

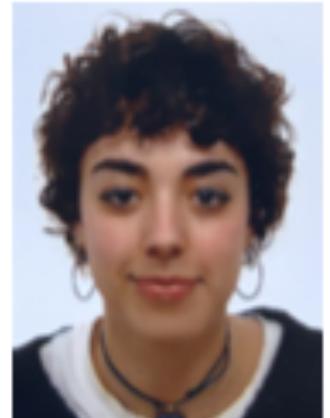
AstroV Observational Cosmology

Prof. Jean-Paul Kneib
jean-paul.kneib@epfl.ch

Assistants: Dr. Rafaella Gopsner, Dr. Antoine Rocher
Aurelien Verdier, Mathilde Guitton, Shengyu He, Ashutosh Mishra

Laboratory of Astrophysics [LASTRO], EPFL
<http://lastro.epfl.ch>

Who Are You ?



Barroso Nevares Carlota
PH Exchange,
2024-2025, Spring
semester



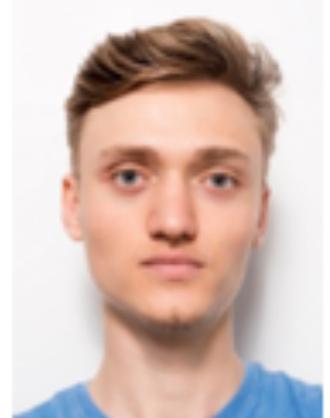
Biel Thibaud Julien
Physics - master
program, 2024-2025,
Master semester 2



Blattner Niklas Julian
GM Exchange,
2024-2025, Spring
semester



Bourrat Lissandre Alois
Physics - master
program, 2024-2025,
Master semester 2



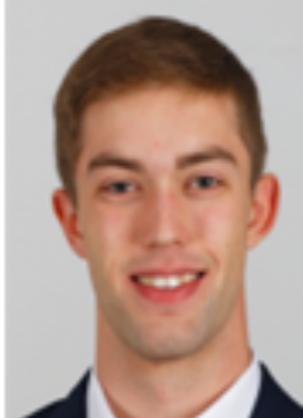
Corbaz Alexandre Paul
Microengineering,
2024-2025, Master
semester 4



De Pinho Oliveira
Apolinario Joao
Physics - master
program, 2024-2025,
Master semester 2



Fugazza Maxence
Robert Amédée
Physics - master
program, 2024-2025,
Master semester 4



Genin Aurélien

Robotics, 2024-2025,

Master semester 2



Gond Adrien
Physics - master
program, 2024-2025,
Master semester 2



Govers Stanislas Shiva
Marie
Physics - master
program, 2024-2025,
Master semester 2



Kornbeck Michael Harry
Physics - master
program, 2024-2025,
Master semester 2



Kuruppu Nethan Mikil

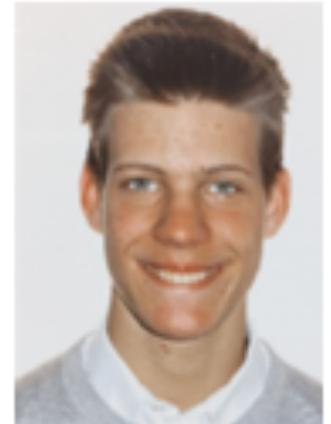
PH Exchange,

2024-2025, Spring

semester

- Your name, where are you from?
- Which Astrophysics course have you taken, or are taking this semester?
- Why are you following this course?
- Which question on the Cosmos you would like to be answered?
- What are your goals for the future?
PhD? Others?
- Do you have other courses overlapping during the lecture/exercises session?

Who Are You ?



Leal Losa Joaquim
PH Exchange,
2024-2025, Spring
semester



Leurent Gauthier
Physics - master
program, 2024-2025,
Master semester 4



Liu Yijun
GM Exchange,
2024-2025, Spring
semester



Manoli Elliot
Physics - master
program, 2024-2025,
Master semester 2



Occhipinti Alexis
Physics - master
program, 2024-2025,
Master semester 2



Rey Zachary Yann
Physics - master
program, 2024-2025,
Master semester 2



Rocamora Martorell
Ferran
PH Exchange,
2024-2025, Spring
semester



Seydoux Théau Clément
Physics - master
program, 2024-2025,
Master semester 4



Taddei Alba Miren
Physics - master
program, 2024-2025,
Master semester 2



Thums Huckleberry
David Gaston
Physics - master
program, 2024-2025,
Master semester 2



Veilhan Marguerite Marie
Jeanne
Physics - master
program, 2024-2025,
Master semester 2

- Your name, where are you from?
- Which Astrophysics course have you taken, or are taking this semester?
- Why are you following this course?
- Which question on the Cosmos you would like to be answered?
- What are your goals for the future?
PhD? Others?
- Do you have other courses overlapping during the lecture/exercises session?

Astrophysics course @ EPFL

Structure of the course

- 14 weeks [NO COURSE on April 21 - Easter]
- 2h lecture, 2h of exercises [PHH331]
- **Exercises - applications of lectures**
 - 4 pencil exercises
 - 7 Jupyter notebook exercises => **assignments** => evaluation
 - 2 or 3 reading & presentation of scientific articles
- **Oral exam:** presentation of scientific articles (**0.25 bonus from assignments**)
- For course, material/assignments check on the Moodle PHYS-402: <https://moodle.epfl.ch/course/view.php?id=14922>
- Notifications will generally be sent by email/moodle
- **Small group, so if you have questions, don't be shy: ASK !!!**

Other Materials

- **Astro/Cosmo Publications!** And how to find them:
 - **ArXiv (pdf accessible from anywhere)**
 - <http://arxiv.org/form/astro-ph?MULTI=form+interface>
 - **ADS-service (pdf accessible from epfl network)**
 - <https://ui.adsabs.harvard.edu/classic-form>
 - **Annual Reviews (pdf accessible from epfl network)**
 - <https://www.annualreviews.org/journal/astro>
 - **Prophy**
 - <https://www.prophy.science/>
- **Researchgate/Wikipedia/even Google search...**
- **Chatbot: AstrroOne - register an account through this link:**
 - <https://astroone.zero2x.org.cn/universal-login/new>

What you will achieve

- **This semester:**
 - Learn about Observational Cosmology
 - What is research?
 - What is a publication?
 - How to read and present a scientific publication?
 - Python/Jupyter notebook
- **Assignments:** Results of the Jupyter notebook to be submitted with comments by Sunday evening
 - Reward: 0.25 bonus for Oral Exam (if all but one notebooks are correct at 70% level or better: *you have 1 joker*)
- **Oral Exam:**
 - Read two publications and prepare their presentation
 - Present **one** publication on the black board (no slides)

Plan of the Course

- **I- Historical perspective of Cosmology**
- 2, 3 - Content and Properties of the Observable Universe
 - Galaxies, Clusters, Quasars, IGM, LSS, CMB
- 4, 5, 6 - Basics of Modern Cosmology
 - Metric, Distances, Einstein Equations, Evolution of the Universe, cosmological tests.
- 7 - The Matter content in the Universe
- 8, 9, 10 - Gravitational Lensing
 - Lensing theory, strong lensing, weak lensing
- 11- The Intervening Universe (Inter-Galactic Medium)
- 12 - The Cosmic Microwave Background
- 13 - Inhomogeneities in the Universe (BAO, RSD)
- 14 - Gravitational Waves

Historical Perspective of Cosmology

Setting the Scene

What is cosmology?

What is the Universe?

How old is the Universe?

Cosmology

- **Definition:** *the study of the formation and evolution of the Universe and its content (mass and energy).*
- **Goal:** *to explain the Universe origin, its evolution, its future*
- Use of the **scientific approach** (physics of cosmology) => *defined by a number of parameters in the context of a theory*
- [observations => modelling => theory that can make predictions => to be tested by observations]

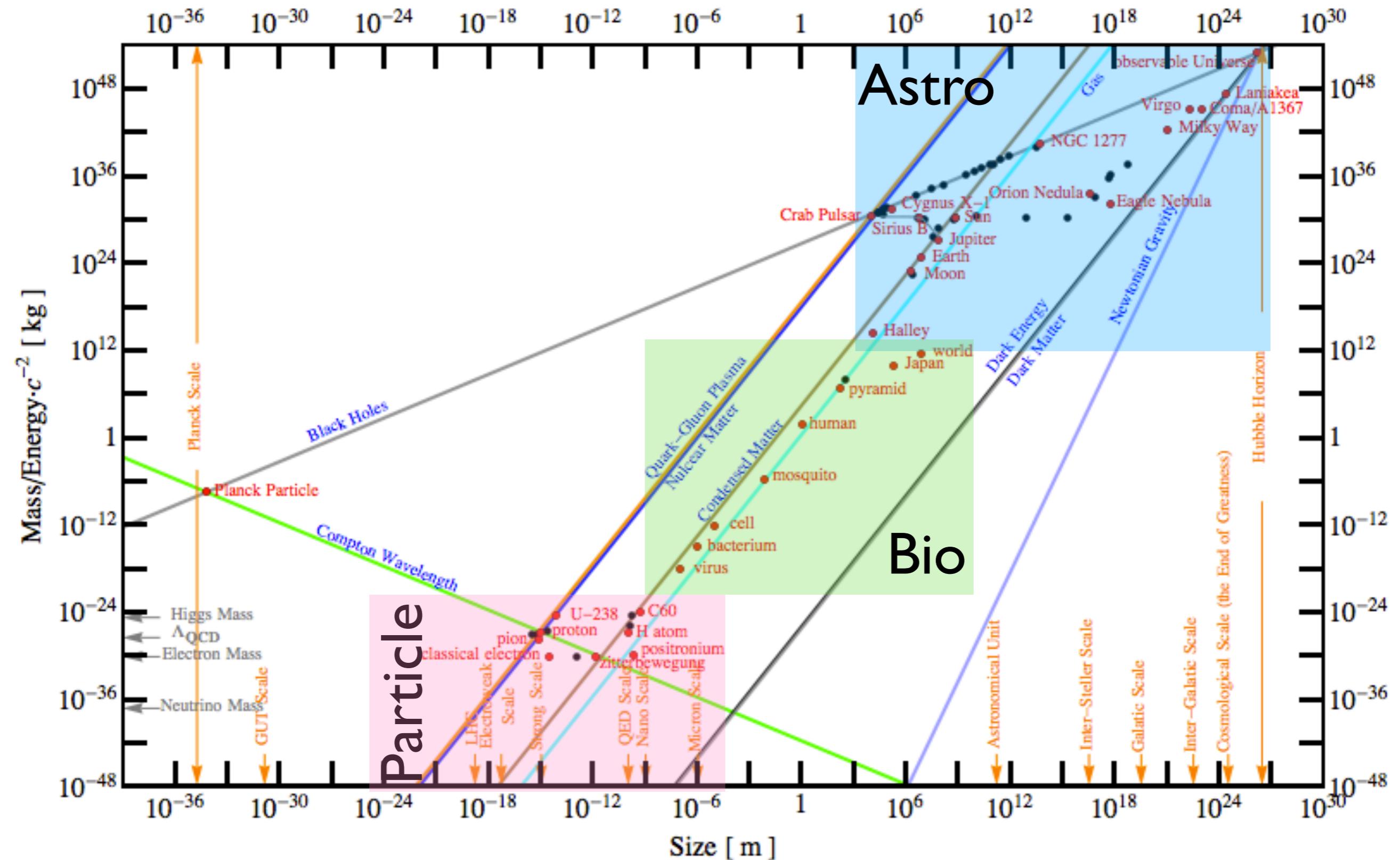
The Universe

- **Definition:** *The world in which we are living. The space-time geometry and the Universe content.*
- Made of material (**mass**) distributed over various size (**distance**) as a function of time (**redshift**):
 - atoms, molecules, dust particles, planets, stars, galaxies, LSS, cosmic web ...
- Movie clip: “*The Powers of 10*”

Power of Ten (1977)

