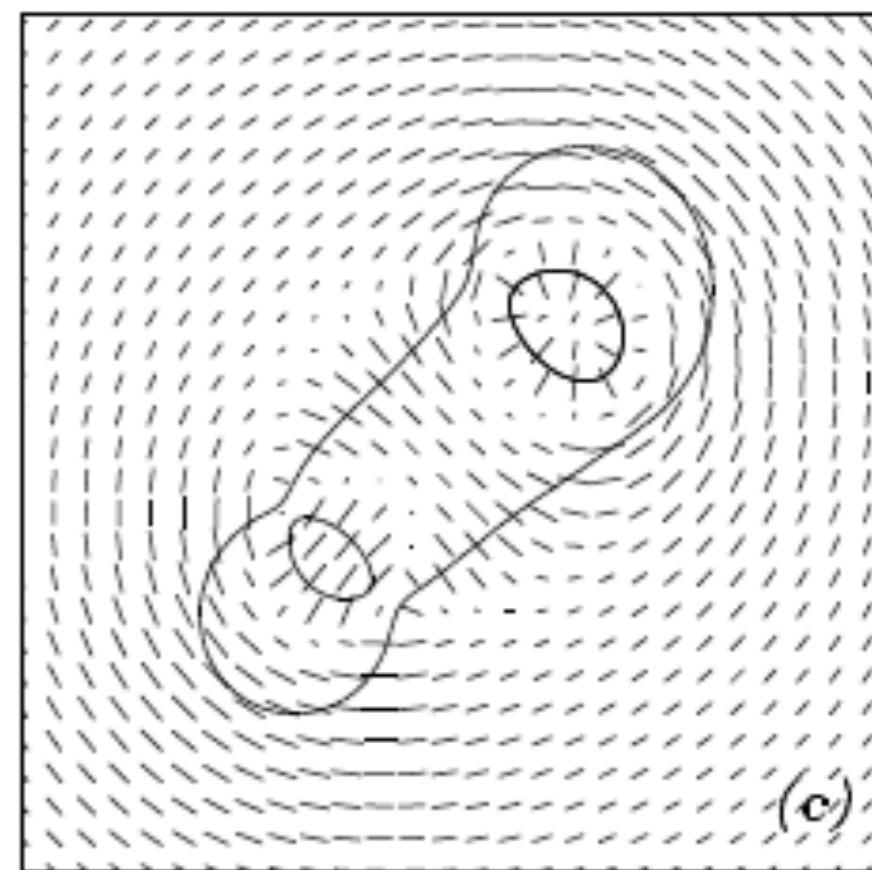
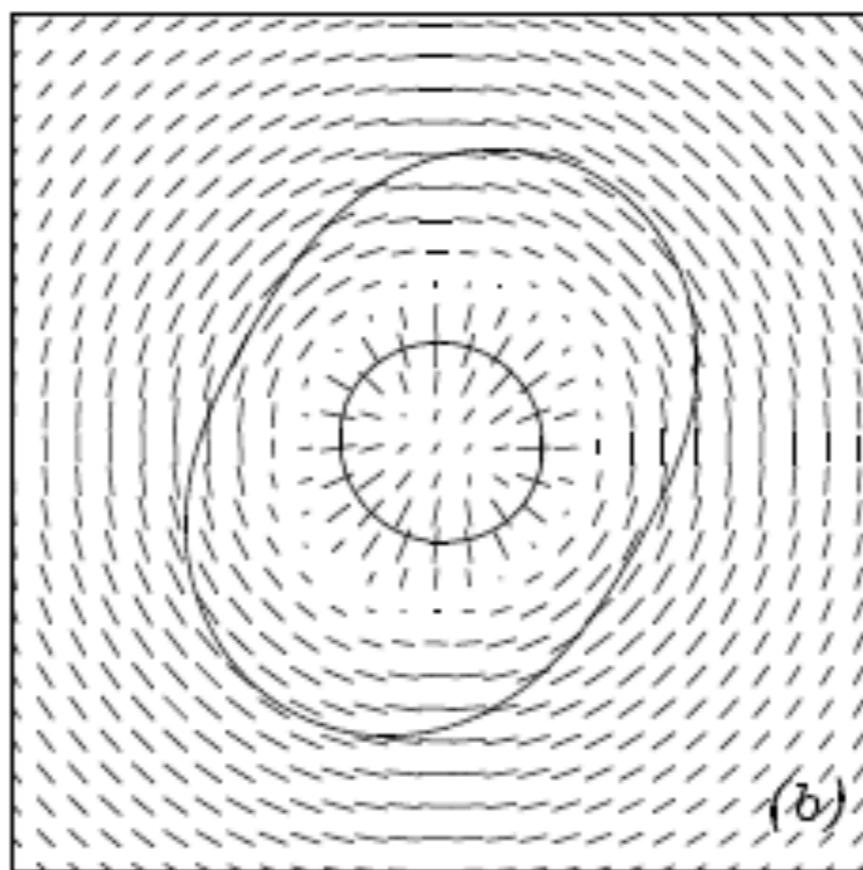
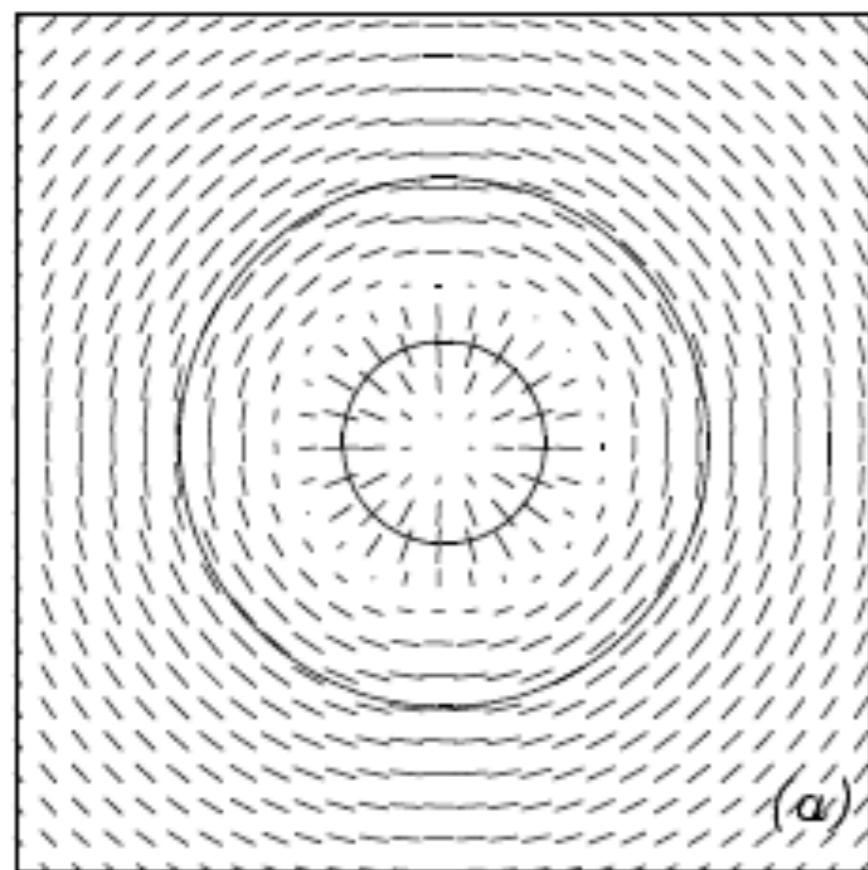
Shear field for:

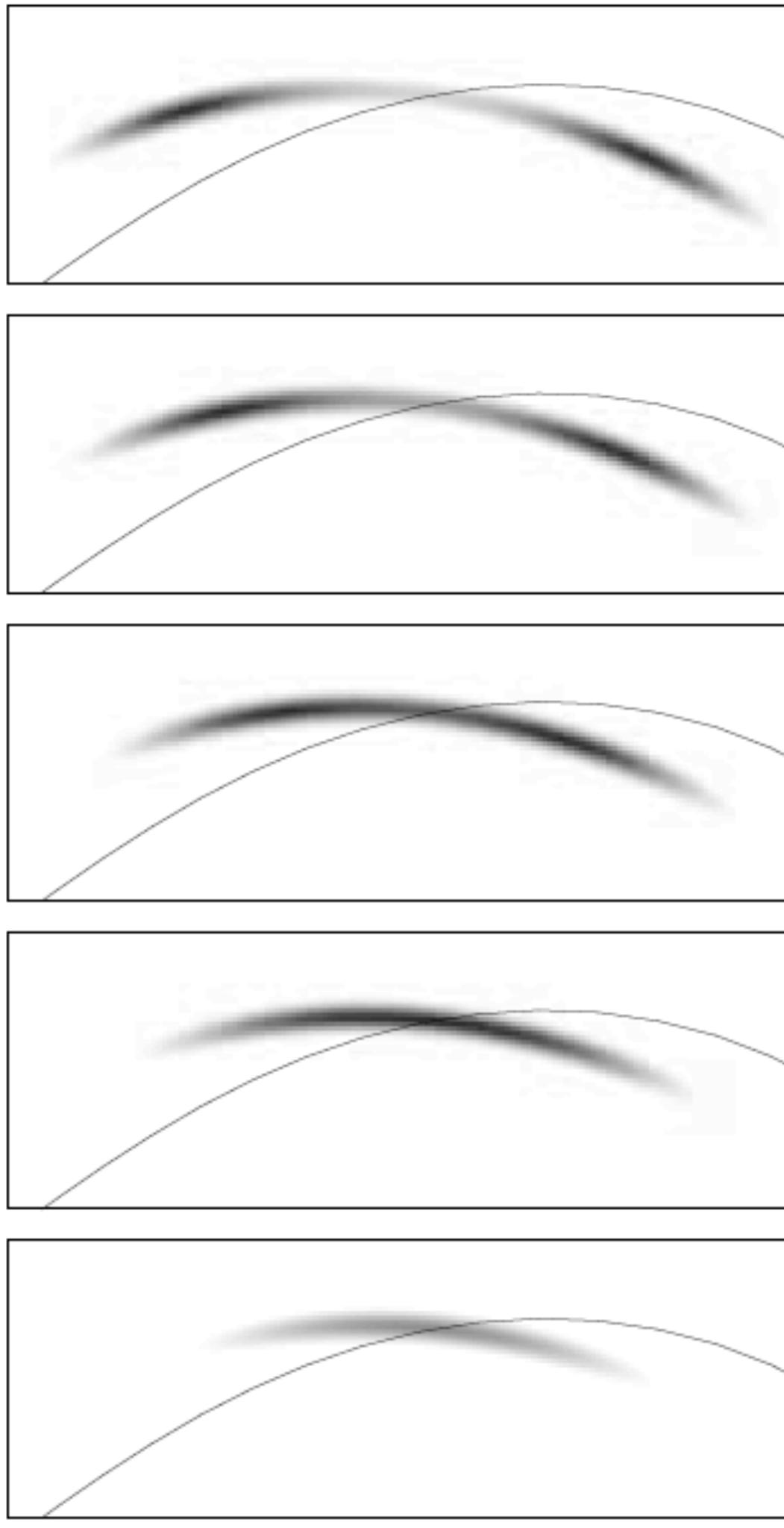
Circular

Elliptical

Bimodal

mass distribution





Strong Lensing

Schematic of image fusion on the critical line

Magnification is due to stretching of the image:
surface brightness is conserved through lensing

Strong Lensing

Image parity

Elliptical case

$(+,+)$

$(+,-)$

$(-,-)$

(a)

Bimodal case

$(+,+)$

$(+,-)$

(b)

Mass & Einstein Radius

Mass within the Einstein ring:

- mass as a function of lensing potential (circular case):

$$M(r) = \frac{c^2}{4G} \frac{D_{OS} D_{OL}}{D_{LS}} r \partial_r \varphi(r)$$

- Mass within the tangential critical line (Einstein ring):

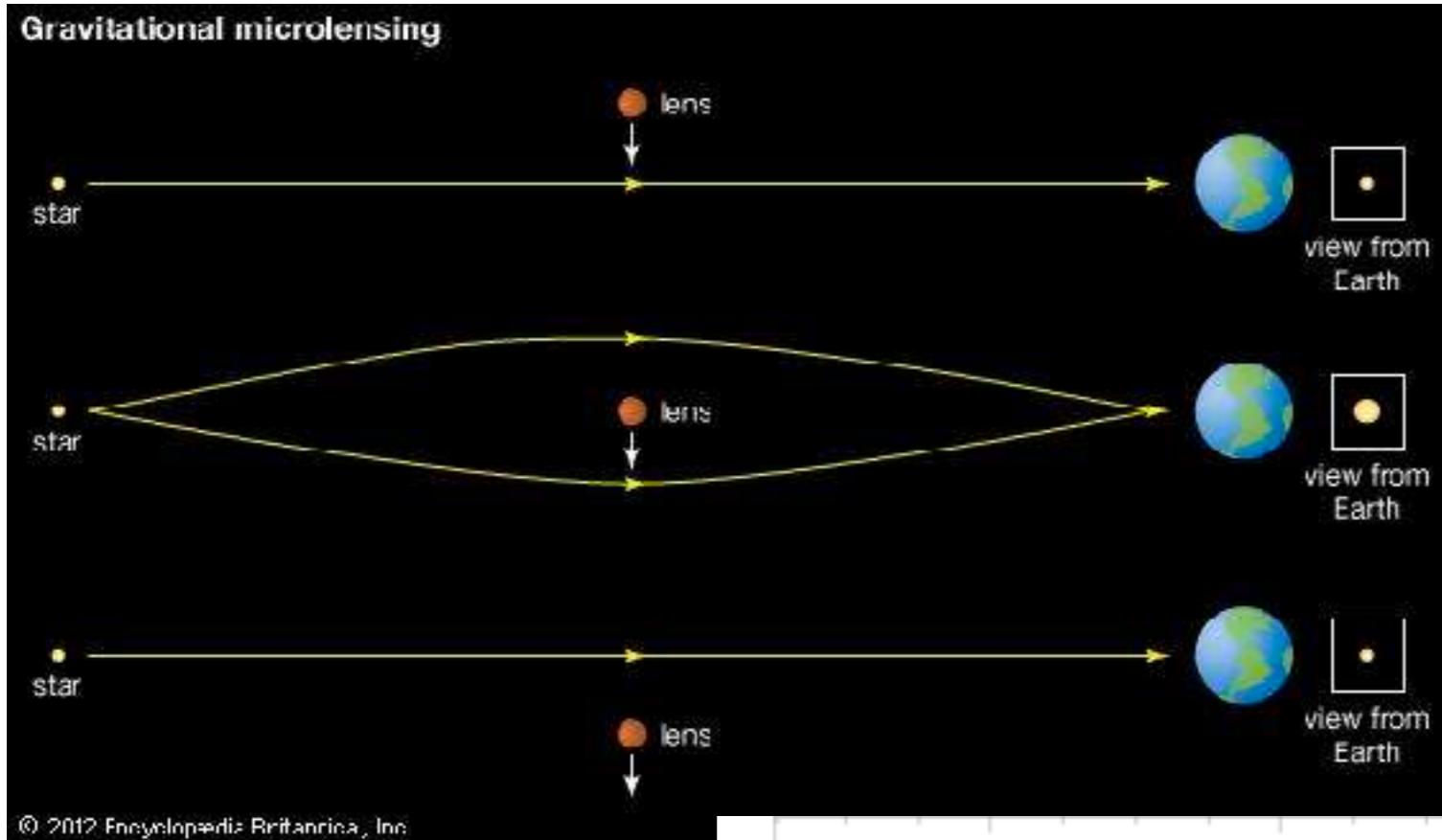
$$M(r_E) = \pi \Sigma_{crit} r_E^2$$

$$\approx 1.1 \times 10^{14} M_\odot \left(\frac{\theta_E}{30''} \right)^2 \left(\frac{D}{1 \text{ Gpc}} \right)$$

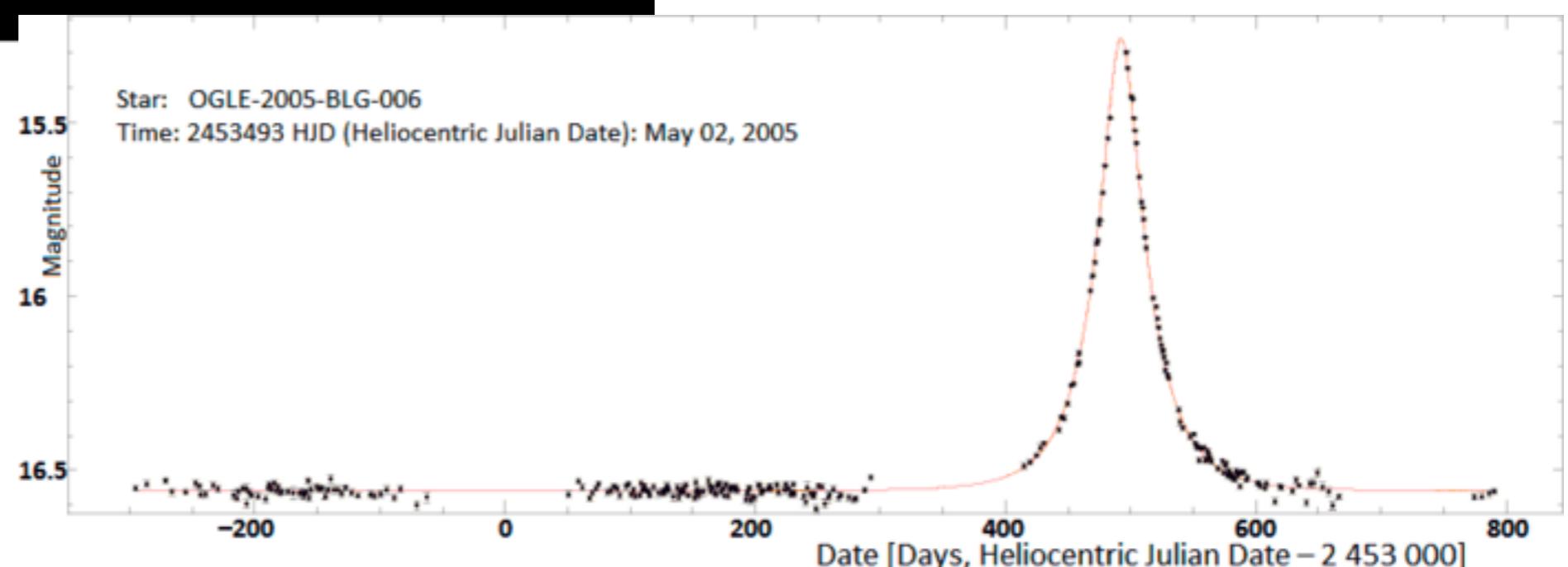
- Value of Einstein radius is not everything!
Anisotropy is also very important.

Micro-Lensing

Micro Lensing



- Moving object lensing a distant star
- Achromatic effect
- search for DM in compact form (EROS, MACHO, OGLE projects)



Micro Lensing - some numbers

- The lensing object is a point mass, Einstein radius is:

$$\theta_E = \sqrt{\frac{D_{LS}}{D_S D_L} \frac{4GM}{c^2}}$$

- Let's define the relative distance:

$$u = \frac{\theta_S}{\theta_E}$$

- The total magnification due to micro-lensing is:

$$A(u) = \frac{u^2 + 2}{u \sqrt{u^2 + 4}}$$

- Time to cross the Einstein radius

$$t_E$$

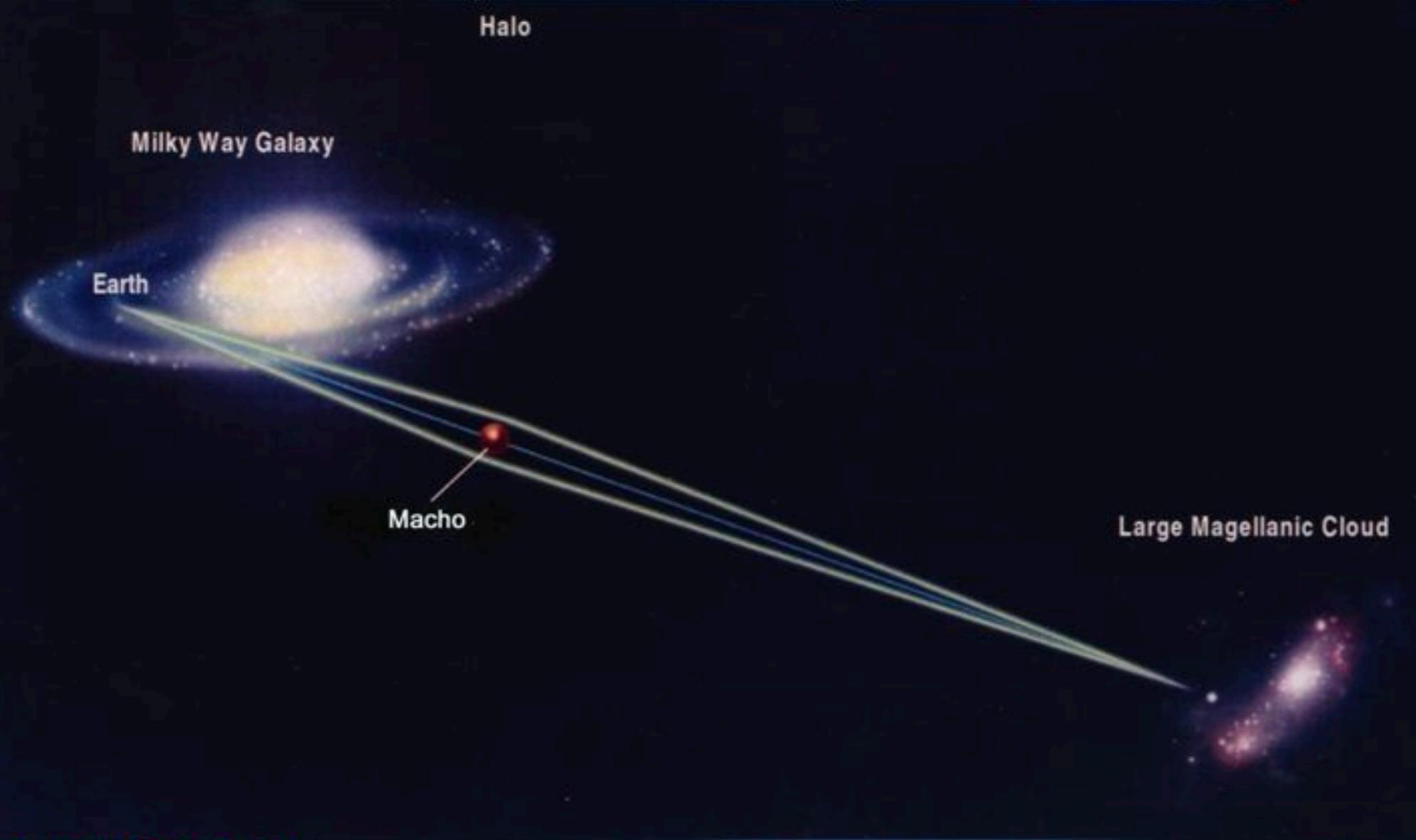
- Variation of the relative distance as a function of time:

