

Basics of Modern Cosmology - 3

Jean-Paul KNEIB

ESO: The European Southern Observatory

- Inter Governmental Organisation funded in 1962 (following the CERN model) - 17 countries.

<https://www.eso.org/>

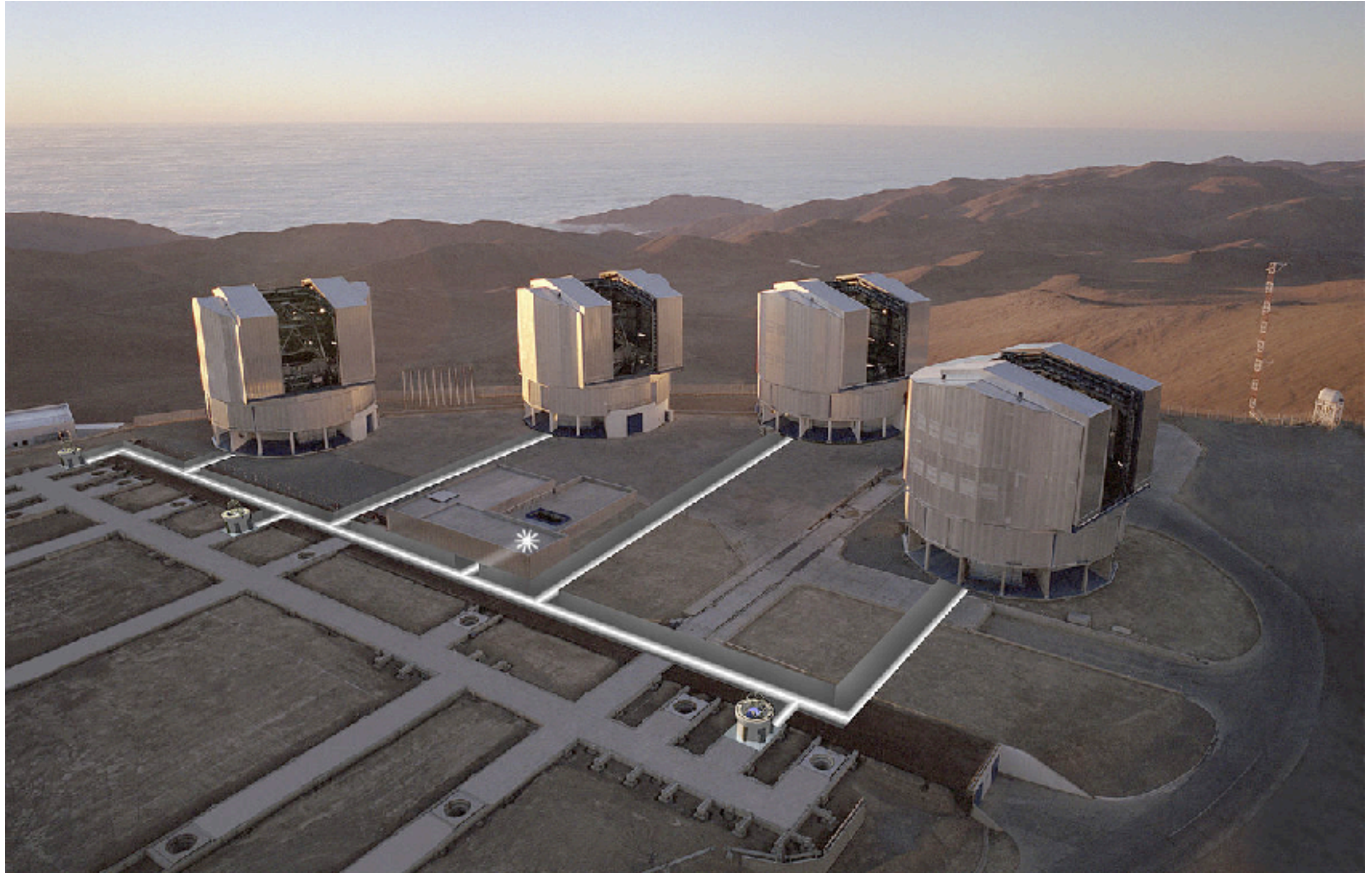
- Based in Garching near Munich
- 4 sites: La Silla, Paranal (VLT), Charnantor (ALMA), Cerro Armazones (E-ELT)
- *ESO PhD studentship* (DL April 20 and Oct 20)

<https://recruitment.eso.org/>

ESO Very Large Telescopes

<https://www.eso.org/>

<http://www.eso.org/sci/activities/FeSt-overview/ESOstudentship.html>



ALMA radio interferometer

<https://www.almaobservatory.org/en/home/>

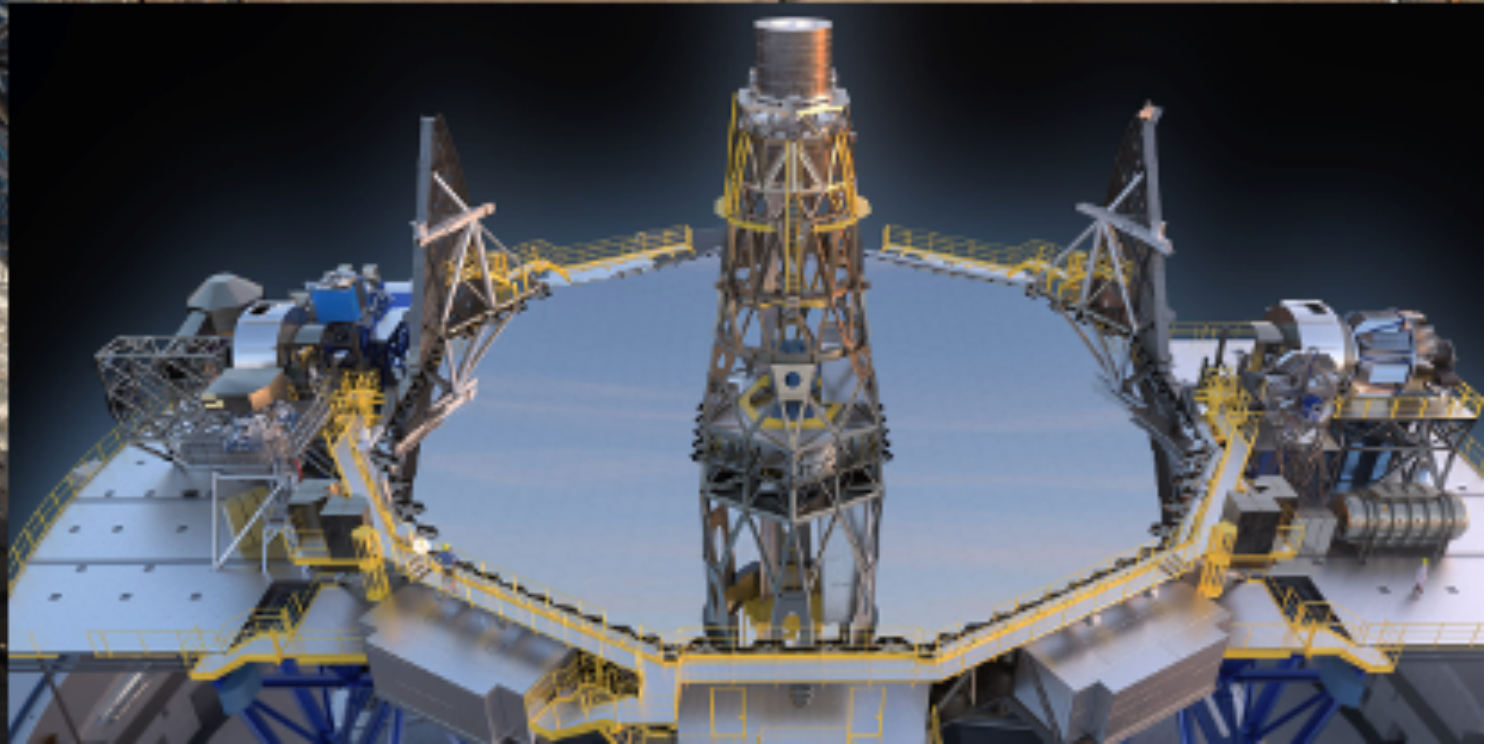
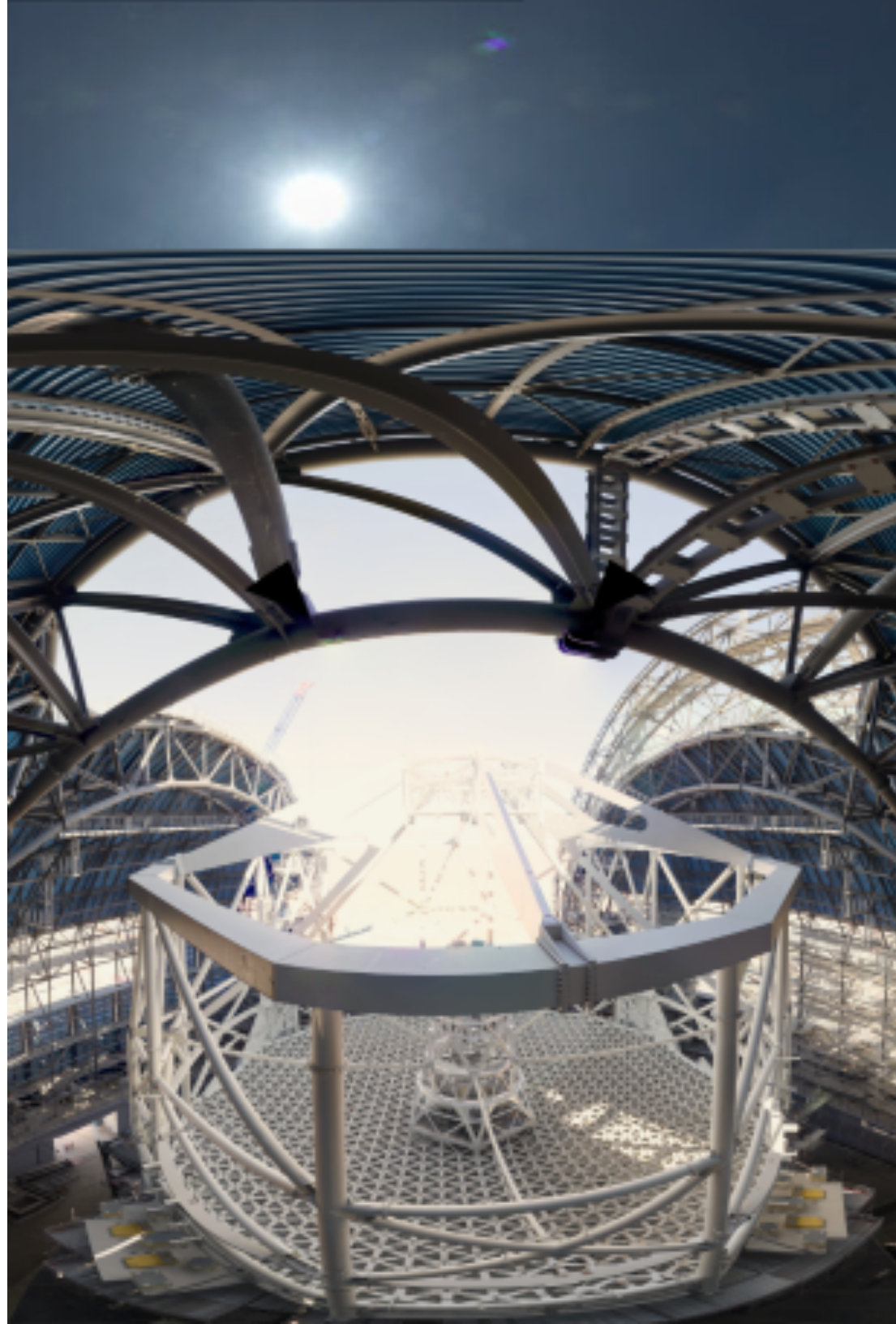


The European Extremely Large Telescope

21 Mar 2025 21:00 CET / 17:00 CLST

<https://elt.eso.org/>

Latest Available Image



Quiz

- What is the unit of Λ ? What is its value?
- What is the equation of state of Matter? of radiation?
- How the density of matter/radiation scales with the scale factor?
- How the scale factor scales with time?
- When the Matter/Radiation dominates the mass/energy density?
- What is the critical density?
- How to write the Friedman equation with Ω egas?

Outline

- More on distances
- Cosmological tests

More on the expression of distances

- Dark Energy parametrization
- $E(z)$
- Comoving Distance
- Proper distance
- Angular diameter distance
- Luminosity distance
- Comoving Volume element
- Lookback time

General Dark Energy parametrization

- The equation of state of DE is *a priori* not a constant (general case):

$$\rho(a) = a^{-3(1+w)}$$

w represent the equation of state:

$w=0$ for matter,

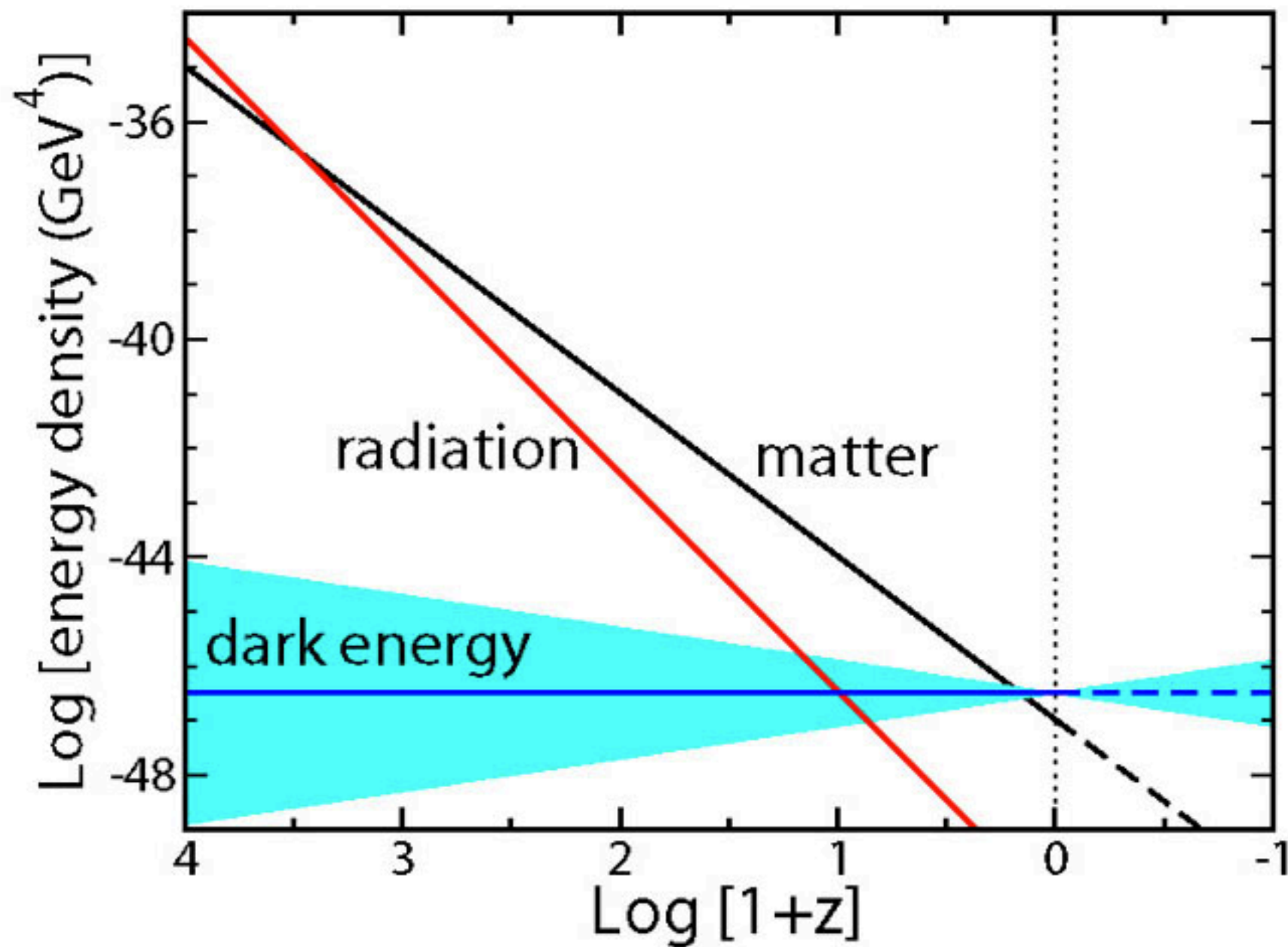
$w=1/3$ for radiation,

$w=-1$ for the cosmological constant

$w=w(a)$ if non constant (Dark Energy)

- Possible parametrisation:

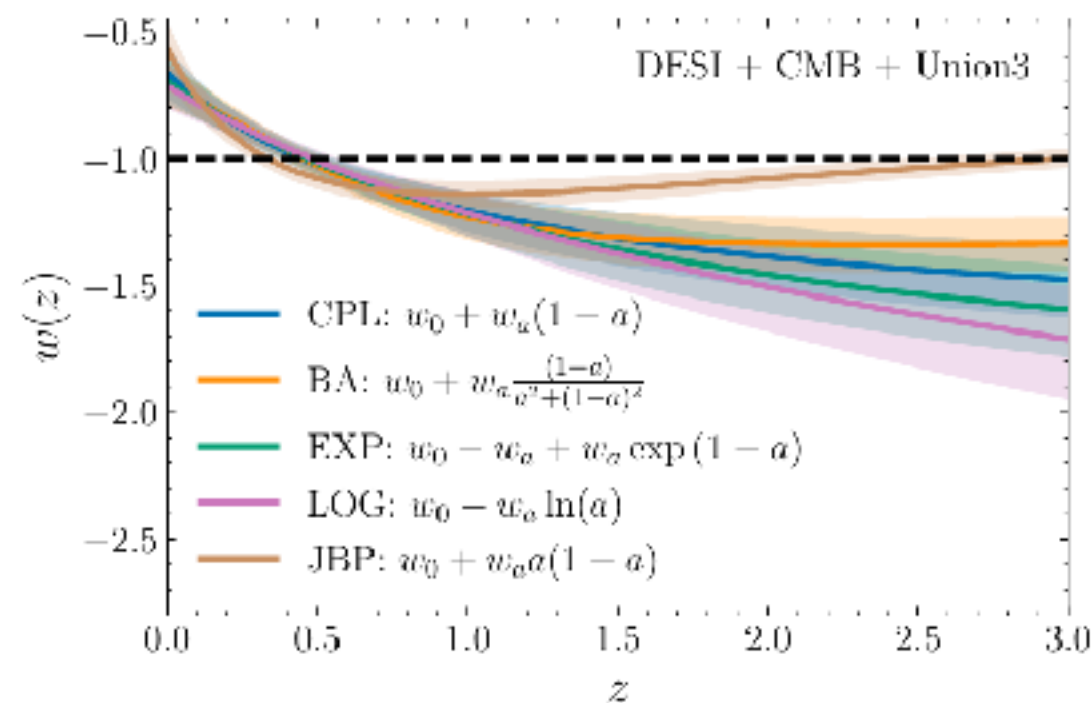
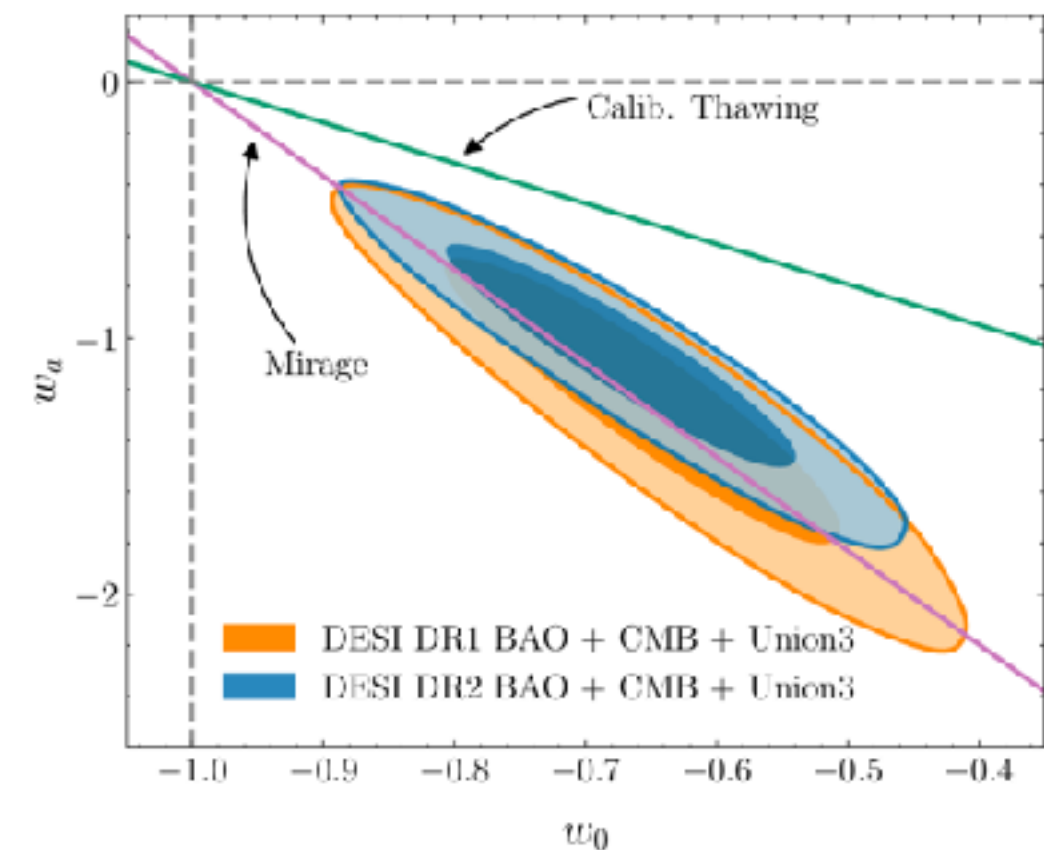
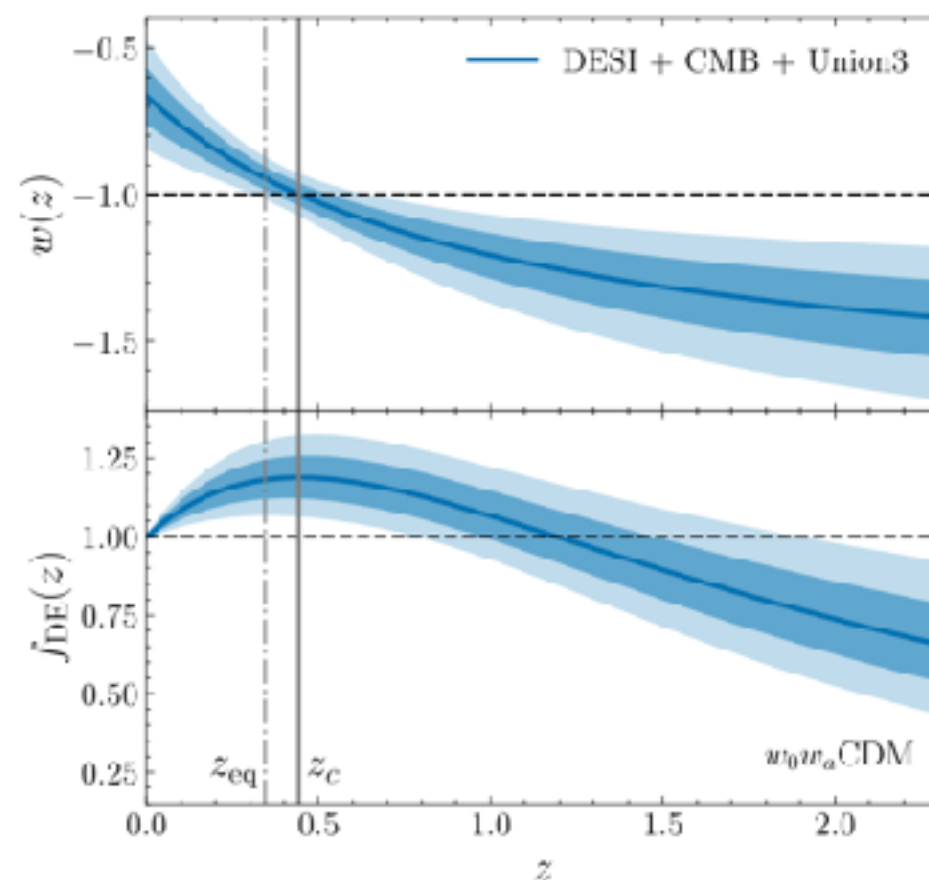
$$w = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$



There are uncertainties of what kind of possible Dark Energy,
Equation of State possible is: $w = -1 \pm 0.2$
But aim to measure this with high accuracy

DESI - DR2 results

March 2025:
Indication of
an evolving
Dark Energy?