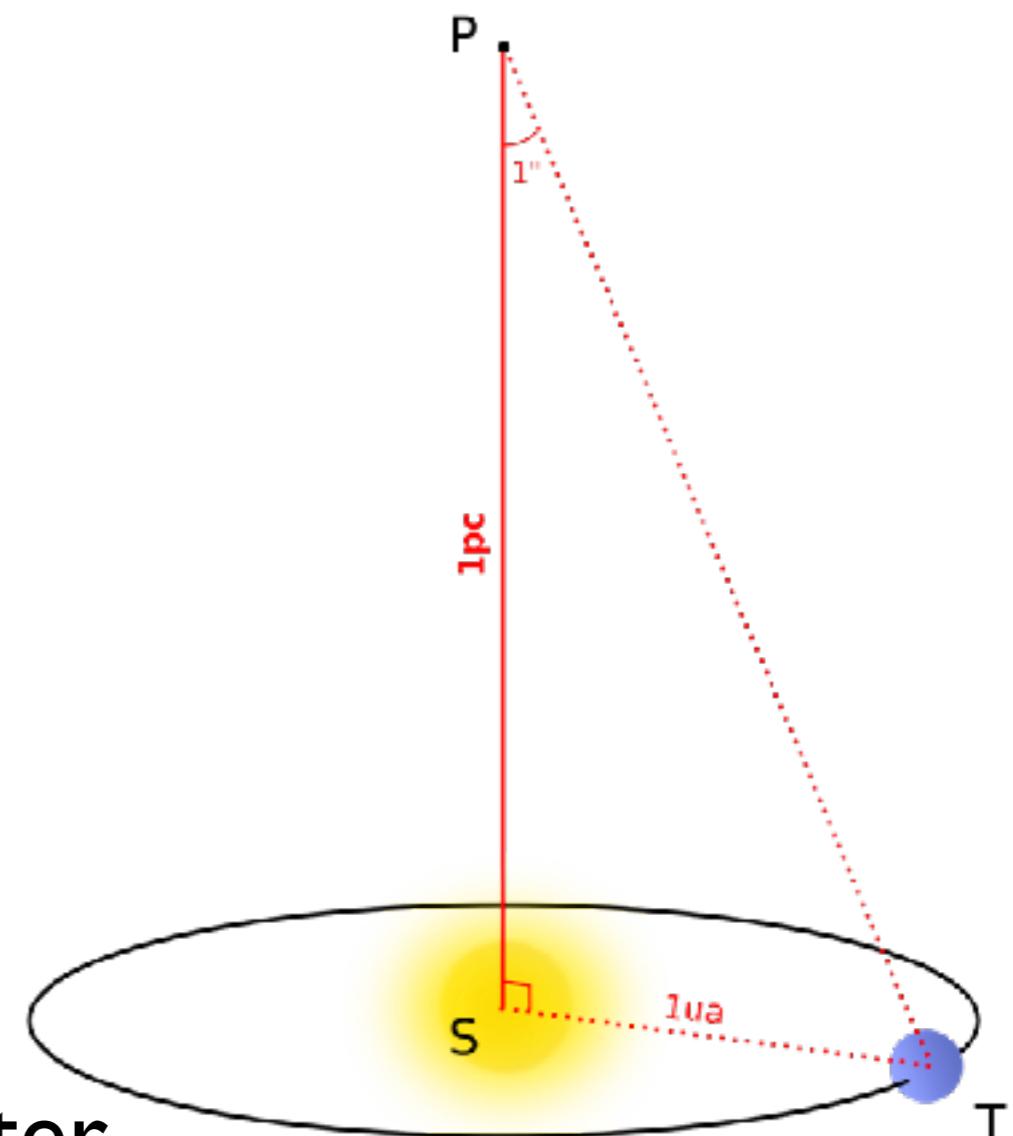


Mass-Size in the Universe



Cosmological Distances

- Units: **Distance = Time**
 - $c = 299\ 792\ 458$ m/s
 - Light Year (ly)
 - **Parsec (pc) = 3.26 Light Year**
 $= 3.10^{16}$ meters
 - $1\ \text{Mpc} = 3.10^{22}$ meters \sim typical distance between galaxies
 - $28\ \text{Gpc} \sim 8.10^{26}$ meters = diameter of the observable Universe



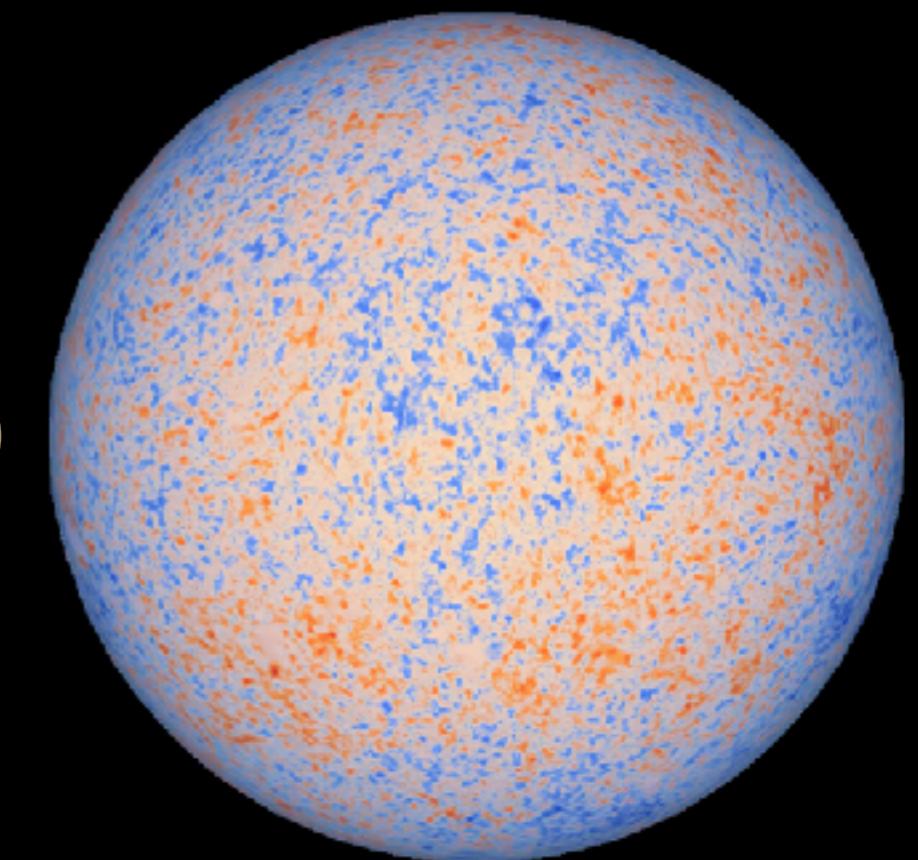
Range of Distance

Earth



Diameter: 12'742 km
Or 0,04 light-second

Observable Universe

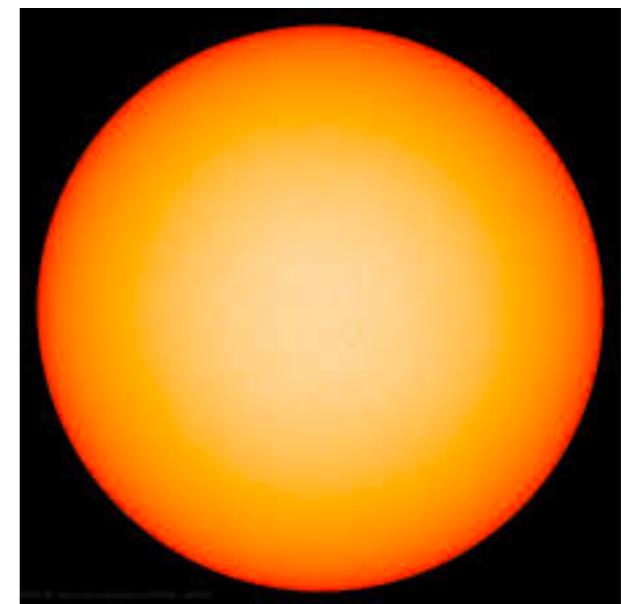


$\times 10^{20}$

Diameter: $\sim 10^{24}$ km
Or 93 billion of light-year

Cosmological Masses

- Units:
 - **Solar Mass** = 2.10^{33} grams
 - $100 \sim$ most massive star
 - $10^{6-8} \sim$ typical SMBH
 - $10^{11} \sim$ typical galaxy
 - $10^{15} \sim$ largest galaxy cluster
 - $10^{22} \sim$ baryonic mass in observable Universe (5%)



Is the Universe:

- A. Finite
- B. Infinite
- C. Don't Know

Is the Universe:

- A. Static
- B. Expanding
- C. Expanding but decelerating
- D. Expanding but accelerating
- E. Don't Know

Historical Perspective

- Our understanding of the Universe is *limited by the **observations*** that are accessible to us.
- The Sun, the Planets, and the Stars *have been known since the beginning of humanity*: because *visible by eye!*
- **But what these objects are telling us about the Universe? Is that all?**

The Scientific Revolutions in Cosmology

- **1500-1650: Copernicus-Galileo Revolution**

- From the fact that the Earth is not the center of the Universe (Copernicus)
- To the study of the Solar System with the first telescope (Galileo)

- **1650: Newtonian Revolution**

- Mathematical approach, **first theory of Gravity** (Newton)

- **1915: Relativity Revolution**

- Special Relativity - the finite speed of light and its impact
- General Relativity: **Gravitation and Geometry of Space-Time are tightly connected** (Einstein)

- **1970: Standard Model of Particles Physics**

- does not explain Gravitation
- nor Dark Matter, nor neutrino oscillations, nor Dark Energy

Key historical books

Nicolas Copernic (1473-1543) published in 1543 “*De revolutionibus orbium celstis*”, where he replaced the Earth by the Sun at the center of the Universe.

Tycho Brahé (1546-1601) published in 1598-1602 “*Astronomiae instauratae progymnasmata*” describing the observation of the supernova in 1572, which cast doubts on the eternity of stars. Within 20 years, he measured with a precision 10 times better than previously the positions of the Sun, the Moon and 777 stars.

Johannes Kepler (1571-1630) published in 1609 “*Astronomia nova*” where he analyzed the positions of planets from Tycho Brahé observations, he replaced the myth of perfect circular orbits by ellipses.

Galileo Galilei (1564-1642) published in 1610 “*Sidereus Nuncios*” where he described the craters of the Moon, the Solar spots, and the discovery of 4 Satellites of Jupiter.

Isaac Newton (1642-1727) published in 1687 “*De philosophiae naturalis principia mathematica*” where he described its gravitational law as the universal attraction - and in 1704 “*Optics*”.

Albert Einstein (1879-1955) published in 1905 “*Special Relativity*” and in 1916 “*General Relativity*”.

The vision of the Universe changed along the history

Ptolémée (90-168): was a great scientist astronomer and geographer. Known for its map of the Earth (it was already known to be spherical).



WHY ?

The vision of the Universe changed along the history

Ptolémée (90-168): was a great scientist astronomer and geographer. Known for its map of the Earth (it was already known to be spherical).

Best argument: during Moon eclipse the projection of the Earth is a circle.



The vision of
the Universe
changed along
the history

For Ptolémée (90-168):

- The Earth is at the center of the Universe
- The Sun is on the 4th orbit

=> This implies complex orbits (which no simple mathematical law can explain !)



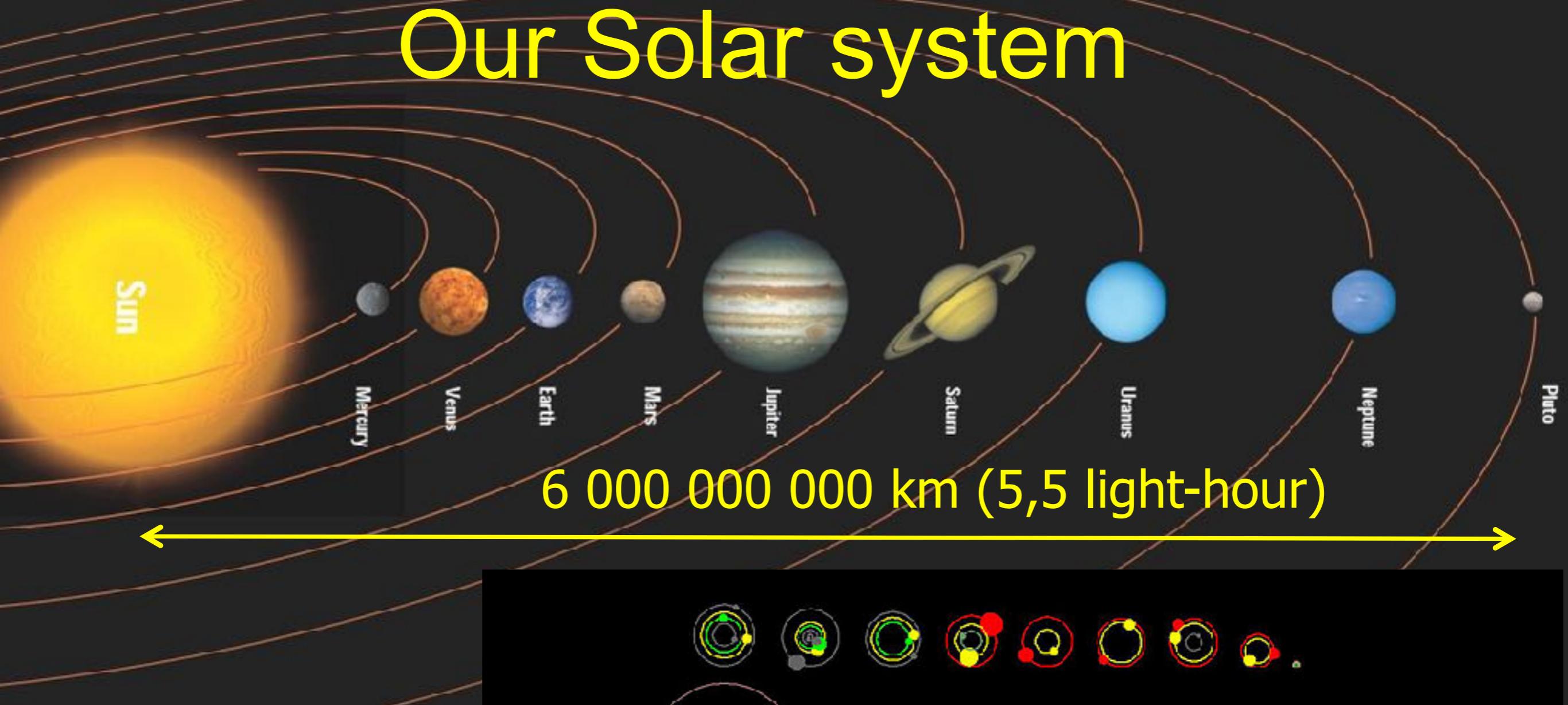
Better observations, means better modelling

For **Copernicus**
(1453-1543):