$$u(t) = \sqrt{u_{min}^2 + \frac{(t-t_0)^2}{t_E^2}}$$

Halo Dark Matter & Paczynski's Idea

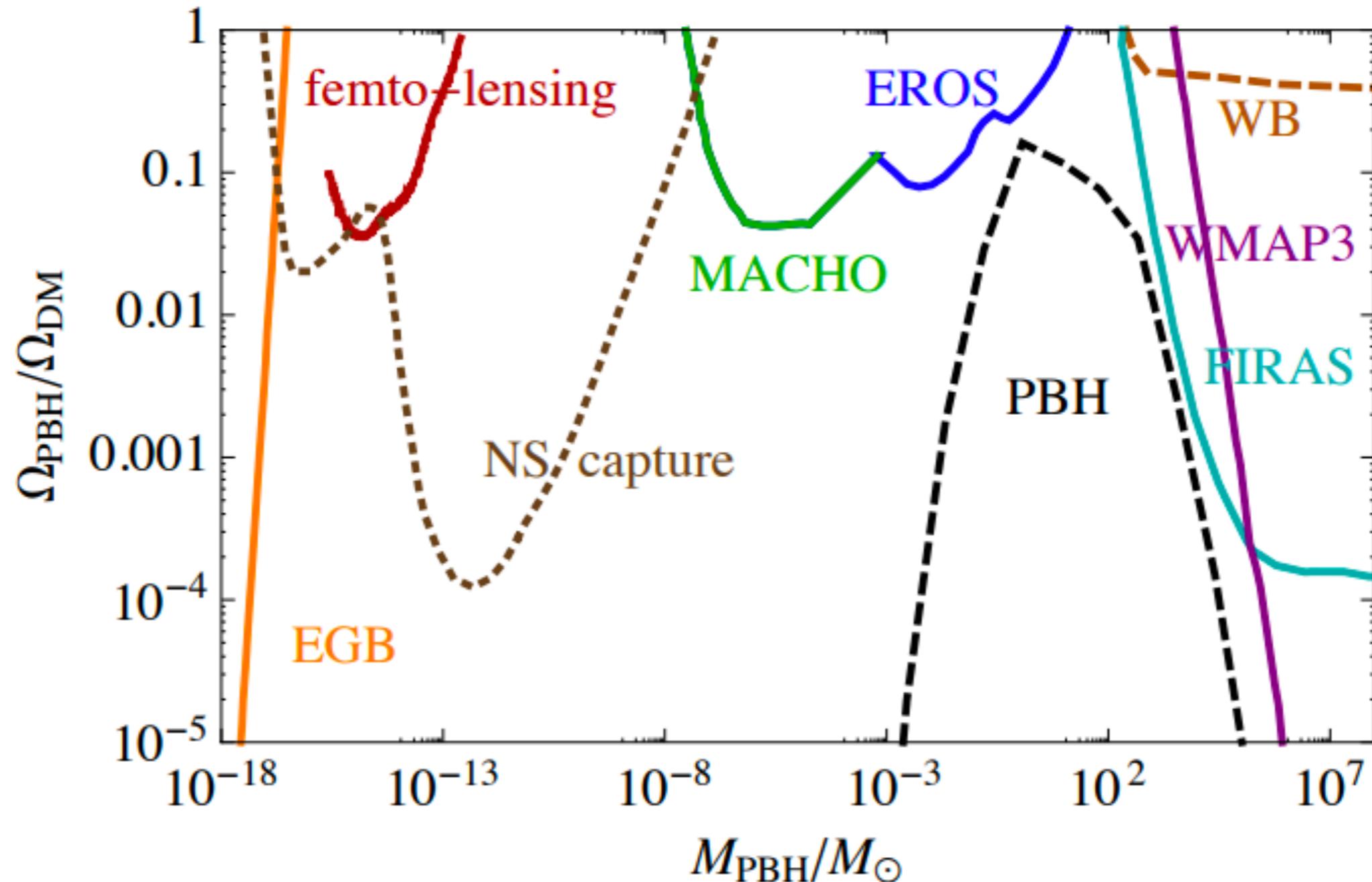
(Paczynski 1986)

- 20~40 times more dark matter than visible mass.
- MAssive Compact Halo Objects (MACHOs) \leftrightarrow WIMPs



- MACHO can be observed by Microlensing.
- $\tau \sim 10^{-6}$ \rightarrow need to observe 1M stars !

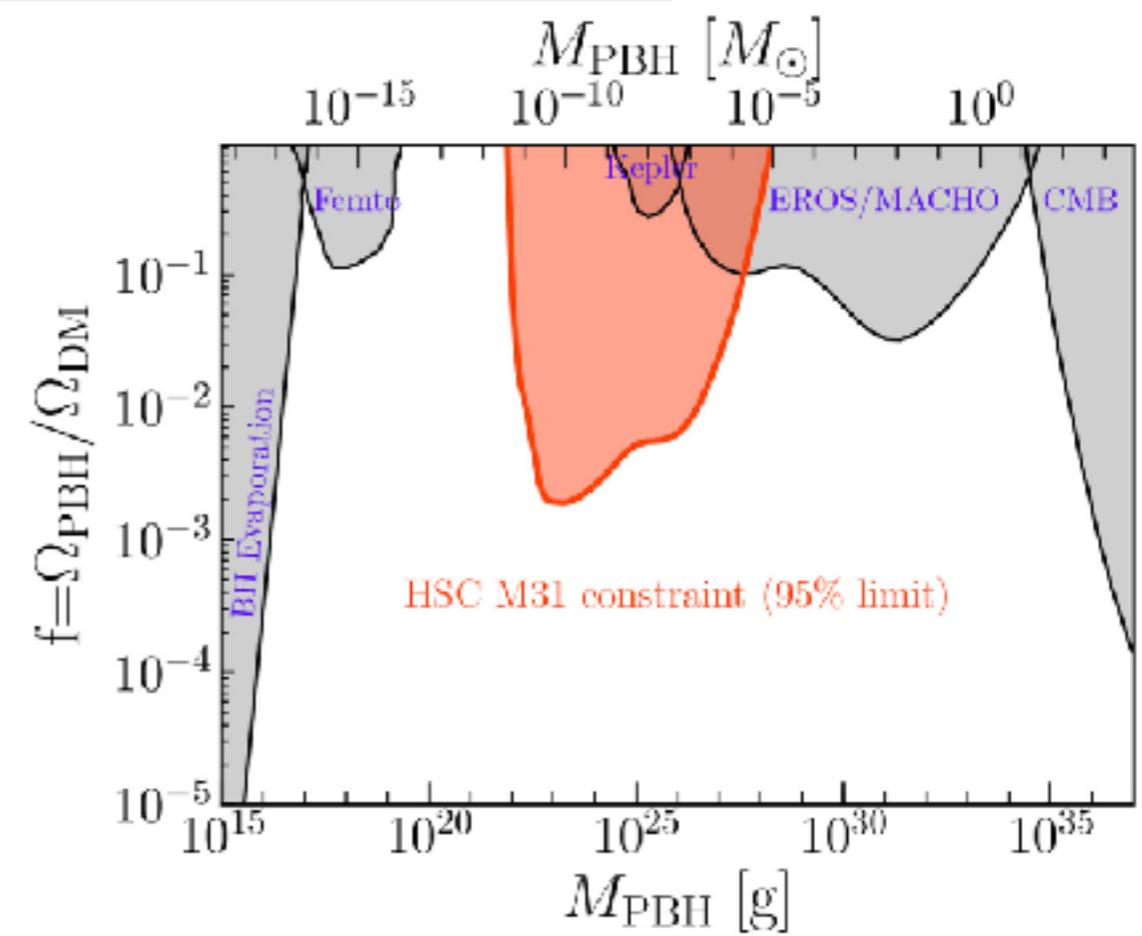
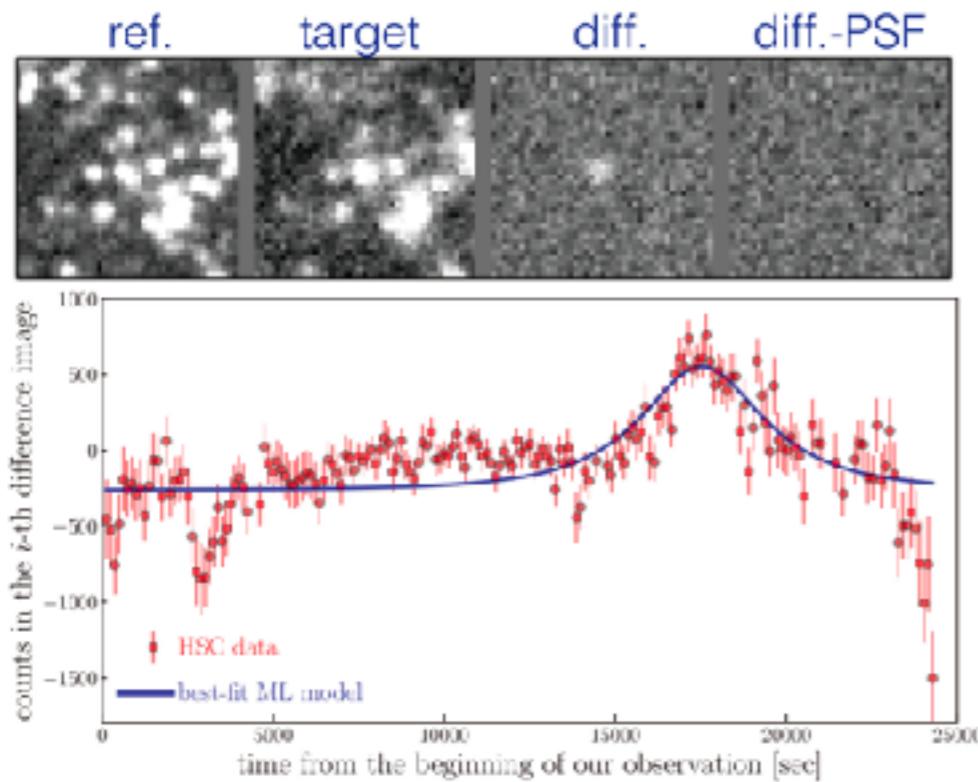
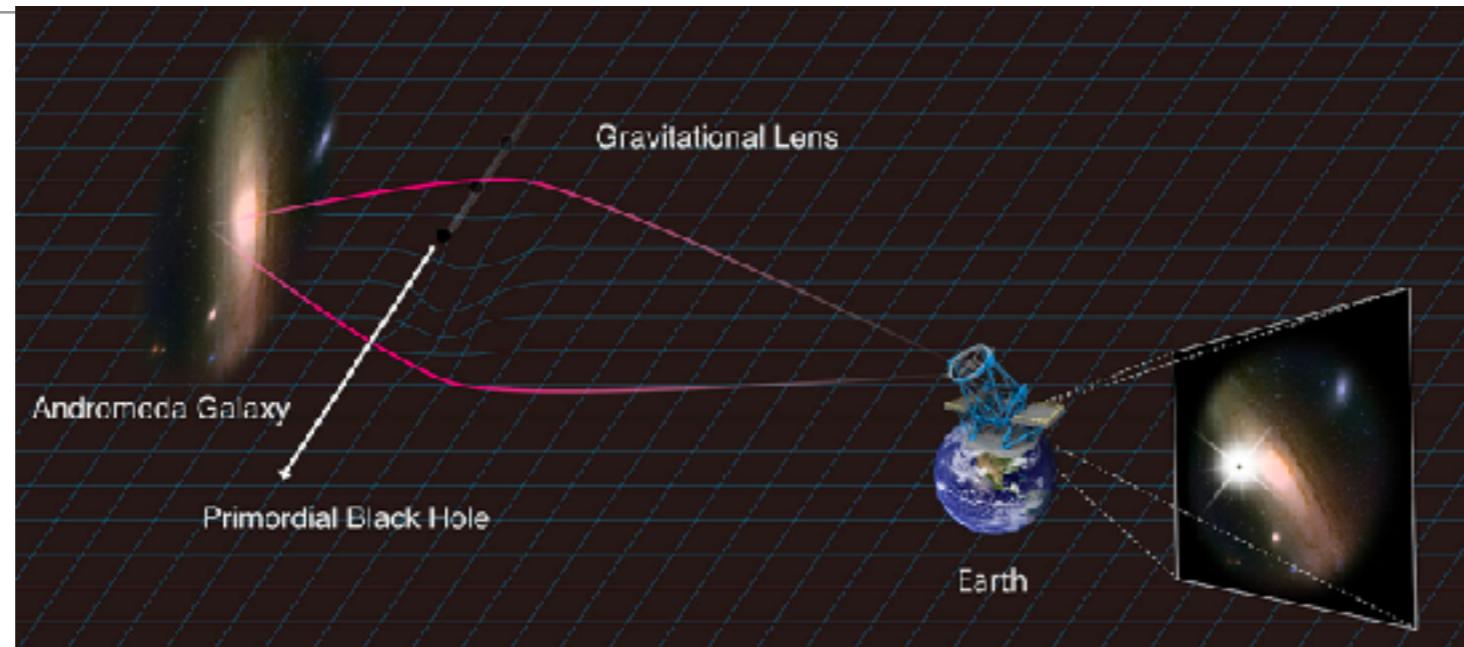
Constraints on the fraction of DM as compact object (BH)



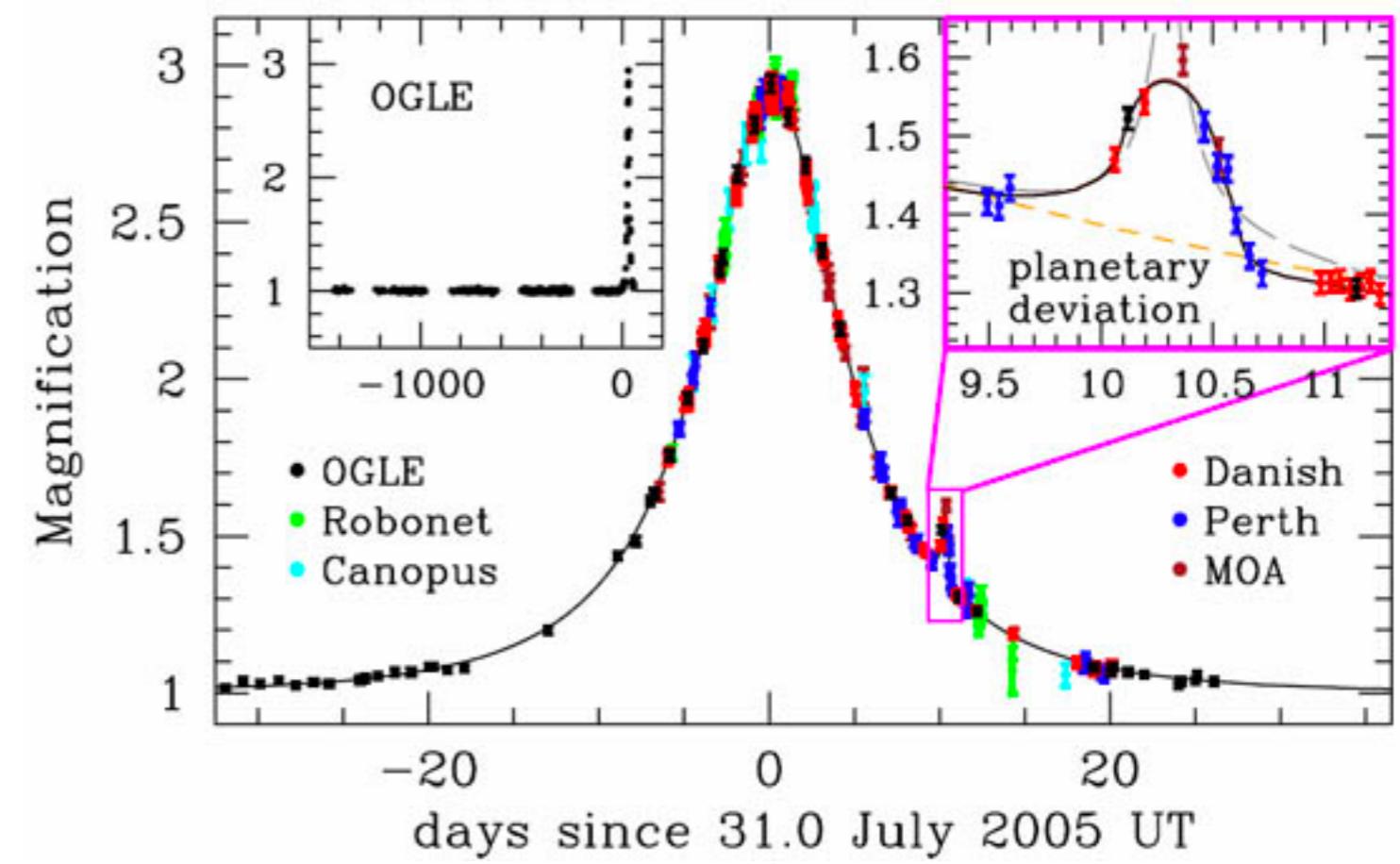
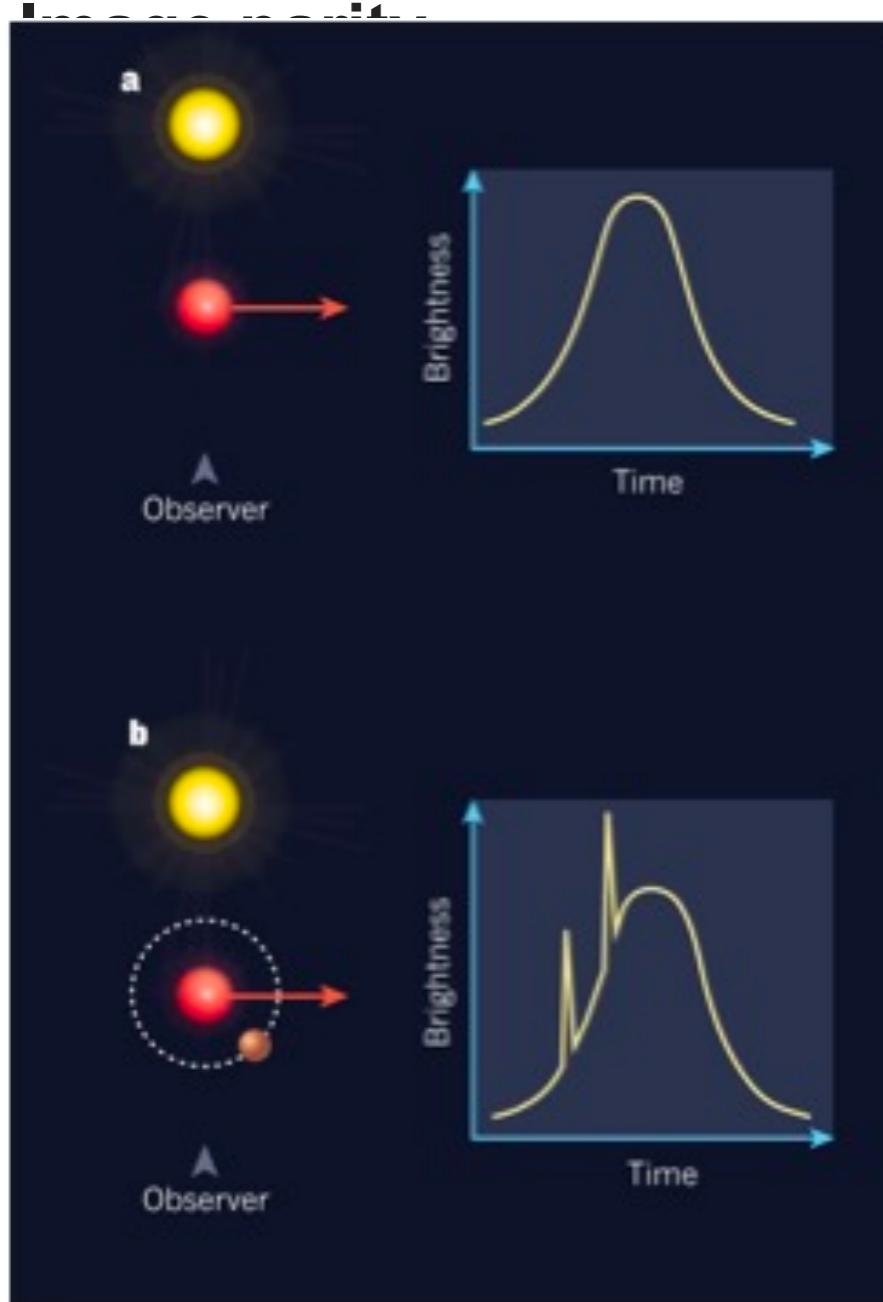
Garcia-Bellido et al 2017

2 April 2019: PBH constraints with HSC Microlensing

Niikural et al 2019 <https://arxiv.org/abs/1701.02151>



Micro Lensing by exo-planets



Lensing fun tools

- Gravitational Lensing iPhone App:

[Plus par cet éditeur](#)

GravLens3
par Eli Rykoff
Ouvrez iTunes pour acheter et télécharger des apps.



[Afficher sur iTunes](#)

 Cette app a été conçue pour iPhone et iPad.

Gratuit
Catégorie: [Éducation](#)
Mise à jour: 19 févr. 2015
Version: 3.2
Taille: 3.3 Mo
Langue: Anglais
Éditeur: Eli Rykoff
© Eli Rykoff
Classé 4+

Compatibilité: Nécessite iOS 6.1 ou une version ultérieure.
Compatible avec l'iPhone, l'iPad et l'iPad touch.

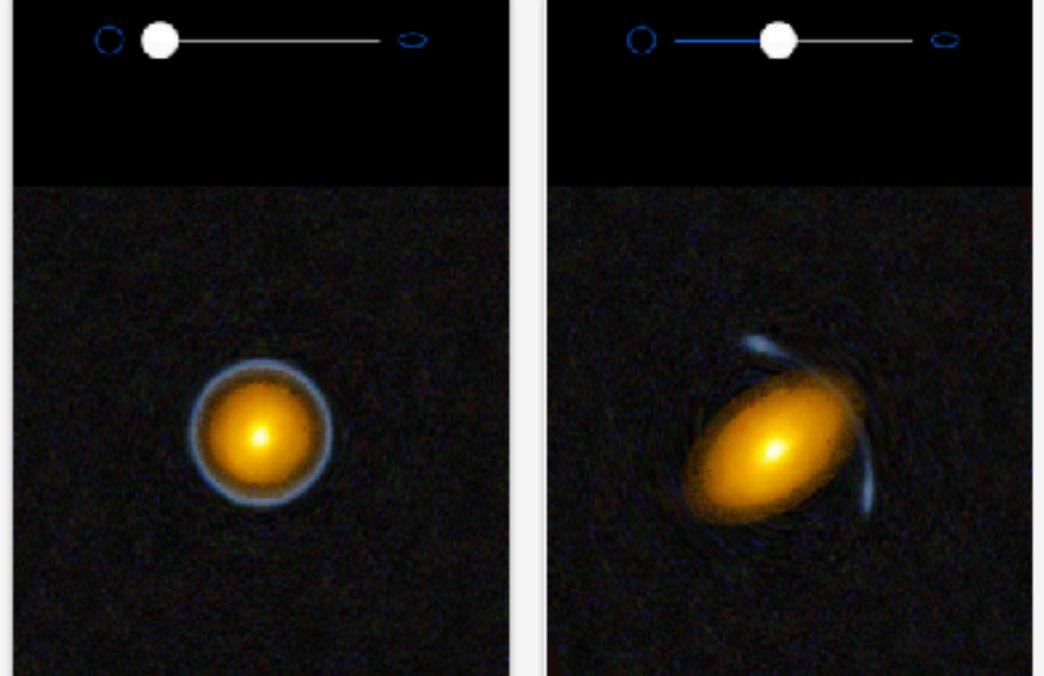
Note
Nous n'avons pas reçu suffisamment de notes pour évaluer la moyenne de la version actuelle de cet article.
Toutes les versions:
* 10 notes

Description
Gravitational lensing is a remarkable astrophysical phenomenon, where the light of a distant galaxy is bent by a massive foreground galaxy.