Normalized Hubble Parameter

- **Definition:** Normalized (or reduced) Hubble parameter $E(z)$:

$$H(z) = H_0 E(z)$$

Expression of $E(z)$ as a function of the reduced densities

$$E^2(z) = \Omega_M (1+z)^3 + \Omega_R (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda$$

$$E^2(z) = (1+z)^2 [\Omega_M (1+z) + \Omega_R (1+z)^2 + \Omega_k + \Omega_{DE} (1+z)^{1+3w}]$$

The Omegas here are those measured today!

Comoving Distance

- The *comoving* distance is the distance between two points measured along a path defined at the present time ($ds=0$):

$$d_{comoving} = \int_{t_0}^t \frac{c dt}{a(t)} = \int_0^z \frac{c dz}{H(z)} = \frac{c}{H_0} \int_0^z \frac{dz}{E(z)}$$

Indeed

$$\frac{dt}{a(t)} = \frac{da}{H a^2} = - \frac{dz}{H(z)}$$

Proper Distance

- r is the proper distance

$$d_{comoving} = \frac{c}{H_0} \int_0^z \frac{dz}{E(z)} = \int_0^r \frac{dr}{\sqrt{1-kr^2}} = f(r)$$

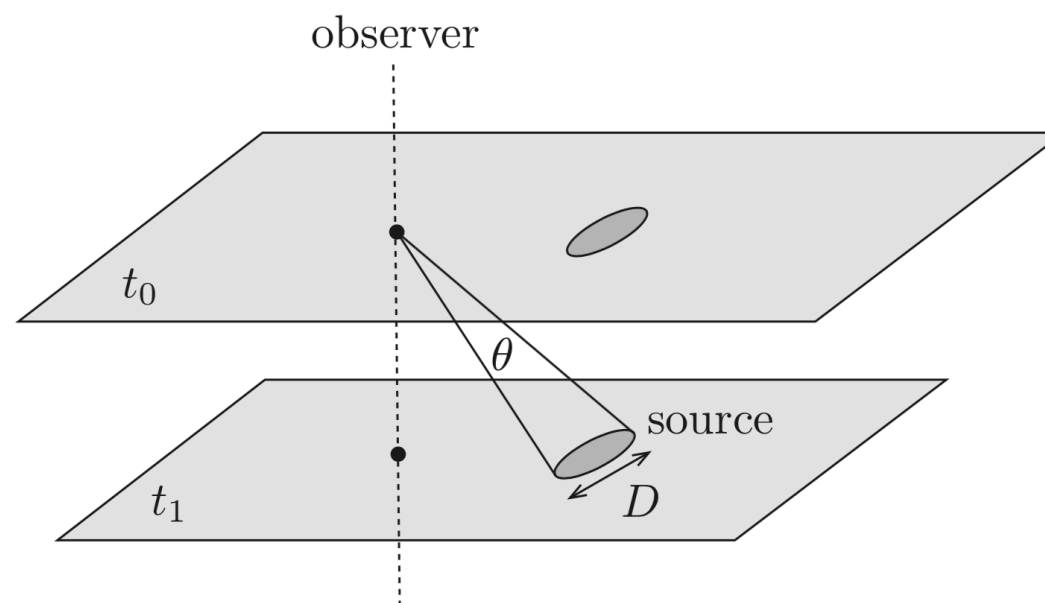
$$f(r) = r \quad (k=0), \quad \sin(r) \quad (k>0), \quad \sinh(r) \quad (k<0)$$

$$r = f_k^{-1}(d_{comoving})$$

Angular Diameter Distance

- Angular diameter distance

$$d_A(z) = r(z)/(1+z)$$



Angular Diameter and Luminosity Distance

- Angular diameter distance

$$d_A(z) = r(z)/(1+z)$$

- Luminosity distance

$$d_L(z) = r(z)(1+z)$$

$$d_L(z) = d_A(z)(1+z)^2$$

Surface brightness

- The surface brightness is the flux received by unit of surface:

$$I_o = \frac{F}{\Omega} = \frac{L}{4\pi d_L^2} \frac{d_A^2}{S} = \frac{L}{4\pi S} \frac{d_A^2}{d_L^2} = \frac{I_e}{(1+z)^4}$$

This is the cosmological **surface brightness dimming**, making high-redshift galaxies difficult to observe

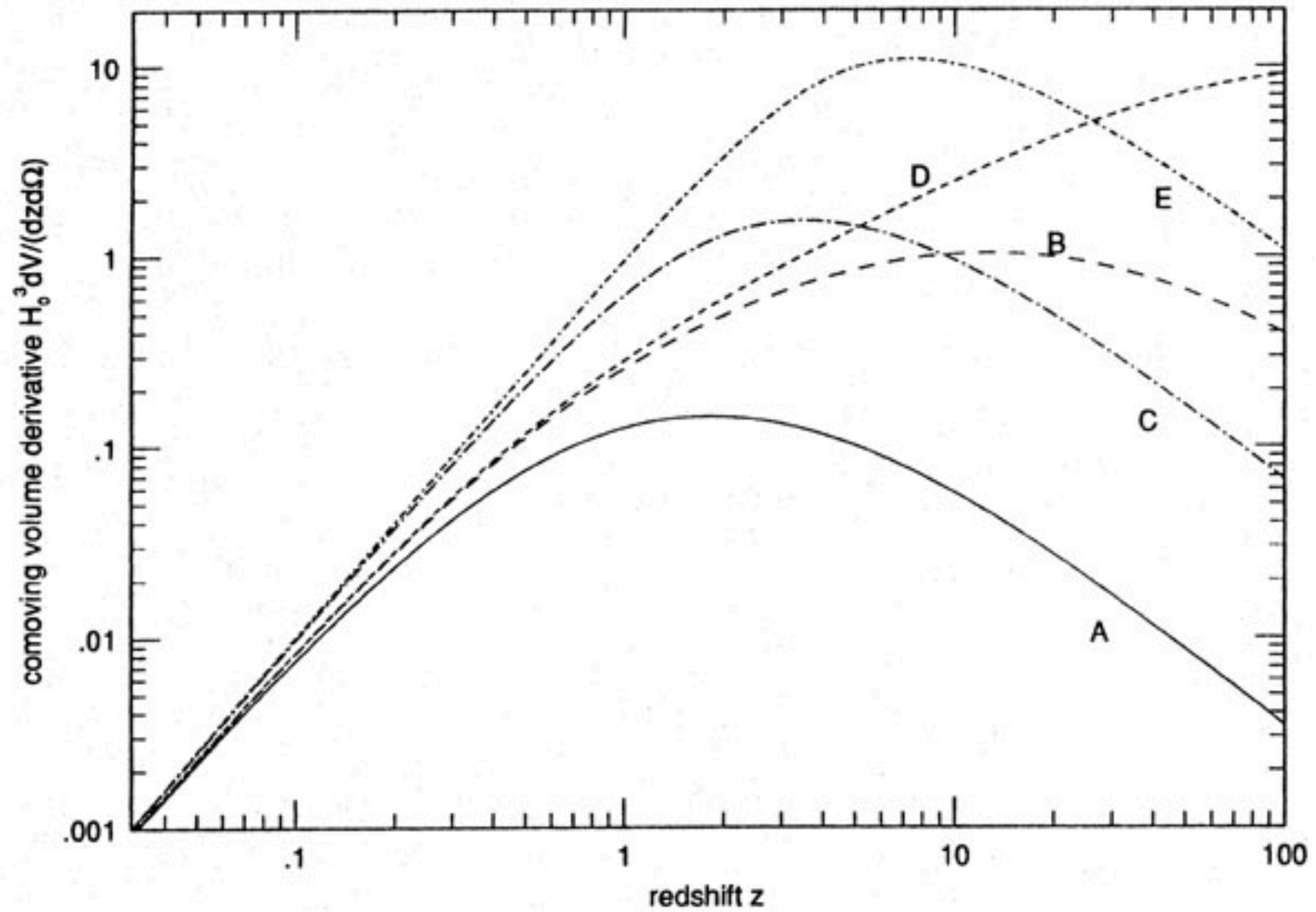
Comoving Volume

- The comoving volume V_C is the volume measure in which number densities of non-evolving objects locked into Hubble flow are constant with redshift.
- It corresponds to the proper volume times: $(1+z)^3$.

$$dV_C = \frac{r^2(z) dr d\Omega}{\sqrt{1 - kr^2}}$$

- Can be rewritten as:

$$dV_C = \frac{c}{H_0} \frac{(1+z)^2 d_A^2}{E(z)} d\Omega dz$$

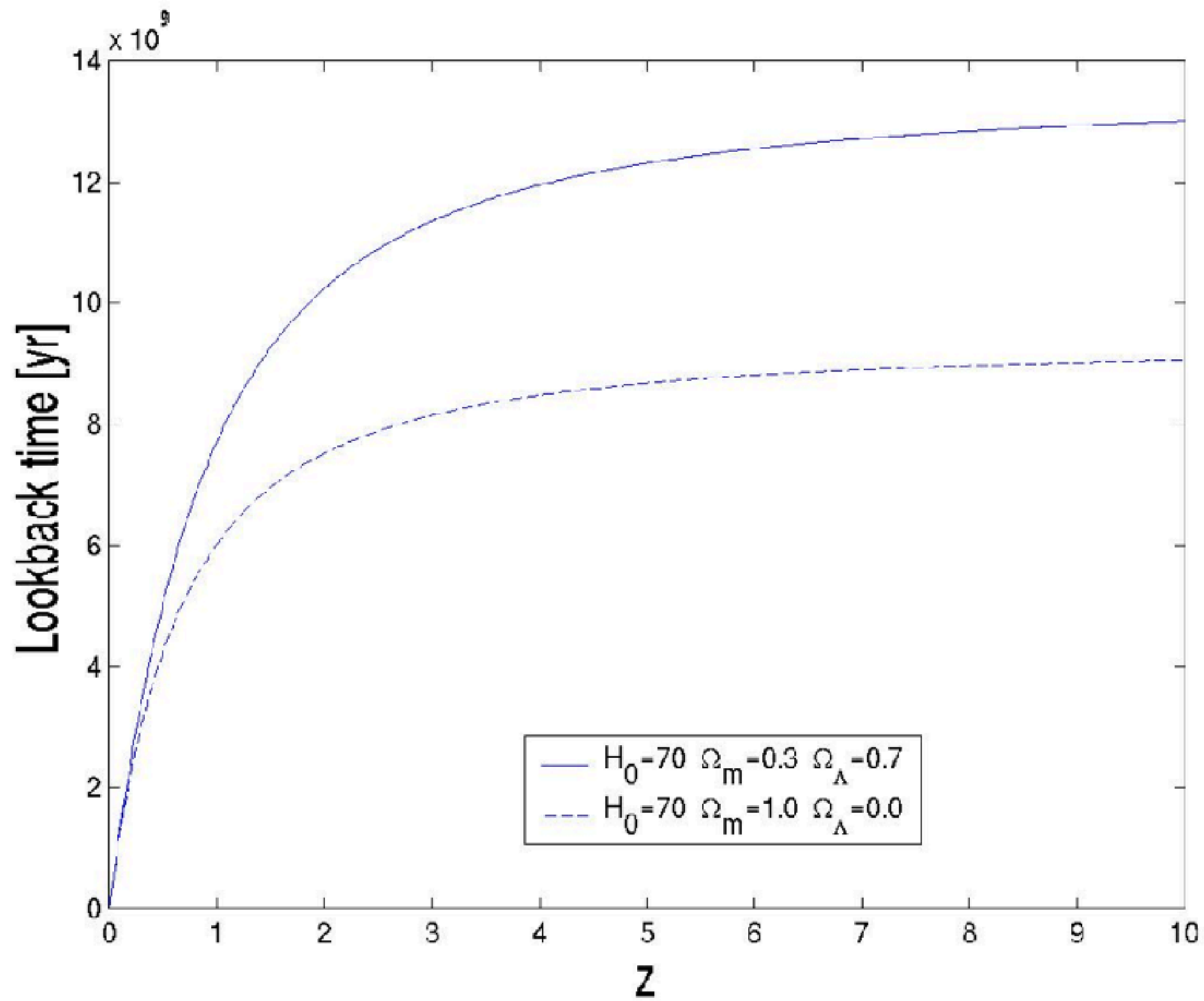


Comoving volume for different cosmology

Look-back Time

- The look-back time to an object: the difference between the age of the Universe now (at observation) and the age of the Universe at the time the photons were emitted:

$$t_L = \frac{1}{H_0} \int_0^z \frac{dz}{(1+z)E(z)}$$



Flatness problem

Friedman equation *without cosmological constant*:

$$H^2 = \frac{8\pi G}{3} \rho - \frac{kc^2}{a^2}$$

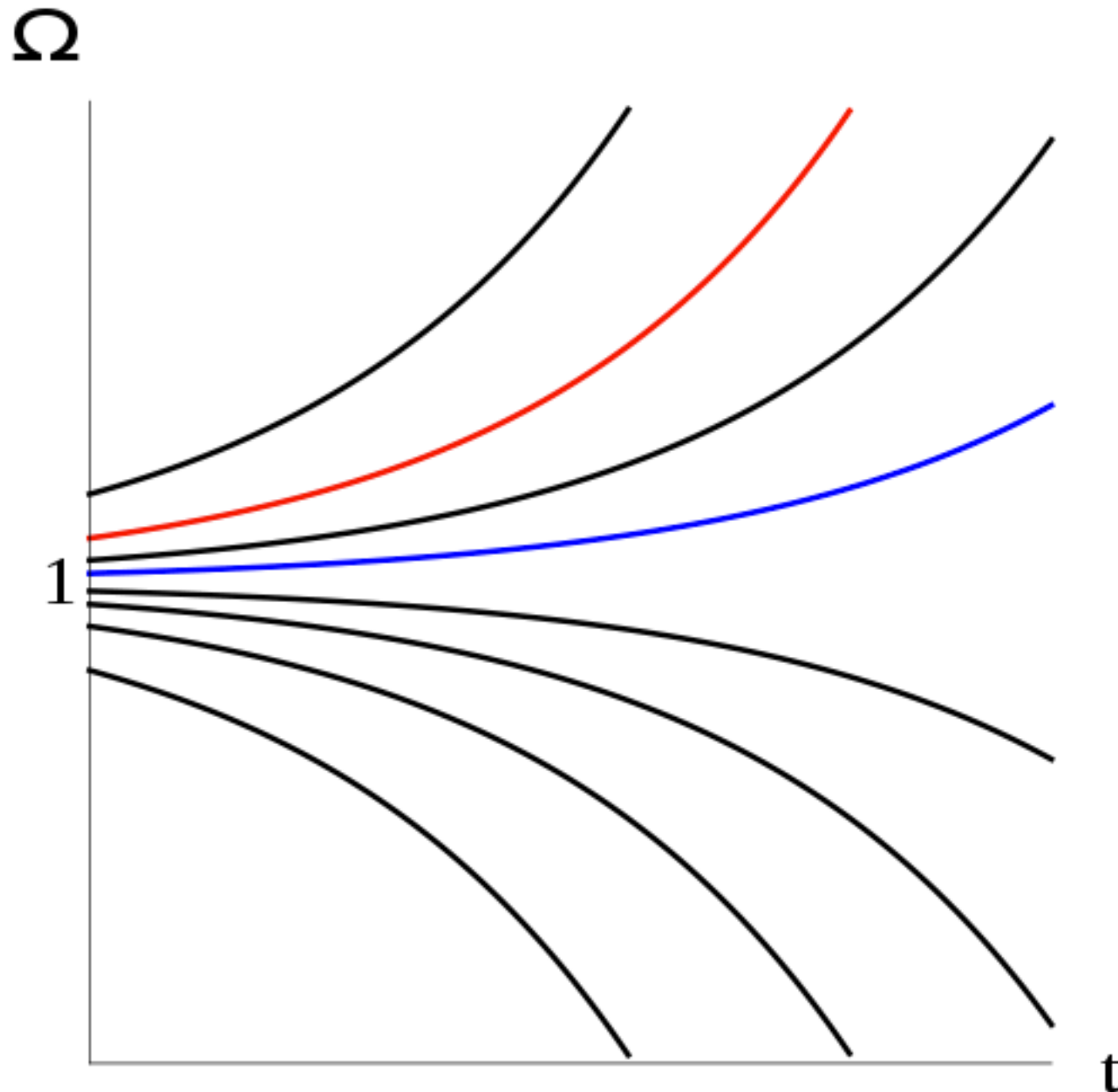
Rearranging the equation, introducing the critical density

$$\frac{3a^2}{8\pi G} H^2 = \rho a^2 - \frac{3kc^2}{8\pi G} \qquad \rho_c a^2 - \rho a^2 = -\frac{3kc^2}{8\pi G}$$

$$(\Omega^{-1} - 1) \rho a^2 = \boxed{\frac{-3kc^2}{8\pi G}} \text{ Constant}$$

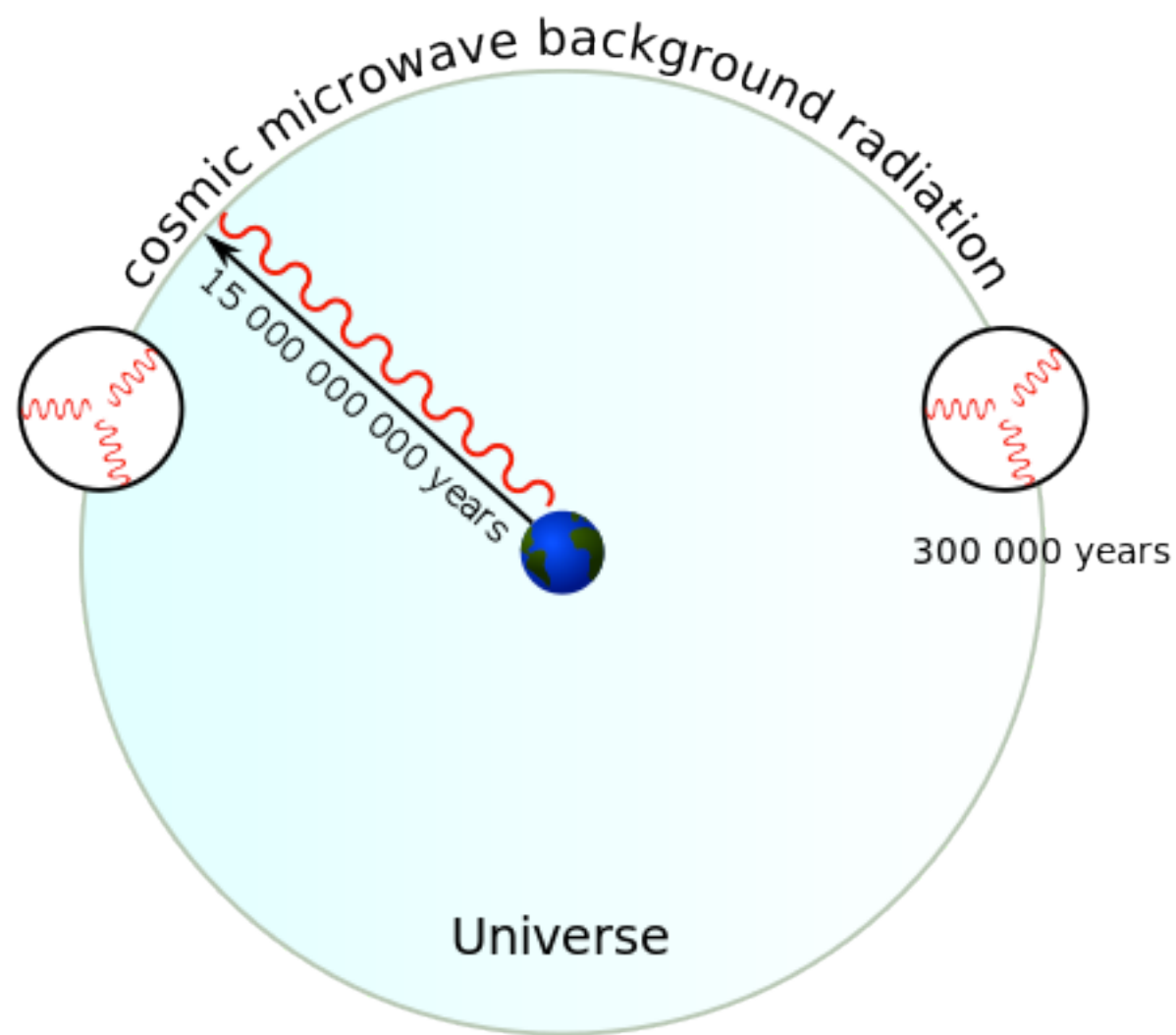
Flatness problem

- The relative density Ω against cosmic time t



The $\Omega=1$ value is repulsive.
So this is considered a good reason to argue that the universe must have been always exactly flat ($k=0$)