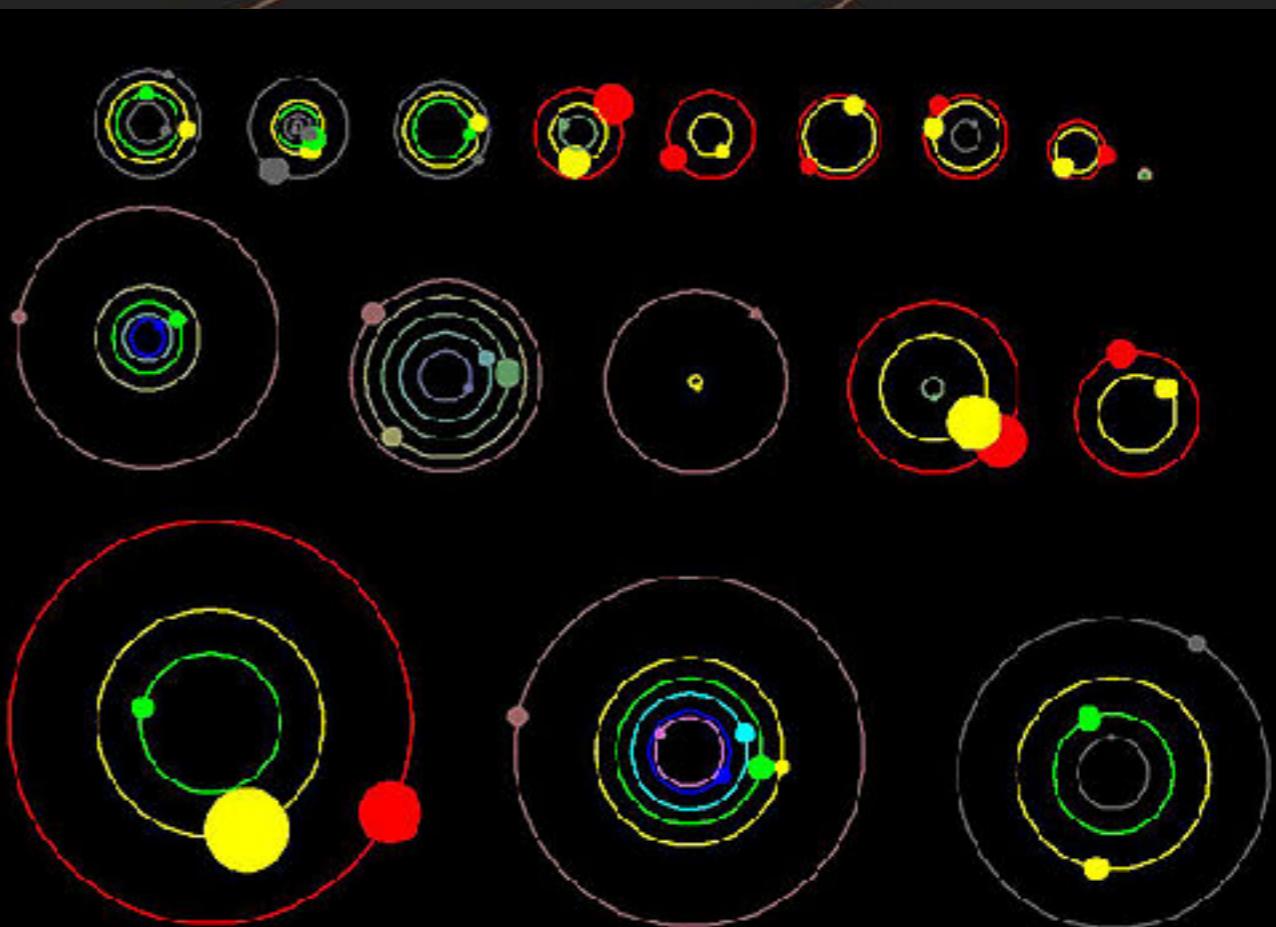
- The Sun is at the center of the Universe
- The Earth is the third planet orbiting the Sun.



Our Solar system

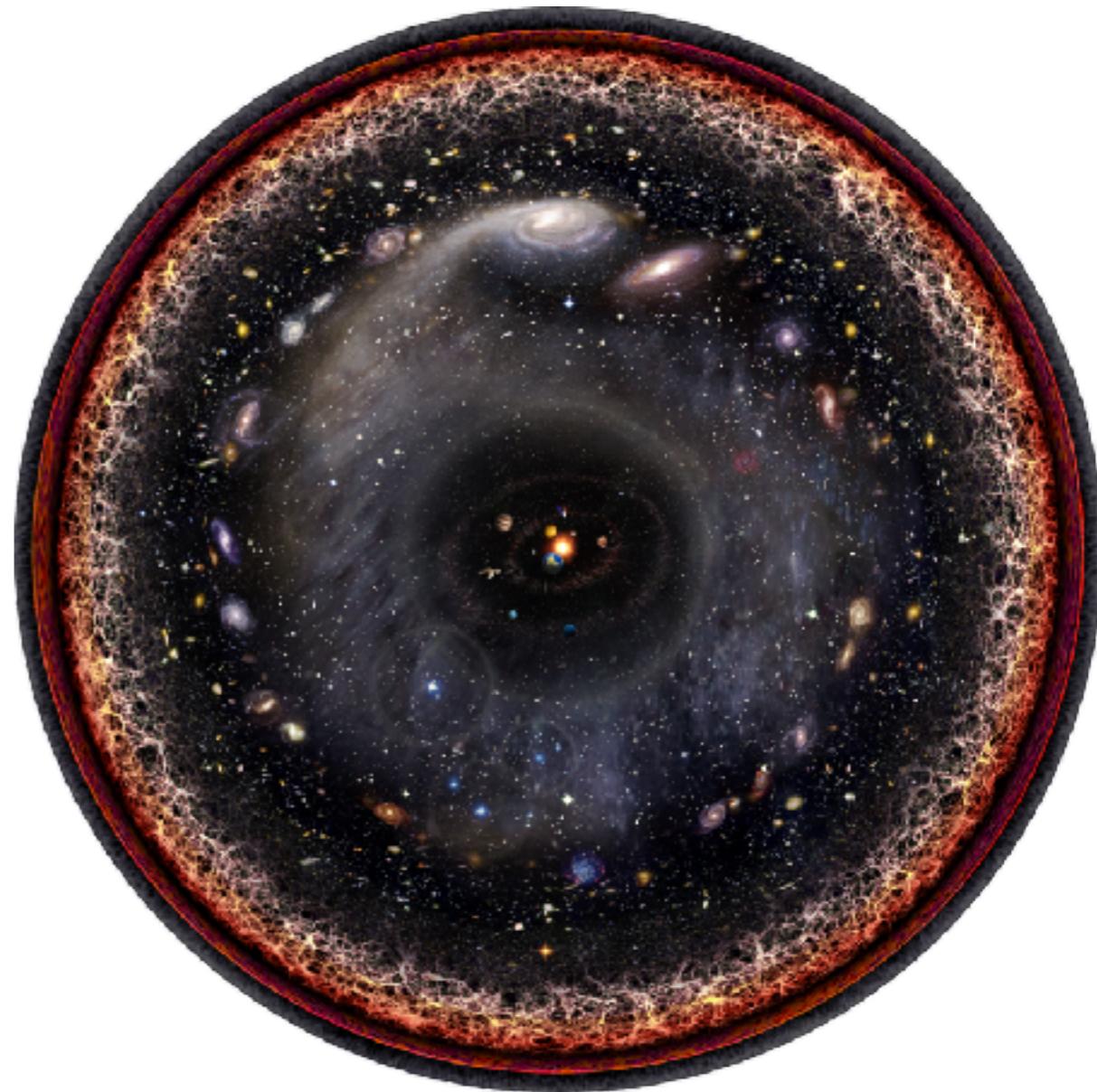


Few of the thousands of exo-planetary systems found by the Kepler space mission



The Big-Bang Universe

- 3 key observations leads to the **Big-Bang Cosmological model**
 - *The expansion of the Universe*
 - *The notion of Singularity*
 - *The thermal history of the Universe with the discovery of the cosmic microwave background*
- The underlying mathematical model follows the precepts of the **General Relativity**



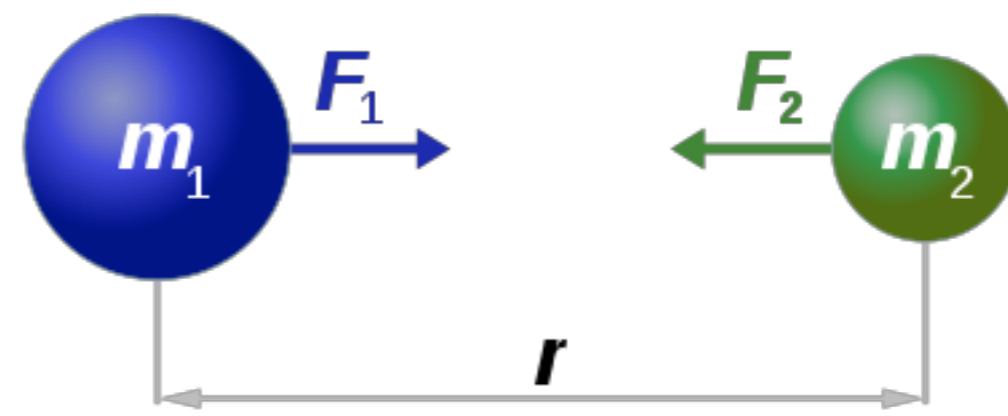
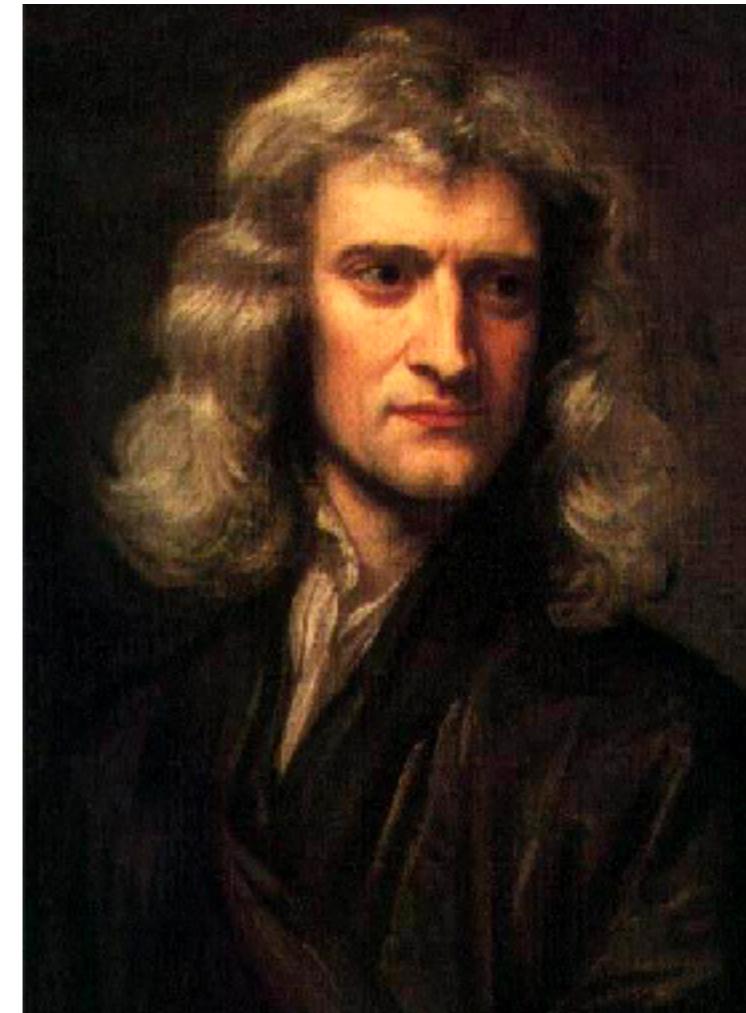
Logarithmic representation of our Universe

Gravitation of Newton [1687]

The gravitation is defined as the force responsible of the free-fall of bodies.

It aims to explain the motion of planets following elliptic orbits (as well as gravity on Earth).