[Assistance à GravLens3](#) [...suite](#)

Nouveautés de la version 3.2
Camera now works on iOS8 + iPad

Captures d'écran



[iPhone](#) [iPad](#)



Welcome to Space Warps - HSC!!! Join our 1M classification challenge with Science Friday, 4/27-5/4. Help us find gravitational lenses.

Searching for strong gravitational lenses in the Hyper Suprime-Cam (HSC) survey

[Learn more](#)

[Get started](#)

<https://www.zooniverse.org/projects/aprajita/space-warps-hsc>

The DarkEnergy Camera of the 4m Blanco Telescope

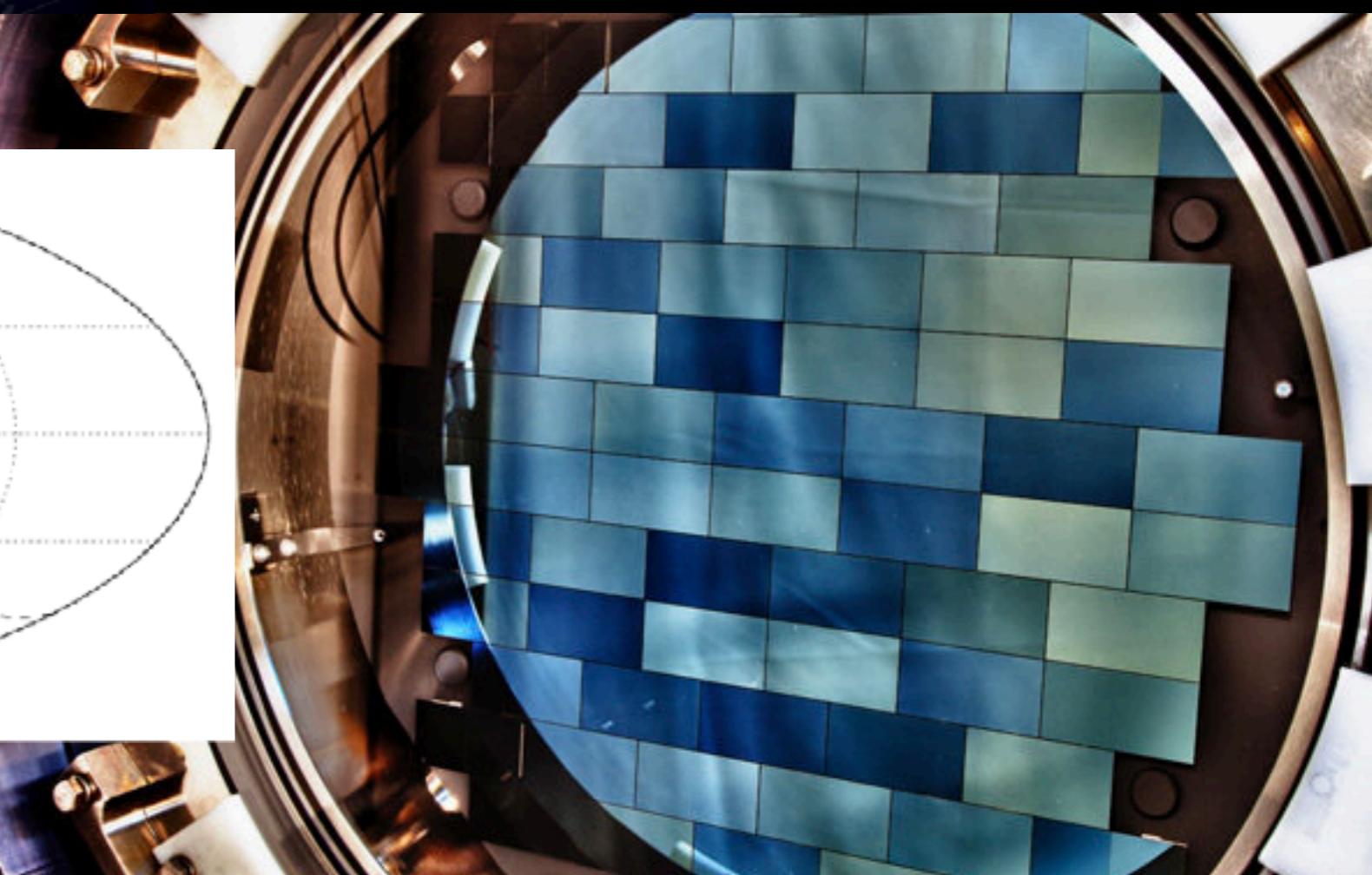
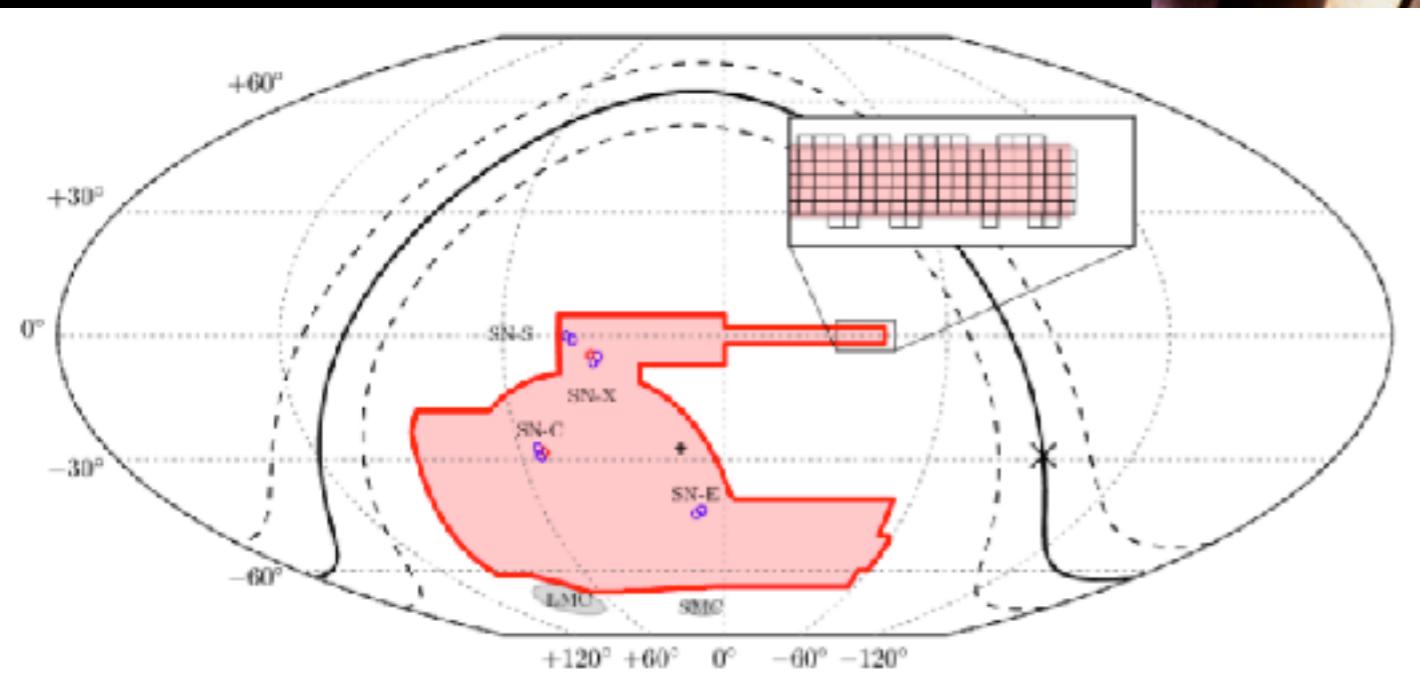
74 2k x 4k CCD detectors => 570 MPixel Camera



<https://www.darkenergysurvey.org/>

Data Release:

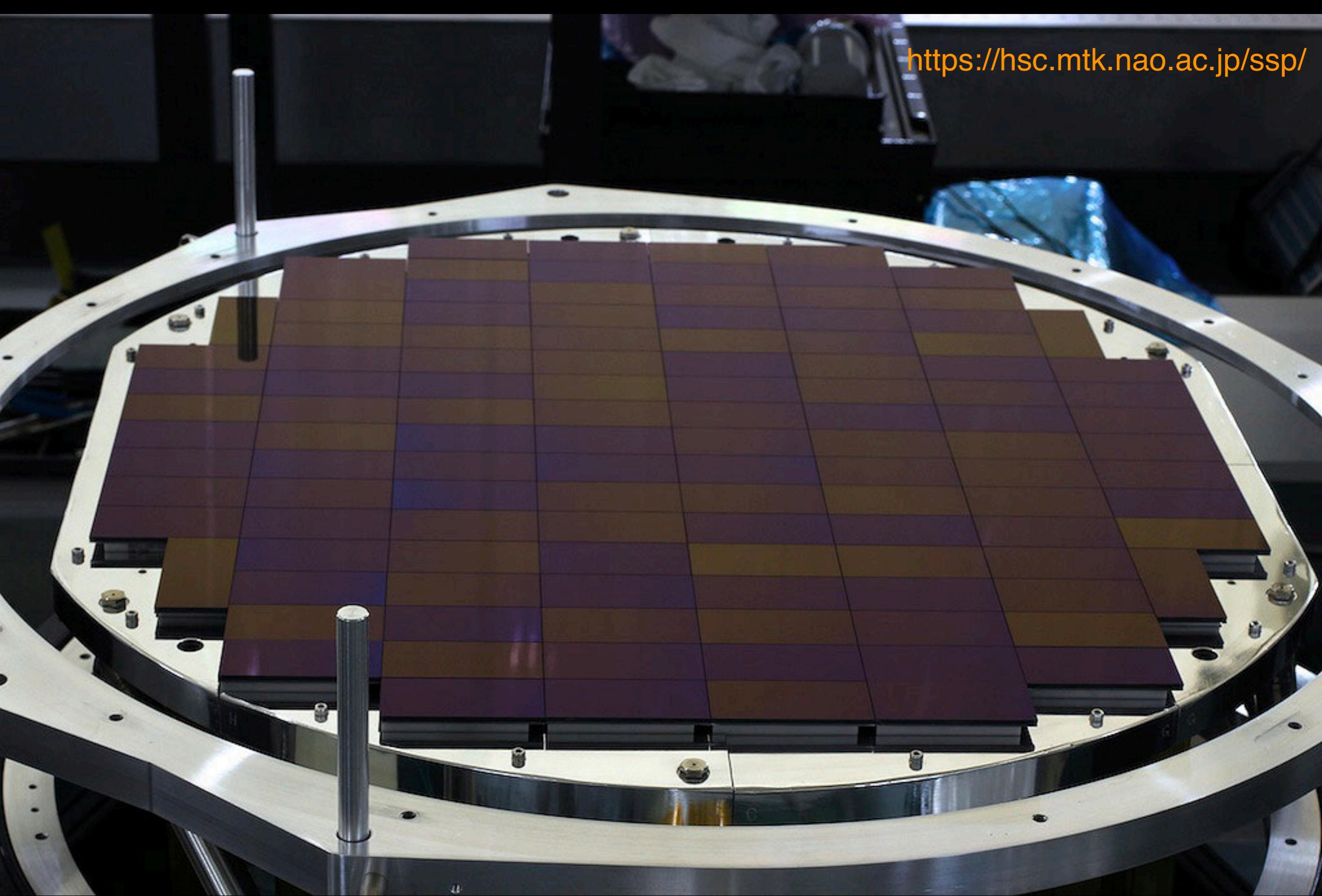
<https://des.ncsa.illinois.edu/releases/dr2>



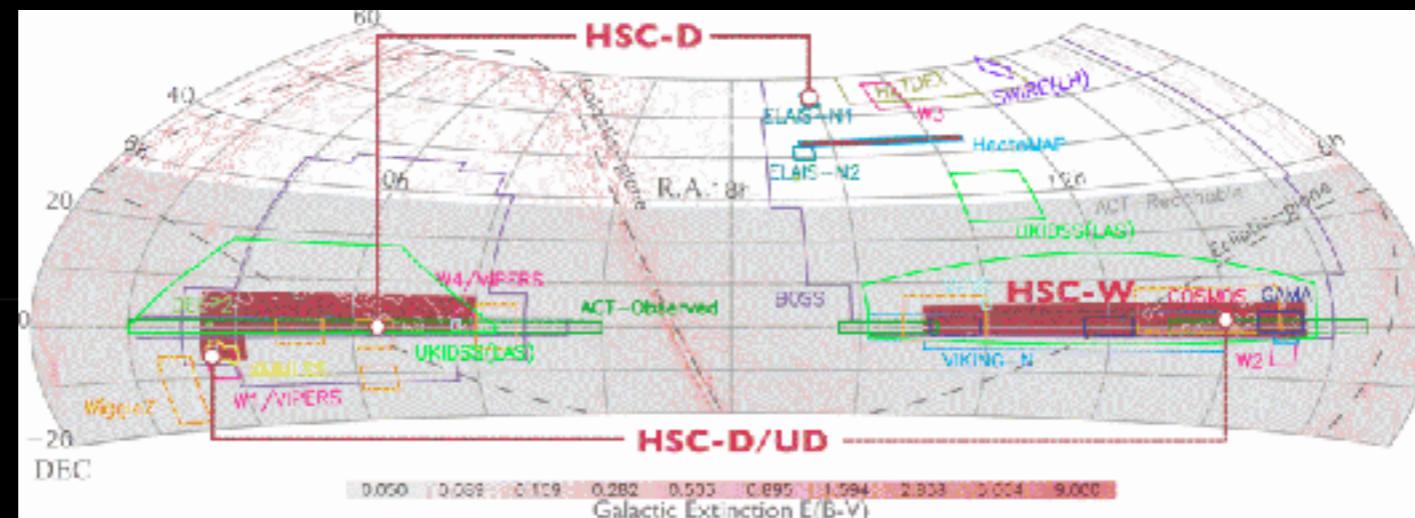
The Hyper Suprime Camera of the 8.2m Subaru Telescope

116 2k x 4k CCD detectors => 870 MPixel Camera

<https://hsc.mtk.nao.ac.jp/ssp/>



The Hyper Suprime Camera of the 8.2m Subaru Telescope

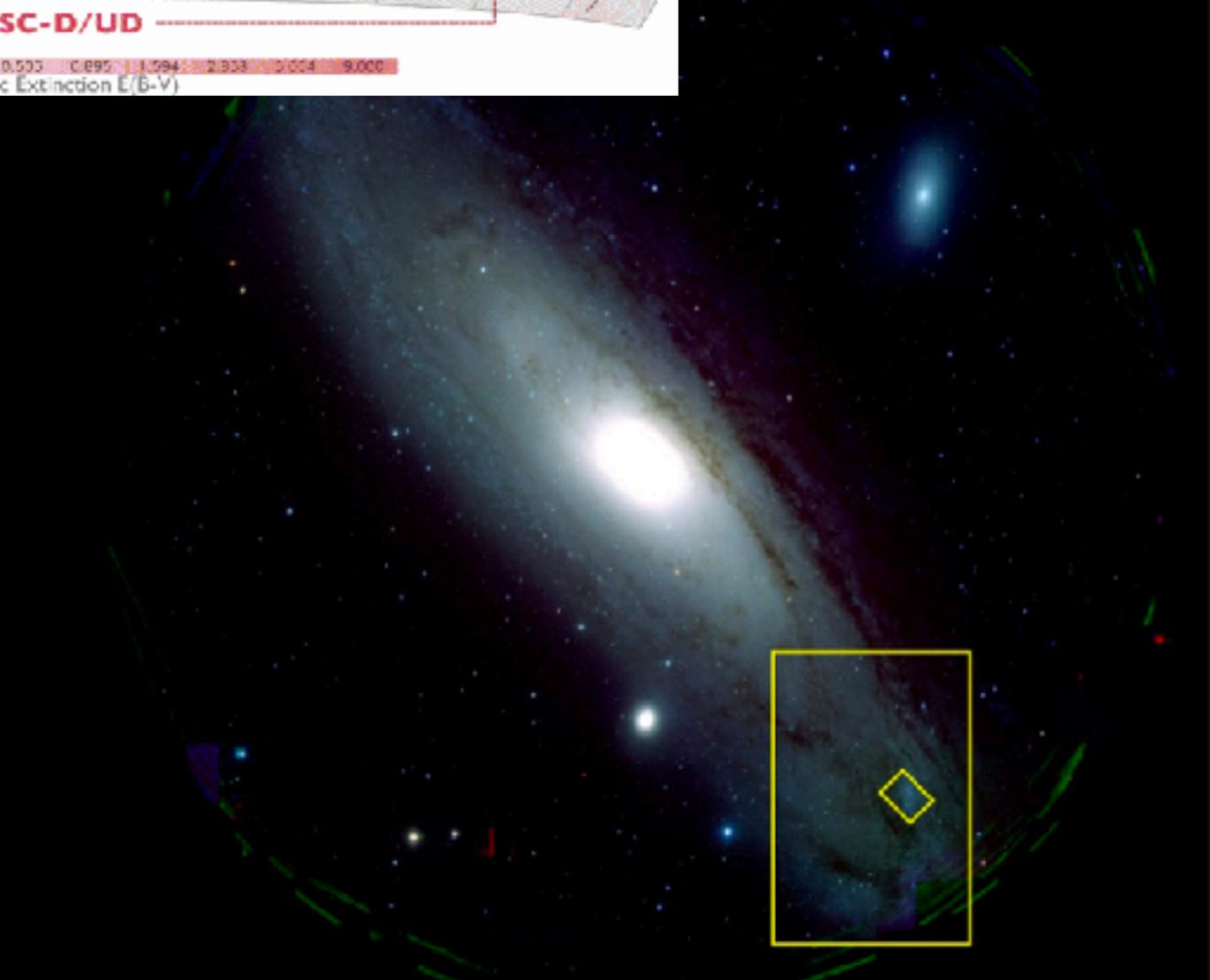


Typical Apparent
Diameter of the
Moon (0.5 degrees)



Suprime-Cam
First Light Release
January 1999

Suprime-Cam
Image Release
September 2001

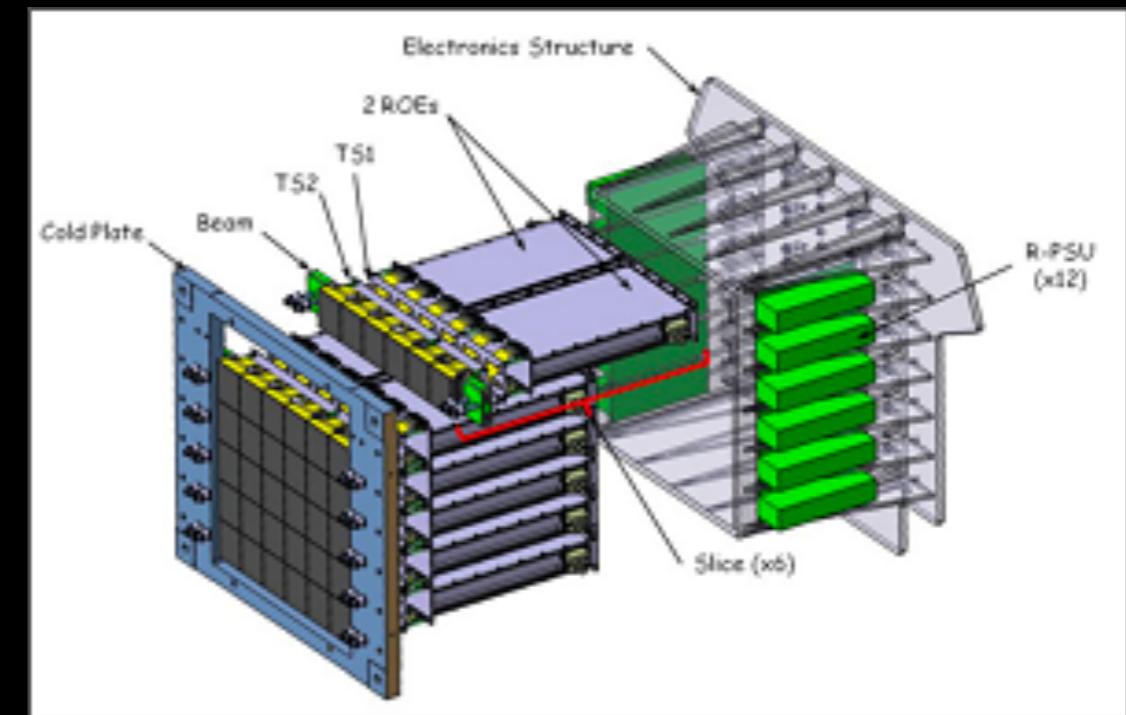
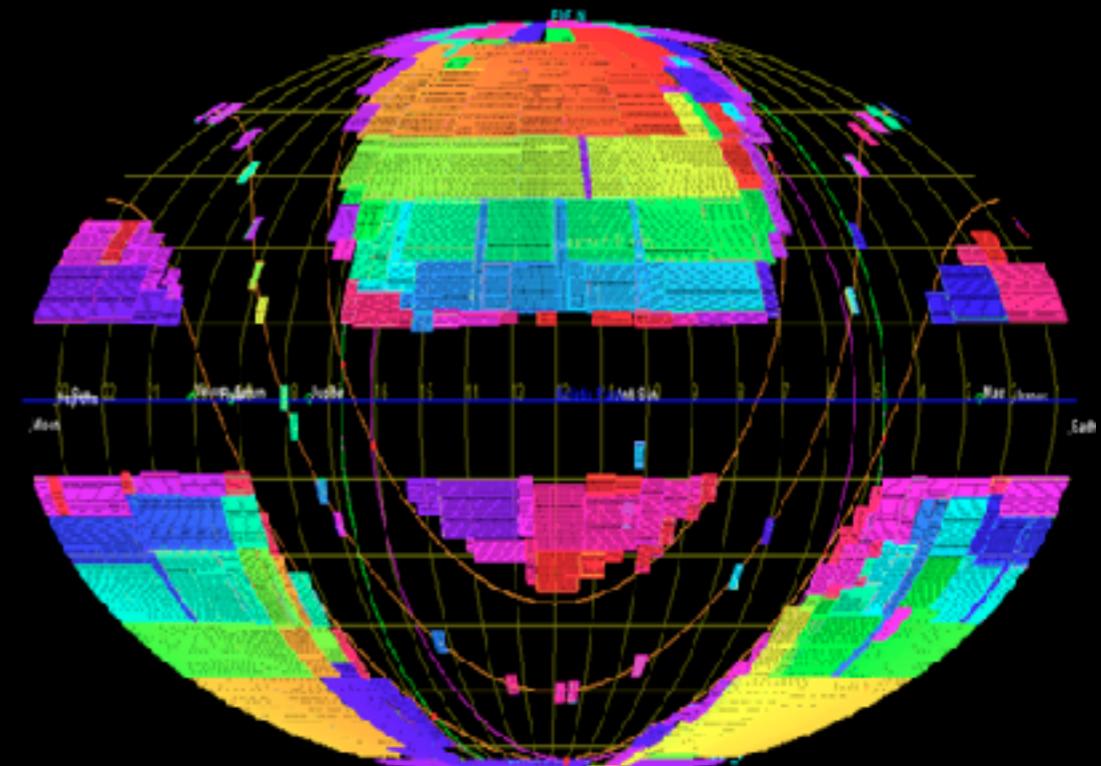
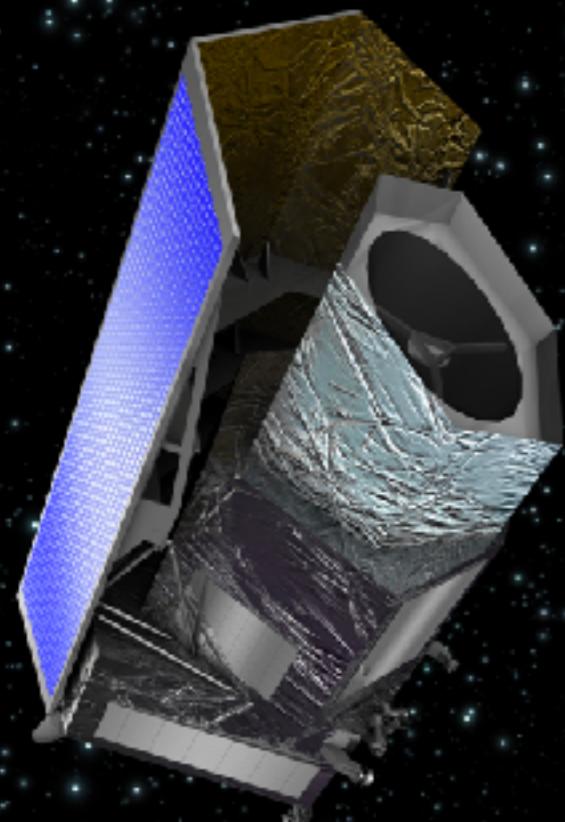


Hyper Suprime-Cam
Image Release
July 2013

The largest camera in Space for the 1.2m Euclid Space Mission

36 4k x 4k CCD detectors => 576 MPixel Camera

<https://www.euclid-ec.org/>



Launch this July 2023 !!!