Horizon Problem

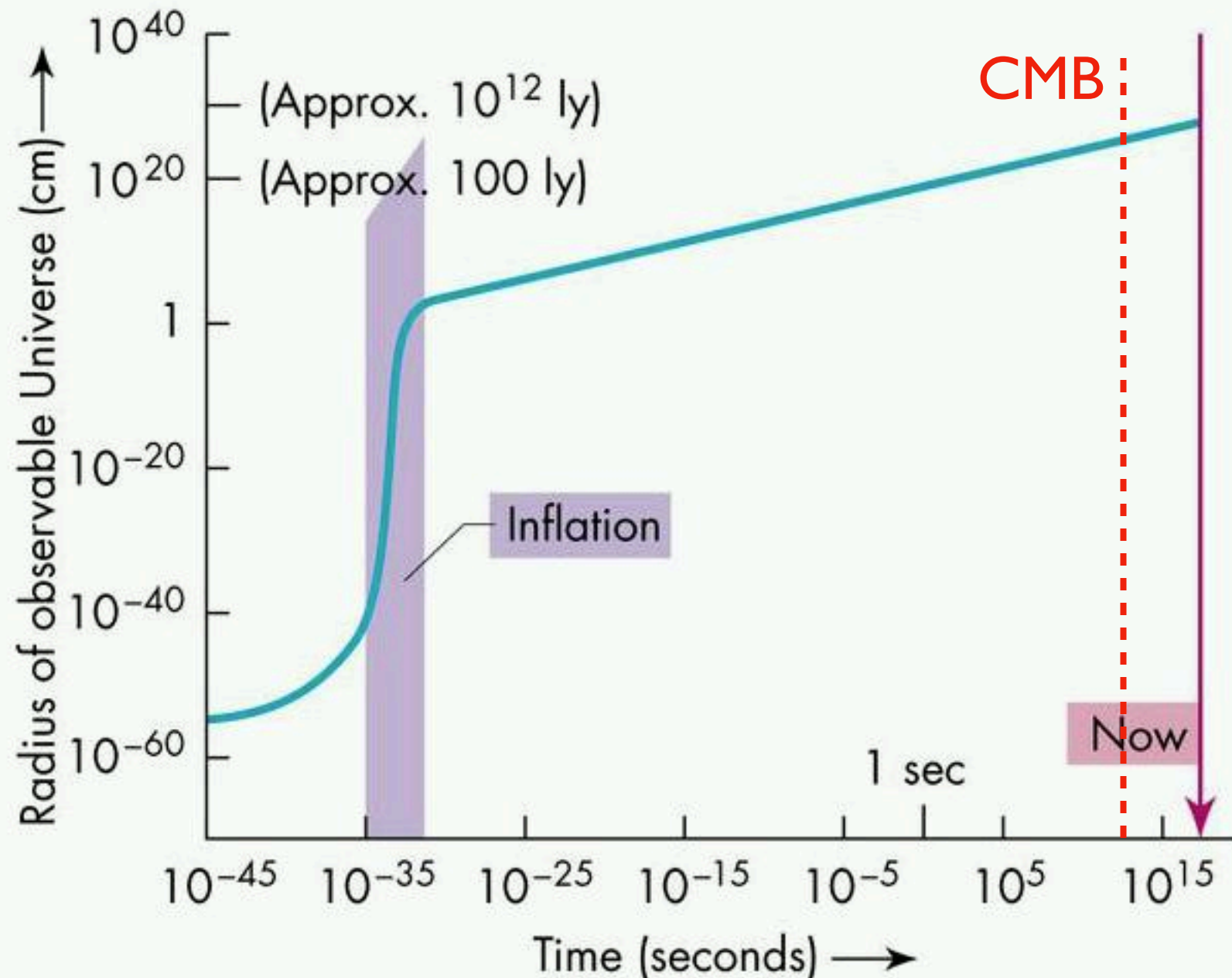


When we look at the CMB it comes from ~45 billion comoving light years away. However, when the light was emitted the universe was much younger (380,000 years old).

In that time light would have only reached as far as the smaller circles. The two points indicated on the diagram would not have been able to contact each other because their spheres of causality do not overlap.

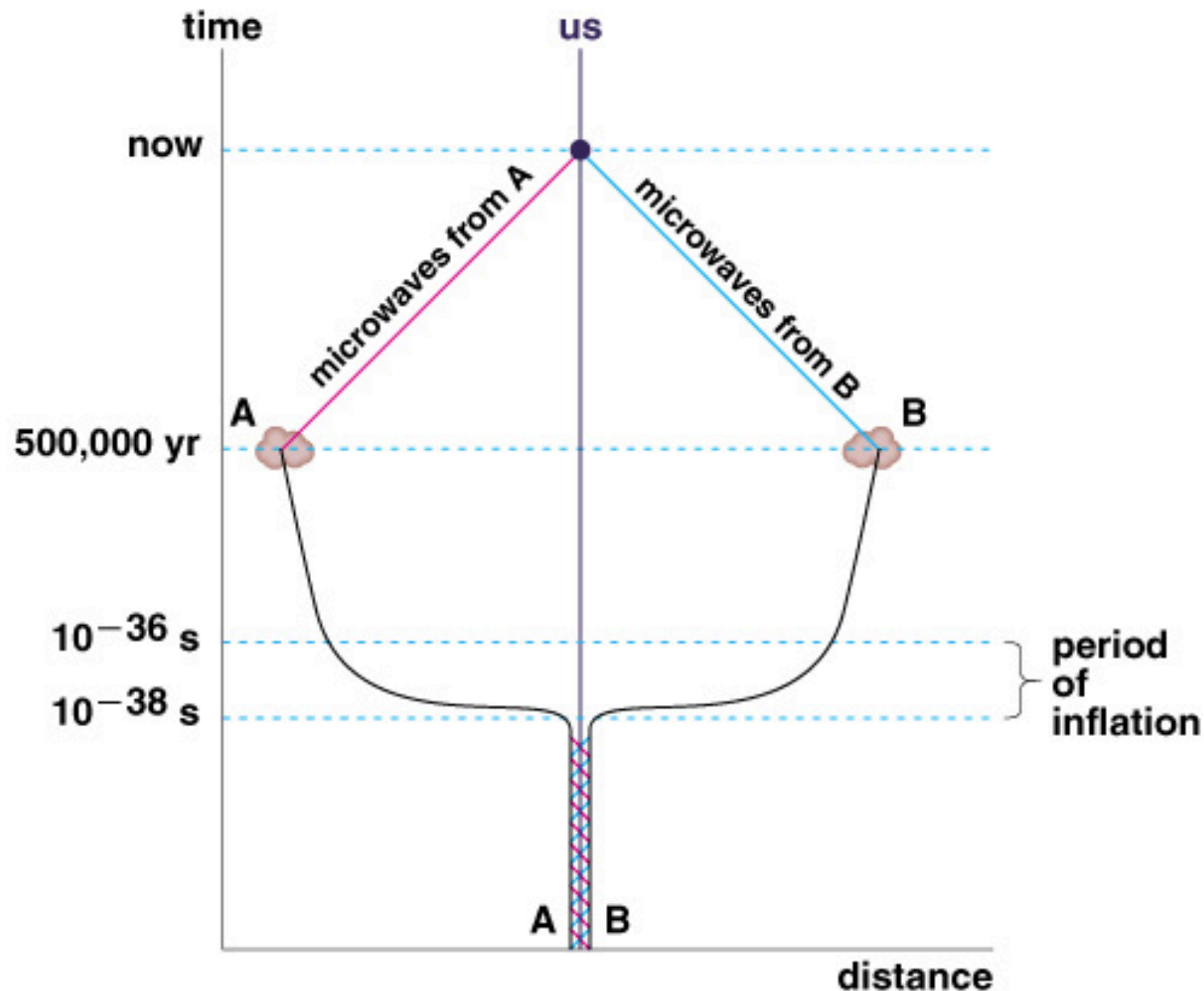
Yet, those 2 points look like identical?

A solution: Inflation



This is a concept that aims to solve the Horizon problem by considering a very fast expansion in the early Universe.

Inflation



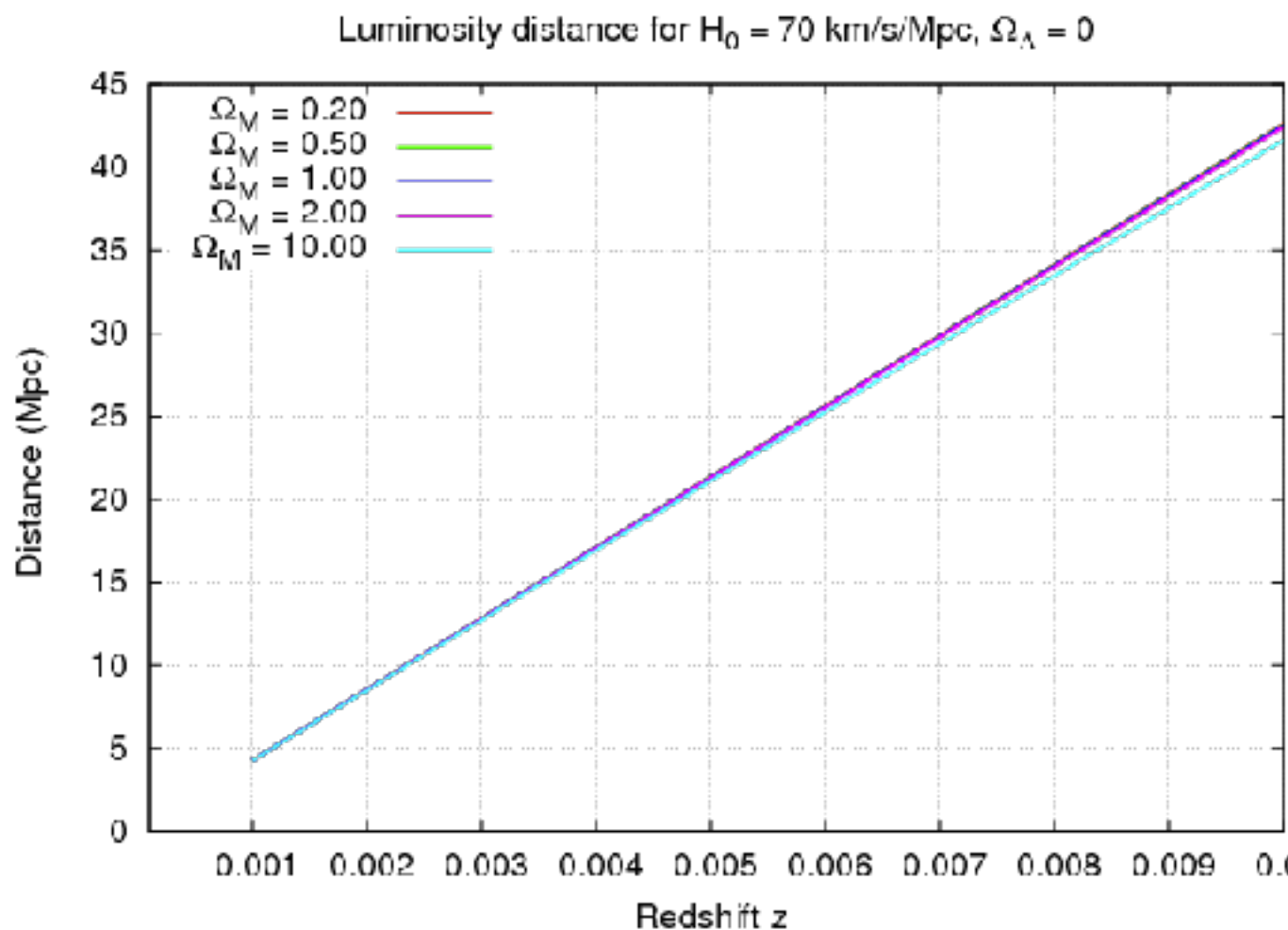
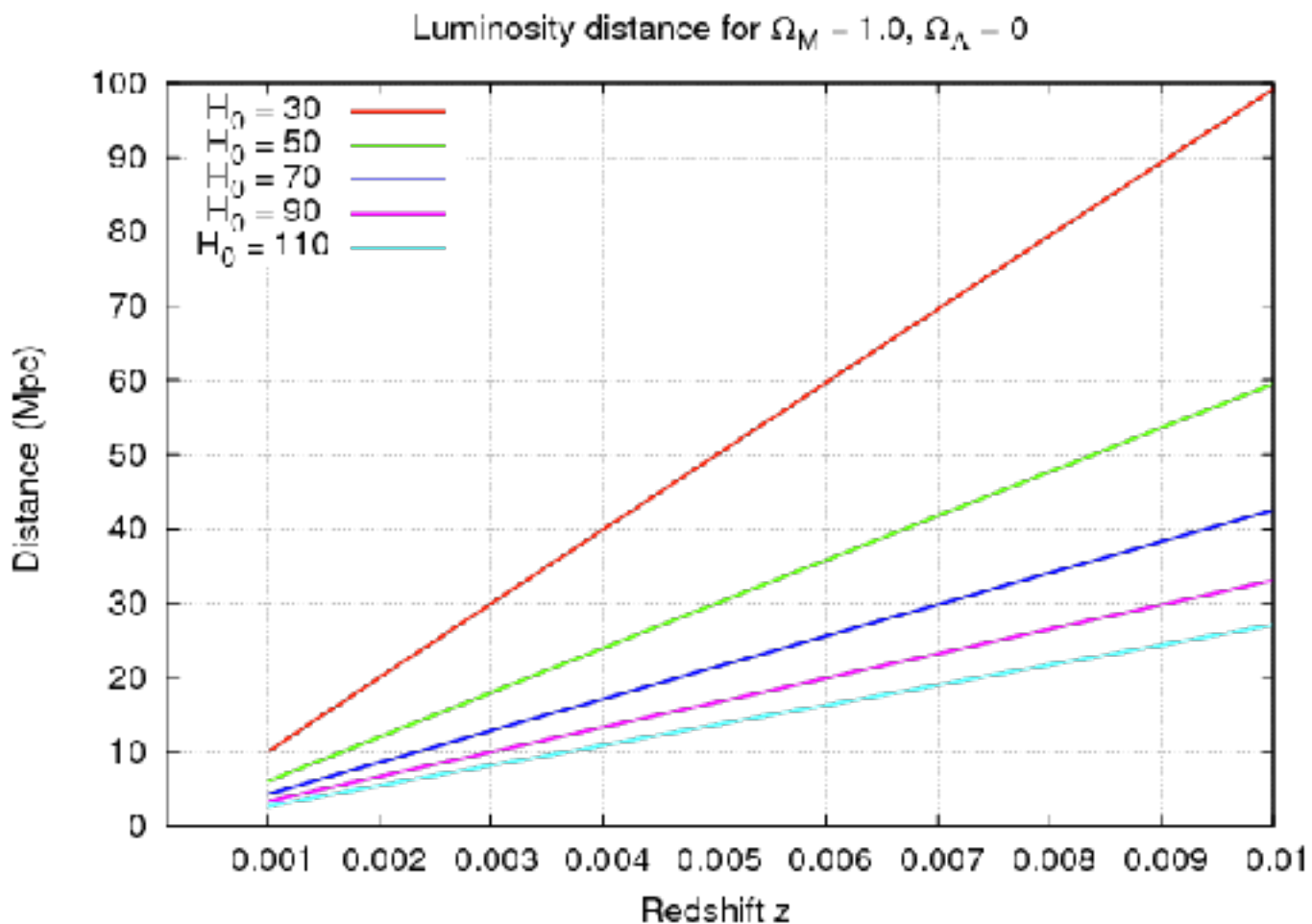
This is a concept that aims to solve the Horizon problem by considering a very fast expansion in the early Universe.

Classical Cosmological Tests

- Luminosity distance at low redshift
- Apparent size versus redshift (size test)
- surface brightness versus redshift (Tolman test)
 - *very hard as there is a big fluctuation of SB from galaxy to galaxy*
- Conservation of number of galaxies (Loh and Spillar test) - **subject to galaxy evolution**
- Counting galaxies as a function of redshift (Covolume test) - **subject to galaxy evolution**

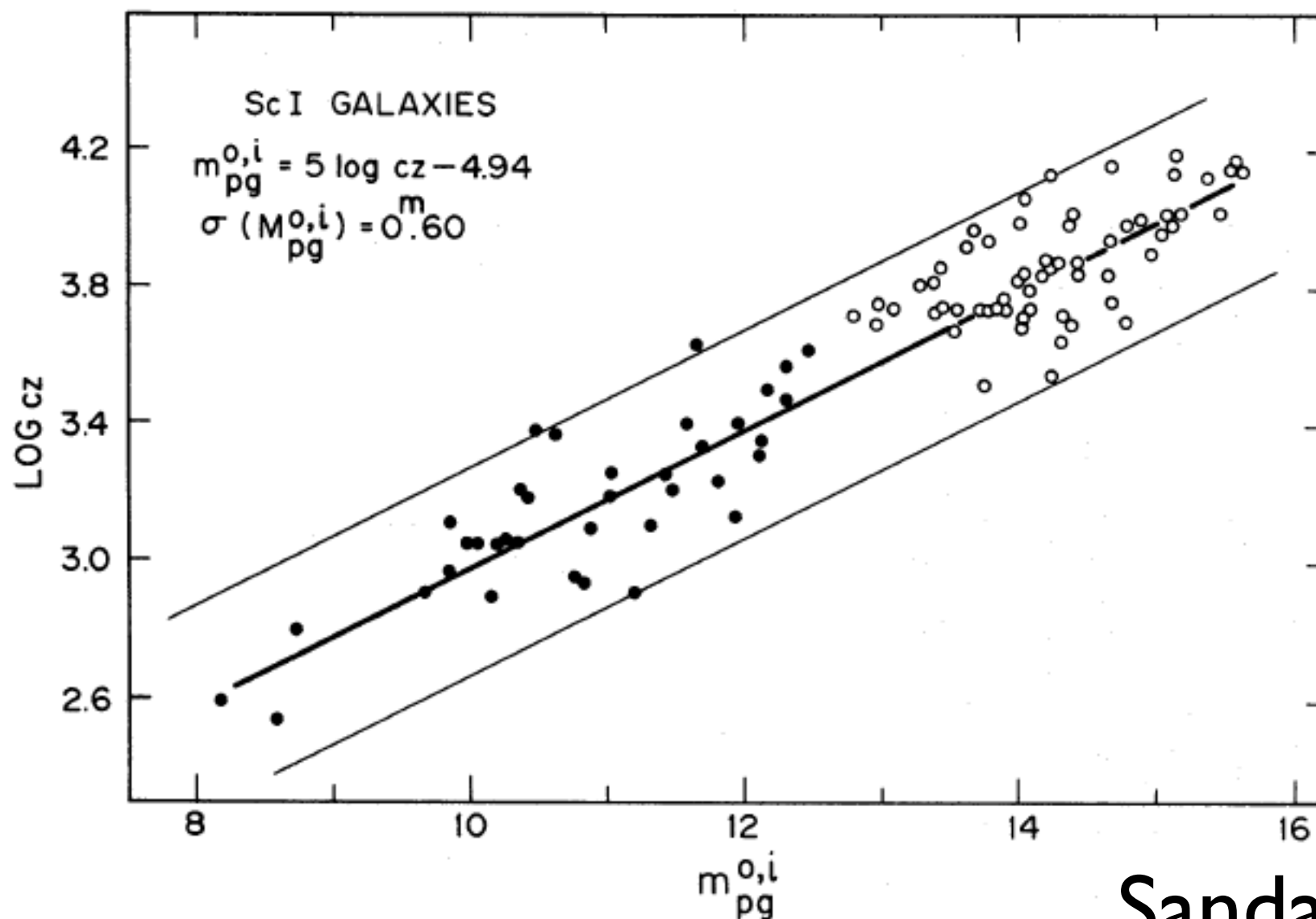
Luminosity distance at low redshift

At low- z , mainly sensitive at H_0 and not cosmology



Luminosity distance at low redshift

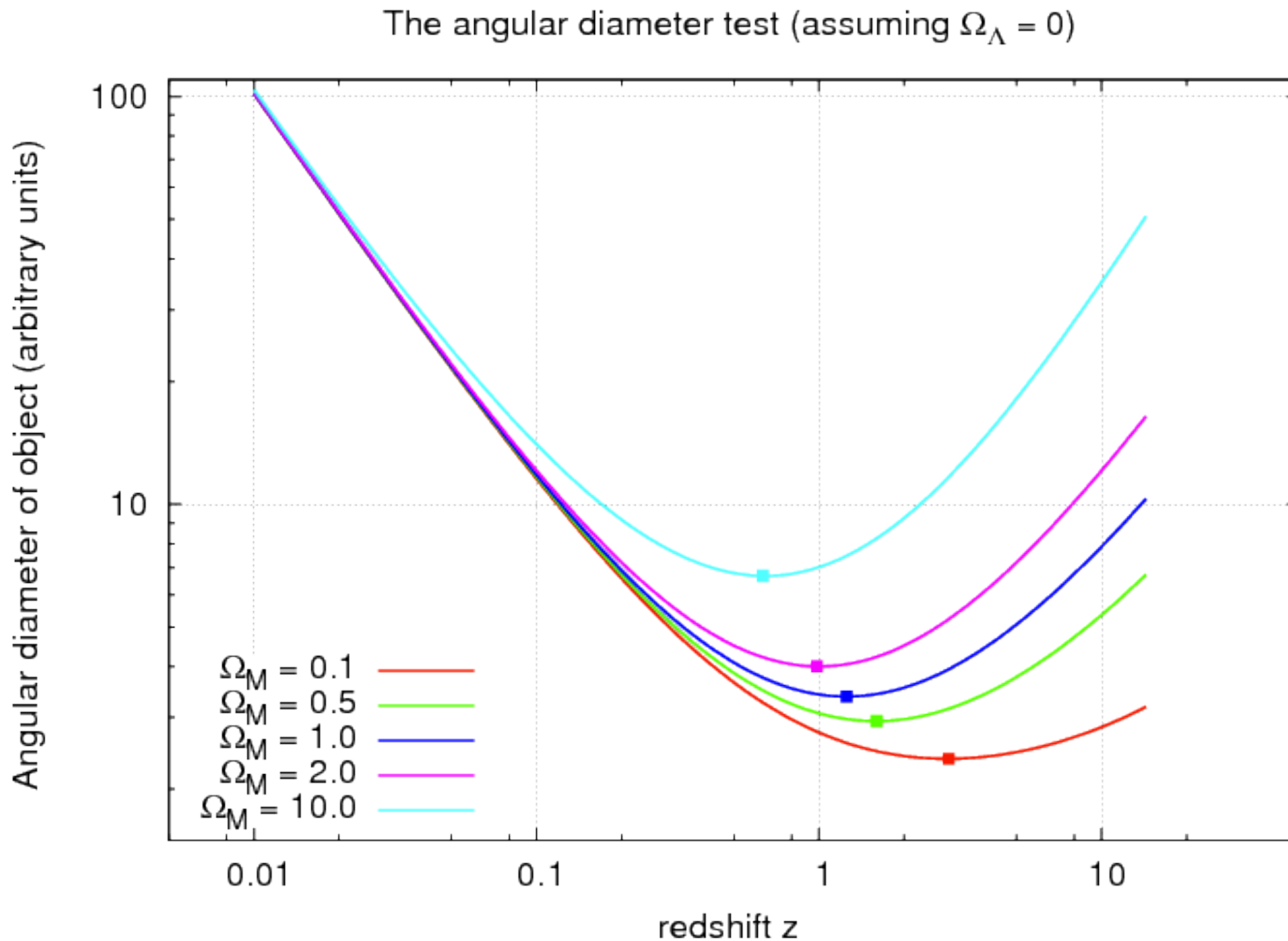
At low- z , mainly sensitive at H_0 and not cosmology



Galaxies are not
standard candle
Large variation of
size and luminosity.
Cannot even
measure H_0 .