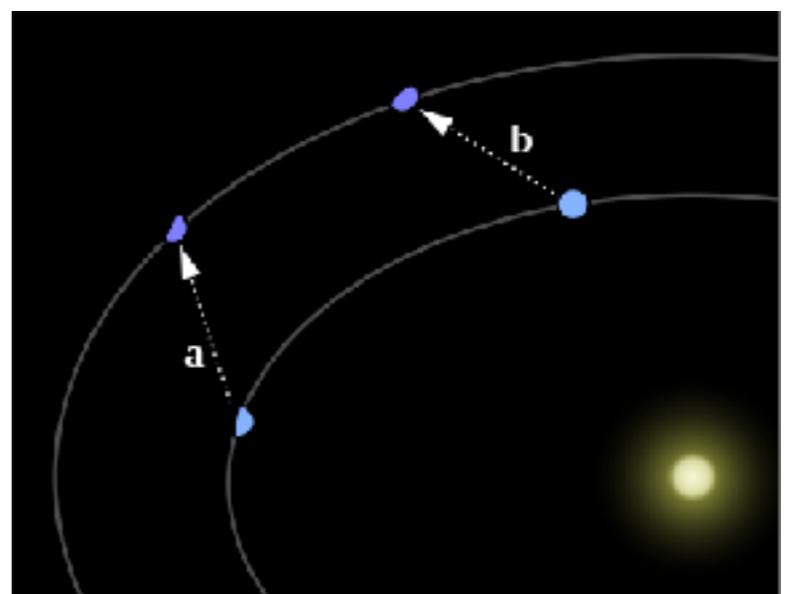
G: constant of gravitation



$$F_1 = F_2 = G \frac{m_1 \times m_2}{r^2}$$

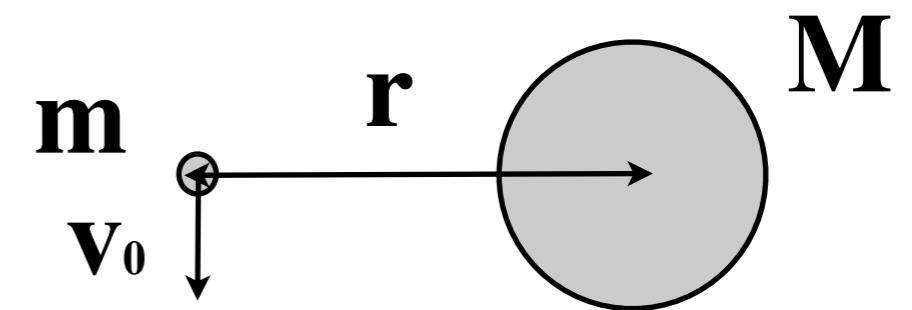
The success of Newton's law of gravitation

- **1736**: The Earth is a (prolate) sphere as shown by Pierre Louis Moreau de Maupertuis (1698-1759)
- **1759**: The return of the Halley comet as predicted by Edmund Halley (1656-1742).
- **1845-6**: The discovery of Neptune after the prediction of its existence through the newtonian perturbation of Uranus by John Adams (1819-1892) and Urbain Le Verrier (1811-1877).



$$v_0 = \sqrt{\frac{GM}{r}}$$

$m \ll M$



Limits of the Gravitation by Newton

- When $r \rightarrow 0$ then, the circular velocity is becoming infinite (but the maximum speed is the speed of light) ... \Rightarrow precession of the Mercury orbit (43 seconds of difference per century).
- Deviation of light by the Sun, is twice smaller in Newton theory than in General Relativity
- Origin of gravitation is not explained (ad-hoc formula)!
- Gravitation is instantaneous!

Special Relativity

by Einstein
[1905]:



Based on 2 postulates:

- * Laws of physics are invariant in all inertial systems (non accelerating)
- * Speed of light (in vacuum) is constant (independent of the speed of the light source)

Lorentz boots

$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$

$$x' = \gamma (x - vt)$$

$$y' = y$$

$$z' = z$$

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Dilatation of time and space

The Lorentz Transformation
Hendrik Lorentz (1853-1928)

Special Relativity

by Einstein

[1905]:



Some consequences:

Length contraction,

Time dilatation,

Relativistic mass

A universal speed limit

Relativity of simultaneity.

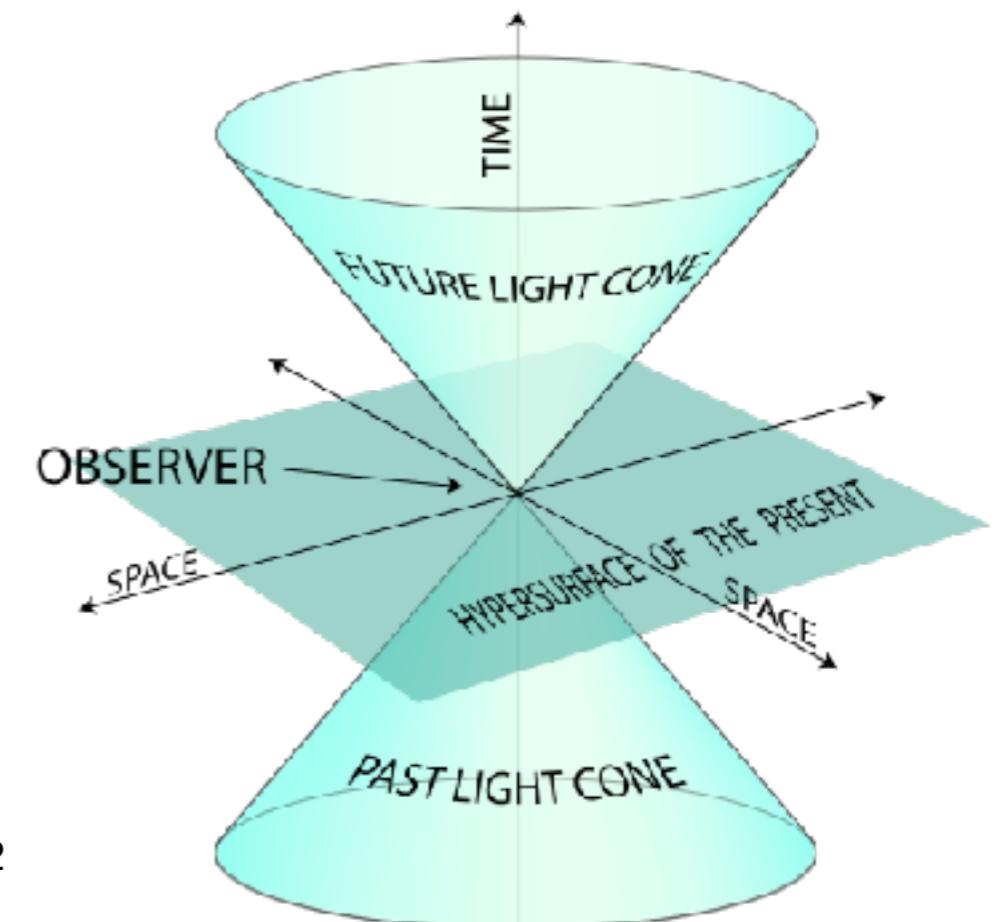
Notion of a time that is dependent on reference frame and spatial position.

Invariant space-time interval.

Equivalence of mass and energy, as expressed in the mass-energy equivalence formula $E = mc^2$

Space-time interval

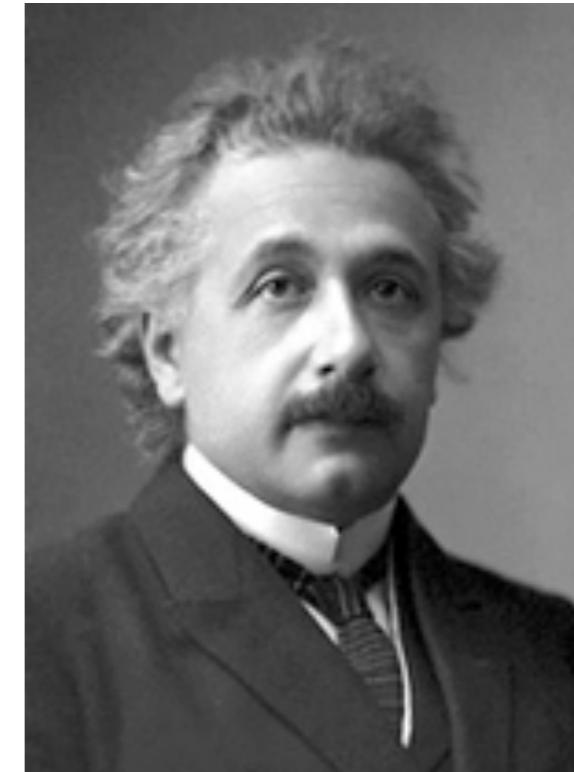
$$s^2 = \Delta r^2 - c^2 \Delta t^2$$



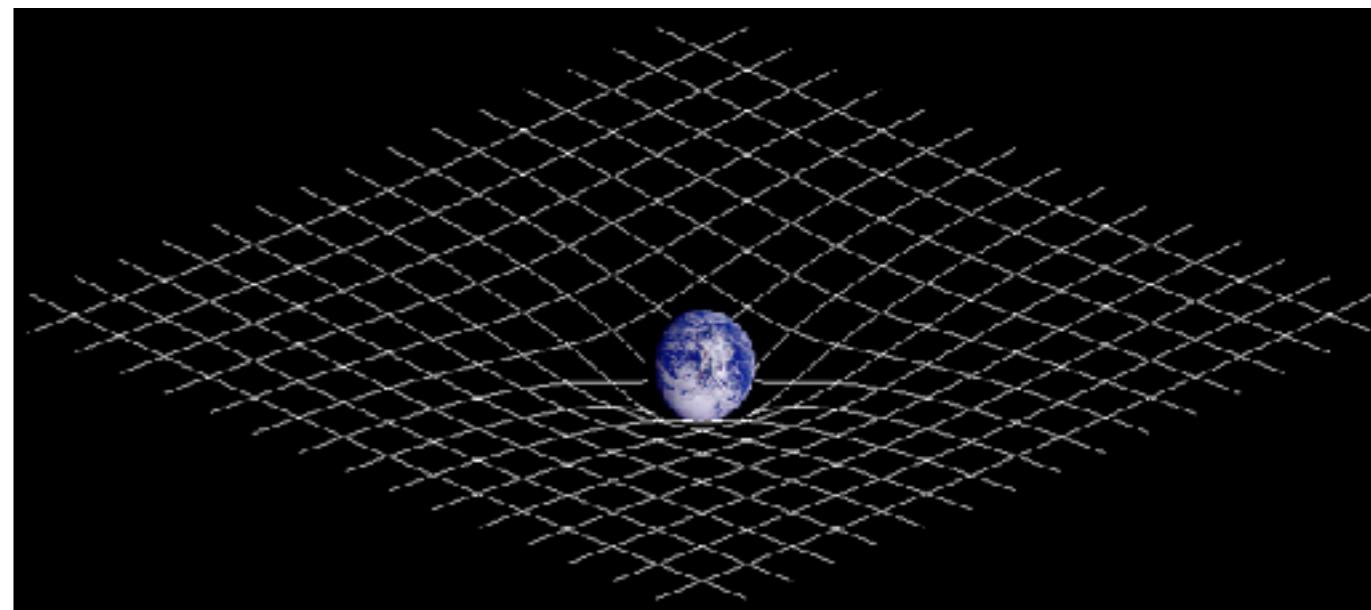
From *special* relativity to *general* relativity

- Special Relativity applies where Gravity can be neglected
- *Bringing in Gravity but keeping the same concept as in special relativity, implies that the presence of Gravity is distorting Space-Time.*

Gravitation by Einstein [1916]: the General Relativity



Gravitation is understood as the deformation of Space-Time by the existence of Matter.



G: constant of gravitation
c: speed of light
 Λ : cosmological constant

$$G_{\mu\nu} + \Lambda \cdot g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Curvature Metric Mass/Energy

The Static Universe

- A **static universe**, is originally a cosmological model in which the *universe is both spatially infinite and temporally infinite*, and space is neither expanding nor contracting.
- Such a universe does not have spatial curvature or in other word: it is 'flat'. A static infinite universe was first proposed by Giordano Bruno (describing the local distribution of stars).
- In contrast to this model, **Einstein** proposed a *temporally infinite but spatially finite model* as his preferred model in 1917, in his paper *Cosmological Considerations in the General Theory of Relativity*. This implies a fixed cosmological constant with: $\Lambda_E = 4\pi G\rho/c^2$ and a curved universe with radius : $R_E = \Lambda_E^{-1/2} = \frac{c}{\sqrt{4\pi G\rho}}$.



Giordano Bruno
(1548-1600)

The Friedmann Equations

- The Friedmann equations are derived from the Einstein equations and assume **the cosmological principle**:
 - *the universe is spatially homogeneous and isotropic*
- Empirically, this is justified on scales larger than **>100 Mpc**. The cosmological principle implies that the metric of the universe must be of the form:

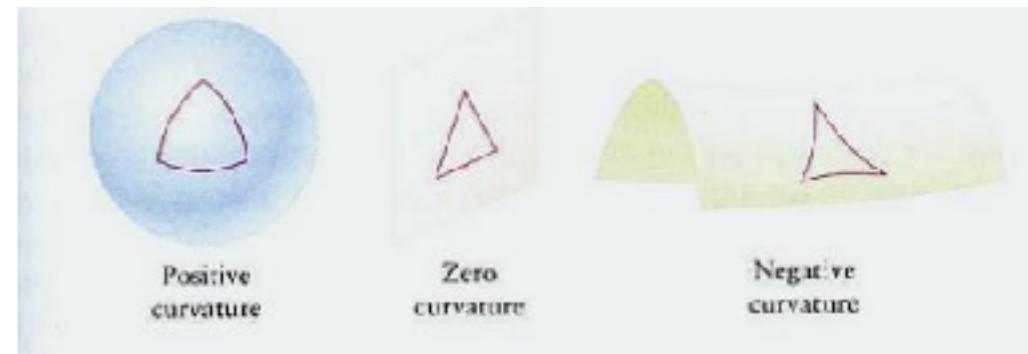
$$ds^2 = a(t)dr_3^2 - c^2dt^2$$

where dr_3^2 is a 3D metric which can be:

- (a) flat space,
- (b) a sphere of constant positive curvature
- or (c) a hyperbolic space with constant negative curvature.

The curvature parameter k takes the value 0. 1. -1 respectively:

$$dr_3^2 = \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d^2\phi)$$



$a(t)$ is the scale factor of the Universe.