The largest ever Camera constructed for the 8.4m LSST

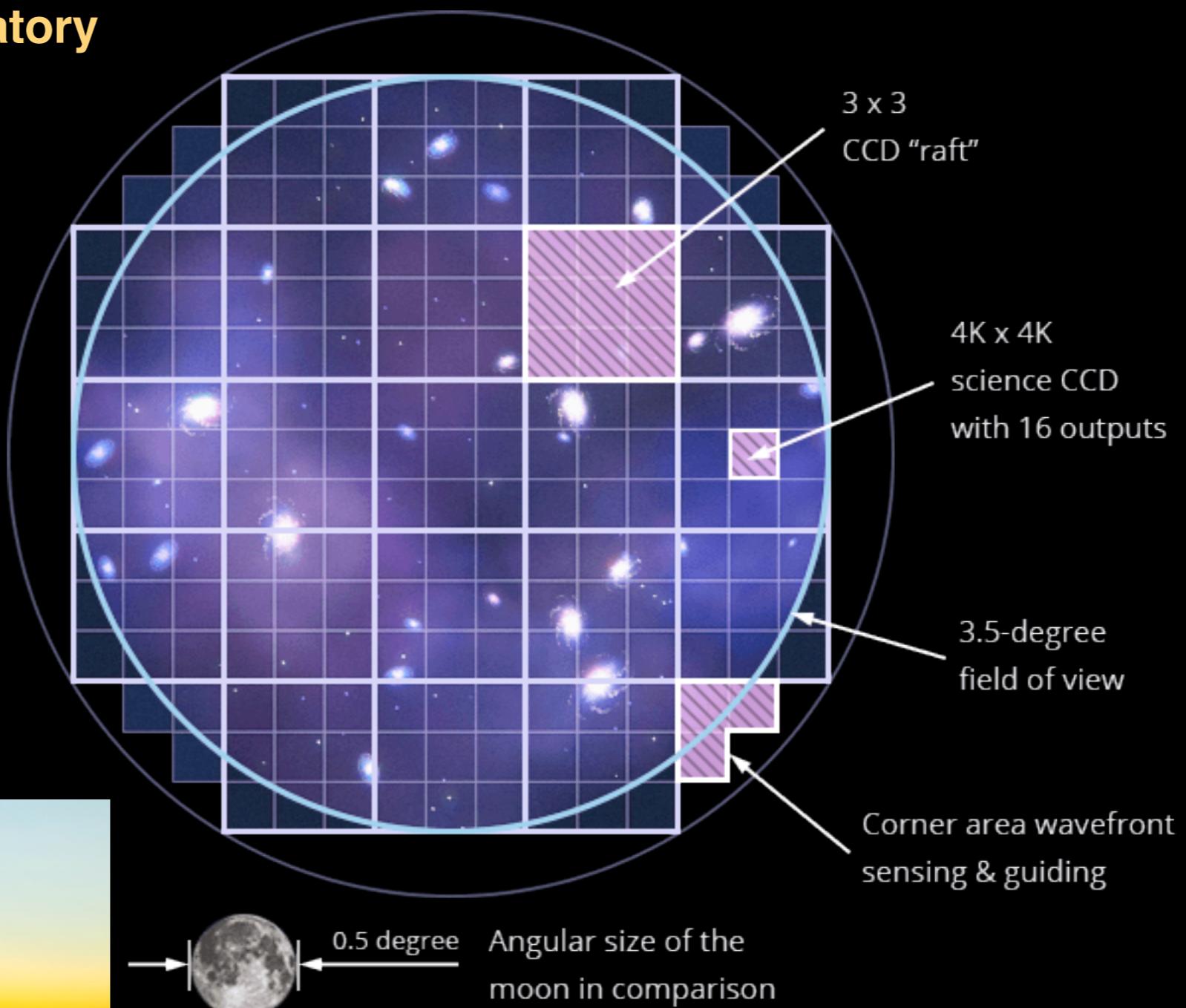
201 4k x 4k CCD detectors => 3.2 GPixel Camera

Renamed: the **Vera Rubin Observatory**

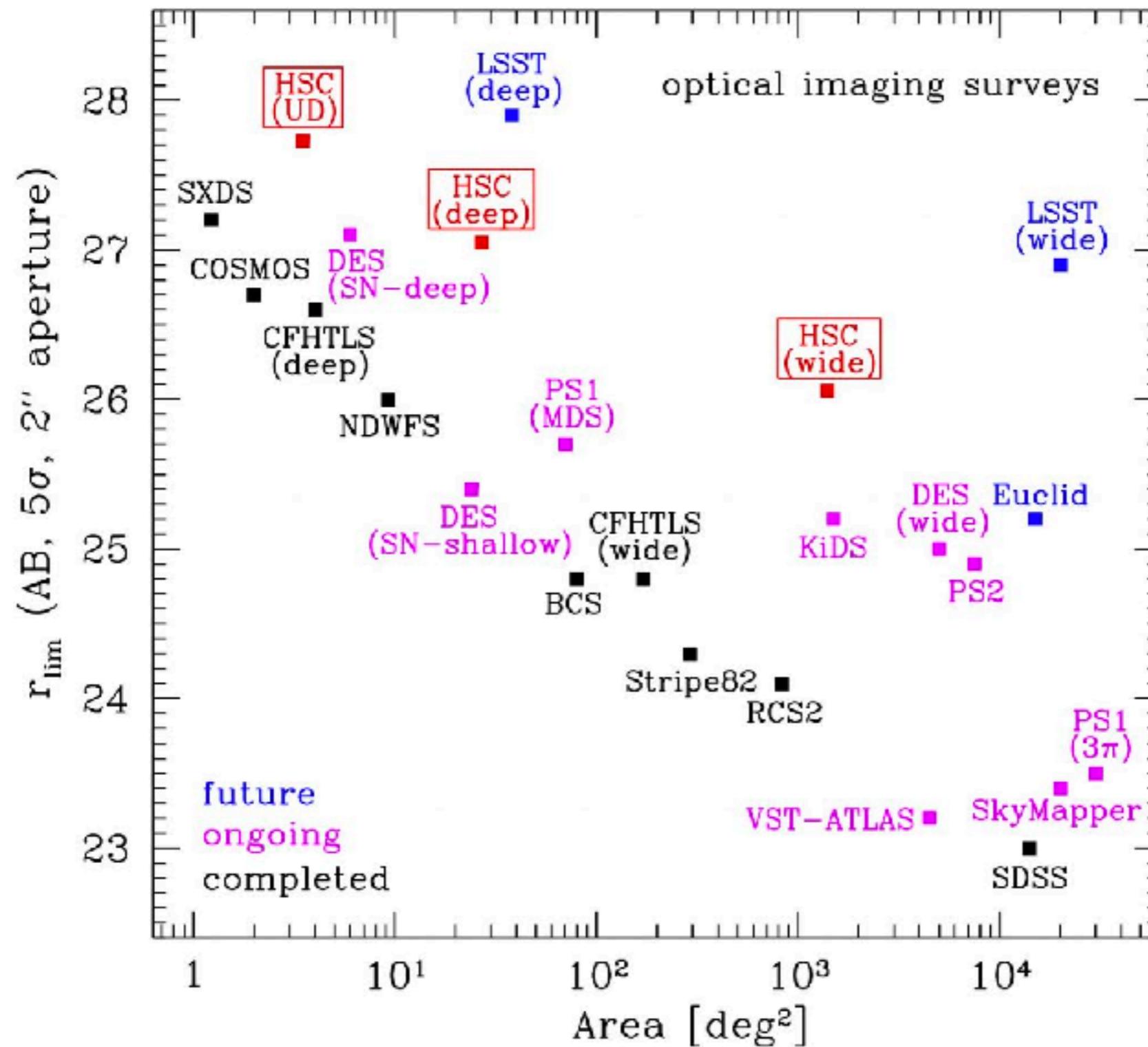
<https://www.lsst.org/>

First light in July 2024 !!!

Picture as April 2021



Weak Lensing Surveys



Outline

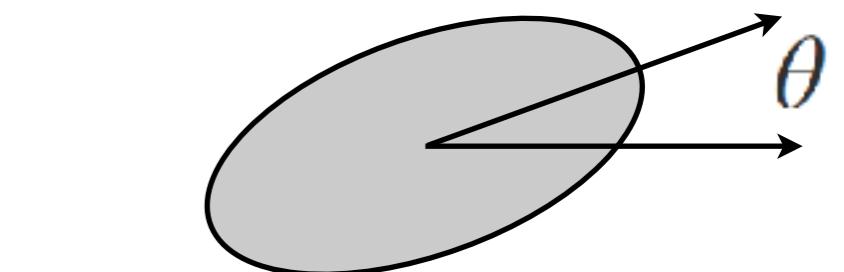
- Weak Lensing
- Modelling strategy of strong lens systems
(focusing on cluster of galaxies)

Weak Lensing Basics

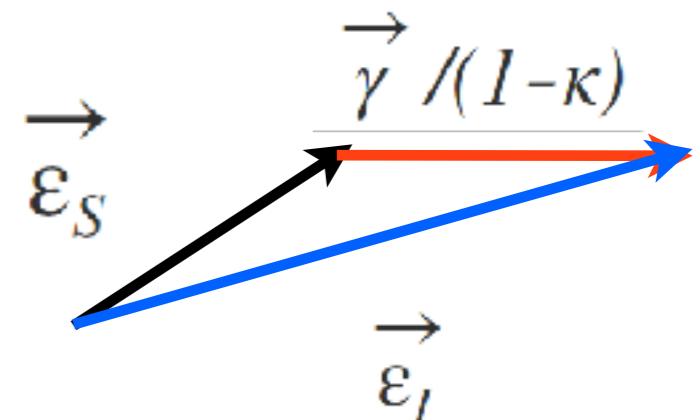
Weak Lensing

- Background galaxies are distorted.
- To first order mass distribution is “shearing” the shape of a galaxy:
- a circle becomes an ellipse
- an ellipse is transformed in an other ellipse
- Can define a vector transform:

$$\vec{\varepsilon}_I = \vec{\varepsilon}_S + \frac{\vec{\gamma}}{1-\kappa}$$



$$\vec{\varepsilon} = \begin{pmatrix} \varepsilon_1 = \varepsilon \cdot \sin 2\theta \\ \varepsilon_2 = \varepsilon \cdot \cos 2\theta \end{pmatrix}$$



Weak Lensing

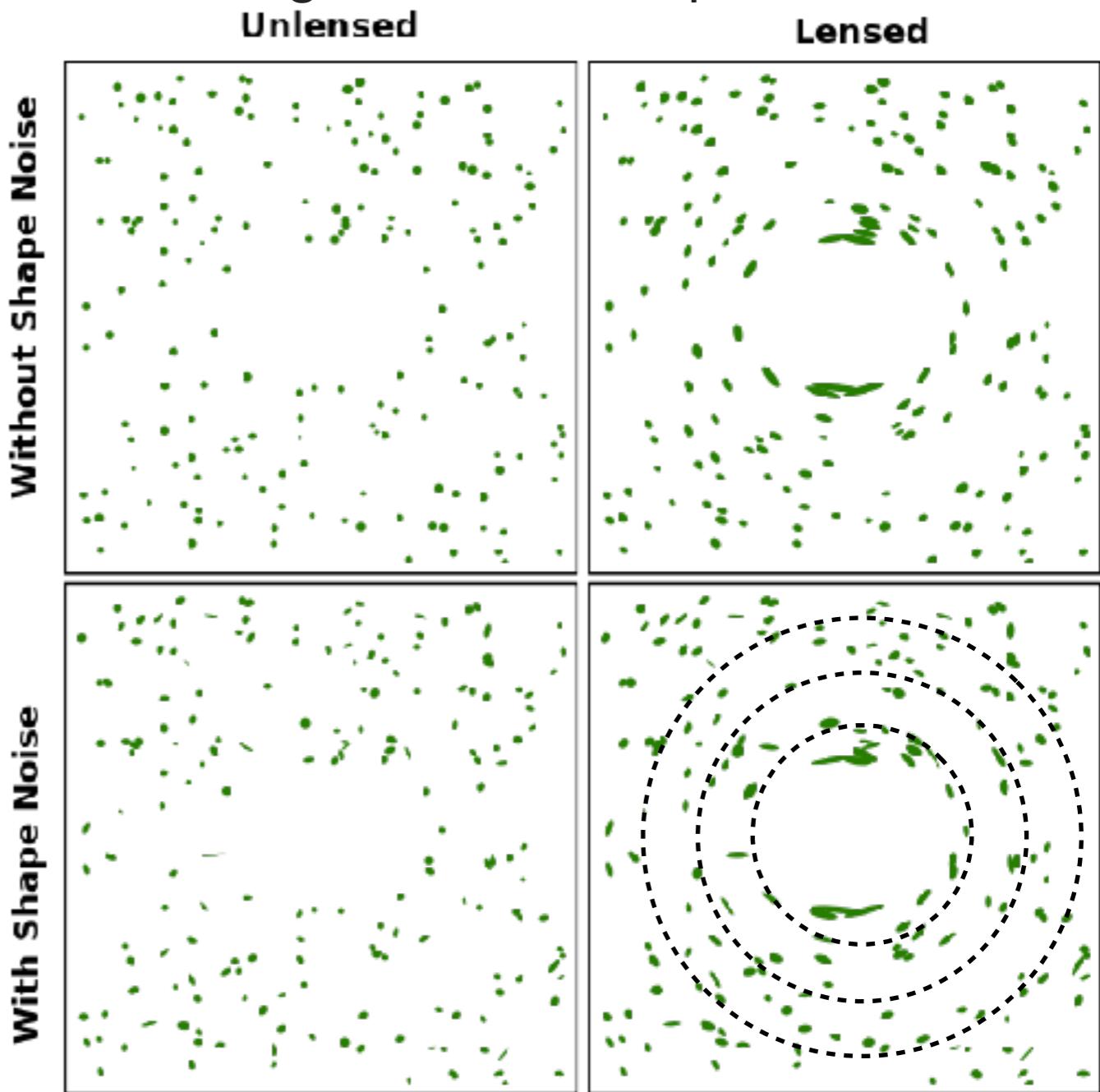
- Averaging over a number of galaxies and assuming the source ellipticities are randomly distributed:

$$\langle \vec{\varepsilon}_I \rangle = \vec{0} + \langle \vec{\gamma} / (1-\kappa) \rangle$$

- For a singular isothermal sphere:

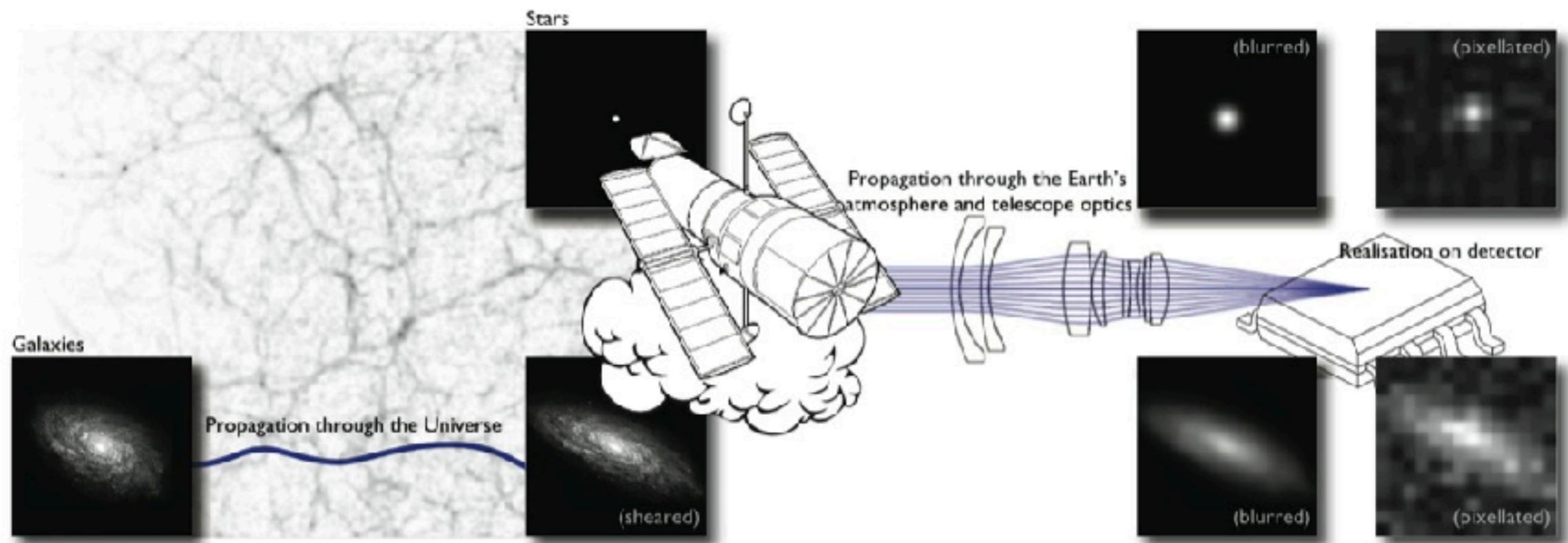
$$\kappa(r) \sim \frac{1}{r}$$

$$\gamma(r) \sim \frac{1}{r}$$



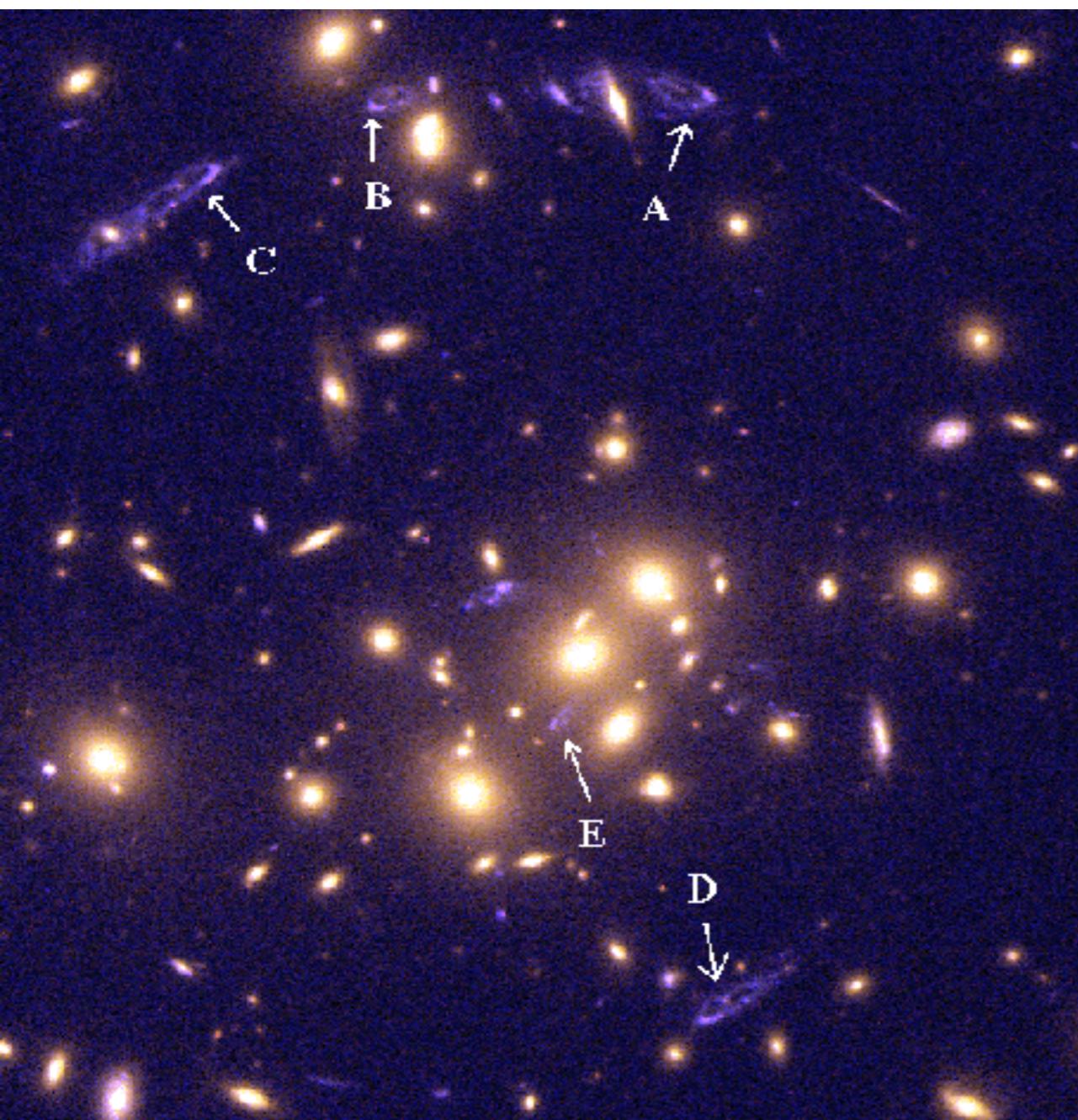
Weak Lensing measurements

- The challenge in weak lensing is to measure the ellipticities of galaxies with no bias
- m is the multiplicative factor ($<10^{-3}$)
- C is the additive constant ($<10^{-6}$)