Sandage & Tamman (1975)

Apparent size versus redshift



Apparent size versus redshift

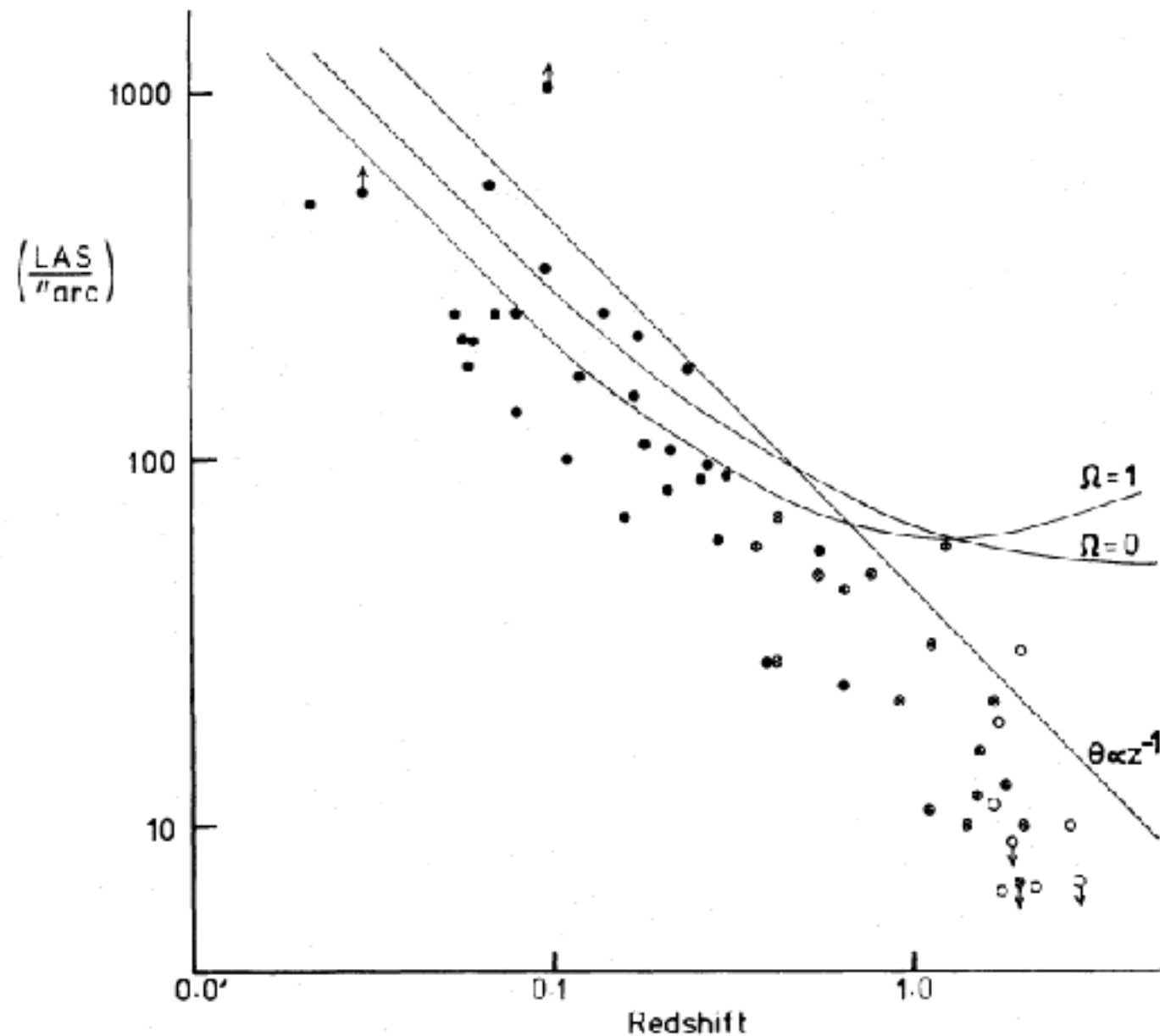


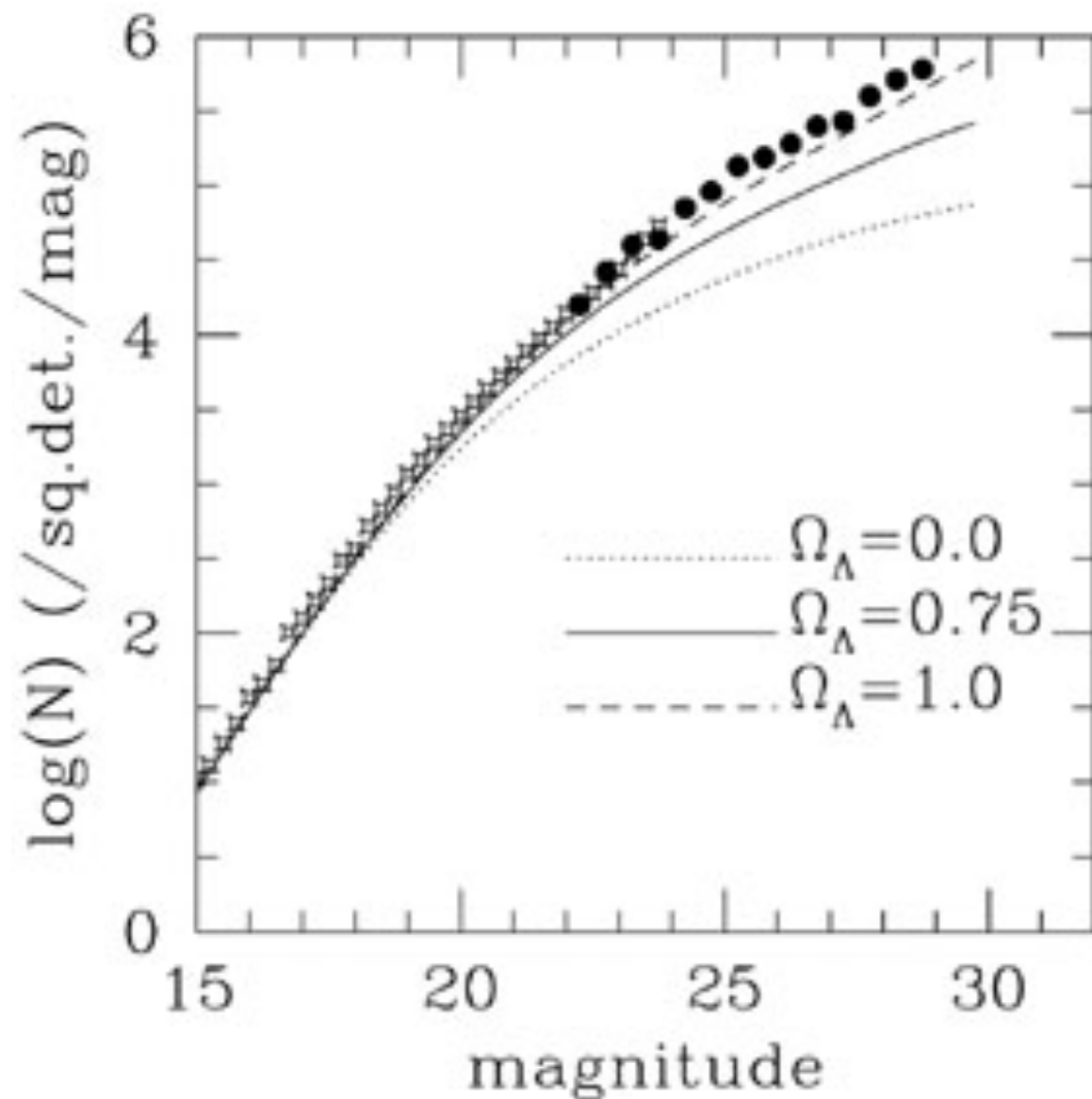
Figure 2. The angular diameter-redshift diagram as in Fig. 1 but omitting sources below a certain $\theta \propto z^{-1}$ line for clarity. The expected relations for homogeneous world models with $\Omega = 0$ and $\Omega = 1$ and the relation $\theta \propto z^{-1}$ are plotted.

Observation of
radio galaxy size by
Hooley et al (1978)

...

but galaxy size is
evolving with
redshift

Galaxy number counts



The solid points are the faint galaxy number counts from the Hubble Deep Fields and the star shaped points are the number counts from ground based data.

The curves are the no-evolution predictions from 3 flat cosmological models.

Galaxy evolution is dominating.

Modern Cosmological Tests

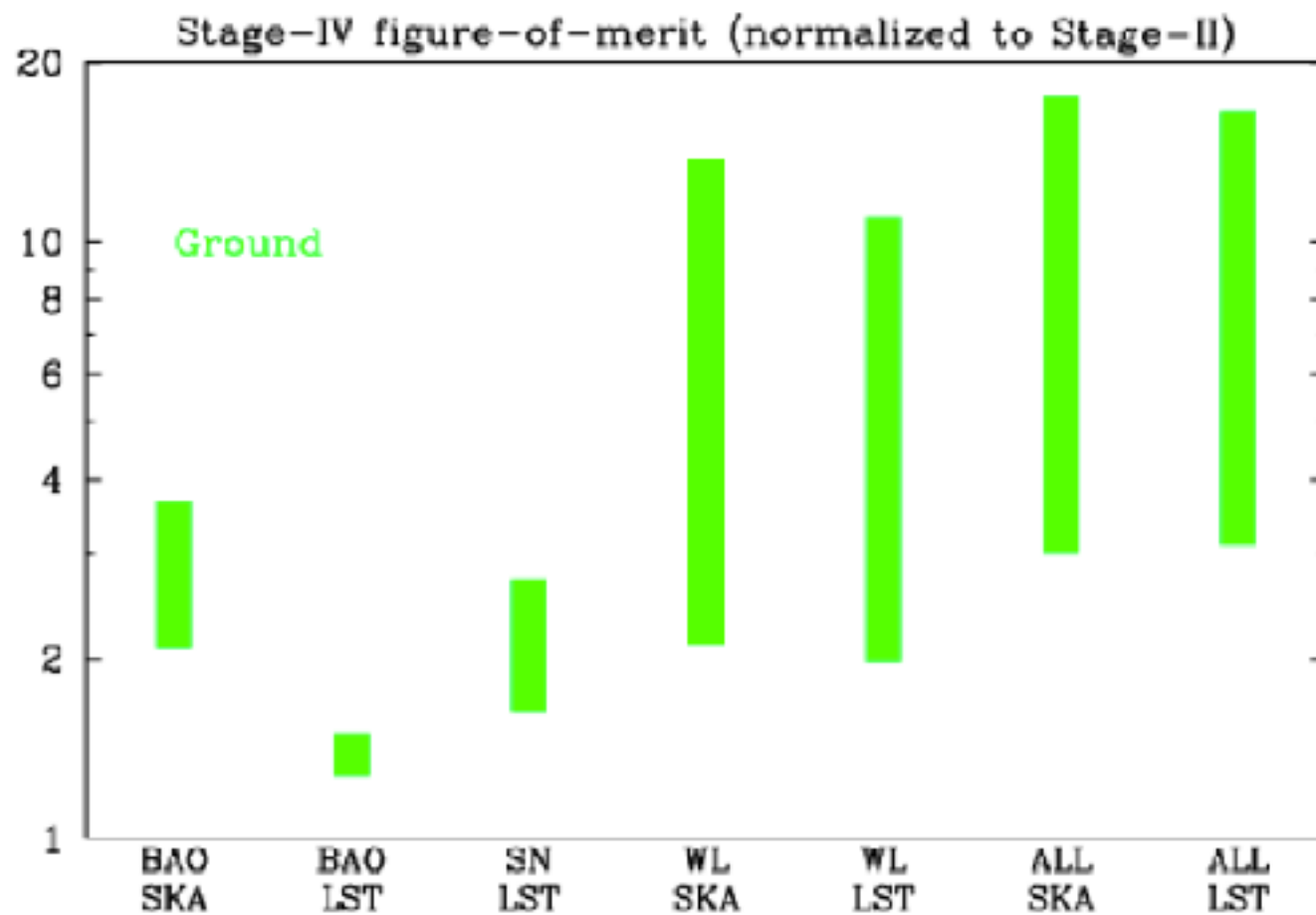
- CMB (flatness)
- Standard candle with SNIa (Luminosity test)
- Standard ruler with BAO (Angular diameter distance test)
- Counting of cluster of galaxies (Volume test+ Growth of structure)
- Cosmic shear signal (Volume test+Growth of structure)

DETF report (2005)

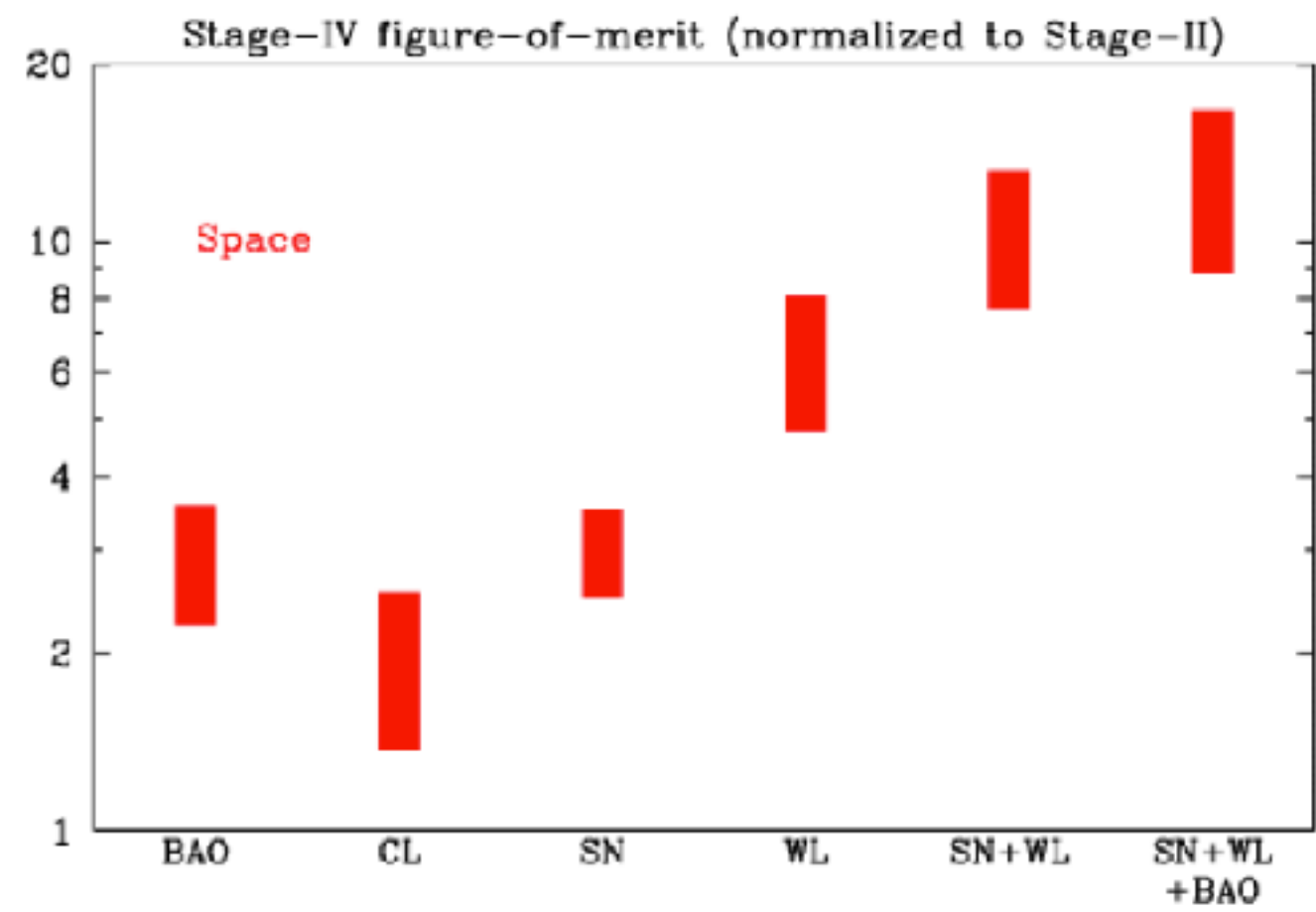
http://www.nsf.gov/mps/ast/aaac/dark_energy_task_force/report/detf_final_report.pdf

In February 2005 a Dark Energy Task Force (DETF) was organized as a joint subcommittee to advise NSF, NASA, and DOE on the future of dark energy research, to help the agencies to identify actions that will optimize a near- and intermediate-term dark energy program and ensure rapid progress towards understanding the nature of dark energy.