The Friedmann Equations [1922-1924]

Friedmann equations are derived from the Einstein equations. **They determine the evolution and the geometry of the Universe.**

k: curvature

H: Hubble parameter (time dependent)

G: gravitational constant

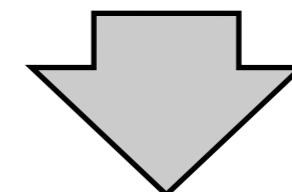
ρ : density (mass, radiation)

Λ : cosmological constant



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$



$$\dot{\rho} = -3H\left(\rho + \frac{p}{c^2}\right),$$

Mass conservation

$$\rho_c = \frac{3H^2}{8\pi G}$$

Critical density

$$\Omega = \frac{\rho}{\rho_c} = \frac{8\pi G \rho}{3H^2}$$

Reduced density

Density Parameters

$$\frac{H^2}{H_0^2} = \frac{\Omega_R}{a^4} + \frac{\Omega_M}{a^3} + \frac{\Omega_k}{a^2} + \Omega_\Lambda$$

Friedman equation,
expressed with reduced densities

The reduced Friedmann equations form the cosmological parameters. This parameters control the evolution of the Universe.

H: Hubble constant (time dependent)

Ω_k : curvature density

Ω_R : radiation density (photon/neutrino)

Ω_M : matter density

Ω_Λ : cosmological constant density (dark energy?)

$$p = w \rho c^2$$

Equation of State

w=0 : matter

w=1/3 : radiation

w=-1 : energy/
cosmological constant

Curvature and the Age of the Universe

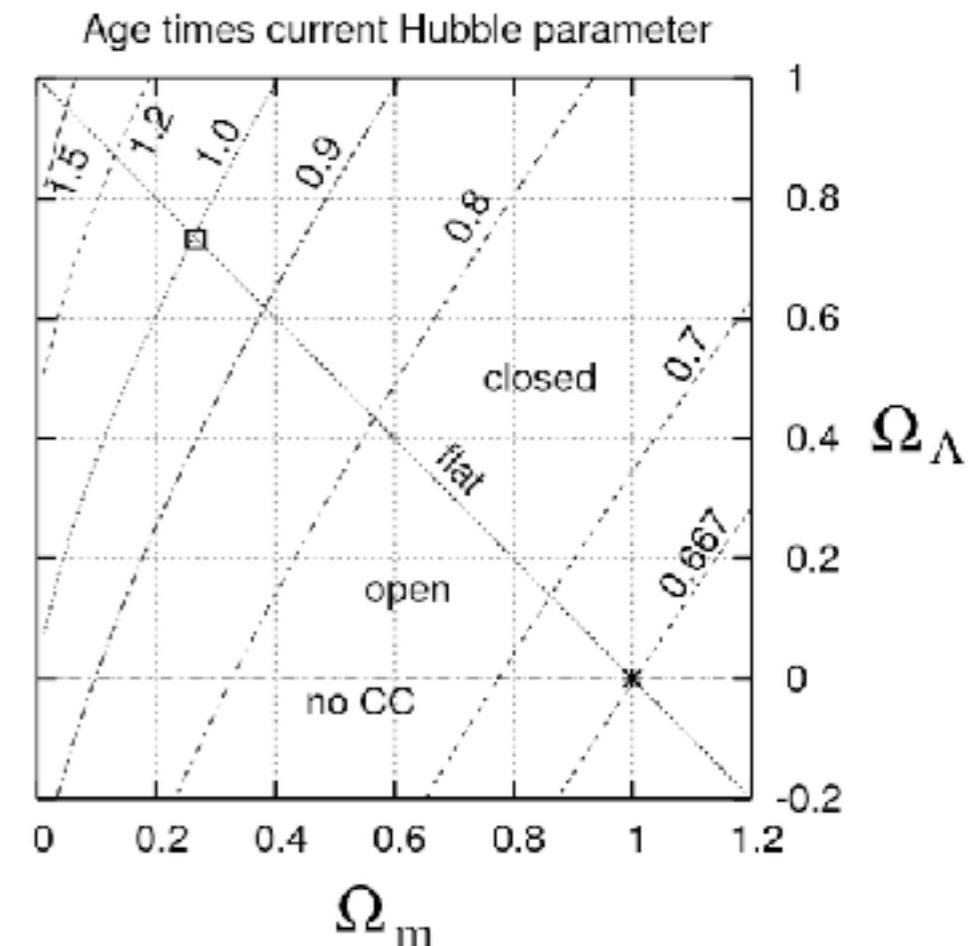
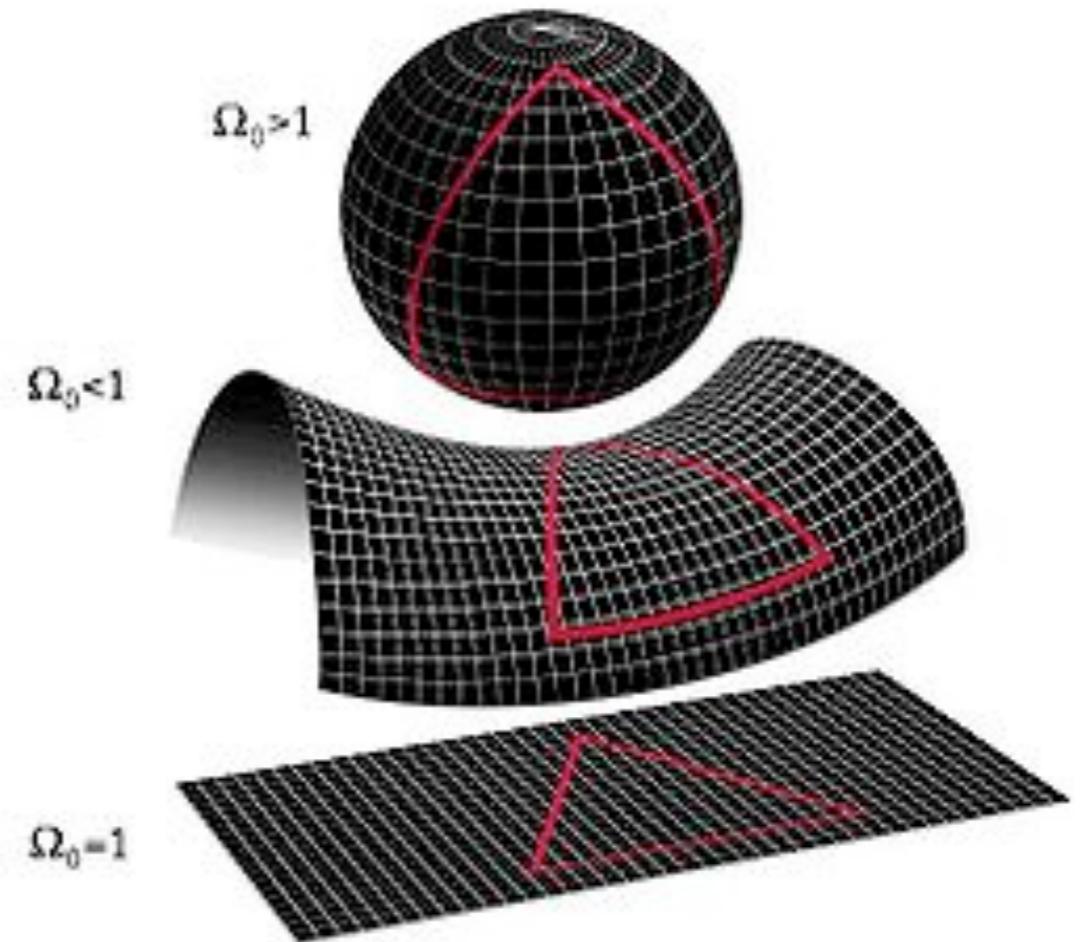
The Universe is flat if the sum of the Ω is equal to 1, if not it is curved (closed if >1 or open if <1)

The age of the Universe does depend on the cosmological parameters.

Ω_R : **radiation density (photon/neutrino)**

Ω_M : **matter density**

Ω_Λ : **cosmological constant density**



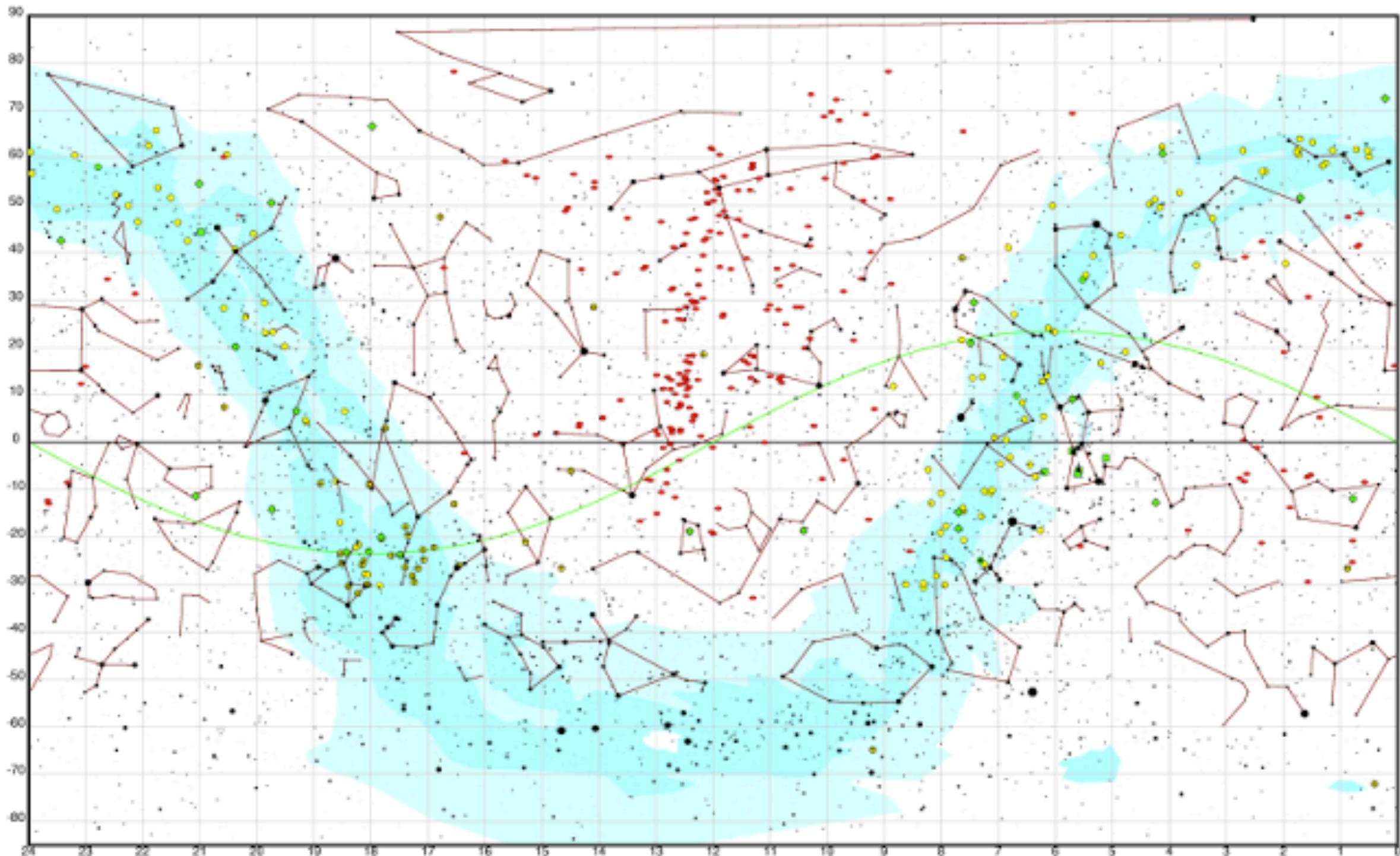


Beyond the Milky-Way

- **Night sky is full of stars - but what is beyond?**
- **Charles Messier (1730-1817)** compiled between 1771 and 1784 a catalog of 107 Nebulae.
- **William Herschel (1738-1822)** observing the Northern Sky published in 1786 a catalog of 1000 new star clusters and nebulae.
- His son **John Herschel (1792-1871)** observing both the Northern and Southern sky published in 1864 the General Catalog of Galaxies (GCG), with 5079 objects.
- **John Dreyer (1852-1926)** published an extension, the New General Catalogue of nebulae and clusters of stars (NGC), with about 15'000 objects.
- In the early 1920, the existence of galaxies (the Universe-Island of **Emmanuel Kant (1724-1804)** in 1755 - similar to the Milky Way) is still controversial, because of the intergalactic nebulae discovered by Williams Huggins in 1864.