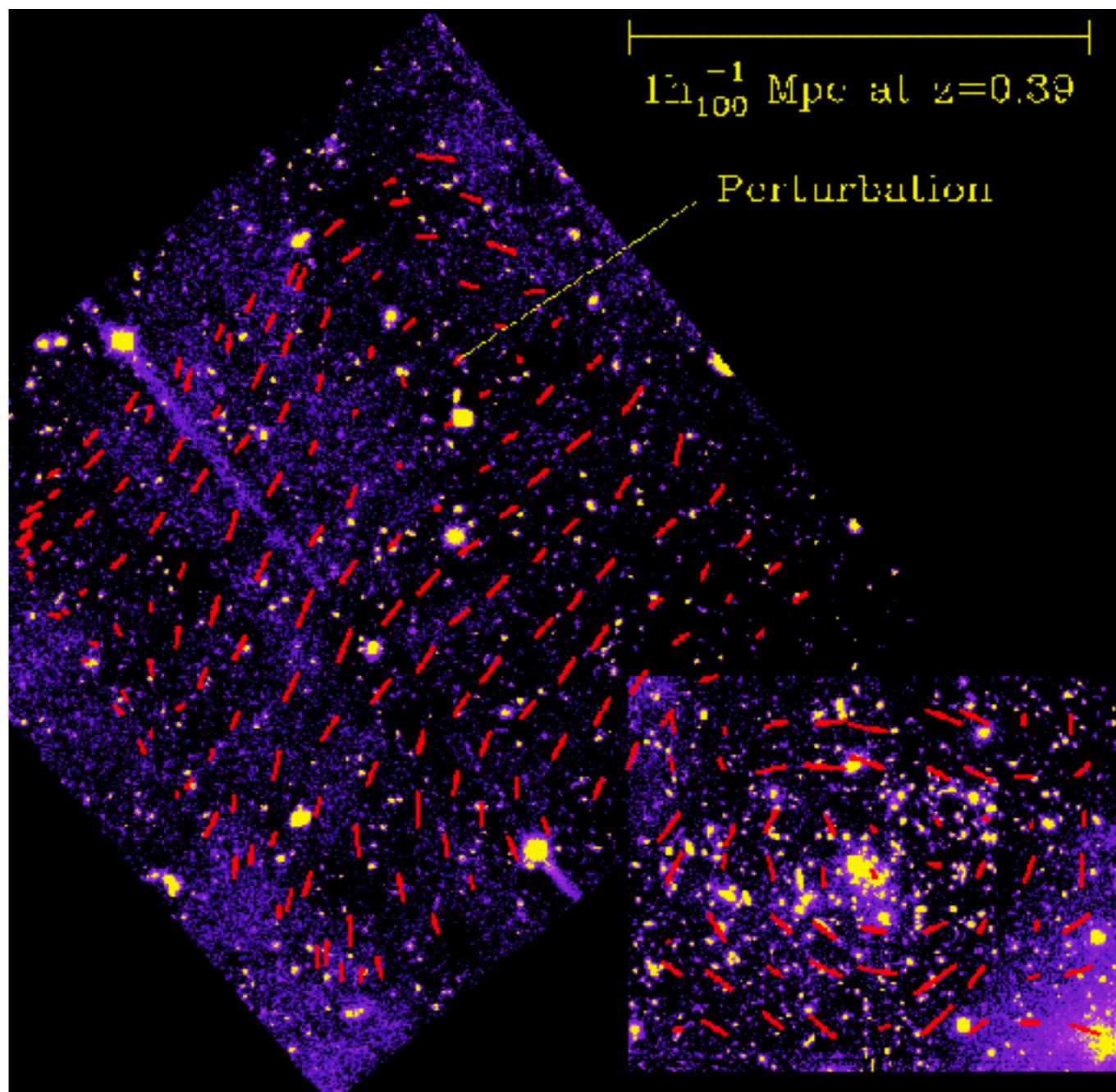
$$\mathcal{E}_{measured} - \mathcal{E}_{true} = m \cdot \mathcal{E}_{true} + C$$


Coupling Strong and Weak Lensing

Absolute central mass



relative total mass and slope

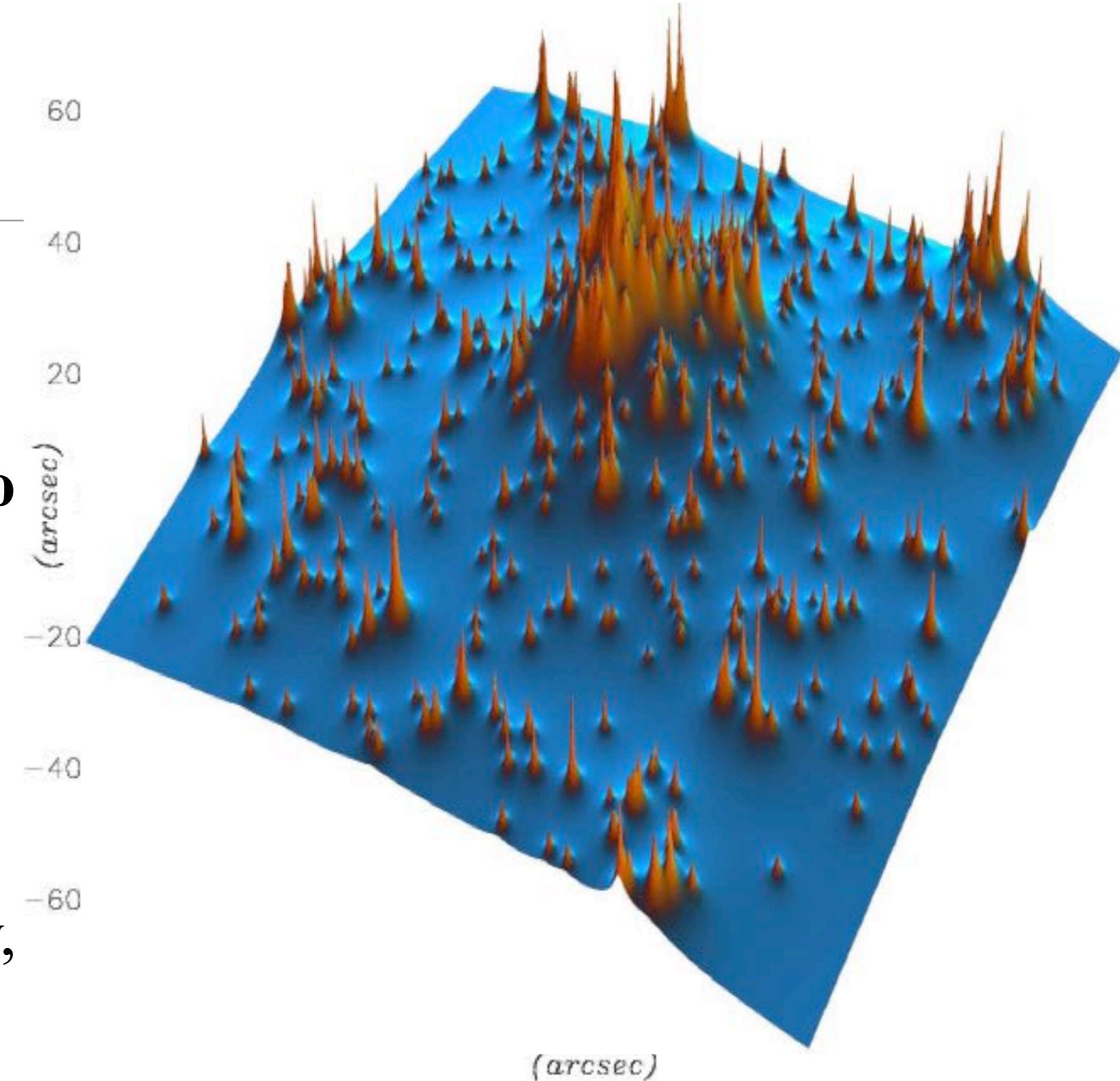


Cl0024+1654

HST wide field sparse mosaic

- 76 orbits, 38 pointings
- Probe regions up to $\sim 5\text{Mpc}$

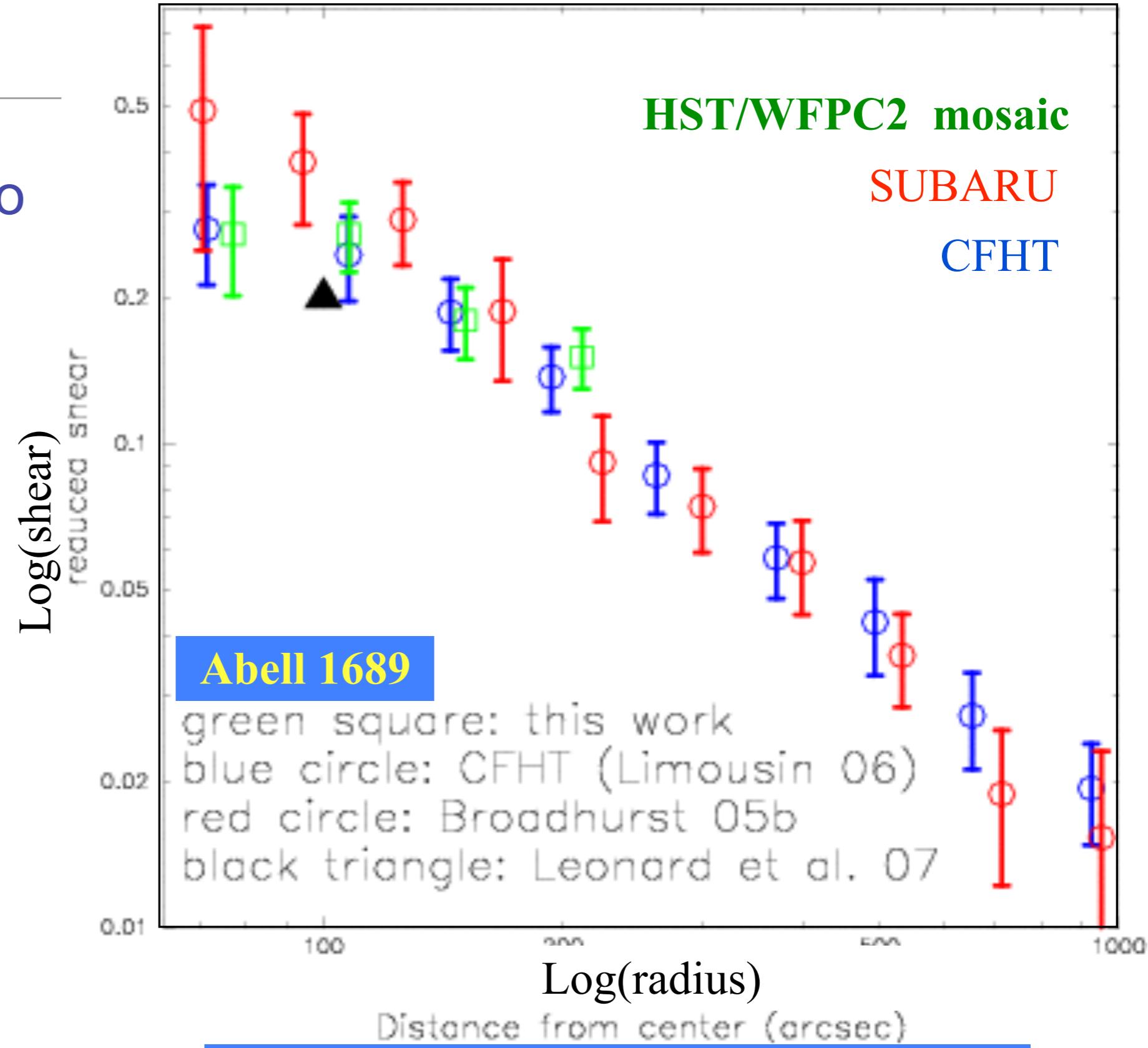
Aim: learn cluster physics of clusters by comparing with other mass estimates: X-ray, dynamics, learn on galaxy halo mass stripping



Treu et al 2003, Kneib et al 2003, Natarajan et al 2008

Mass Profile of Clusters (SL+WL)

- Background source selection is *critical* to accurately measure WL
- Improved lensing constraints, revised concentration from $c \sim 15$ to $c \sim 8$
- Better agreement with numerical simulation predictions



Weak lensing mass map => identify massive clusters



« Bullet Cluster » unusually strong cluster mergers



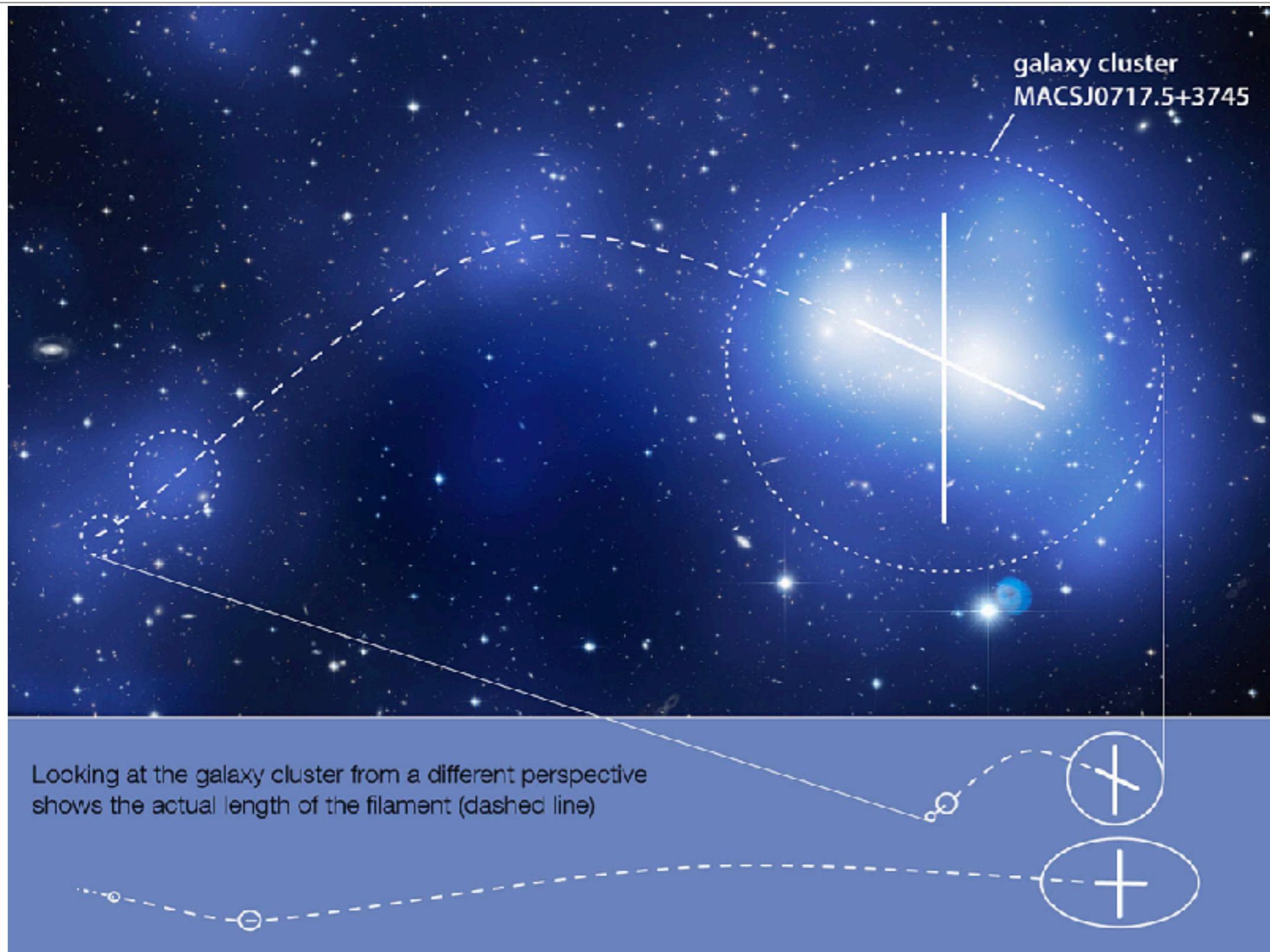
1E0657 alias “the Bullet”

Clowe et al 2006,
Bradac et al 2006

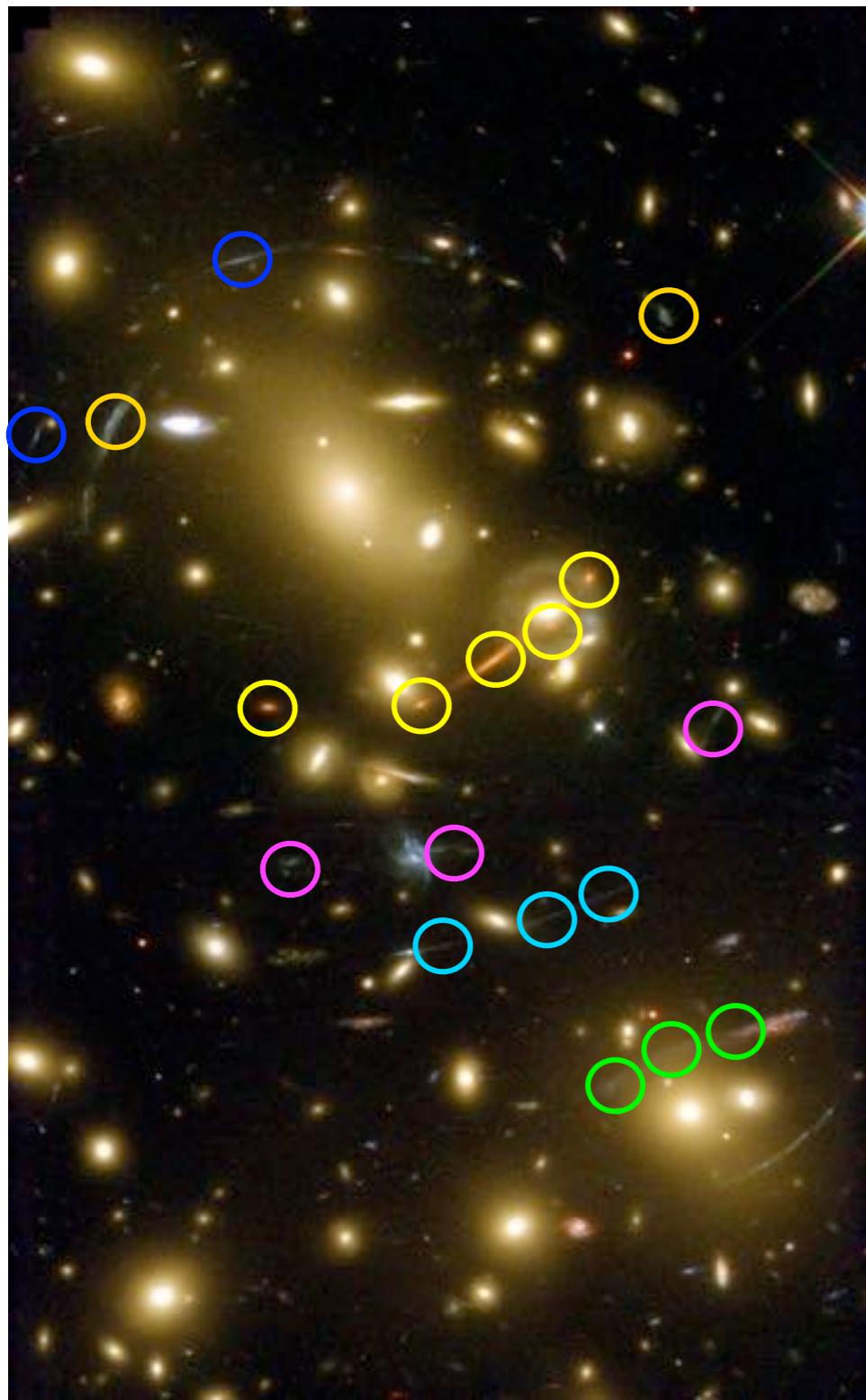
- Encounter of 2 massive clusters
- Significant offset between X-ray gas and lensing mass peaks:
 - ⇒ **best evidence for « collision-less dark matter »**
 - ⇒ put constraints on DM/baryon interactions

Weak lensing filament detection

Jauzac et al 2012



Modeling Strong Lensing



SL Cluster Modeling and Errors

Constraints:

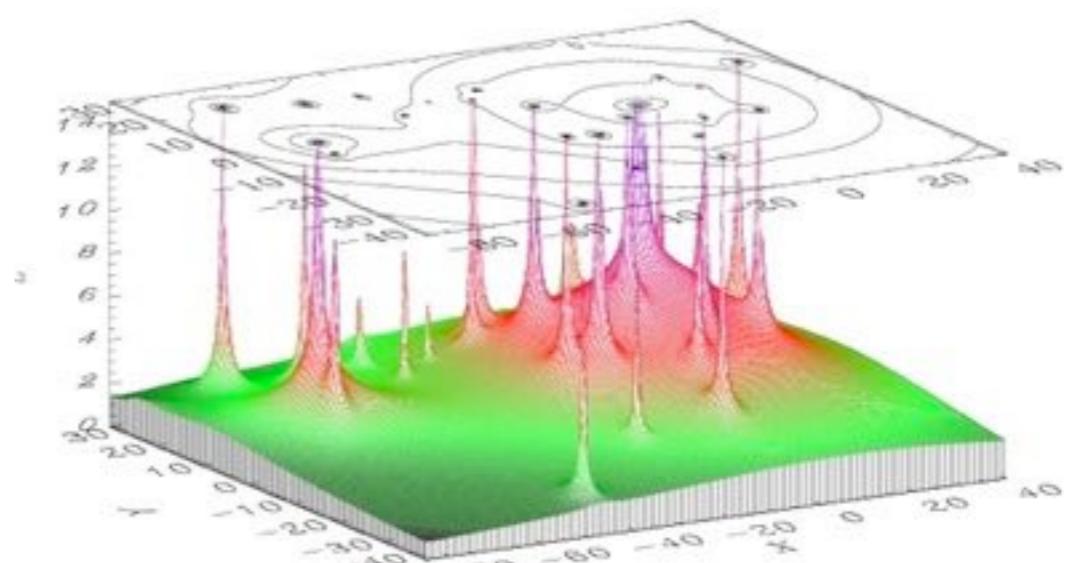
- Multiple images (position, redshift, flux, shape)
- Single images with known redshift
- Light/X-ray gas distribution