DETF report (2005)

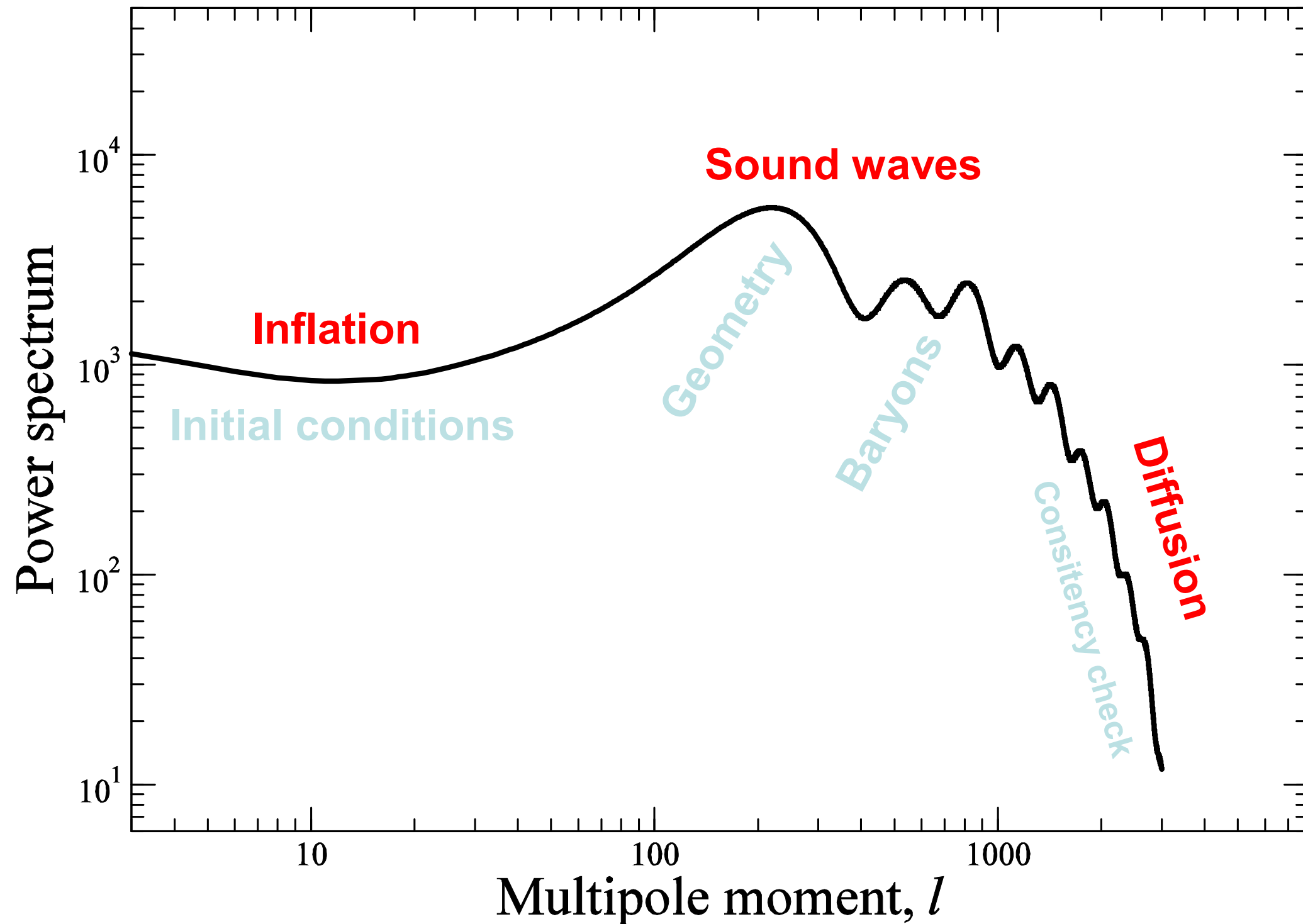


Ground

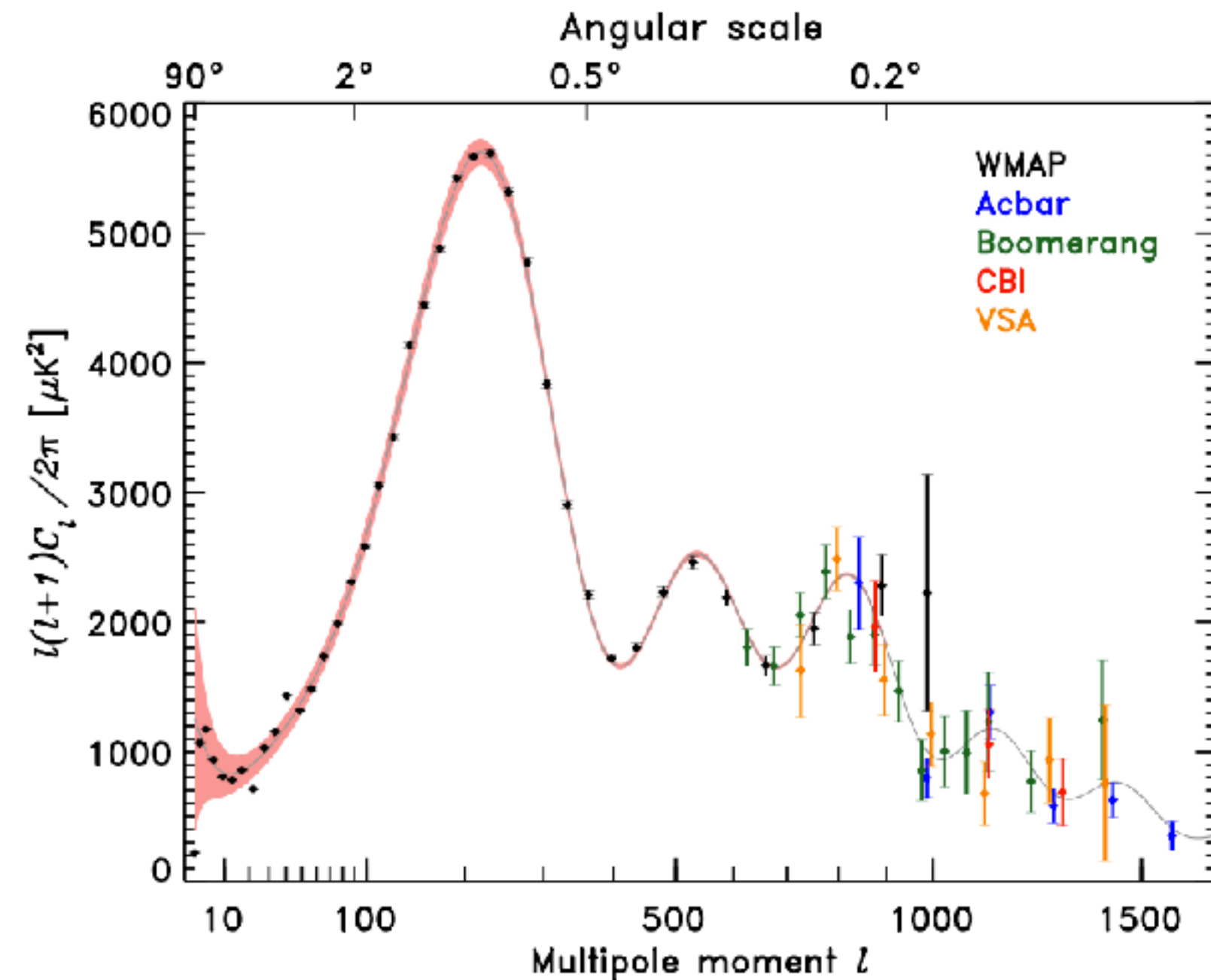


Space

Overview of the CMB spectrum



CMB

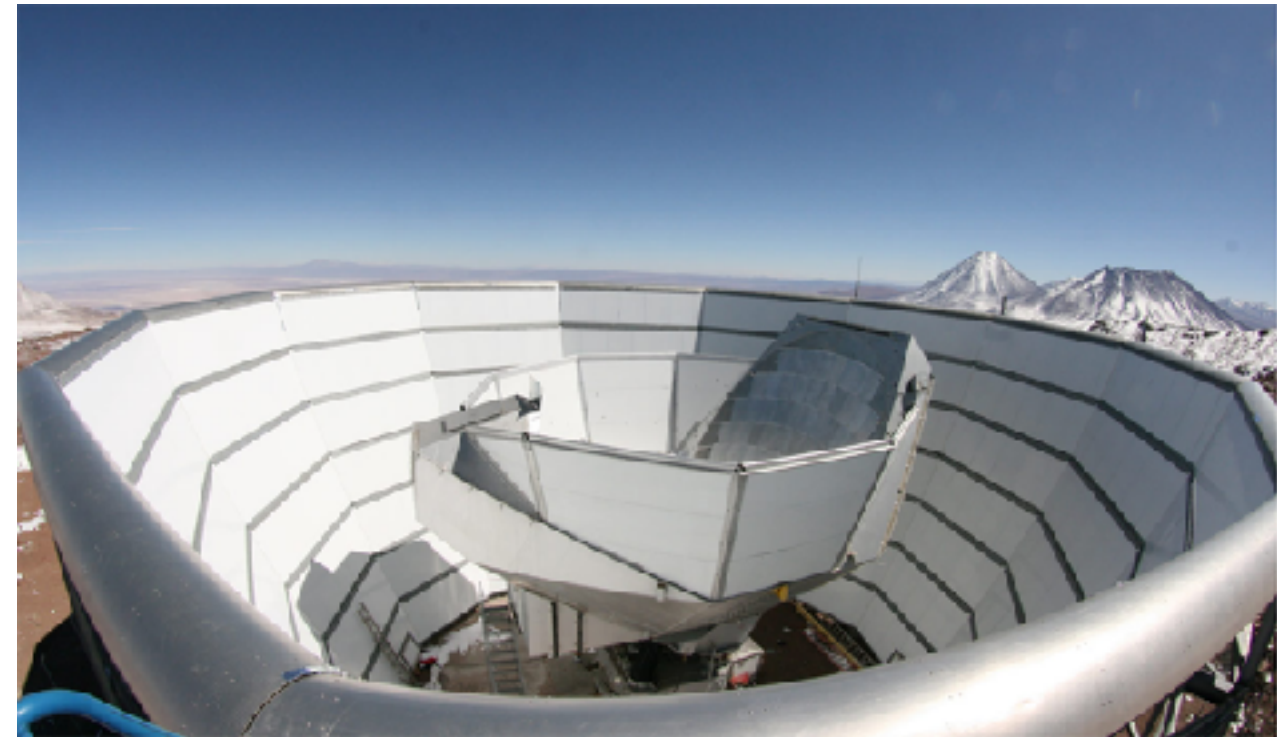
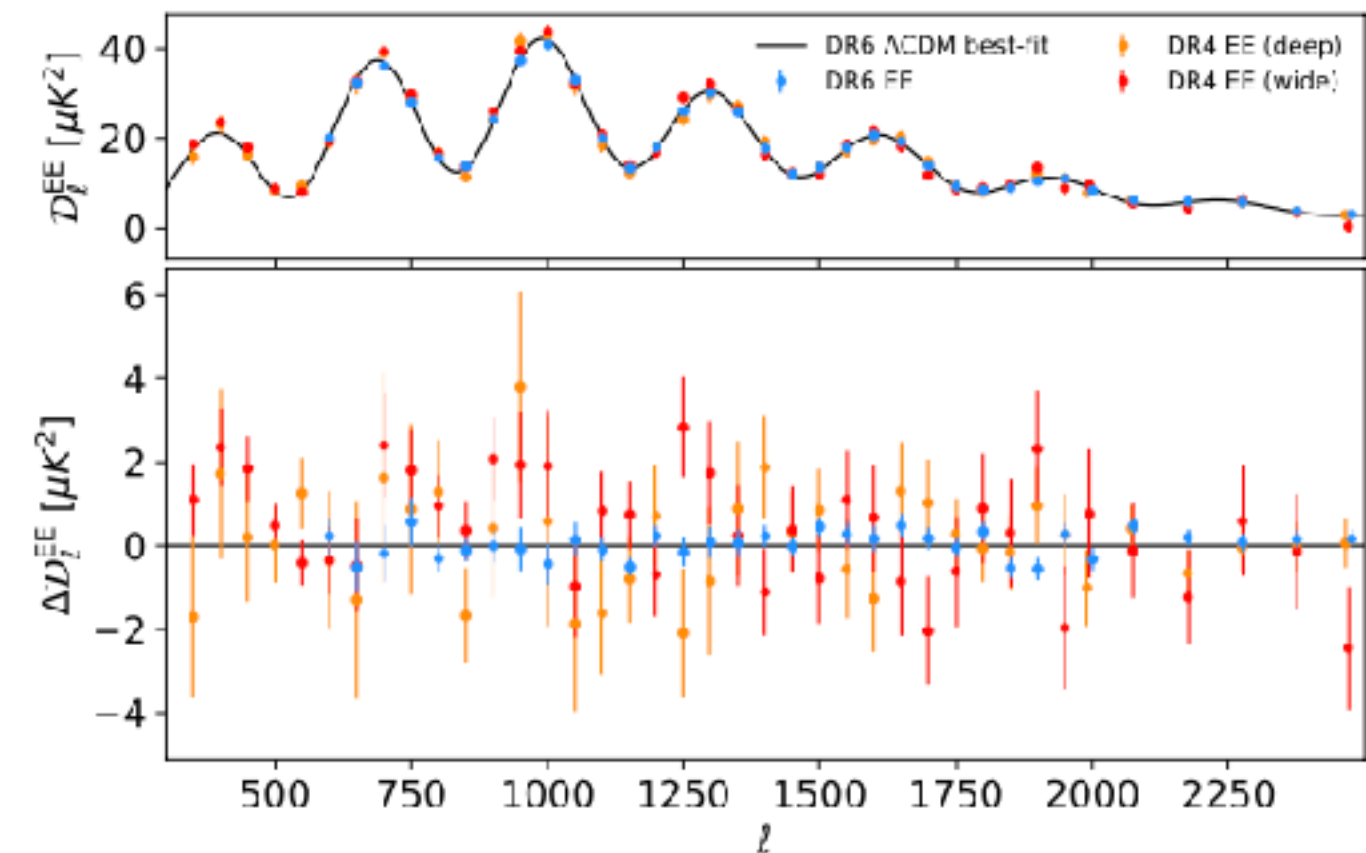
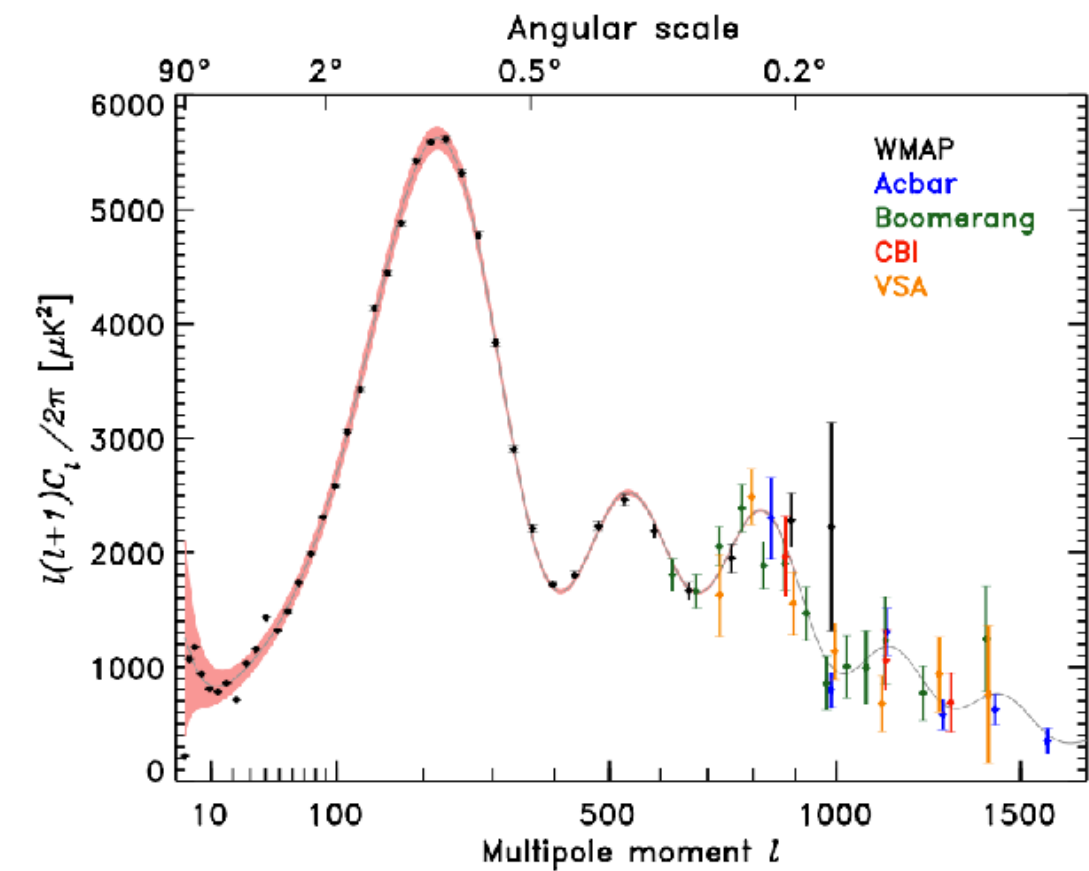


The position of the first peak indicates the universe is flat.

The relative height of the second peak gives the ratio between DM and baryons.

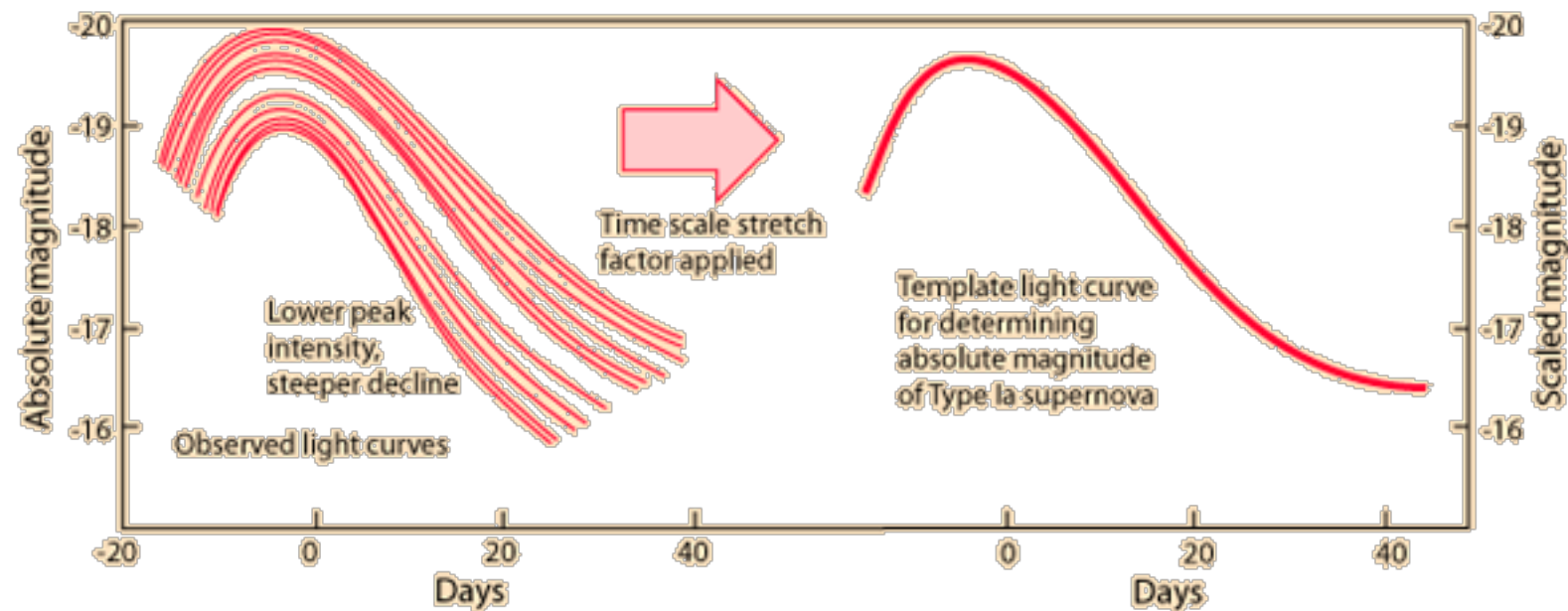
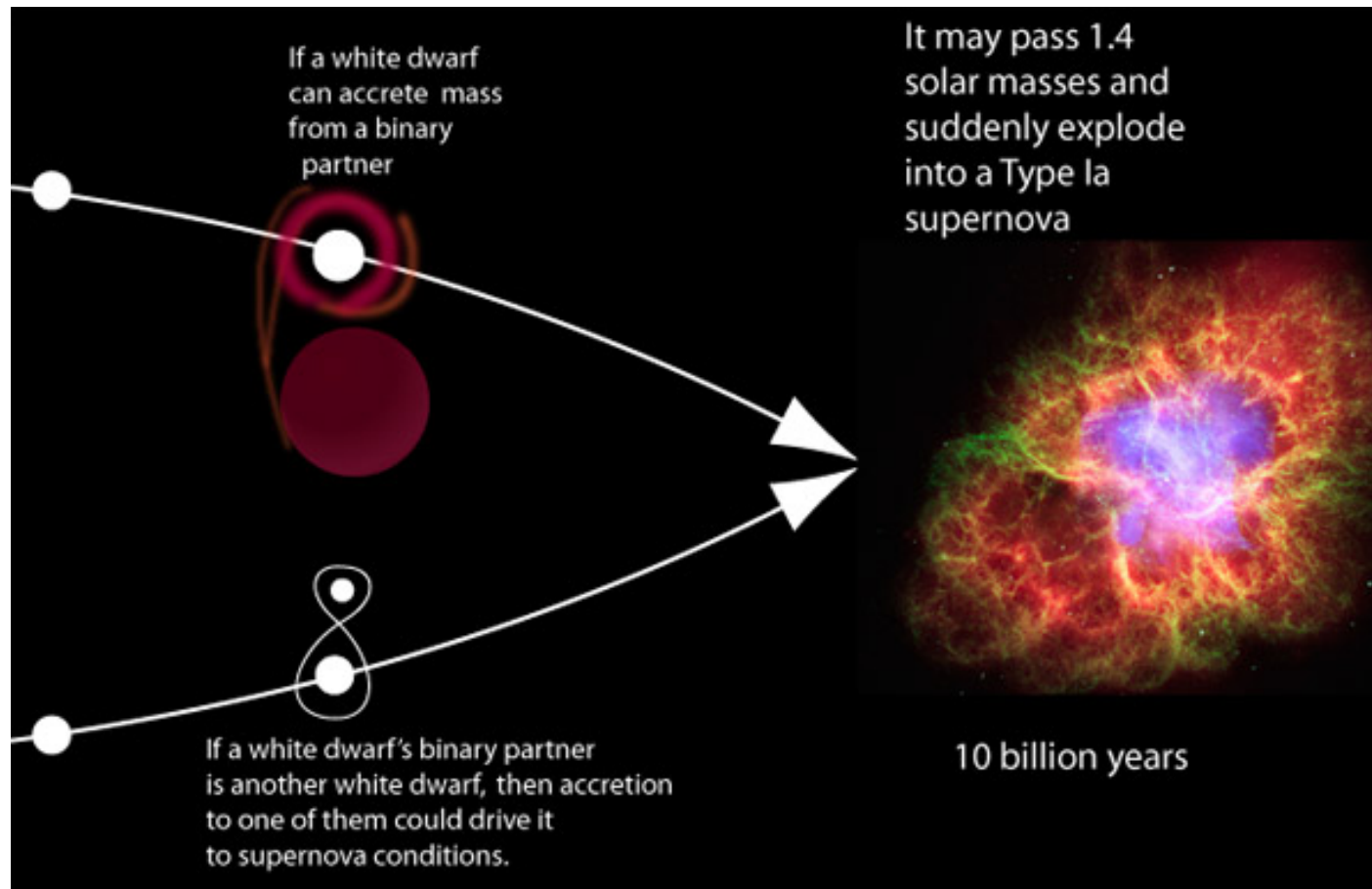
(White et al. 1994)