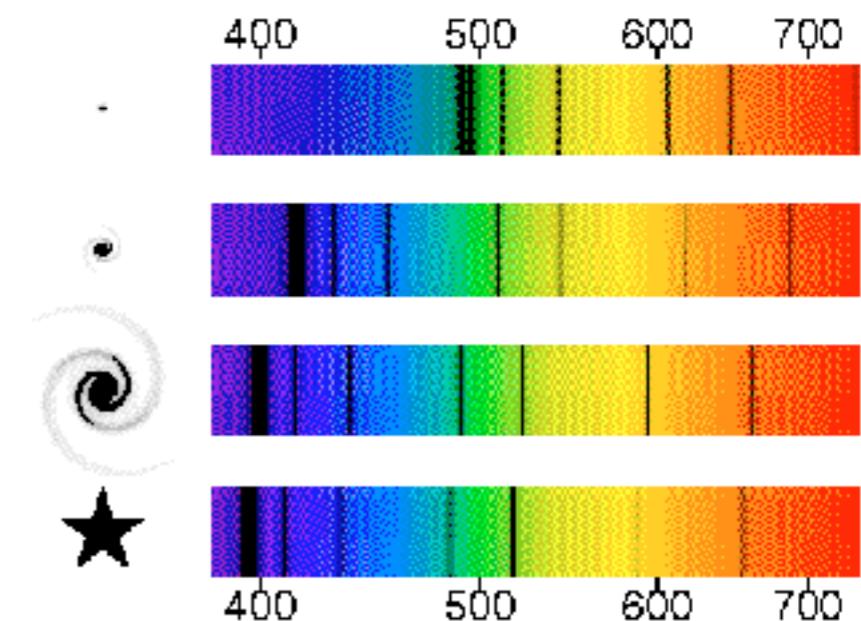
Herschel catalogue

Red: Galaxies, Yellow: Star Clusters, Green: Nebulae



Redshift Measurement

- Slipher and Shapley since 1912 conduct a systematic redshift measurement of bright nebulae.
- In 1920 at Washington, a big discussion take place between those who believes in extragalactic nebulae, and those who don't.
- In 1921 for 41 nebulae measured, 36 display a positive **redshift**. These can be interpreted as a recession (velocity) field.



$$1 + z = \frac{\lambda_{\text{obsv}}}{\lambda_{\text{emit}}}$$

$$z \approx \frac{v}{c} \quad \text{for small } v$$

Proof of existence of Galaxies

Edwin Hubble found Cepheids in 3 Nebulae, allowing to measure their distances:

1925 ApJ 62 409 "NGC 6822, a remote stellar system"

1926 ApJ 63 236 "A spiral nebula as a stellar system : Messier 33"

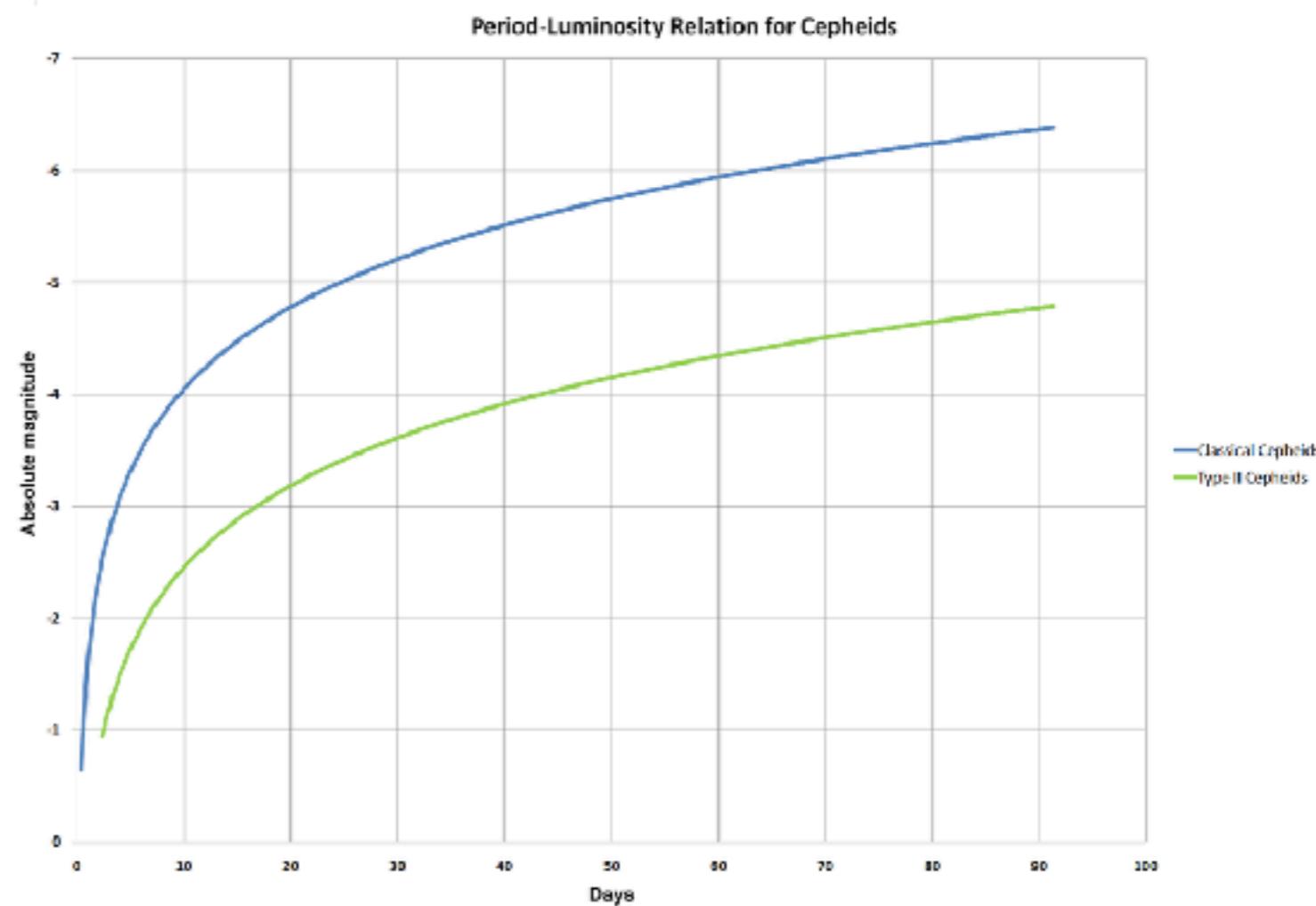
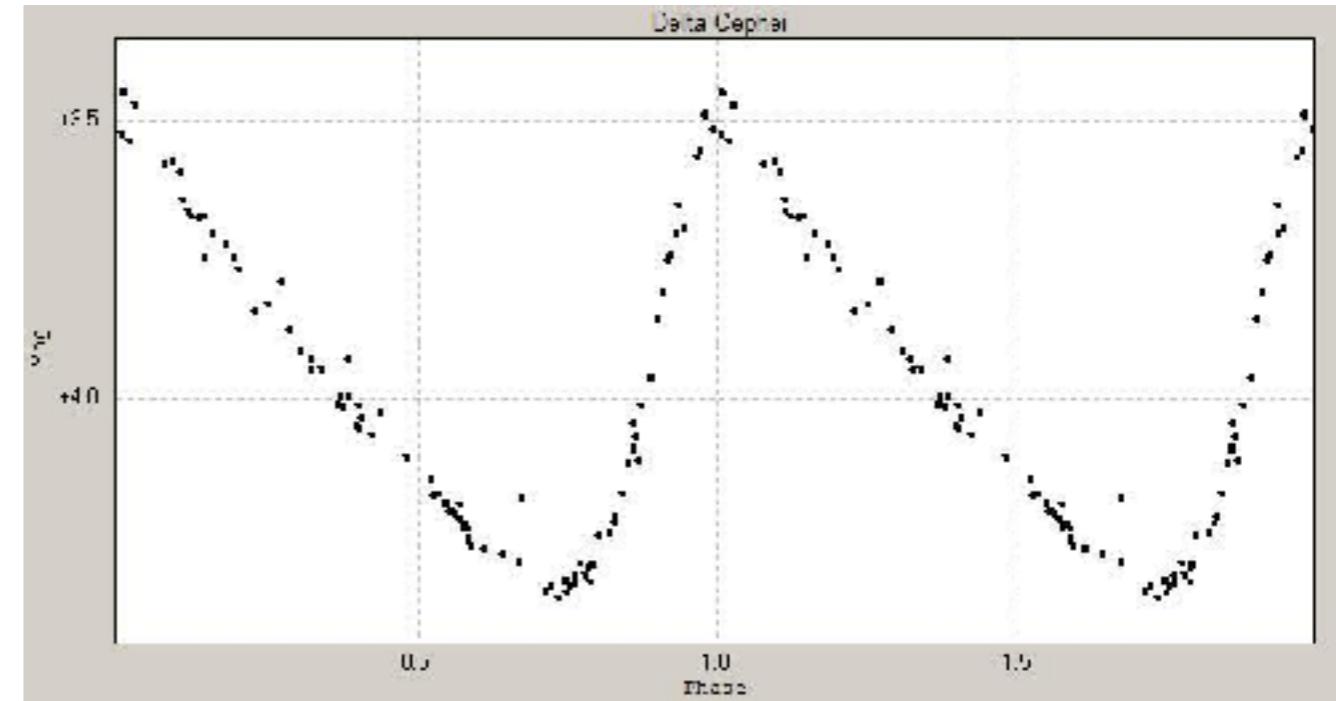
1929 ApJ 69 103 "A spiral nebula as a stellar system : Messier 31"



Proof of the existence of
Universe-Islands

Cepheids

- Cepheids are a particular class of variable stars,
- Cepheid period is proportional to their intrinsic luminosity => *can be a way to measure distances*

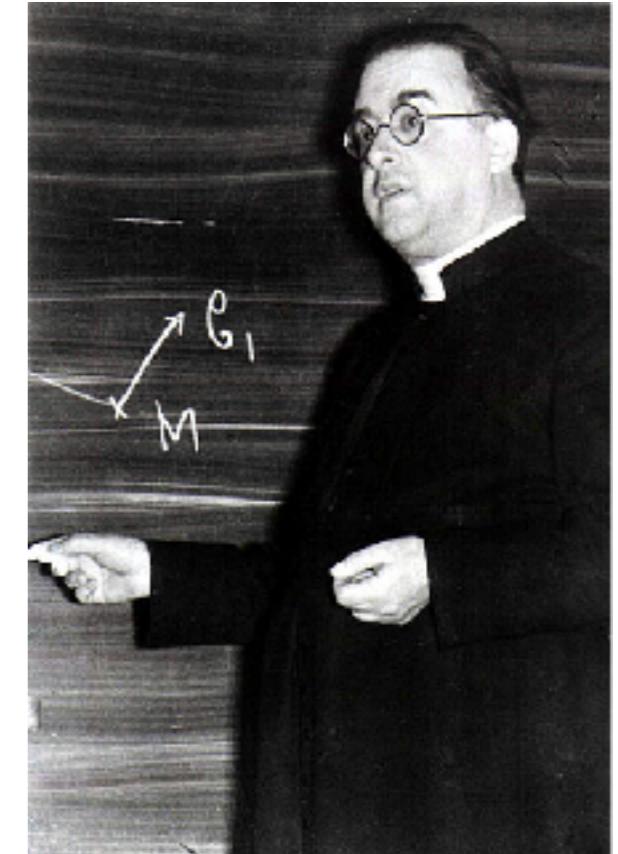


General Relativity and the Mathematical Model

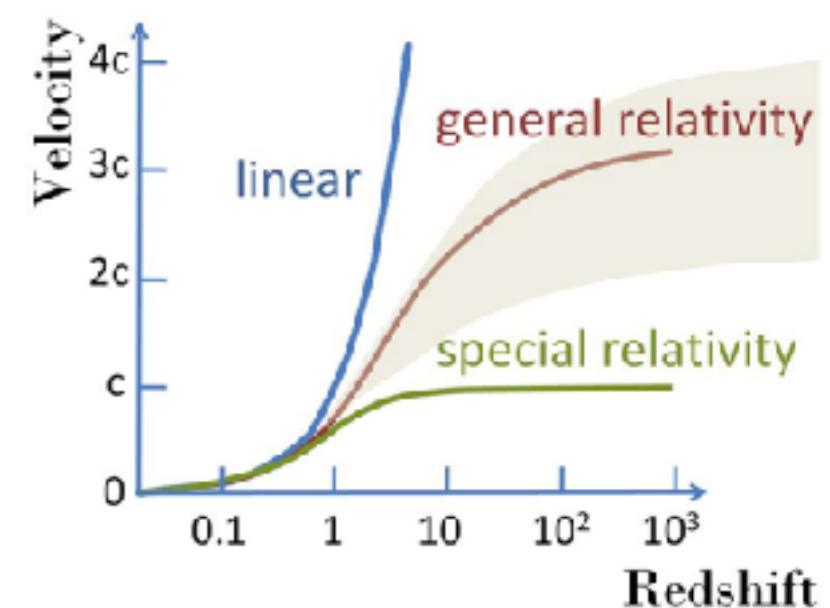
- In a 1927 article, which preceded Edwin Hubble's work, Lemaître derived from the Einstein's Equation what became known as *Hubble's law* and proposed it as a generic phenomenon in relativistic cosmology.