Model parameterization

- Need to include **small scales**: galaxy halos (parametric form scaled with light)
- Large scale: DM/X-ray gas (parametric form or multi-scale grid)

Model optimization

- e.g. Bayesian approach (robust errors)
- **Not a unique solution**: “most likely model and errors”
- *Predict amplification value and errors => cluster as telescopes*



Jullo et al 2007, Jullo & Kneib 2009

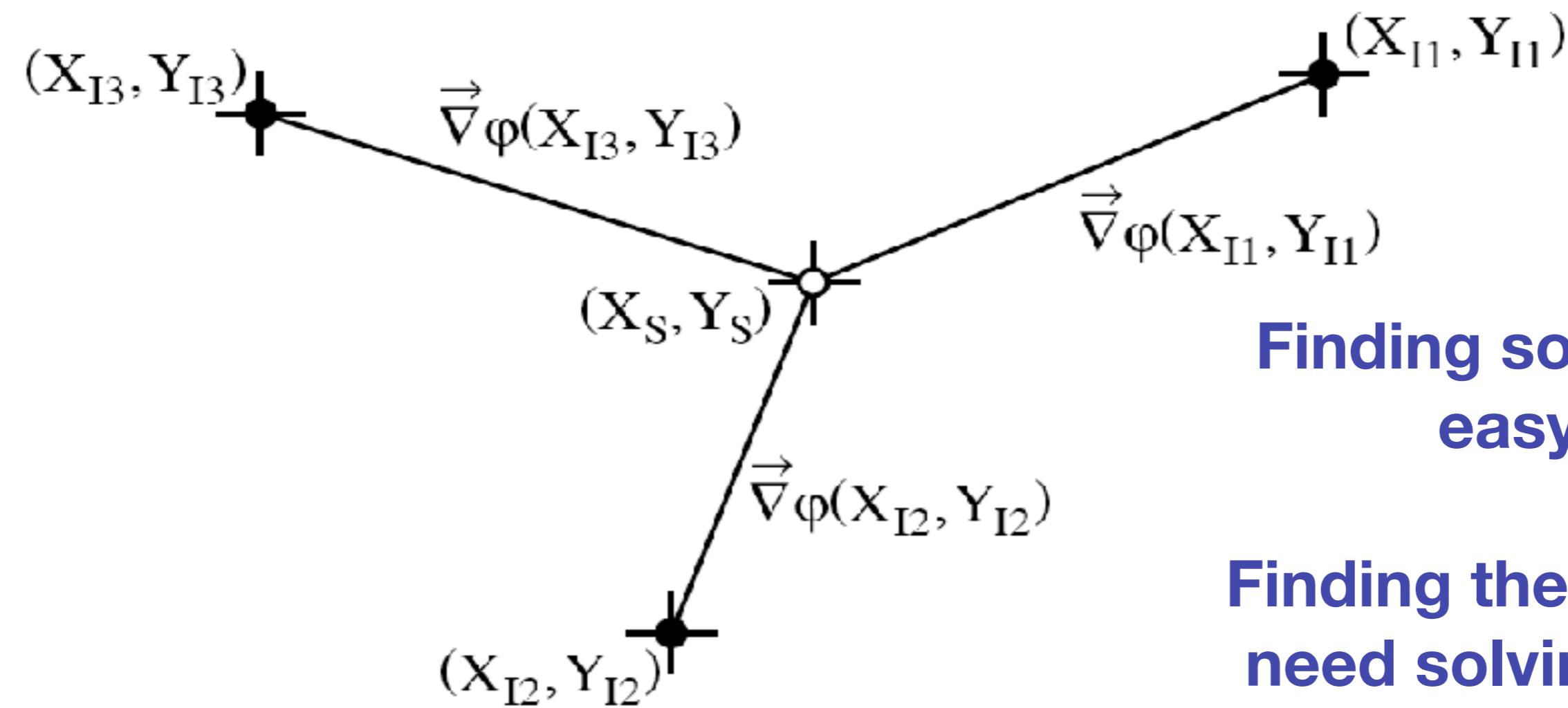
LENSTOOL public software

<https://projets.lam.fr/projects/lenstool/wiki>

Strong Lensing

Lensing equation have multiple solution (Strong lensing):

$$\theta_s = \theta_I - \nabla \varphi(\theta_I)$$

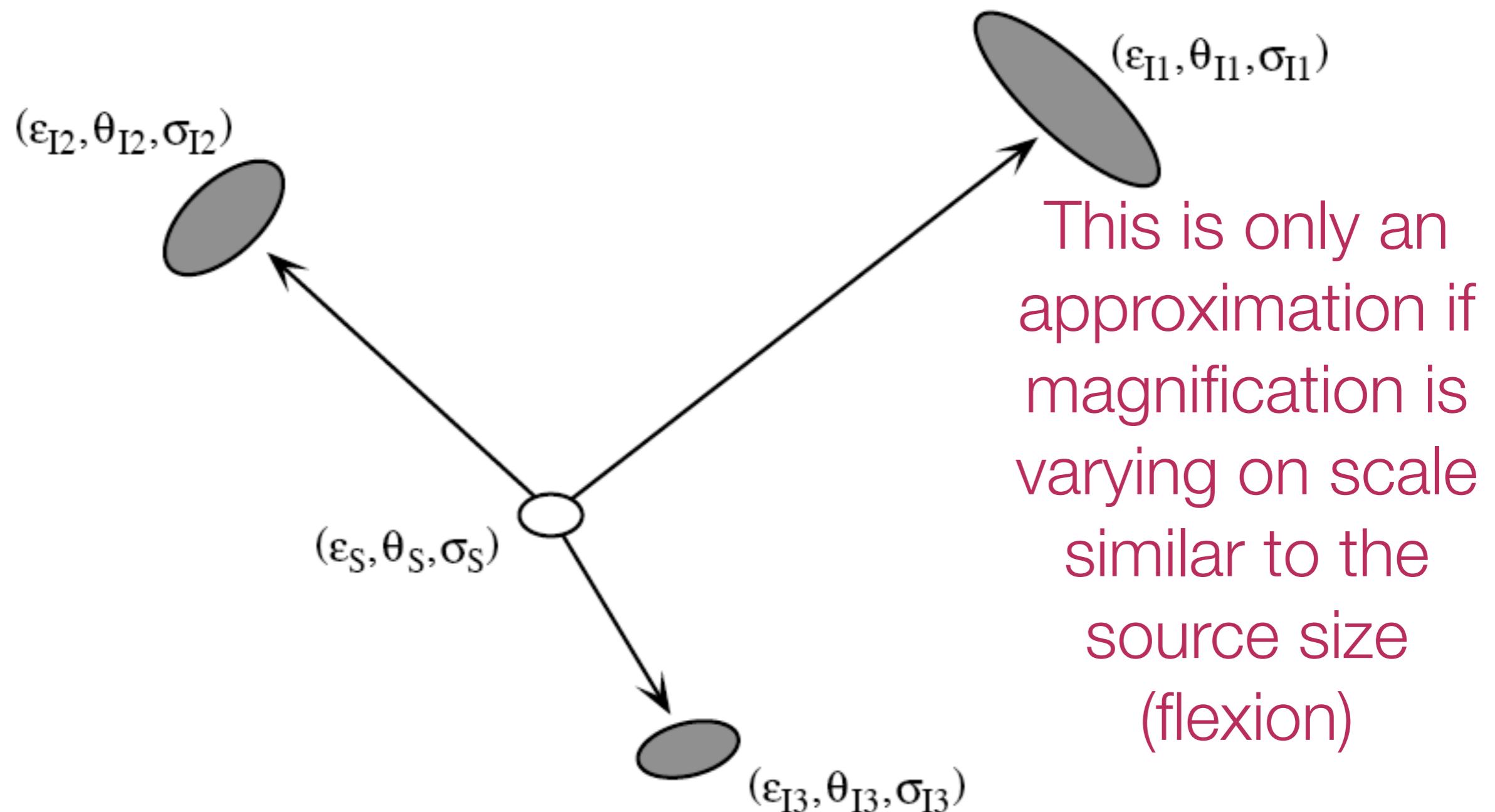


**Finding source is
easy!**

**Finding the images
need solving a 2D
equation (ray tracing)**

Strong Lensing

Image shape are transformed depending on the local shear matrix



Strong Lensing

Image parity

Elliptical case

$(+,+)$

$(+,-)$

$(-,-)$

(a)

Bimodal case

$(+,+)$

$(+,-)$

(b)

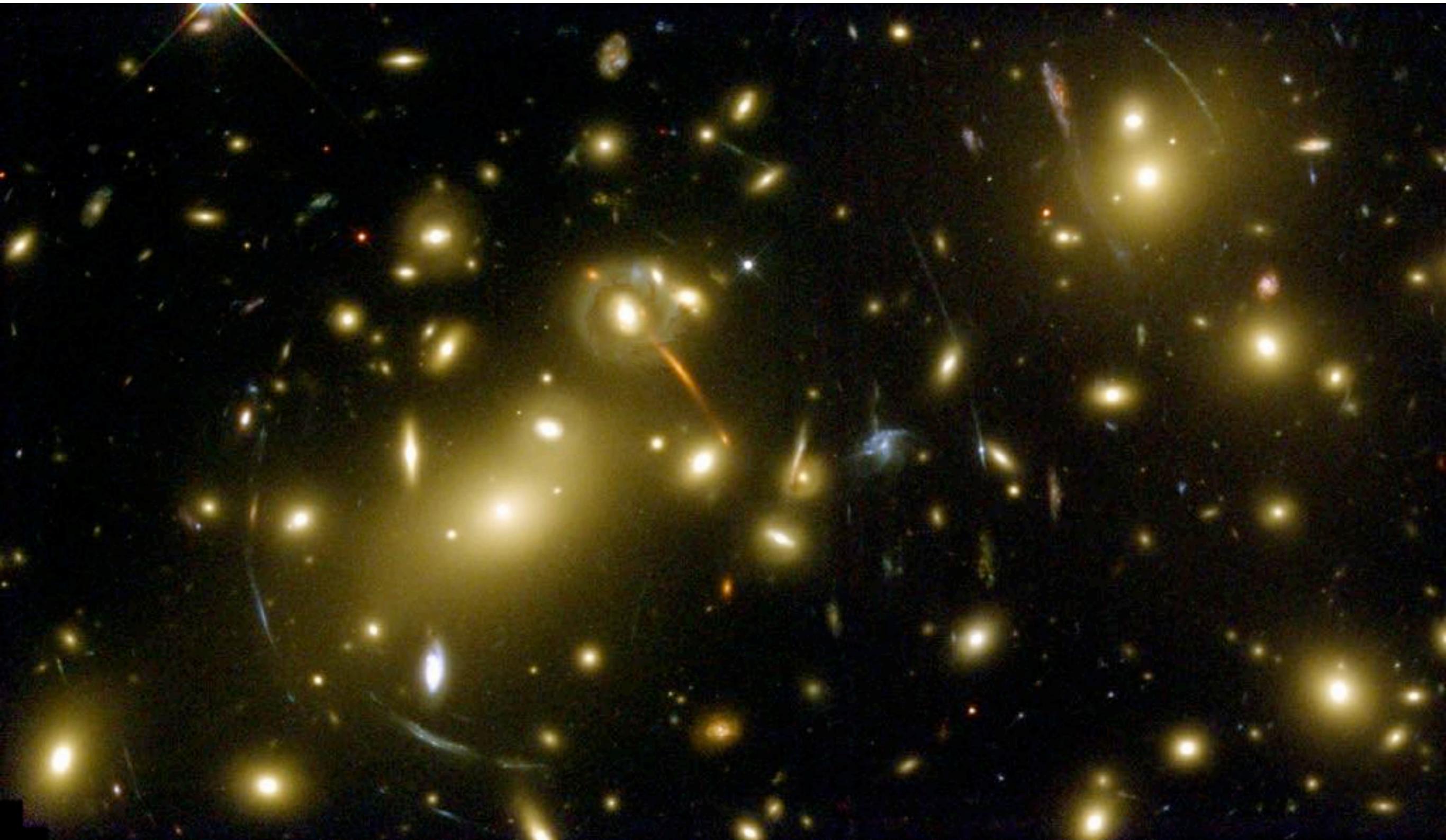
Modeling

- Finding Multiple images

Finding Multiple images

- Need GOOD (high-resolution) data
 - *Really this means HST quality data !*
- Morphology (should agree with rules of **image parity**)
- Color (could do from ground but hard)
- Spectroscopic confirmation (important for lensing strength)
- Modelling confirmation/finding
- Still missing an automatic software for multiple image identification! Human eye is still the best!

Best strong lensing data: Hubble (color) images

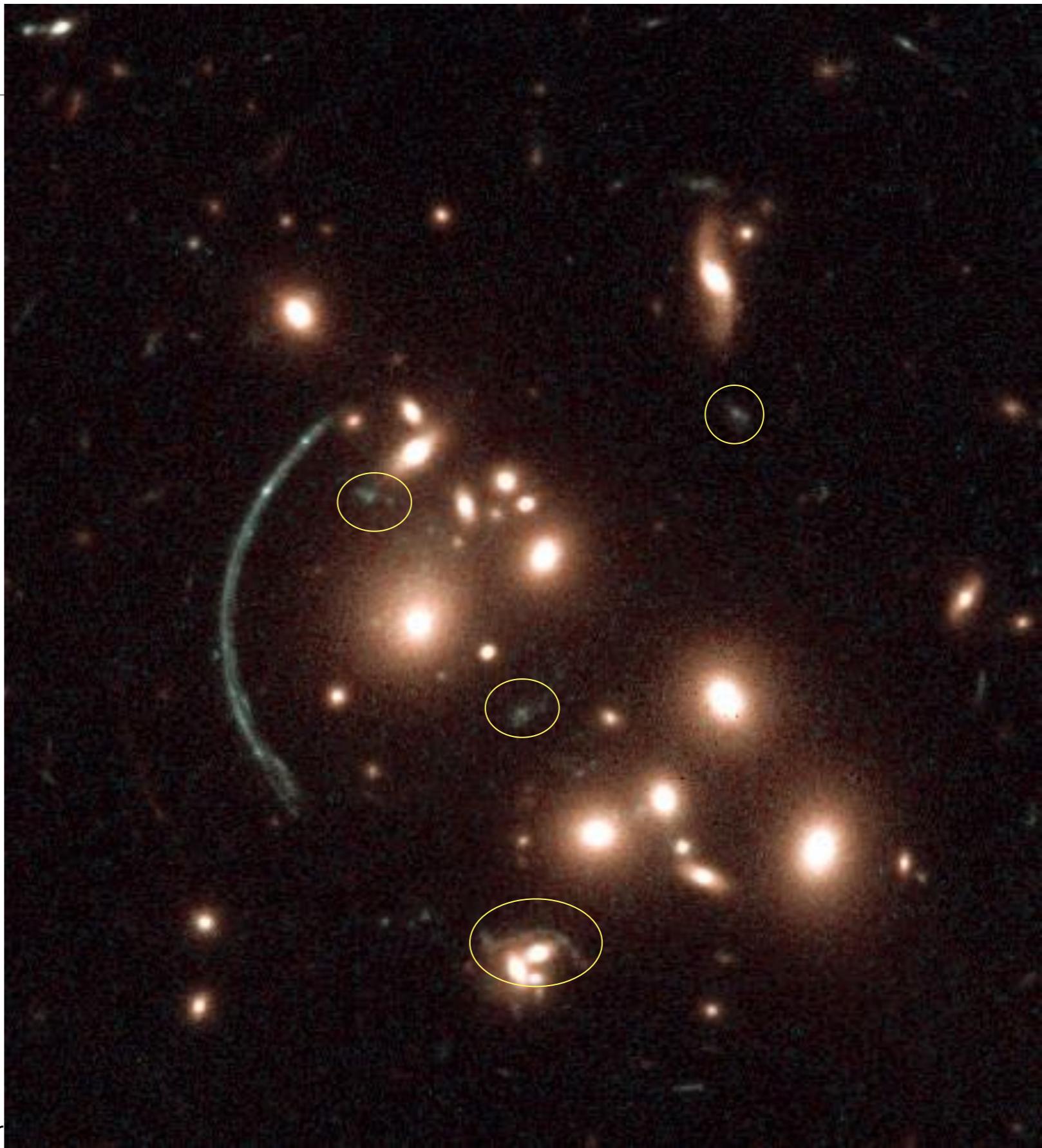


Abell 2218 at $z=0.175$

How to identify multiple images ?

Extreme distortion:
Giant arcs are the
merging of 2 or 3 (or
possibly more)
multiple images

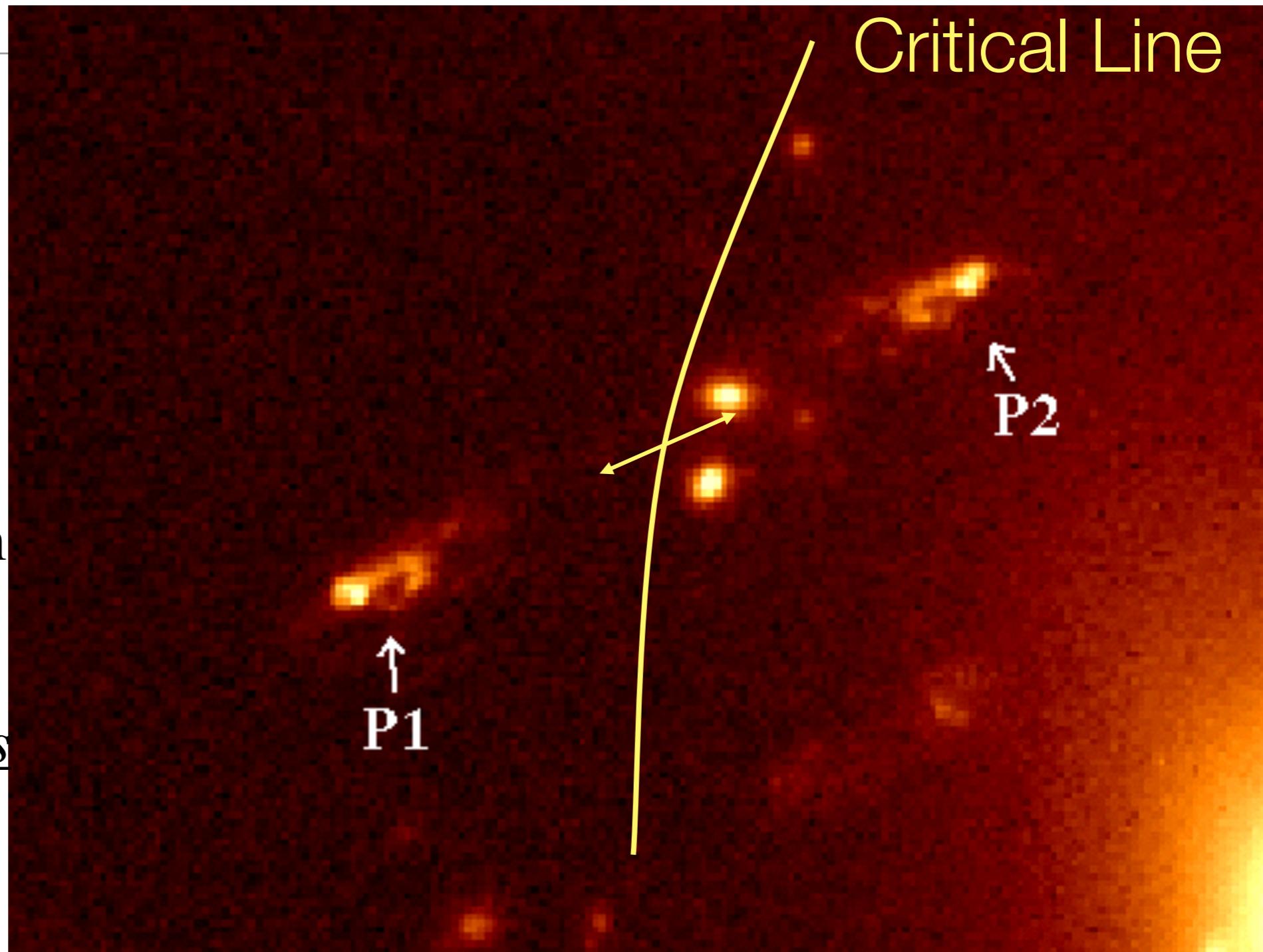
**Giant arc in
Cl2244-04,
 $z=2.24$,
Septuple image**



How to identify multiple images ?

Morphology: *Change of parity across a critical line.*

Note: lensing amplification is a gain in the angular size of the sources. Allow to *resolve distant sources* and study their size and morphologies.