CMB with the Atacama Cosmology Telescope



Data Release 6
(DR6) of ACT

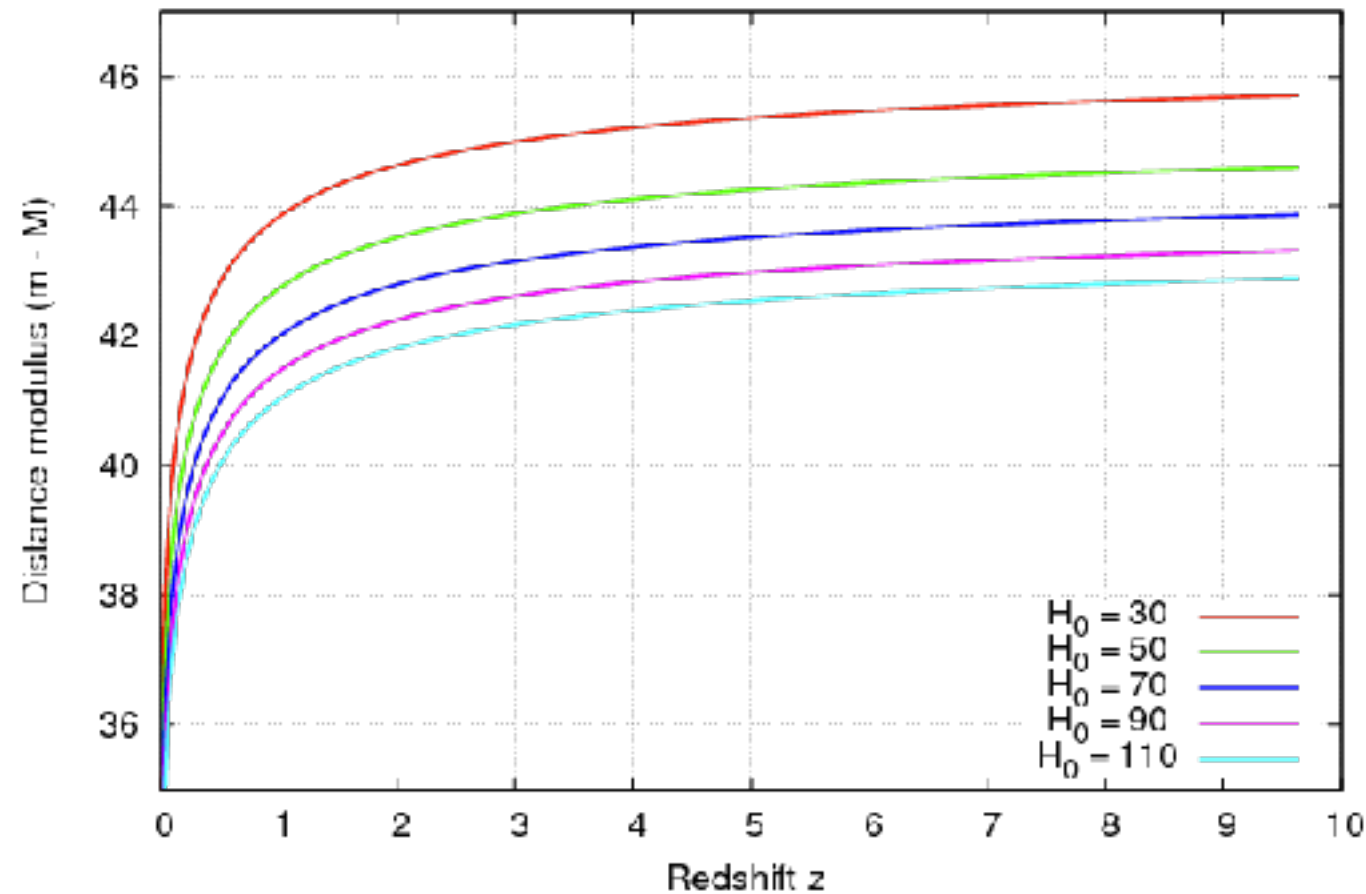
Type Ia supernova



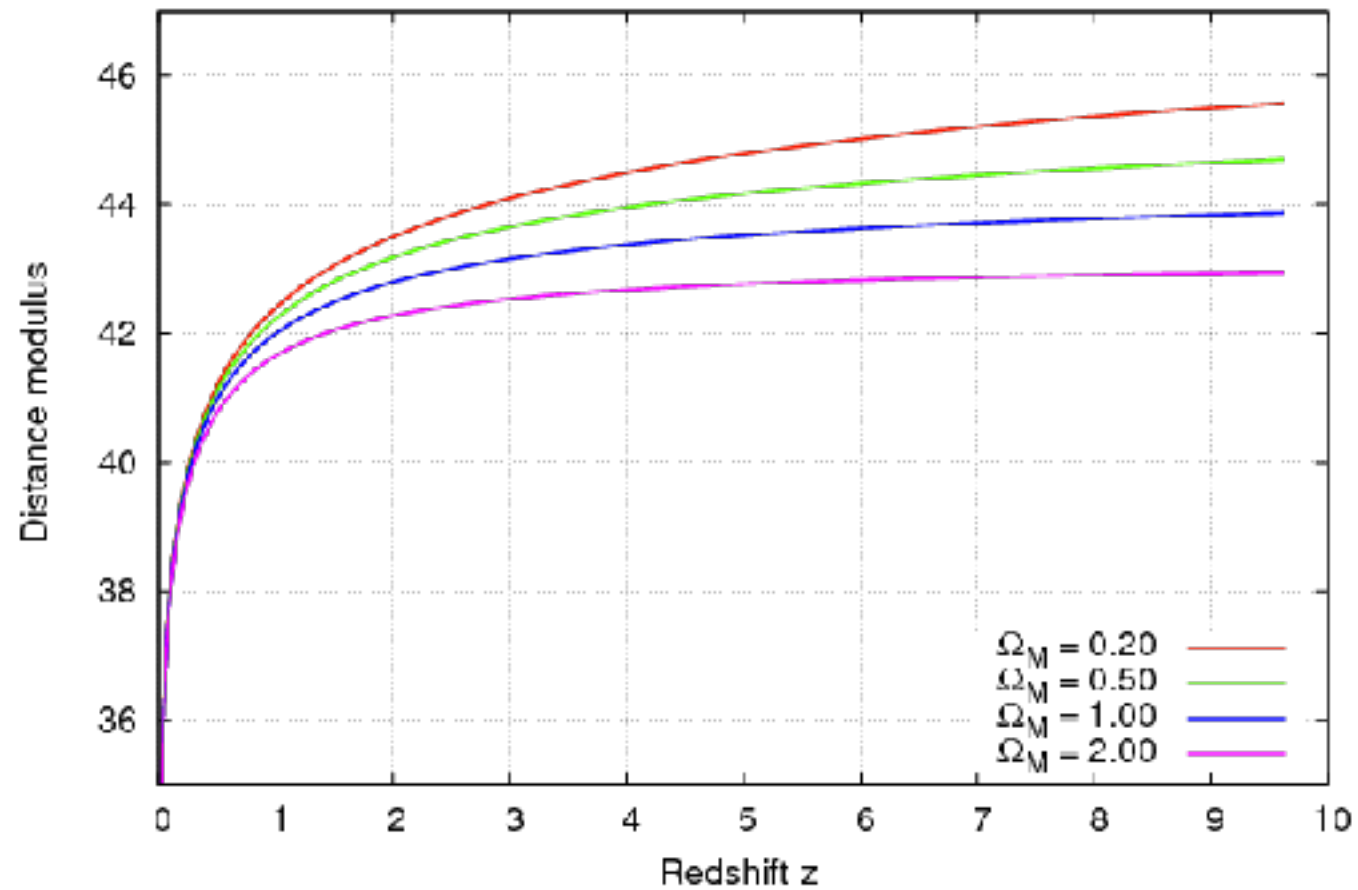
Luminosity distance at high redshift

At high- z , for H_0 well known,
cosmology can be tested

Luminosity distance for $\Omega_M = 1.0, \Omega_\Lambda = 0$

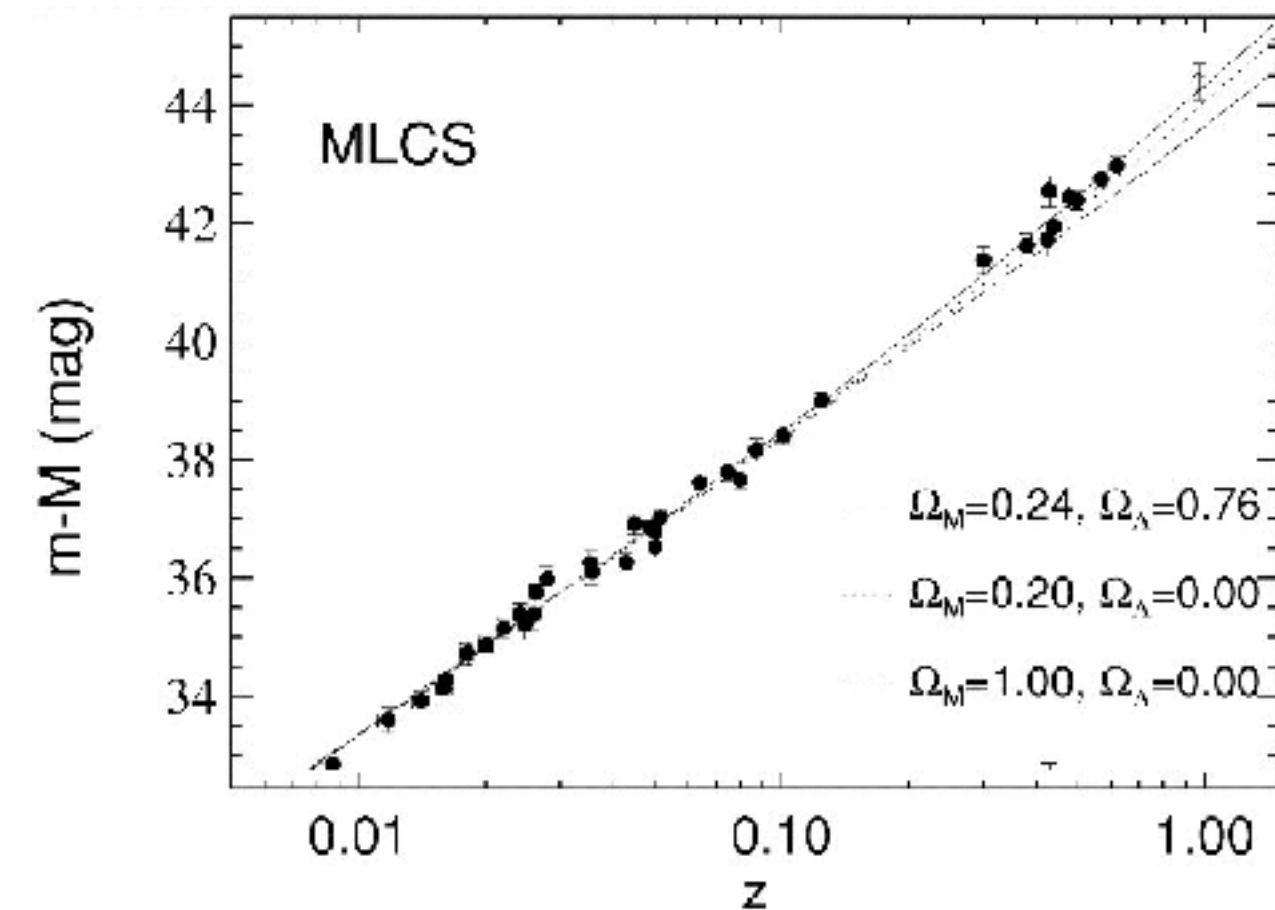


Luminosity distance for $H_0 = 70 \text{ km/s/Mpc}, \Omega_\Lambda = 0$

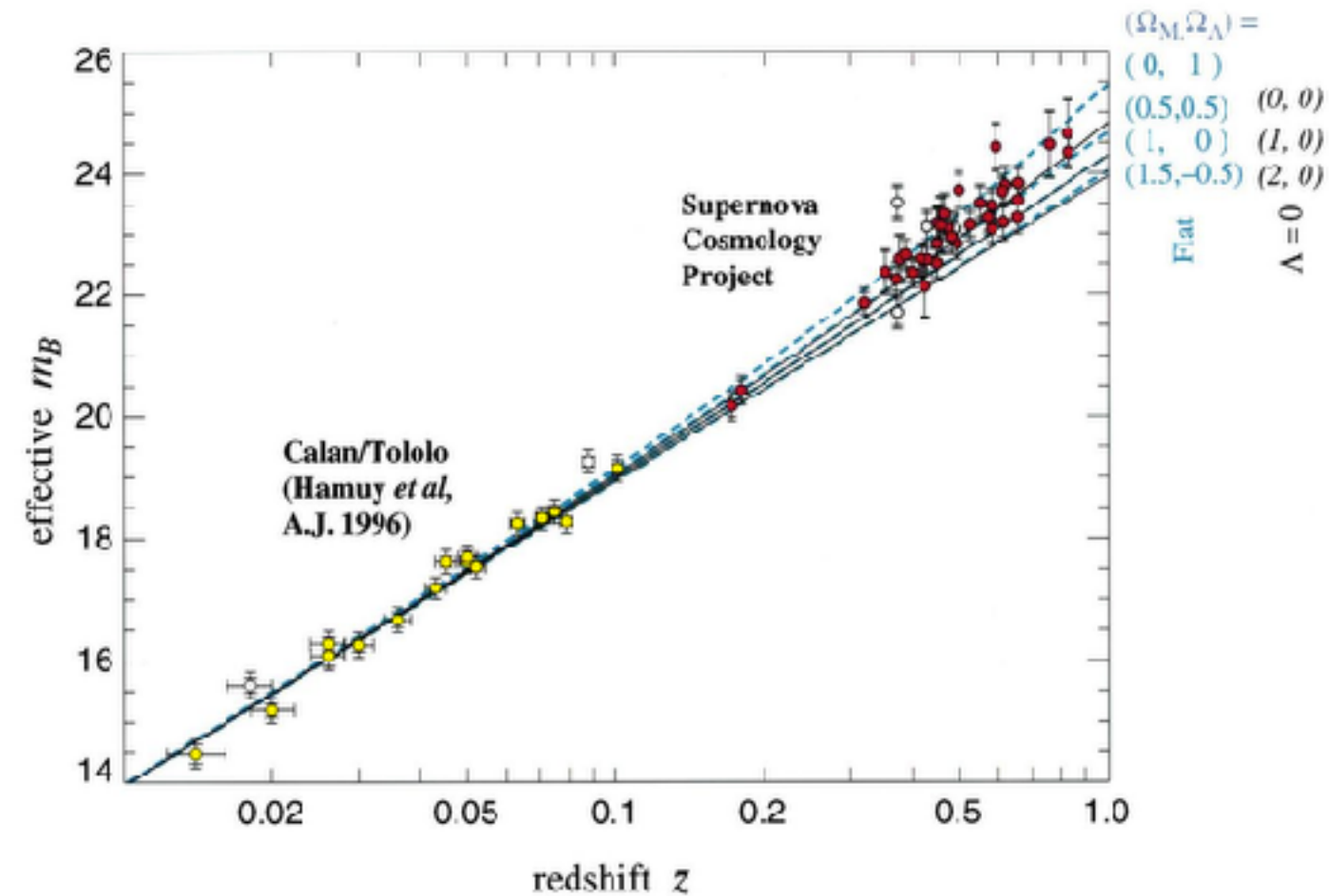


Luminosity distance at high redshift

At high- z , for H_0 well known, cosmology can be tested



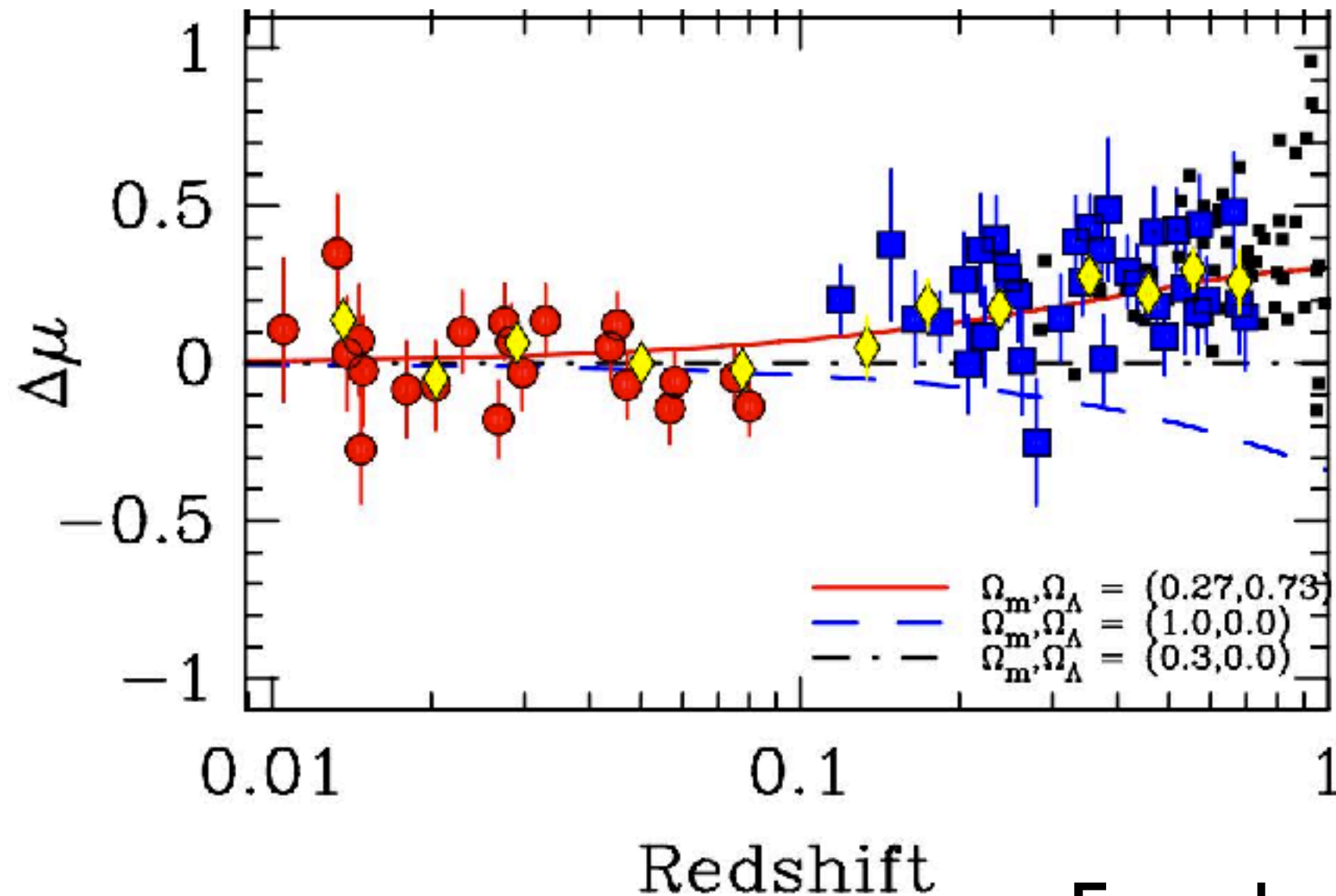
Riess et al 1998