$$v = H_0 \cdot D \quad \text{Hubble's Law}$$

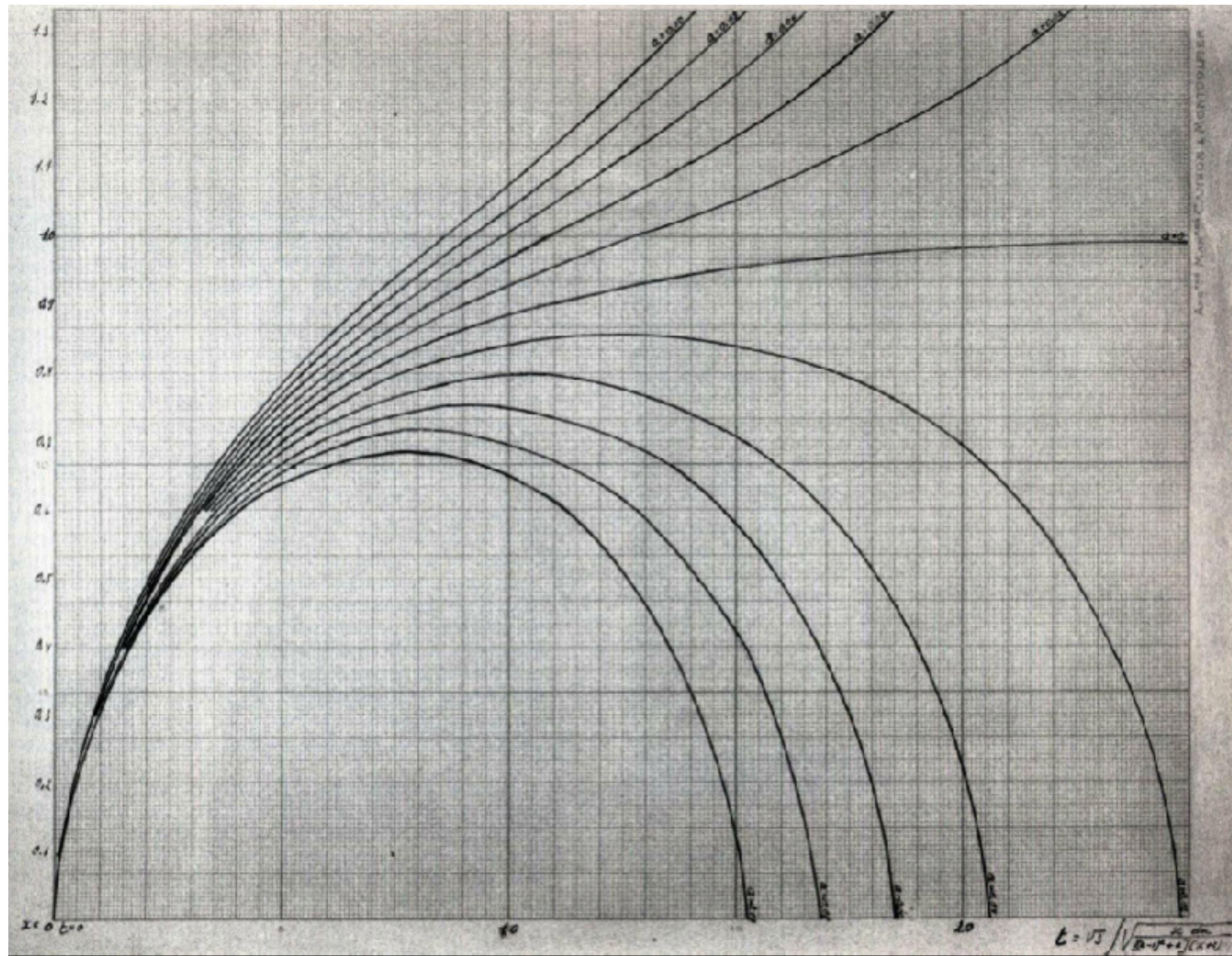
- Lemaître also estimated the numerical value of the *Hubble constant*. However, the data used by Lemaître did not allow him to prove that there was an actual linear relation, which Hubble did two years later.



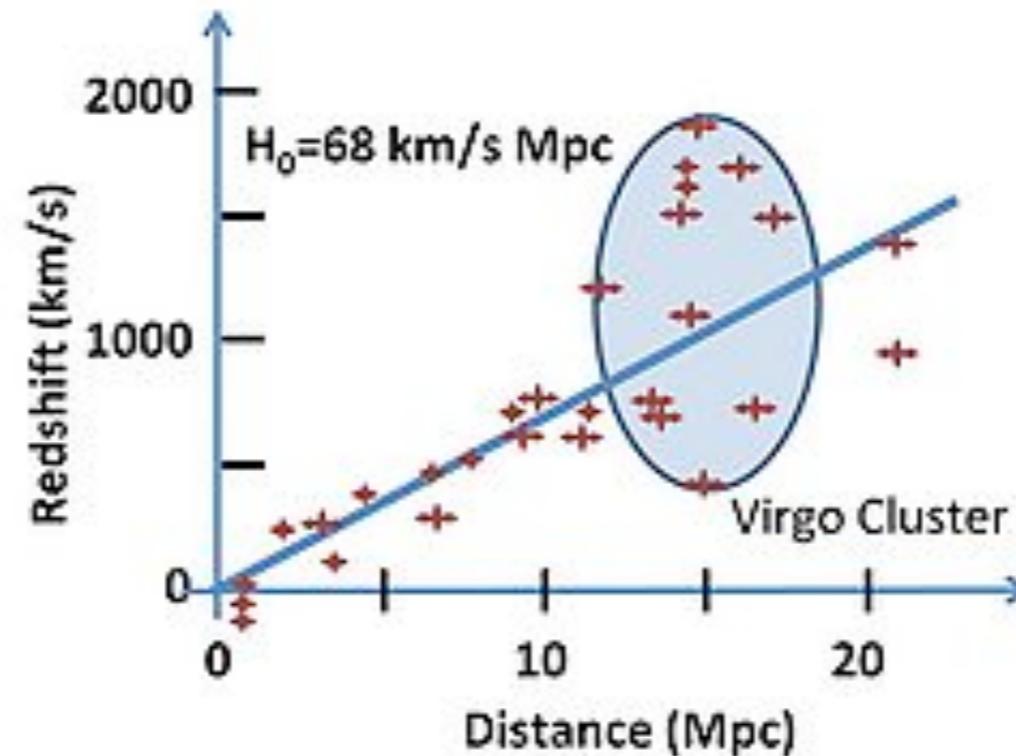
Georges Lemaître
(1894-1966)



Lemaître Model of the Universe



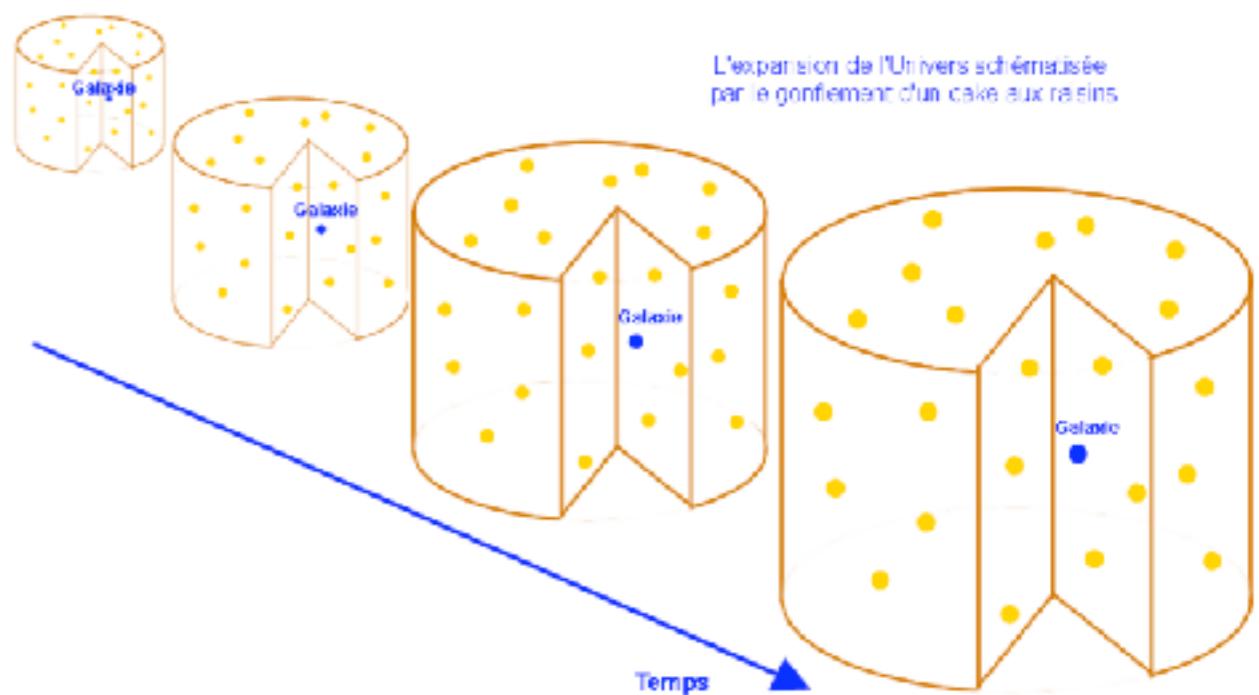
Edwin Hubble: The expanding Universe [1929]



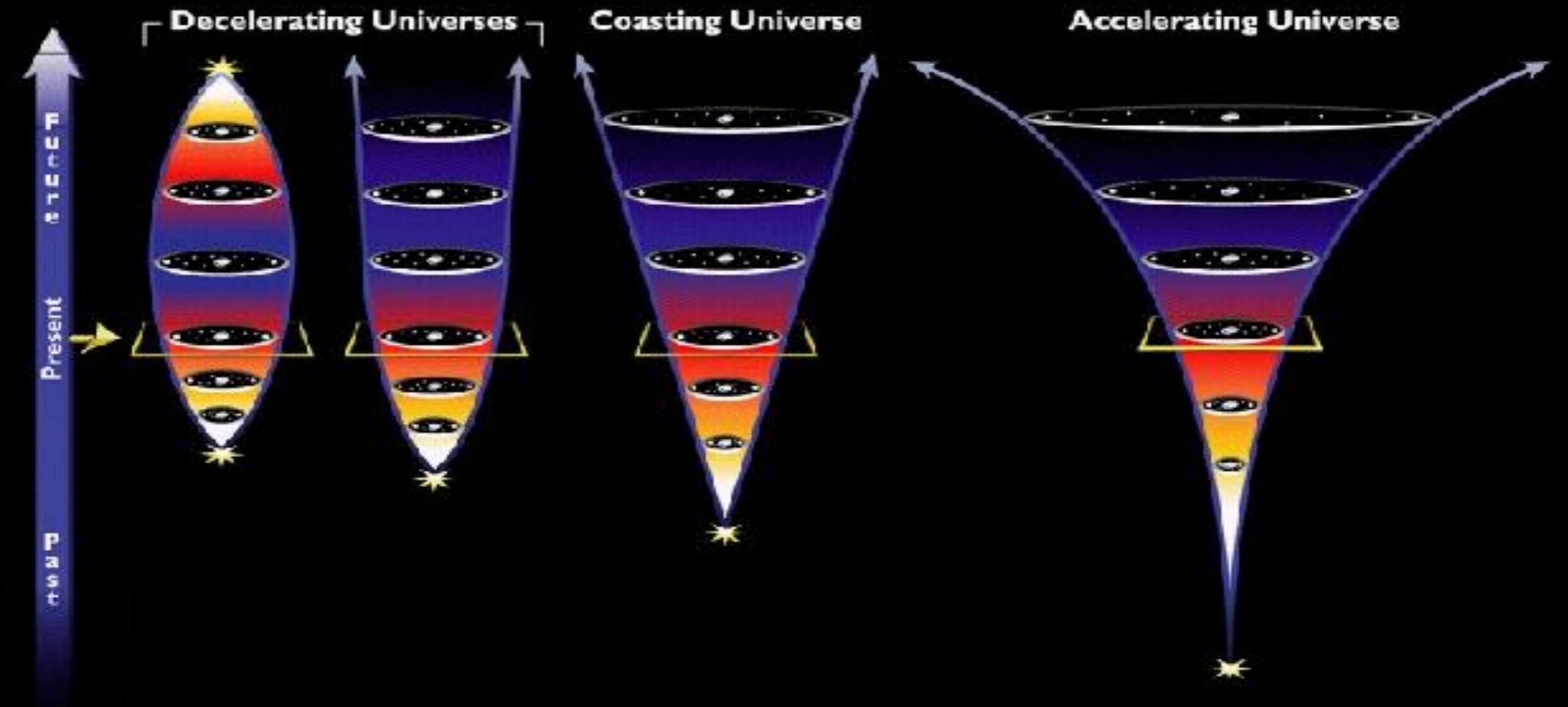
The recession speed of galaxies
is proportional to their distance!

$$V = H_0 D$$

Ho: Hubble constant
[~72 km/s/Mpc] (the inverse is
time => age of the universe
=13.7 billion years)



Possible Models of the Expanding Universe



Evolution of the Universe

Different expansion history of the Universe
For a Friedmann-Lemaître model

Edwin Hubble (1929): Measure of the Universe expansion



Annie Hall (1977):

- He's been depressed
- The Universe is expanding,
the Universe is everything.
What is the point?
- You here in Brooklyn,
Brooklyn is not expanding!