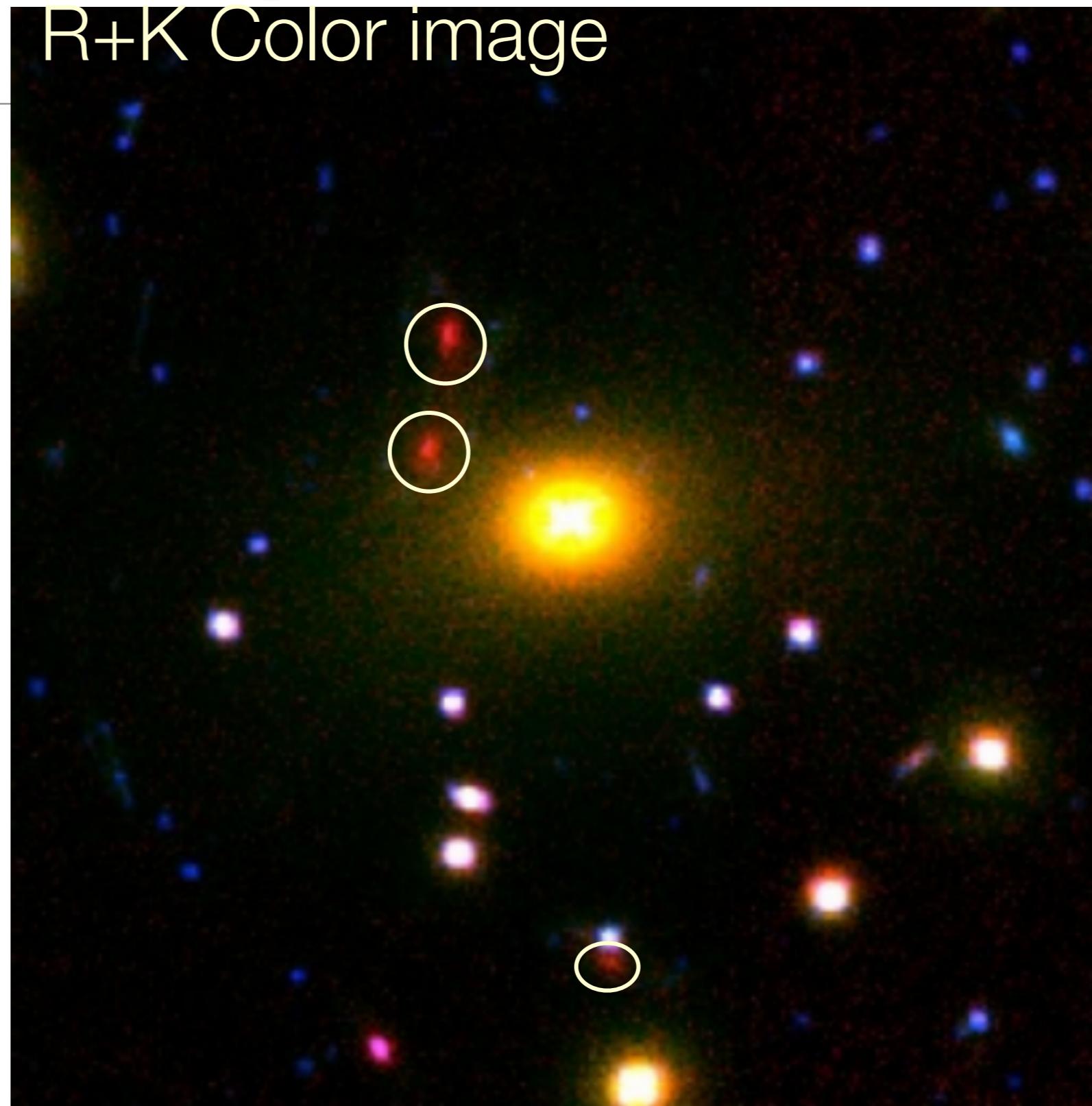
Lensed pair in AC114, $z=1.86$

How to identify multiple images ?

Extreme similar colors:

Example of a triple ERO system at $z \sim 1.6$ (Smith et al 2002) lensed by Abell 68

Interest of magnification is to allow to resolved the morphology of these systems (see **Johan Richard presentation**)



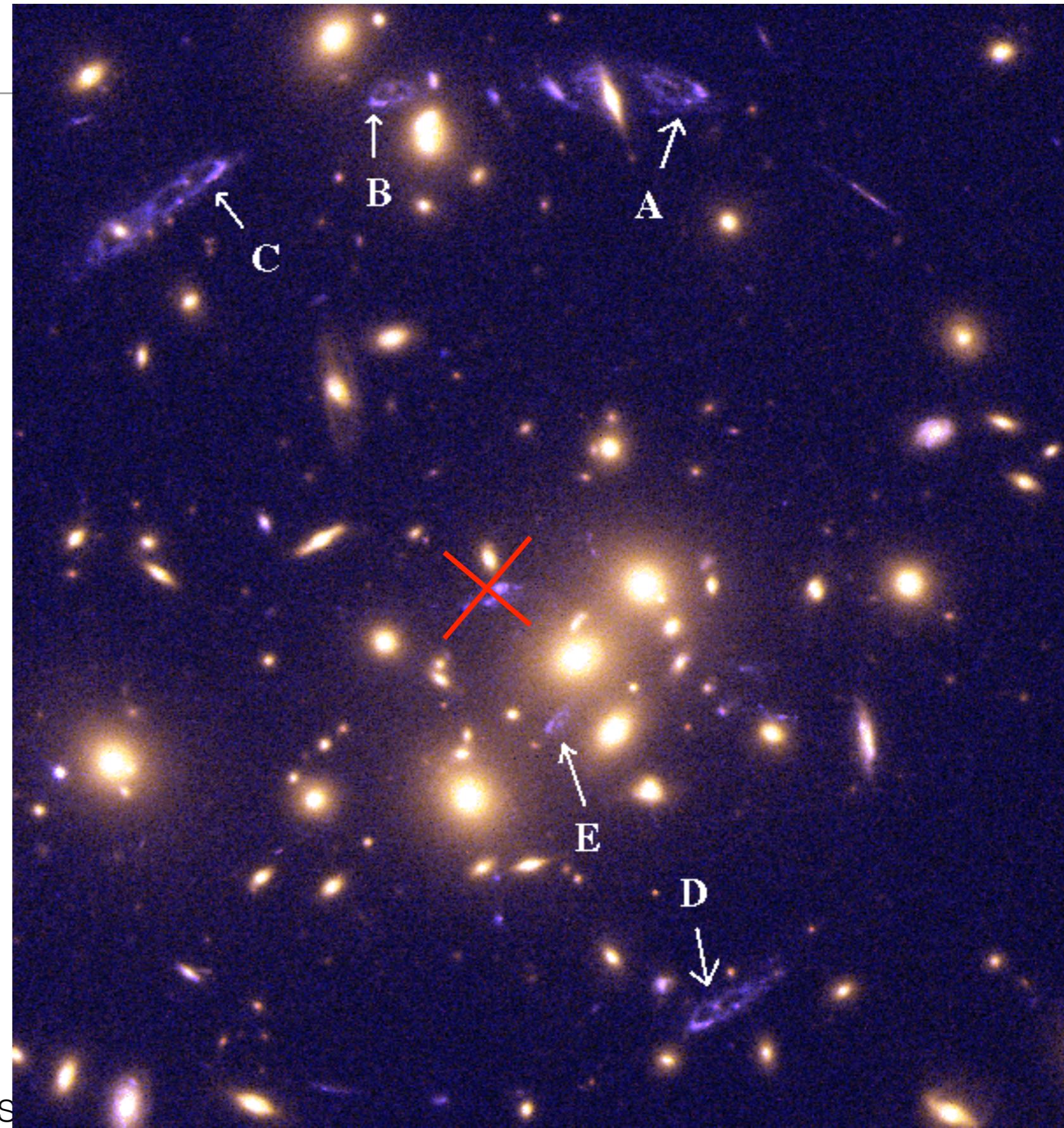
Abell 68: ERO triple image at $z \sim 1.6$

How to identify multiple images ?

Color and
Morphology:

Lens model can help
for the identification
when different
solution are possible

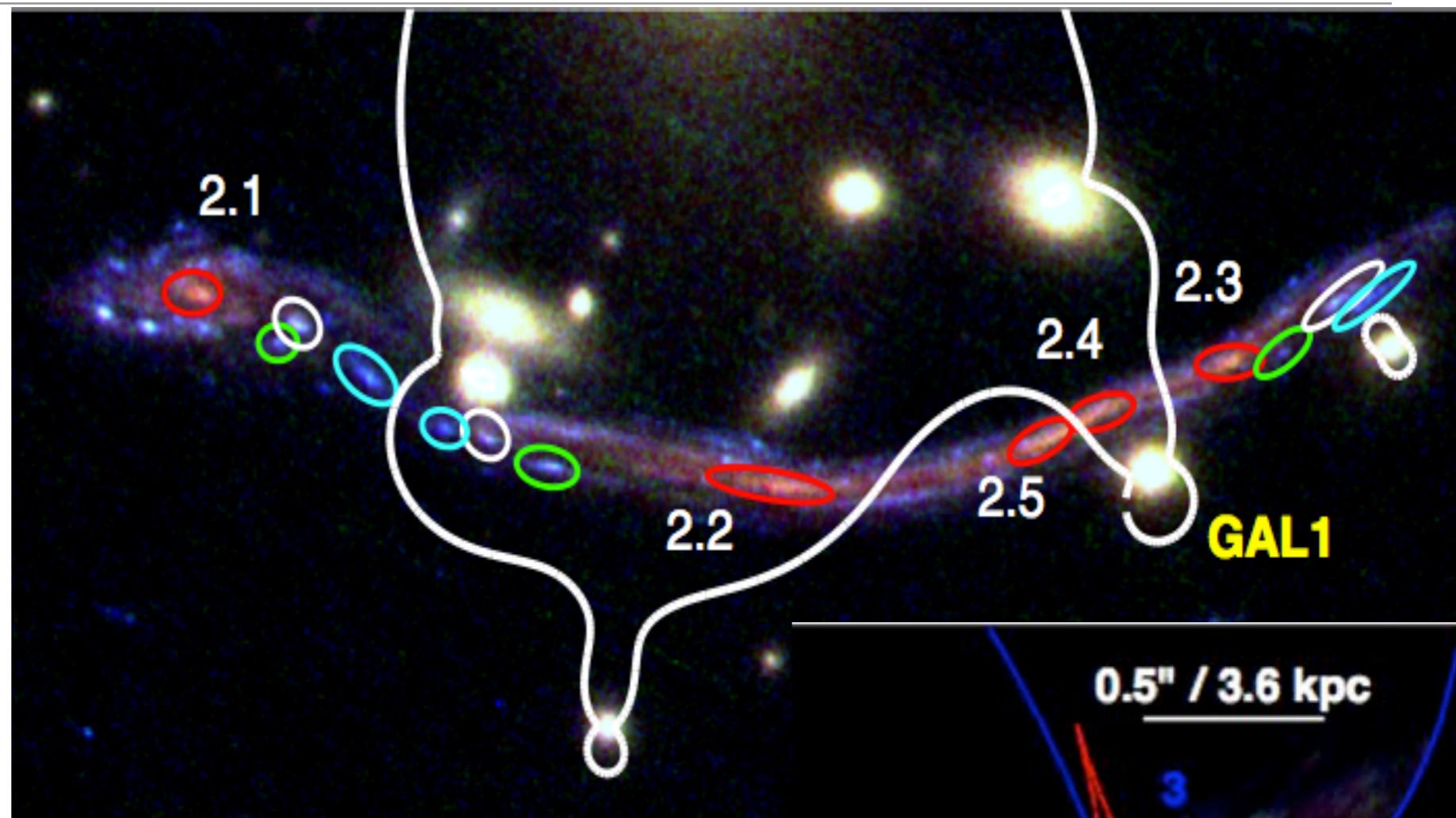
Quintuple arc ($z=1.67$)
in
Cl0024+1654 ($z=0.39$)



How to identify multiple images ?

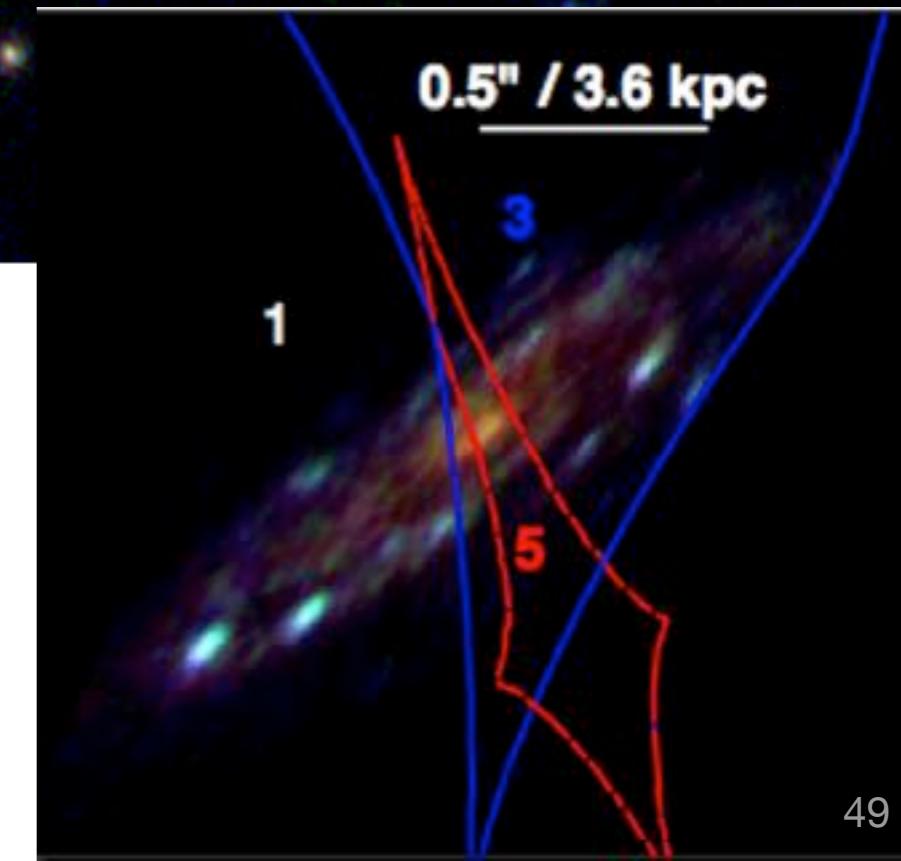
Giant arc in Abell 370 by Richard et al., 2010

HST multi-color images help understand giant arc morphologies...



... and allow unlensed source reconstruction (Richard et al 2010)

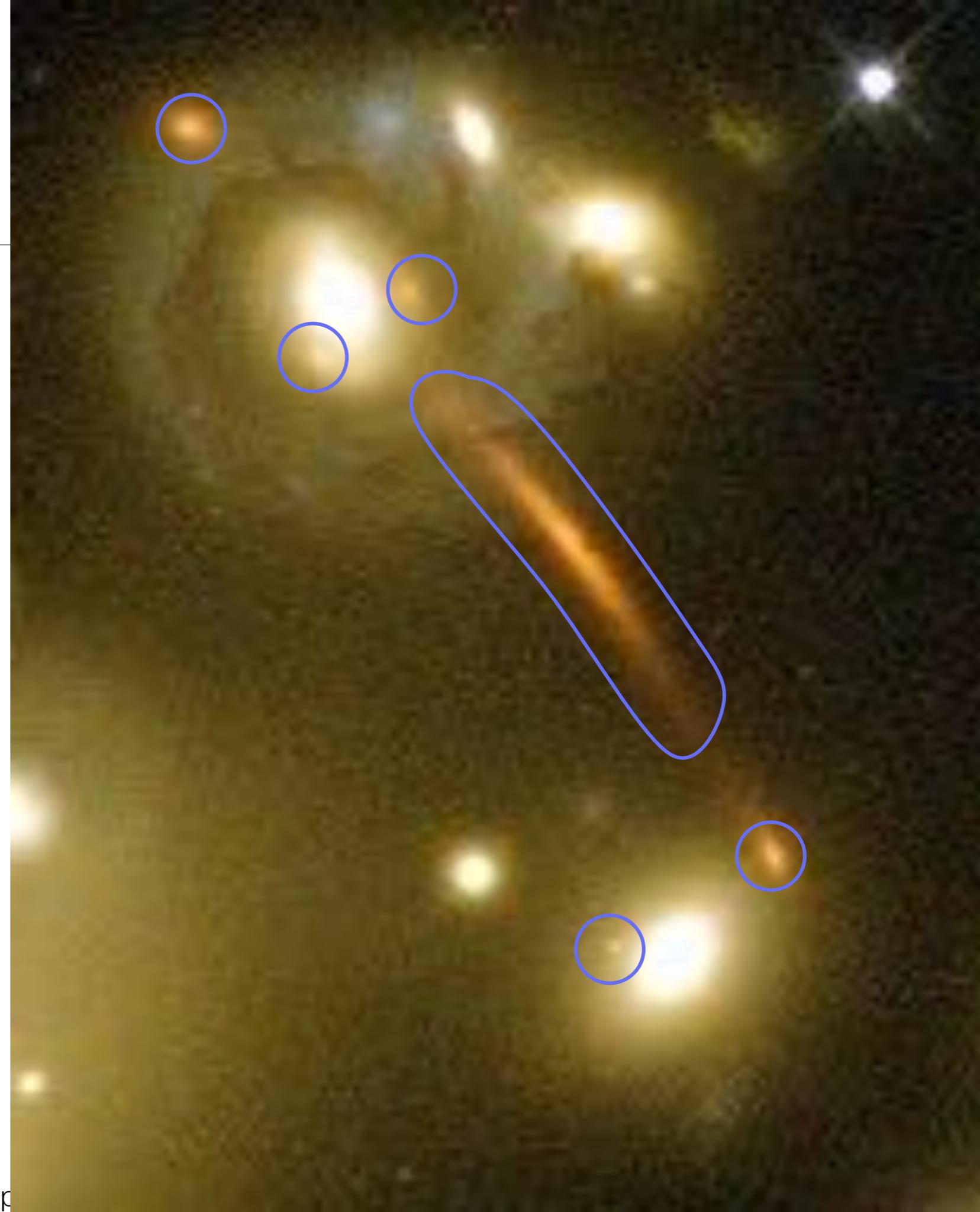
Here the source is at redshift $z=0.725$



Strong Galaxy-Galaxy Lensing in Cluster

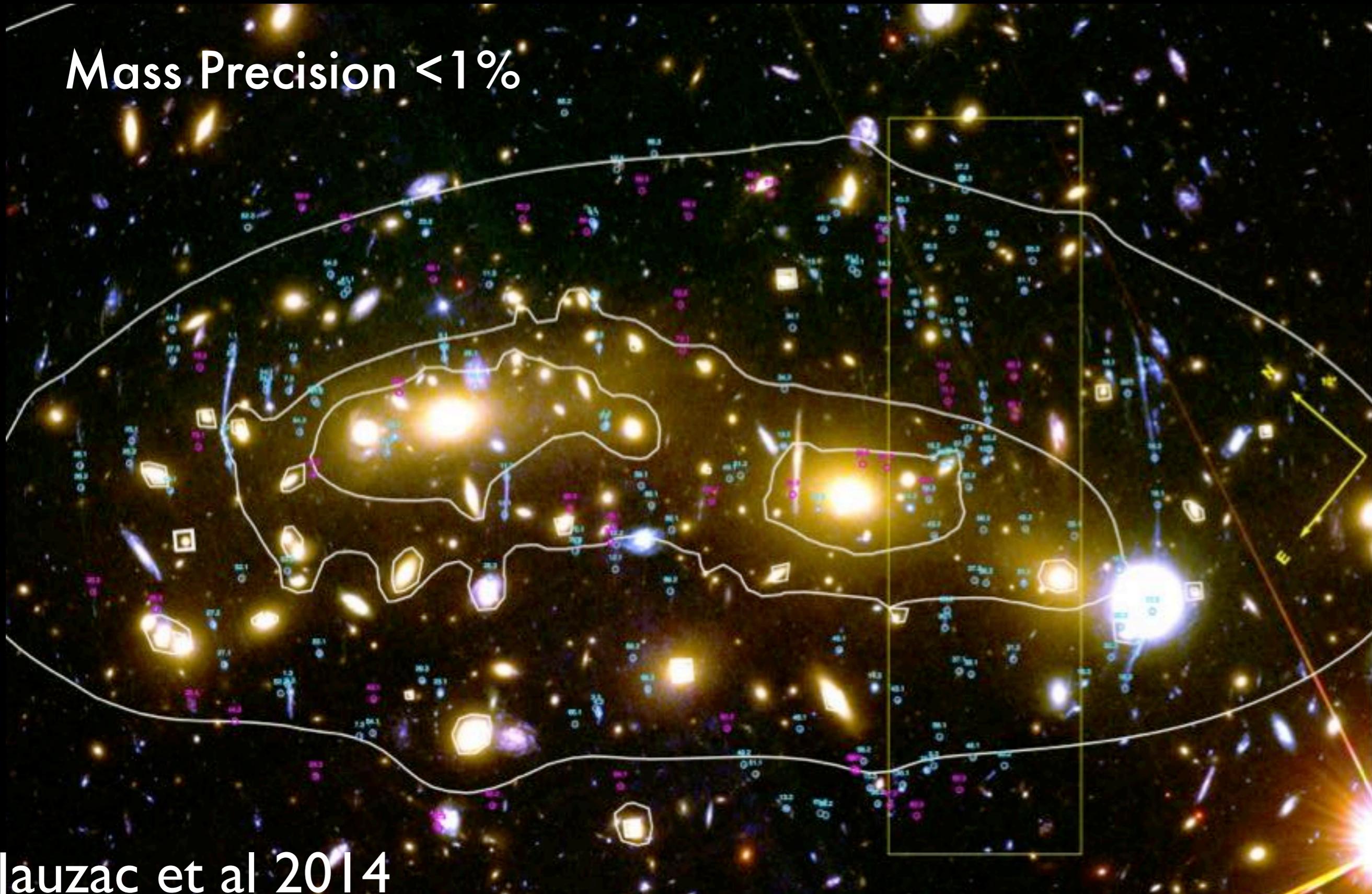
Cluster Galaxies are breaking arcs into smaller ones, adding new images of the lensed galaxy.

Abell 2218, arc at $z=0.702$, with 8 images identified (the arc is the merging of 2 images)



>200 Multiple Images in MACSJ0416

Mass Precision <1%



Modeling

- Which Mass Model?
- What Mass Model Parametrisation?

Singular Isothermal Sphere - 1

- To first approximation stars or other mass components are like particles in a gas.