Perlmutter et al 1999

Luminosity distance at high redshift

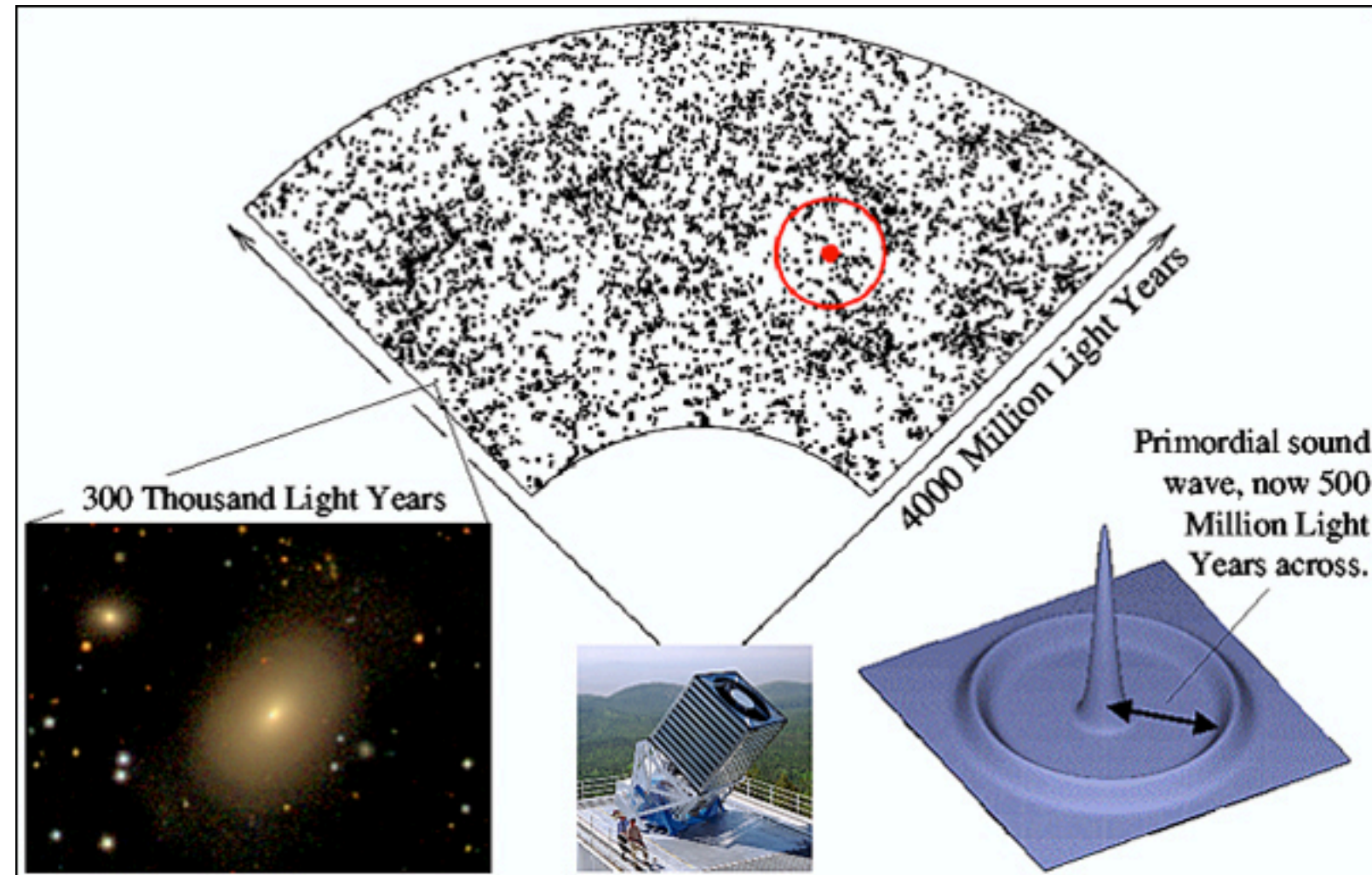
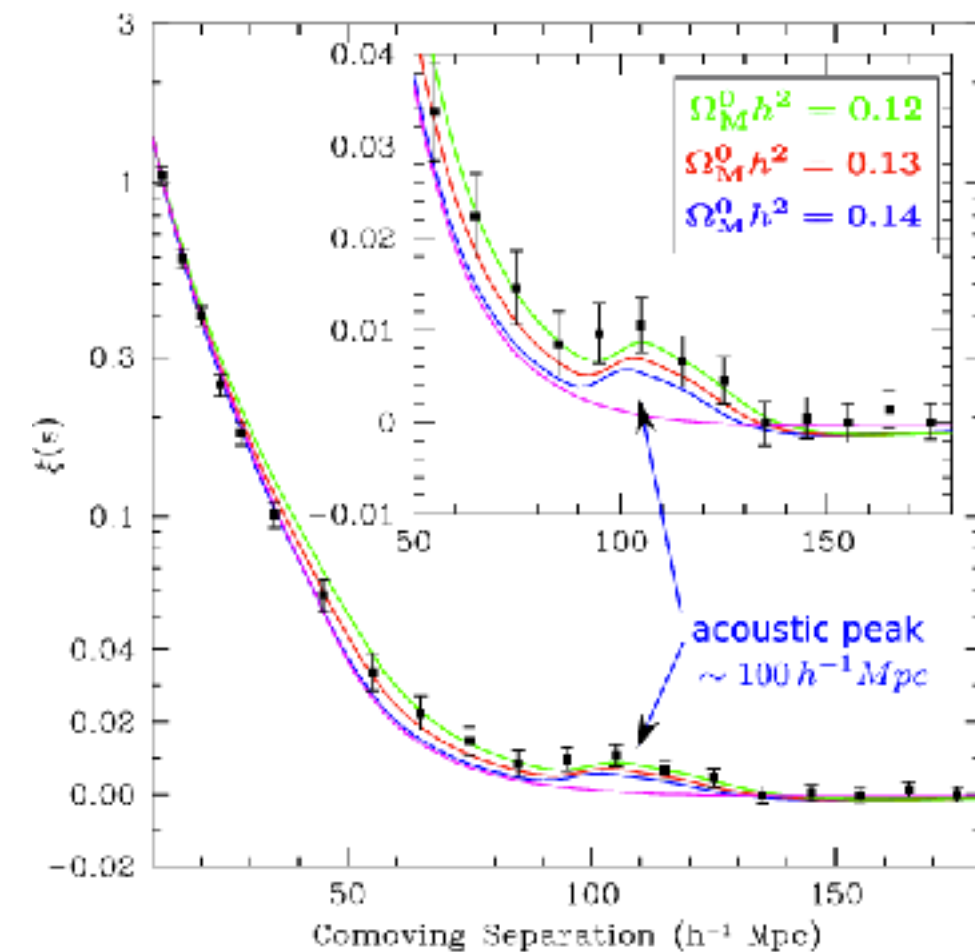
At high- z , for H_0 well known, cosmology can be tested



Freedman et al 2009

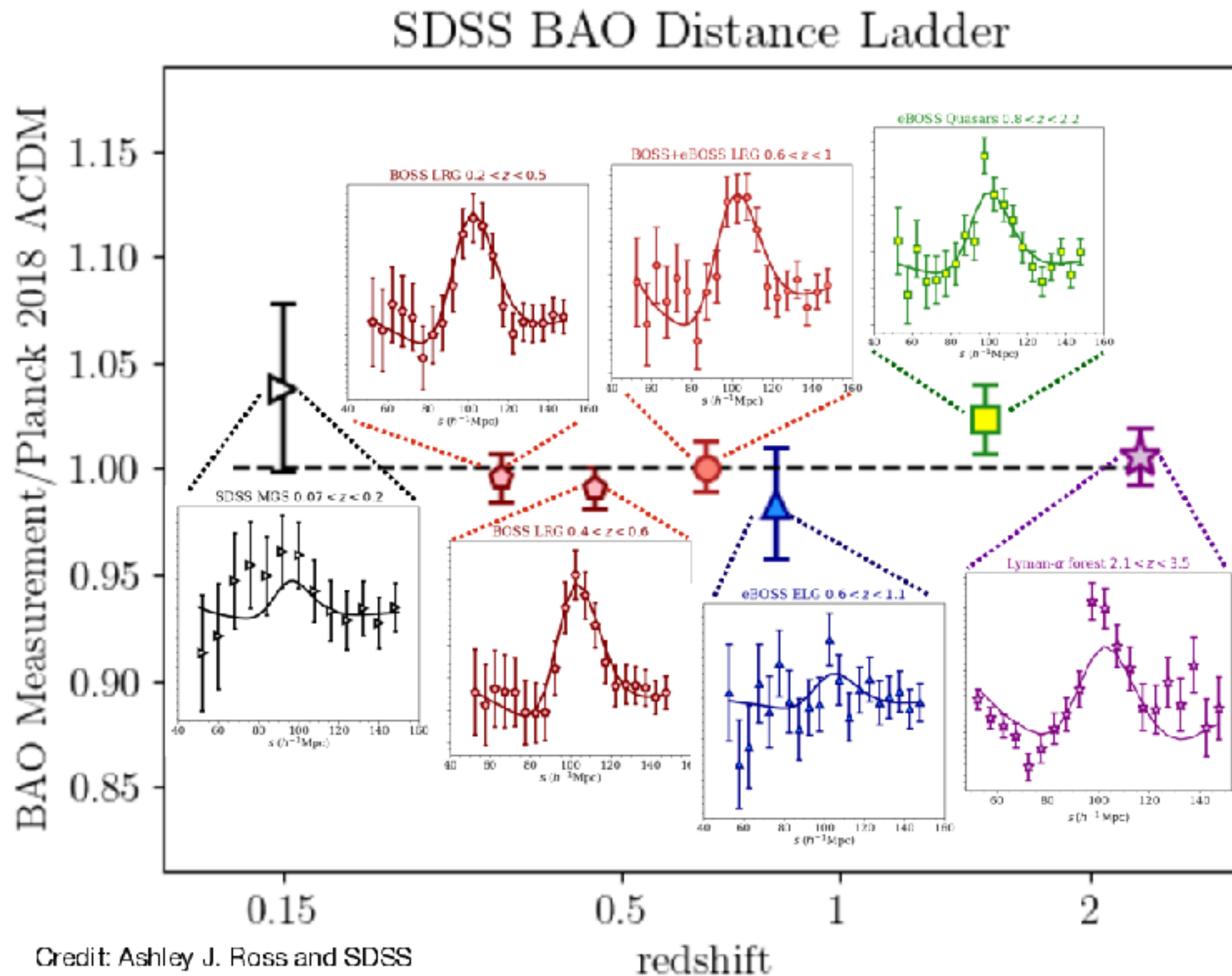
BAO as standard Ruler

First BAO peak measurements in 2005

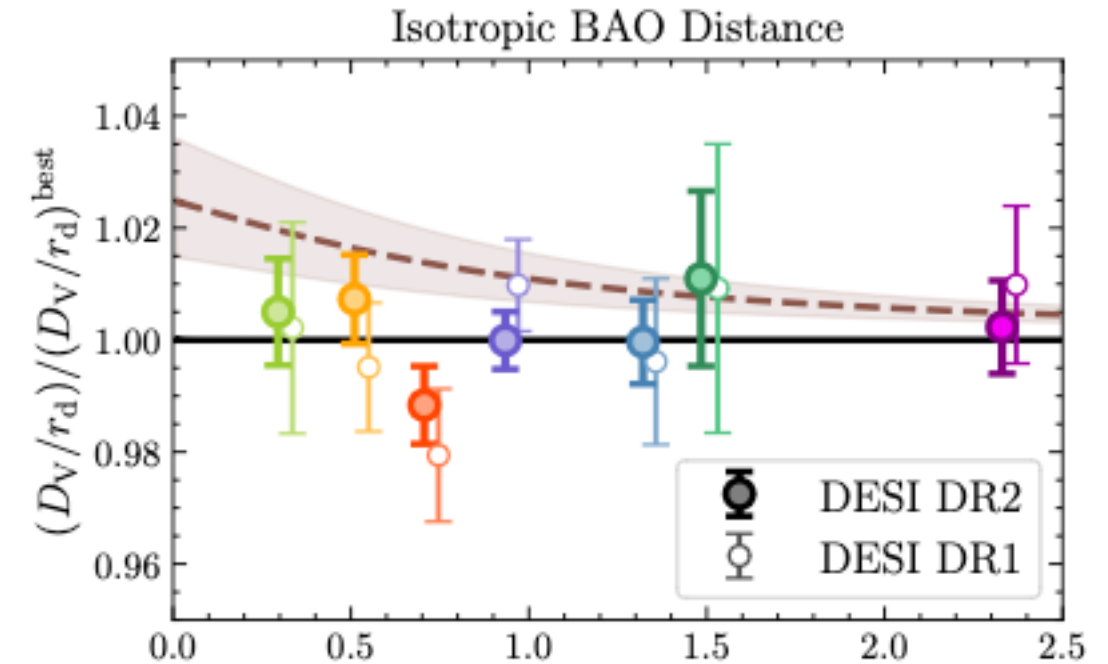
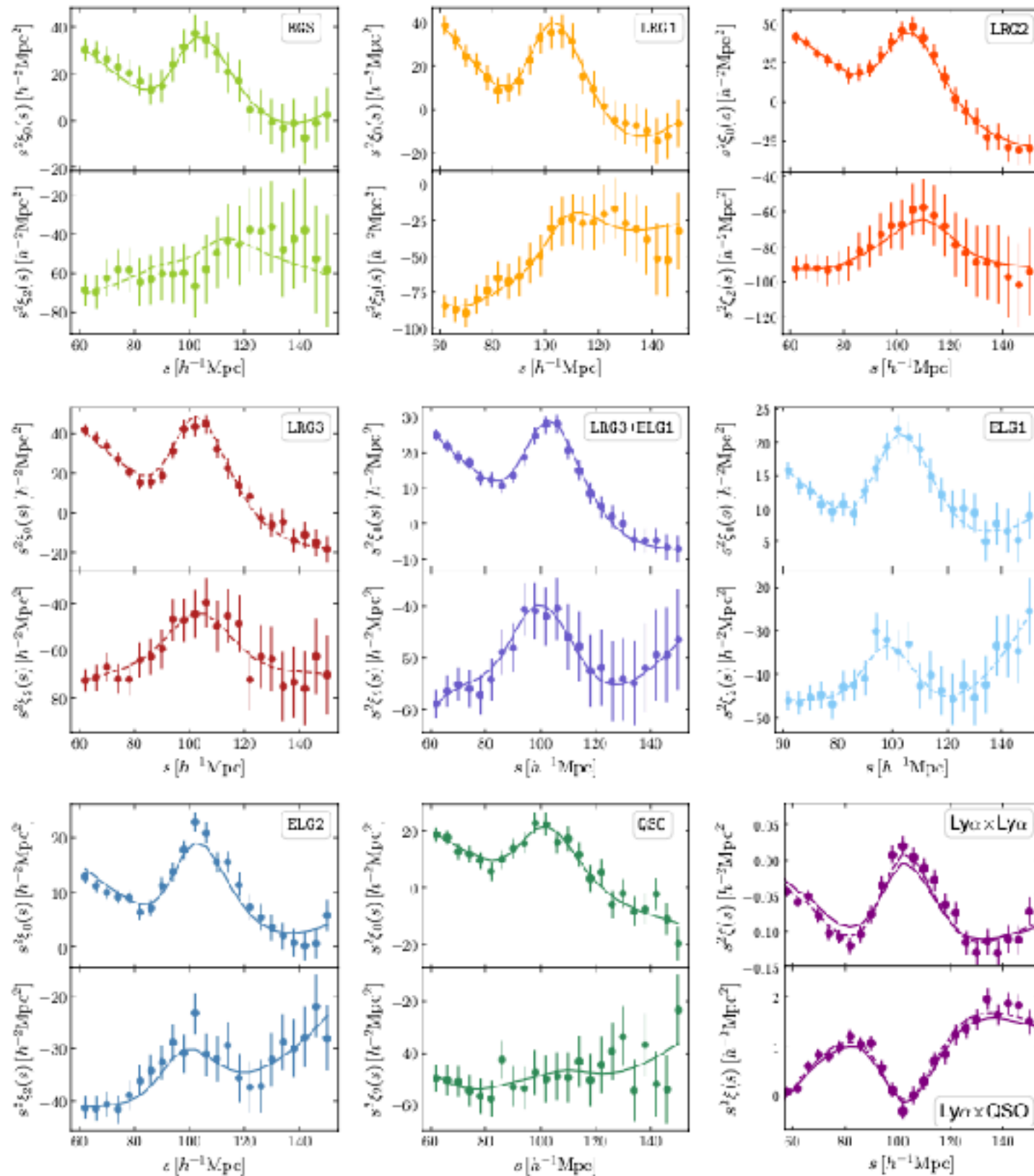


Eisenstein et al 2005

BAO as standard Ruler

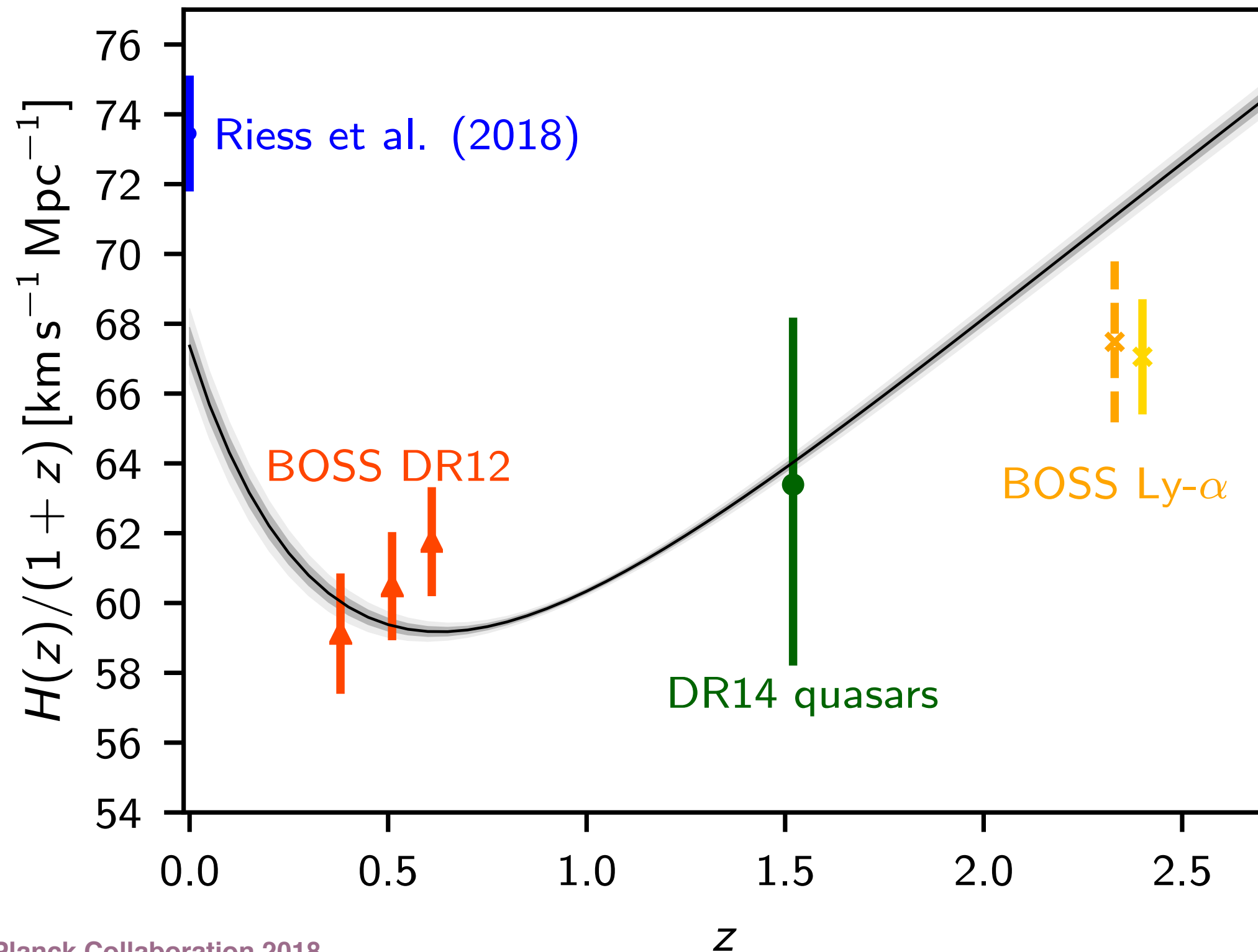


BAO peak from DESI DR2

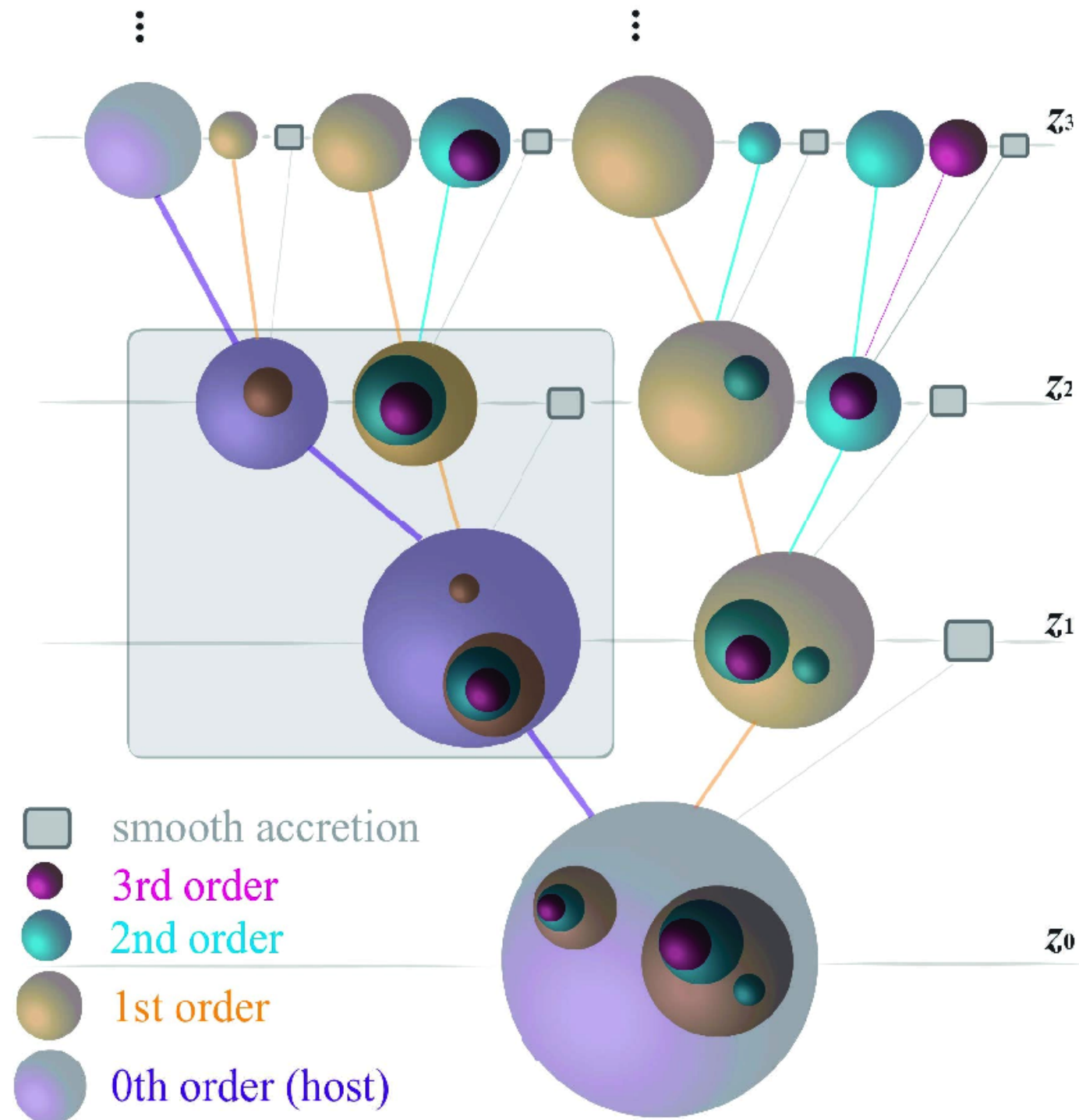


The Planck Λ CDM predictions are shown with brown dashed lines with 68% confidence intervals