- Let's consider an ideal gas:

$$p = \frac{\rho k T}{m}$$

- The temperature T is related to the 1D velocity dispersion of the particles with:

$$m \sigma_V^2 = k T$$

- Thus: $p = \rho \sigma_V^2$

Singular Isothermal Sphere - 2

- Consider a spherical shell, its mass is:

$$dM = 4\pi r^2 \rho dr$$

- The gravitational force between the inside and the shell can be expressed as:

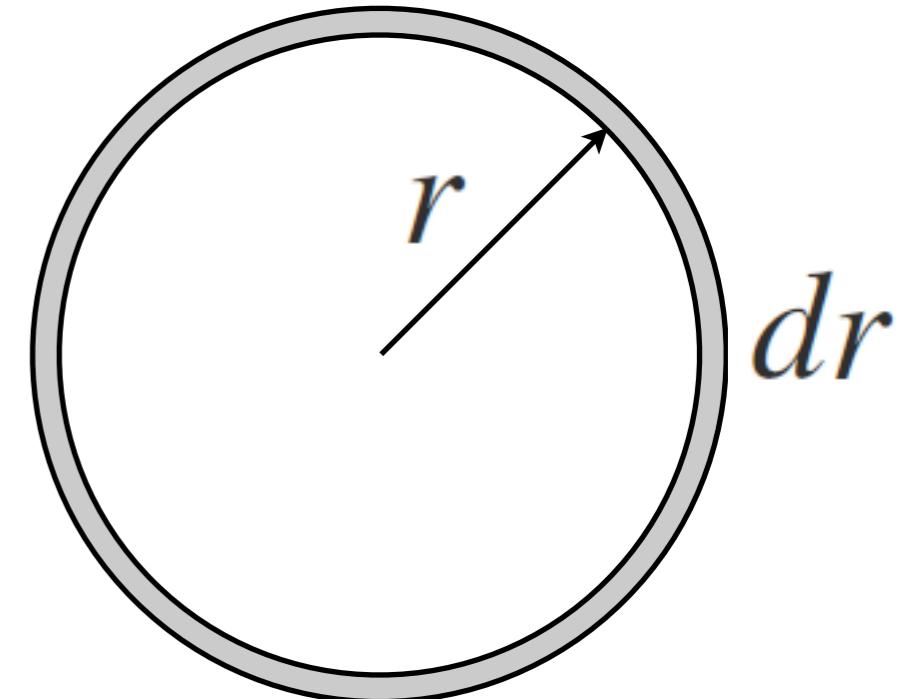
$$dF = -\frac{GM(r)dM}{r^2}$$

- The pressure is thus:

$$dp = -\frac{dF}{4\pi r^2} = -\frac{GM(r)}{r^2} \rho dr$$

- One solution of this equation is:

$$\rho = \frac{\rho_0}{r^2} = \frac{\sigma_v^2}{2\pi G} \frac{1}{r^2}$$



Singular Isothermal Sphere - 3

- Total mass is thus:

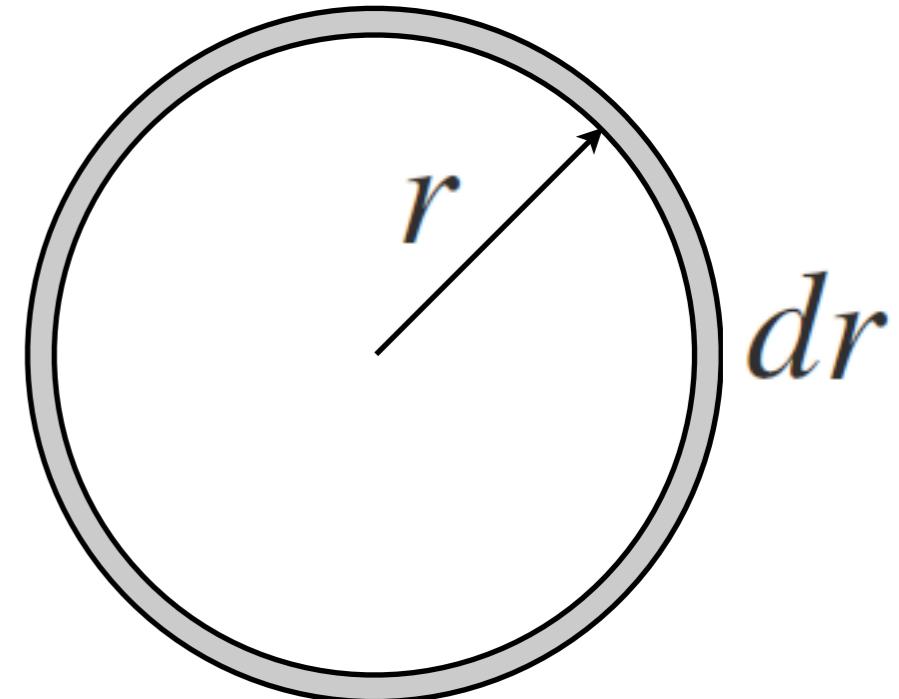
$$M(r) = 4\pi\rho_0 r = \frac{2\sigma_v^2}{G} r$$

- Projected surface density:

$$\Sigma(R) = \frac{\sigma_v^2}{2G} \frac{1}{R}$$

- Issues:

- Mass diverges at large radius
- Mass density diverge at small radius



Truncated Isothermal Sphere with a core

- Adding a core (softening the central density spike):

$$\rho(r) = \frac{\sigma_v^2}{2\pi G} \frac{1}{r_c^2 + r^2}$$

- Truncating the profile at large radius:

$$\rho(r) = \frac{\sigma_v^2}{2\pi G} \left(\frac{1}{r_c^2 + r^2} - \frac{1}{r_{cut}^2 + r^2} \right)$$

- Pseudo isothermal profile:

- Mass converge at large radius $\rho \sim r^{-4}$ $M_{tot} \sim \sigma_v^2 r_{cut}$

- Mass density diverge at small radius $\rho \sim cste$

Navarro-Frenk-White (NFW) mass profile

- The NFW profile correspond to the 3D mass distribution of dark matter fitted to dark matter haloes identified in N-body simulations by Julio Navarro, Carlos Frenk and Simon White (1996, 1997).

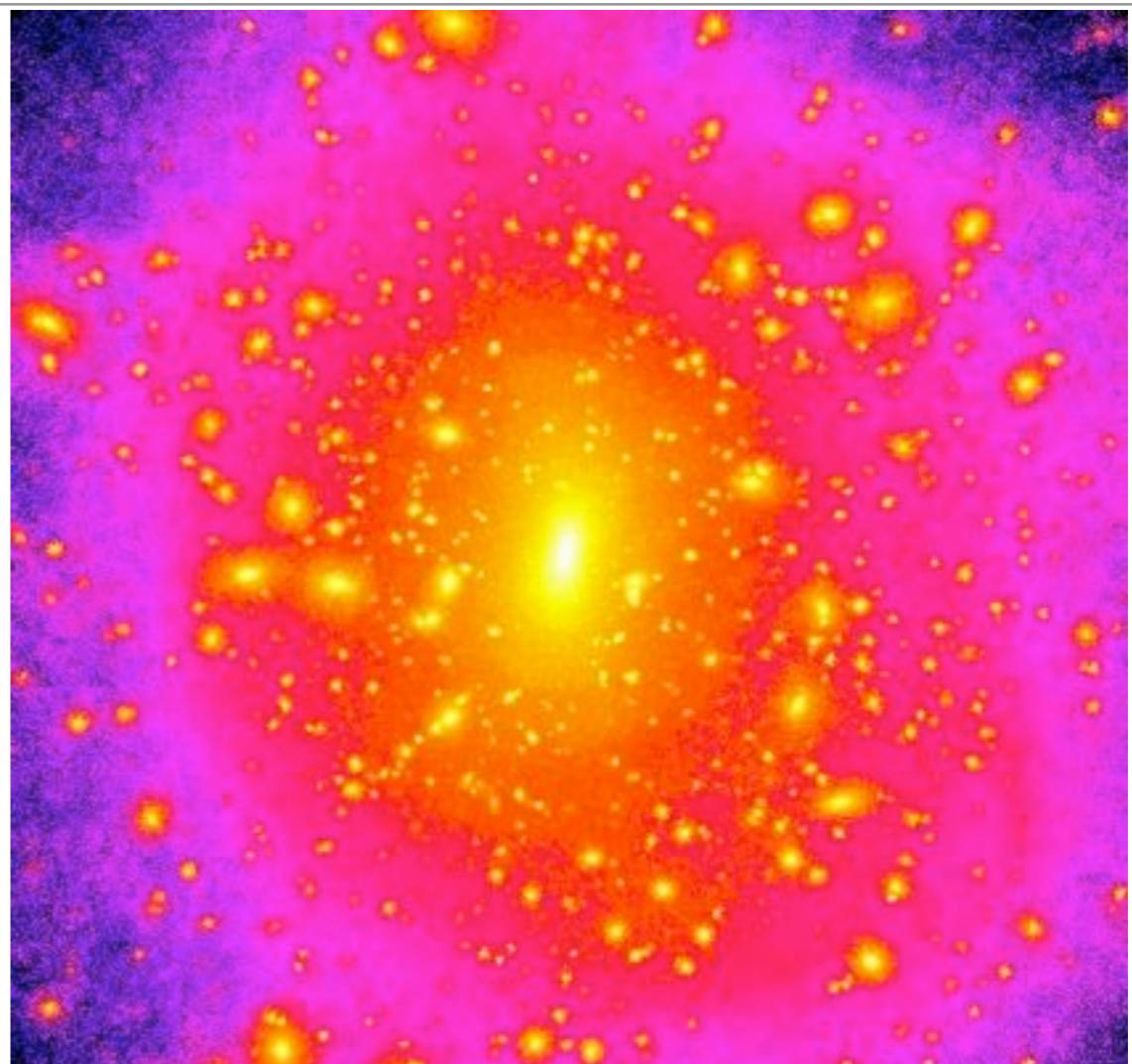
$$\rho = \frac{\rho_0}{r/r_s (1+r/r_s)^2}$$

- The NFW profile is one of the most commonly used model profiles for dark matter halos.
- However:
 - Mass diverges at large radius
 - Mass density diverge at small radius

More recent CDM-only Simulations

(e.g. Navarro et al. 2004; Diemand et al. 2004; Tasitsiomi et al. 2004 + others)

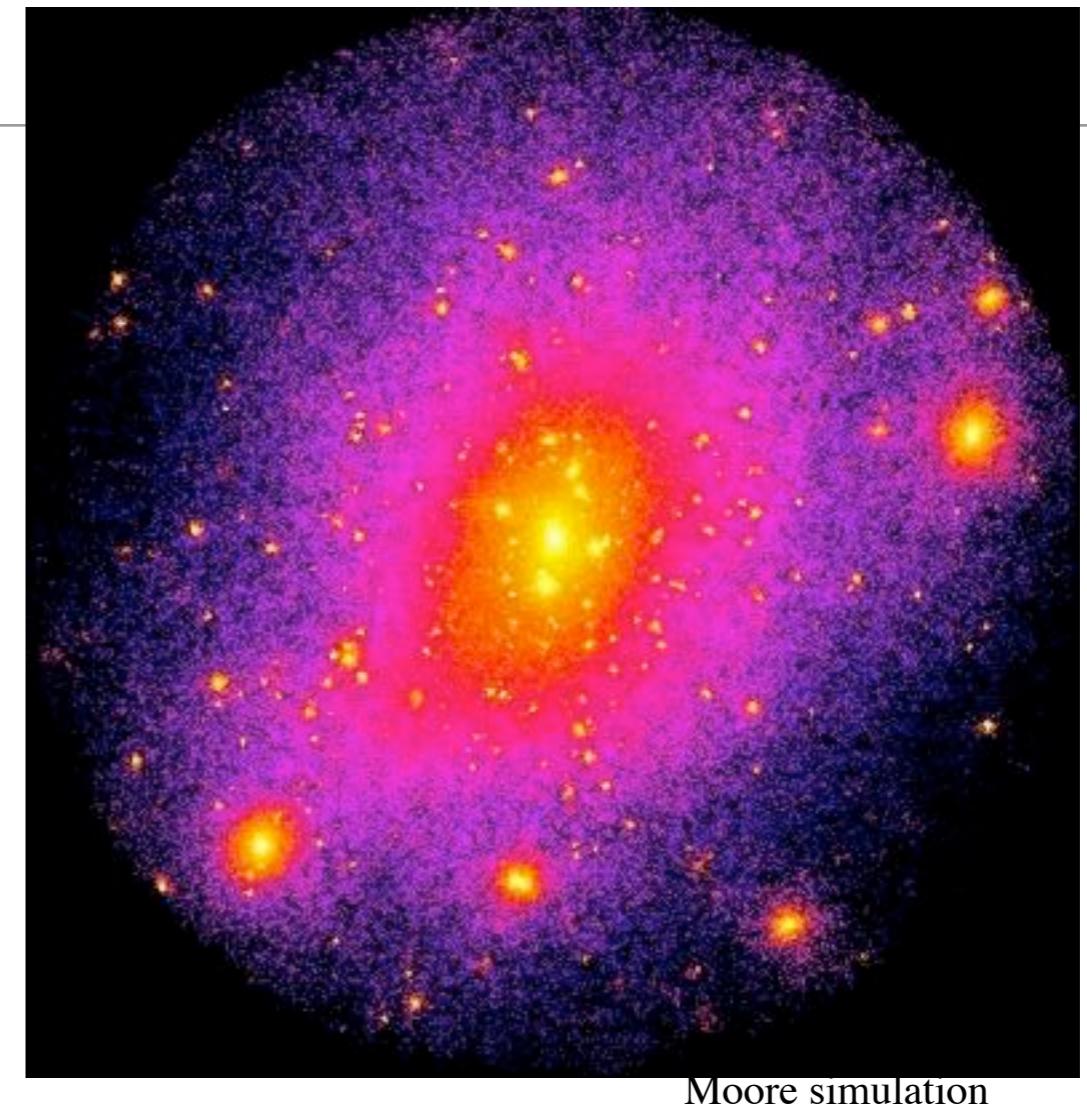
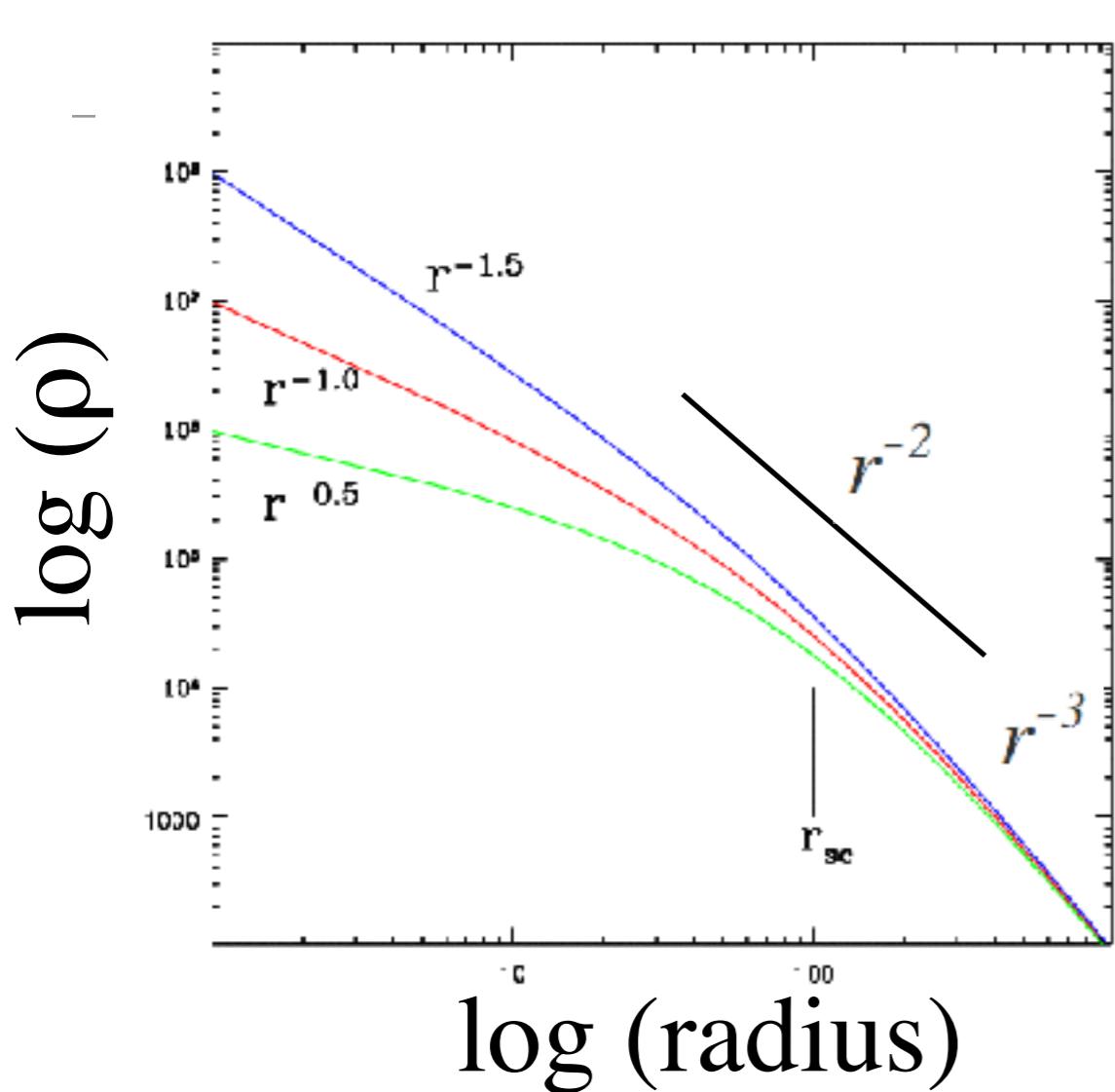
- Convergence achieved down to $\sim 0.003r_{\text{vir}}$...roughly the size of massive galaxies. Baryons are important for progress!!
- Density profiles obtained using different codes and initial conditions agree.
- The generalized NFW density profile is a good fit to simulations with β between 1.0 & 1.5. There is significant scatter.
- Sersic profiles seems to be an alternative to generalized NFW expression (Merritt et al 2005)



Generalized NFW

$$\rho = \frac{\rho_0}{(r/r_s)^\beta (1+r/r_s)^{3-\beta}}$$

Generalised NFW profile



Inner profile: $\rho \propto r^{-\beta}$ NFW: $\beta \rightarrow 1.0$ Moore: $\beta \rightarrow 1.5$

What is the inner slope of cluster DM profiles?

What is the TOTAL density profile?

Mass model representation

- large scale cluster component+sum of galaxy halo components ([DM+gaz]+galaxy halos):

$$\phi_{tot} = \phi_{cluster} + \sum_i \phi_{halos}^i \quad \text{Kneib et al 1996}$$

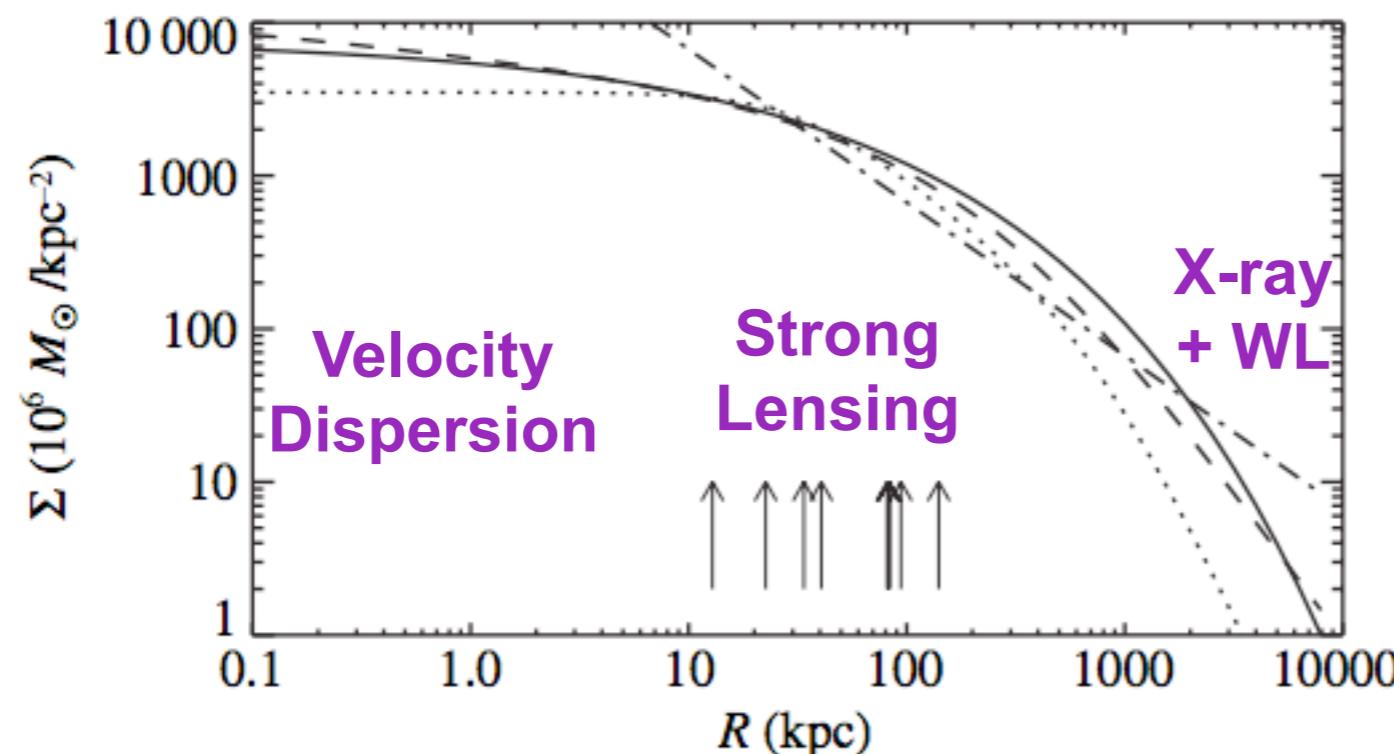
- need to scale the galaxy halo components; for example for a PIEMD mass distribution:

$$\sigma = \sigma_* \left(\frac{L}{L_*} \right)^{1/4} \quad r_{cut} = r_{cut}^* \left(\frac{L}{L_*} \right)^\eta$$

- Hence: $\frac{M}{L} \propto L^{\eta-1/2}$ $\eta = 1/2$ Constant M/L
 $\eta = 0.8$ FP scaling

Mass profile for a mass clump?

- Mass profile should match theoretical or numerical simulations in order to be close to reality (avoid Gaussian function for example):
 - isothermal model (singular => cored & truncated, circular => elliptical): PIEMD
 - NFW model => gNFW, Sersic, Einasto (beware at infinite values, truncate?)

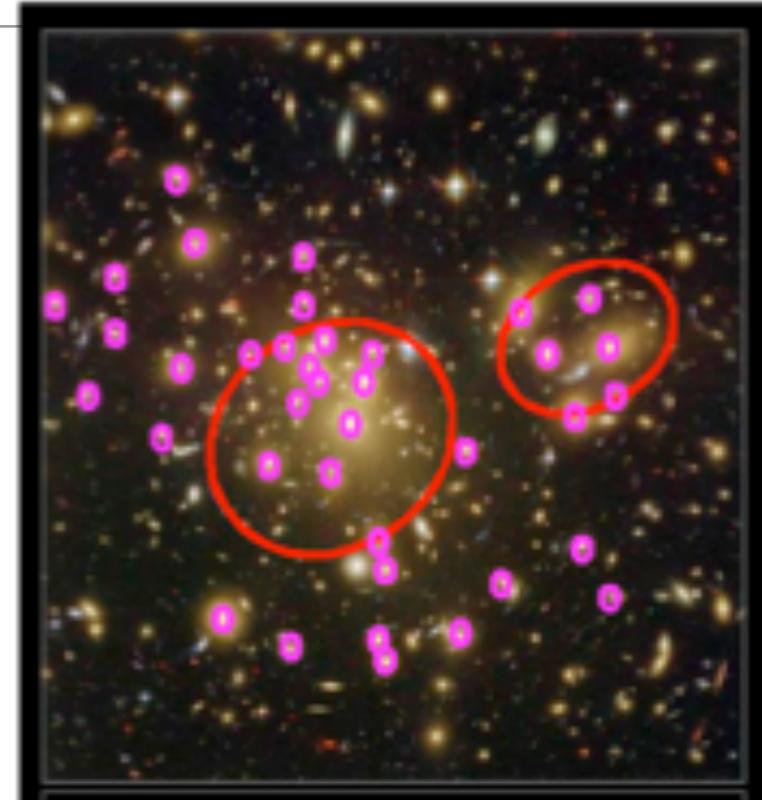


Additional data
useful to constraints
further the mass
profile

Strong lensing modeling strategies

Observationally motivated models

- Decomposition into halos
- Simple clusters
- Few constraints
- Good fit with few constraints



Grid-based models

- Decomposition into pixels/clumps
- Complex clusters
- Lots of constraints
- Better fit with lots of constraints