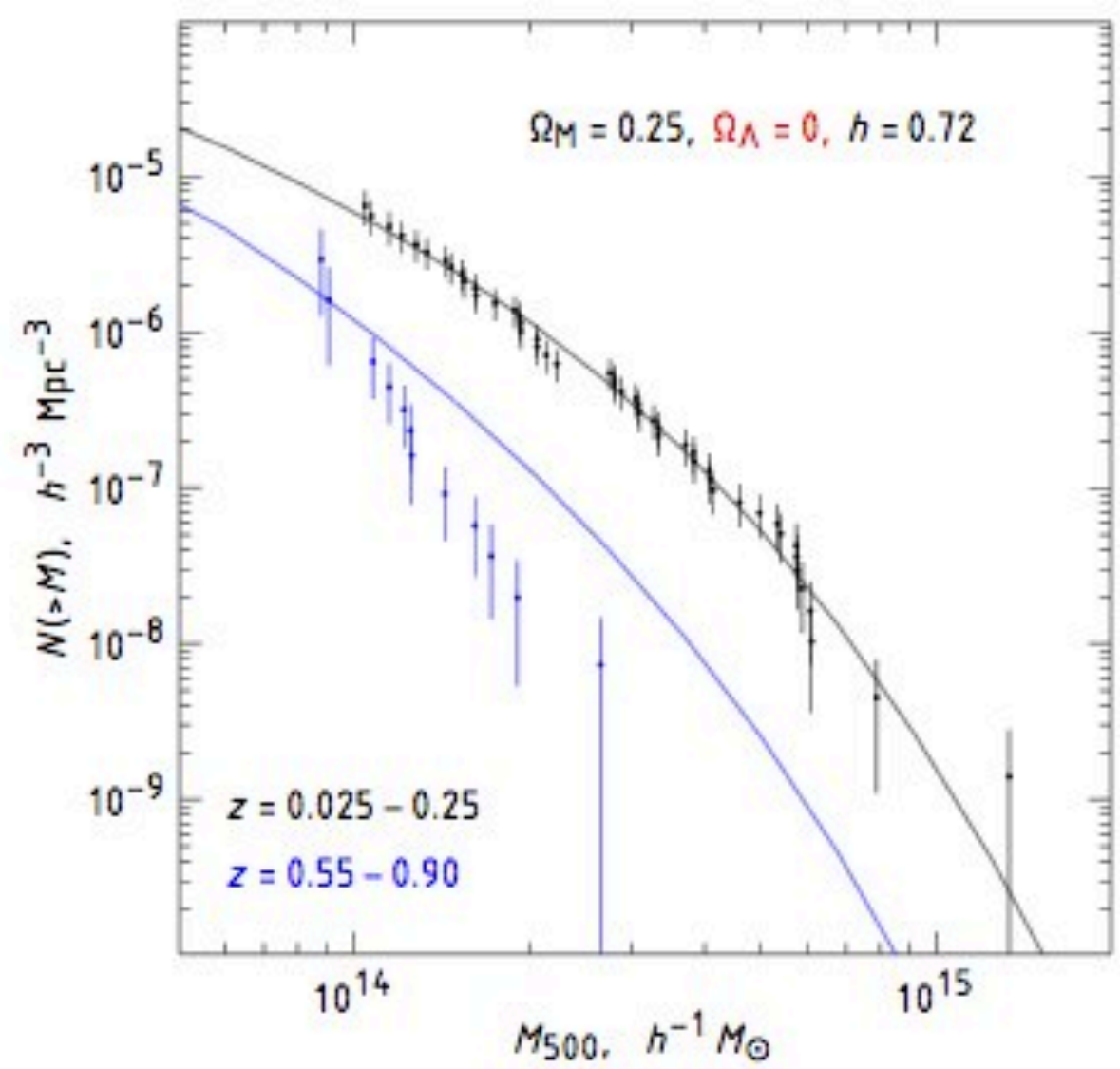
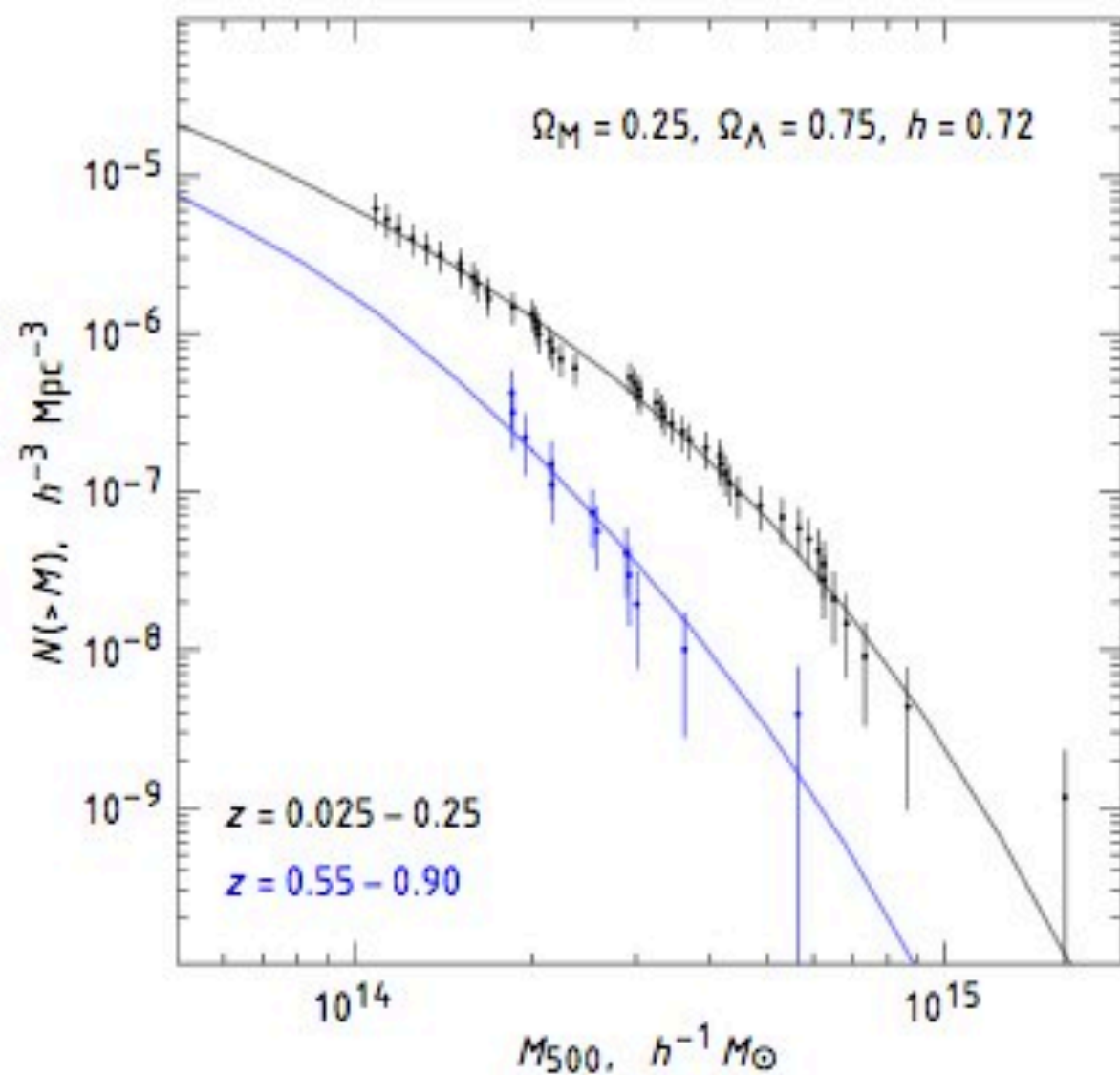
Hubble parameter



Cluster Counts (X-ray)

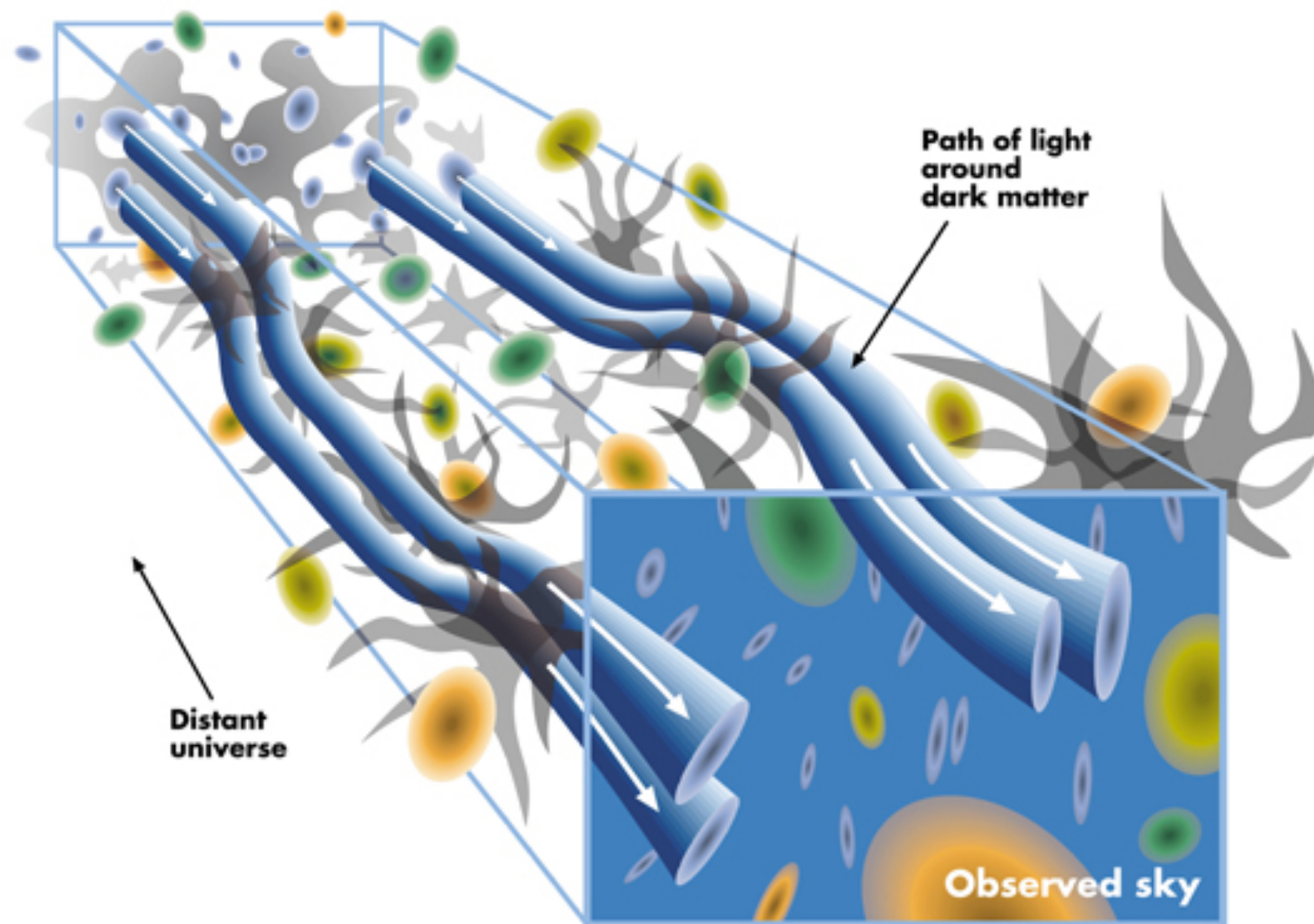


Cluster Counts (X-ray)

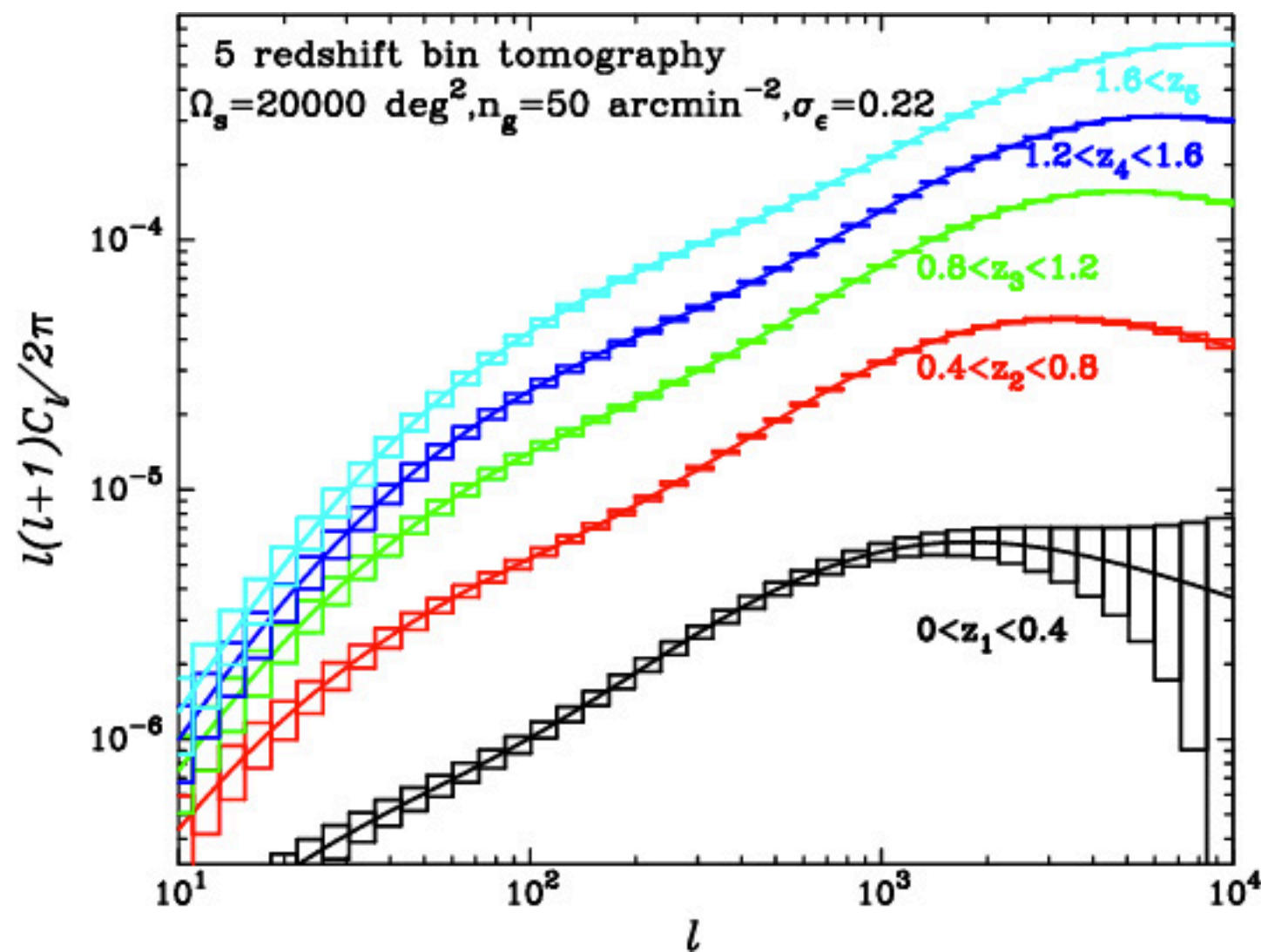


Caveats are: X-ray cluster evolution and selection function

Cosmic shear signal



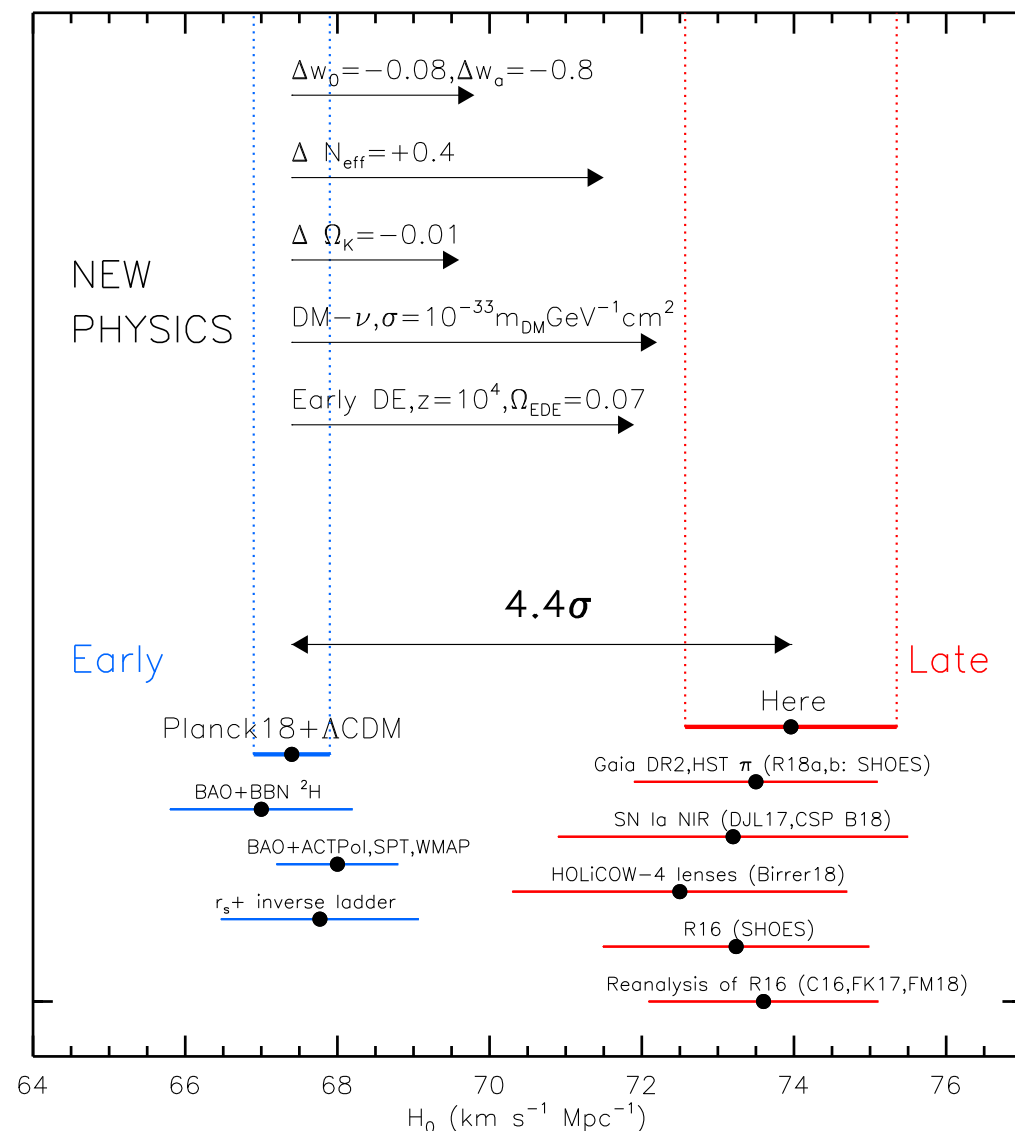
Cosmic shear signal



Cosmic shear may provide in the near future precise cosmological test.

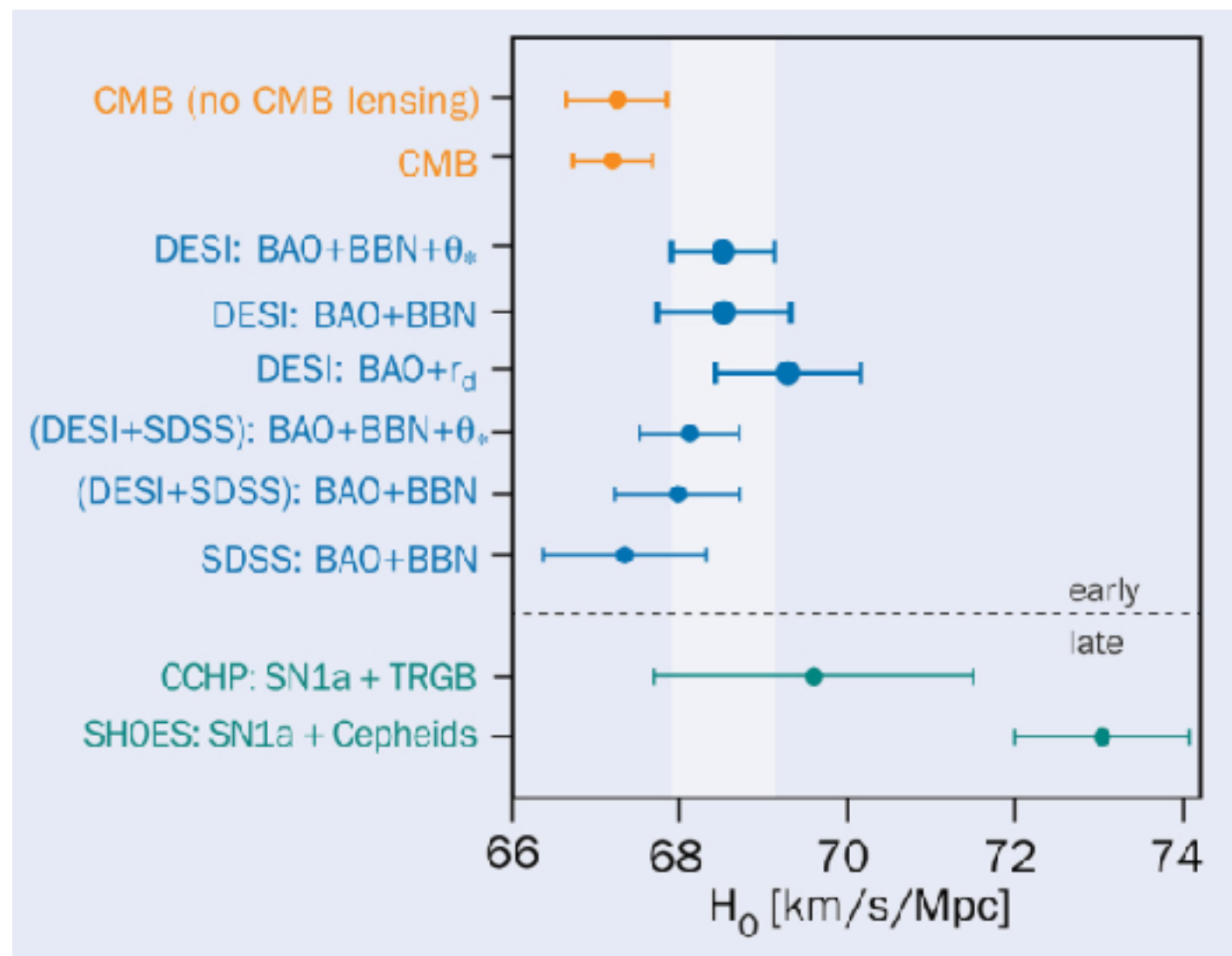
Shown here: the amplitude of the shear strength as a function of size and redshift.

Flat- Λ CDM in trouble?



Arxiv: 1903.07603

H0 from DESI DRI



An emerging problem in Physics