Nobel Prize of Physics (2011):

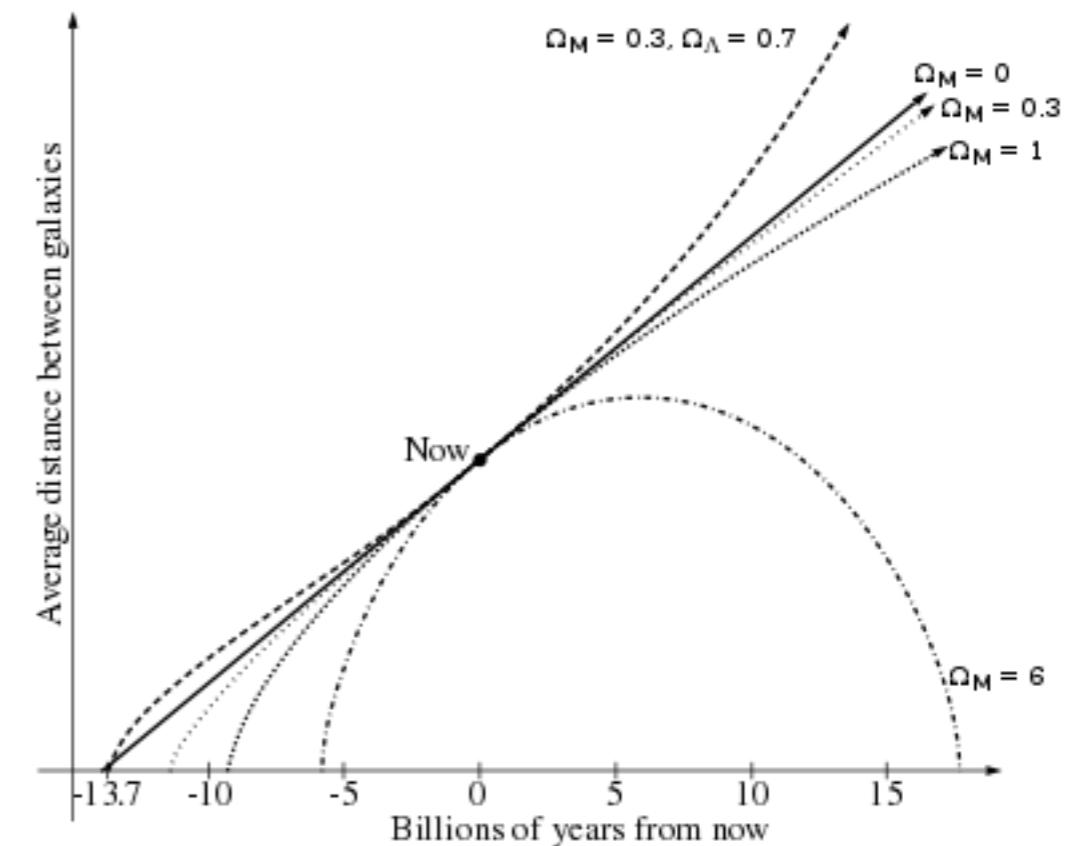
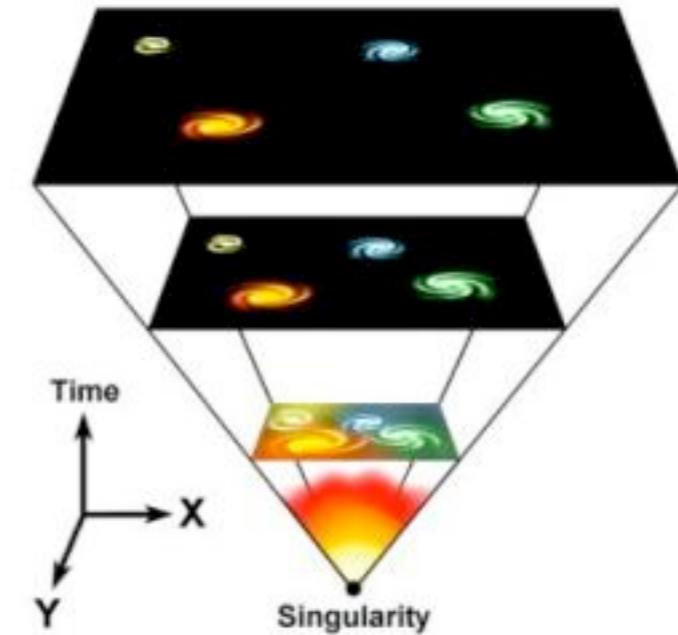
The universe **expansion** is accelerating
confirmed by the measure of SuperNovae Type Ia



The expanding Universe: => idea of an initial singularity

Going back in the past, the Universe was **smaller**, but also **denser**, and **hotter**.

Notion of a singularity:
The **Big-Bang model [1950]**



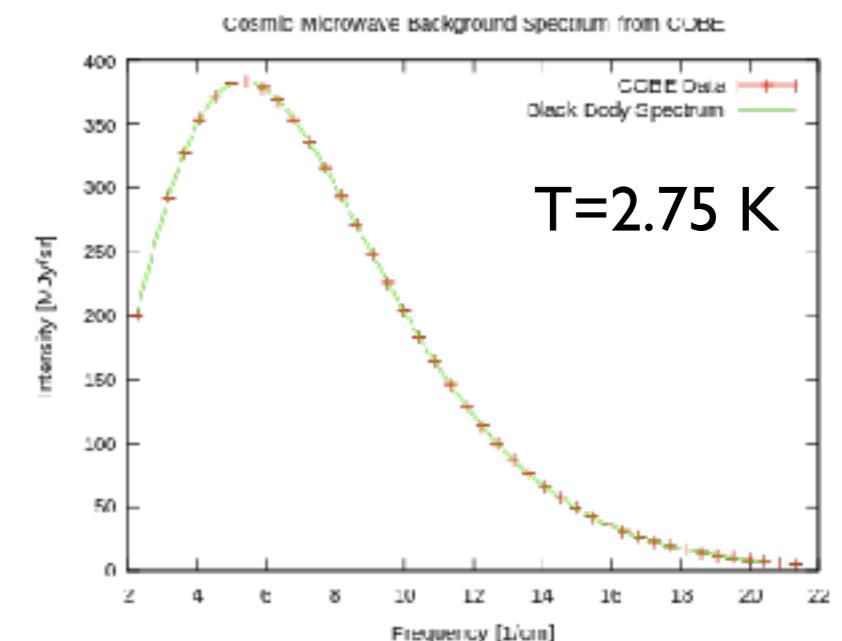
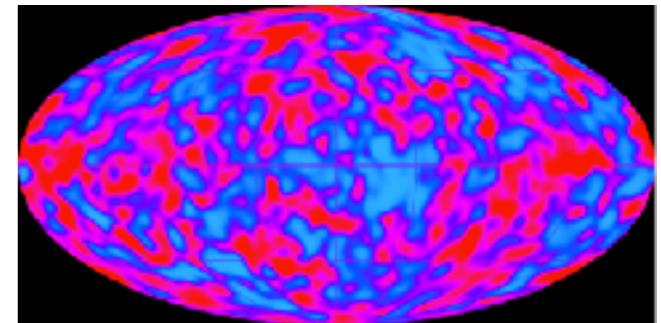
The thermal history and the cosmic microwave background

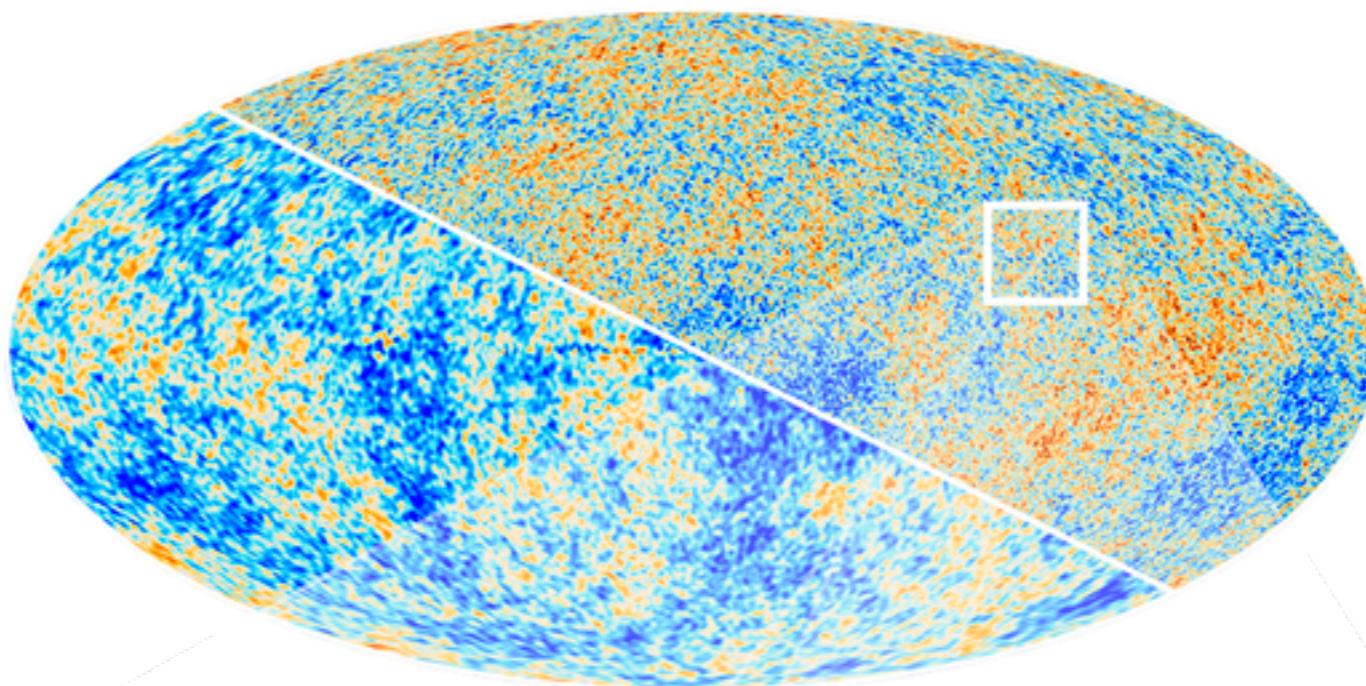
- **1946** Georges Gamow estimated a temperature of 50K and presented a theory where all elements in the Universe are produced at the beginning.
- **1946** Robert Dicke predicted a microwave background radiation temperature of “less than 20K”, but later revised to 45K
- **1948** Ralph Alpher and Robert Herman re-estimated Gamow's estimate at 5K. After the nucleosynthesis, they mentioned that the universe then cooler had become “transparent” - photons can cross the Universe.
- **1949** Alpher and Herman re-re-estimated Gamow's estimate at 28K.
- **1960s** Robert Dicke re-estimated an MBR (microwave background radiation) temperature of $\sim 10K$
- **1964** Arno Penzias and Robert W. Wilson measured the temperature to be approximately 3K using a microwave telescope.
- Lemaître learned about these measurement just before his death in 1966



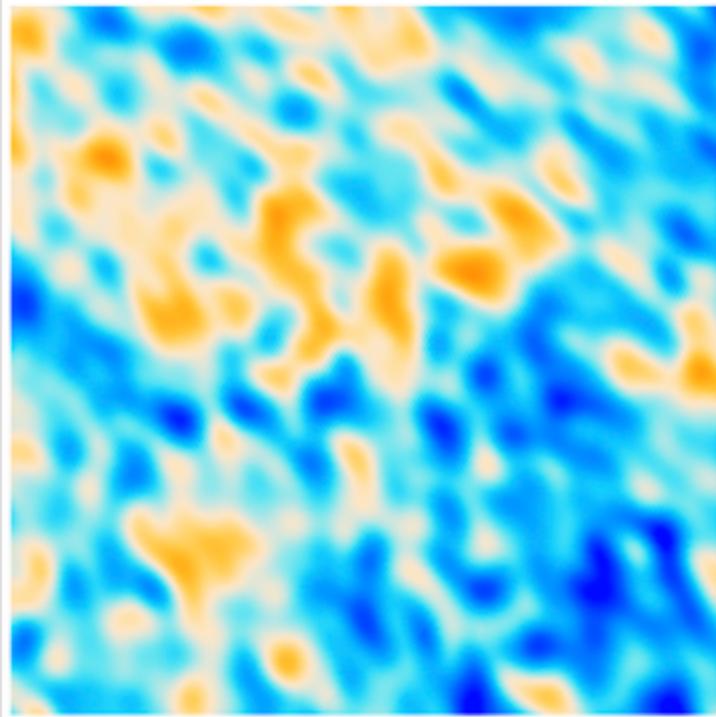
COBE and the measurement of the CMB

- In 1974, NASA issued an Announcement of Opportunity for a small/medium missions.
- Out of the 121 proposals received, 3 dealt with studying the cosmological background radiation.
- These 3 proposals lost out to the Infrared Astronomical Satellite (IRAS).
- In 1976, NASA formed a committee of members from each of 1974's 3 proposal teams to put together their ideas for such a satellite. A year later, COBE was born.
- In 1989, COBE was launched.
- In 1992 the CMB anisotropy map was published. It displayed small variations of temperature across the sky: $\Delta T/T \sim 10^{-5}$. These variation are now known to be linked to matter density fluctuation in the early Universe, fluctuations that are the seeds of the structures in today's Universe.



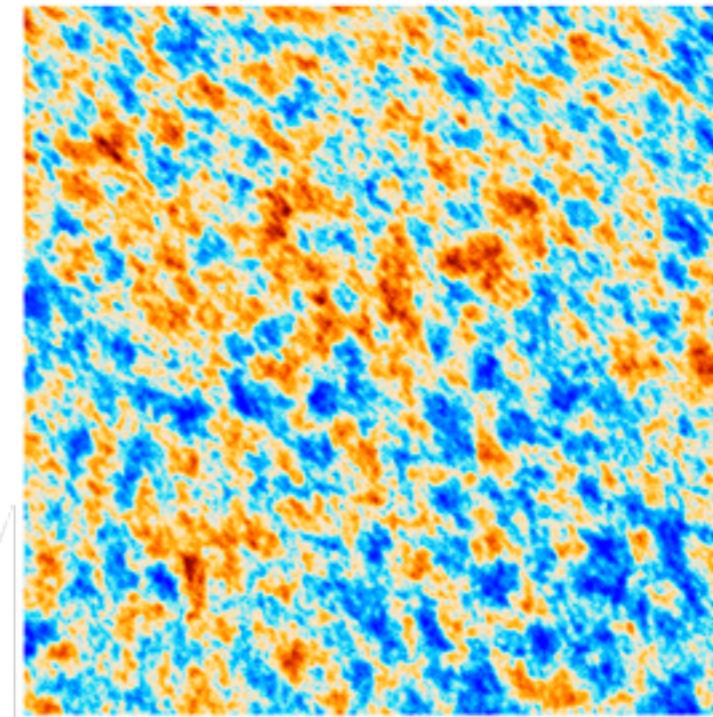


WMAP



WMAP

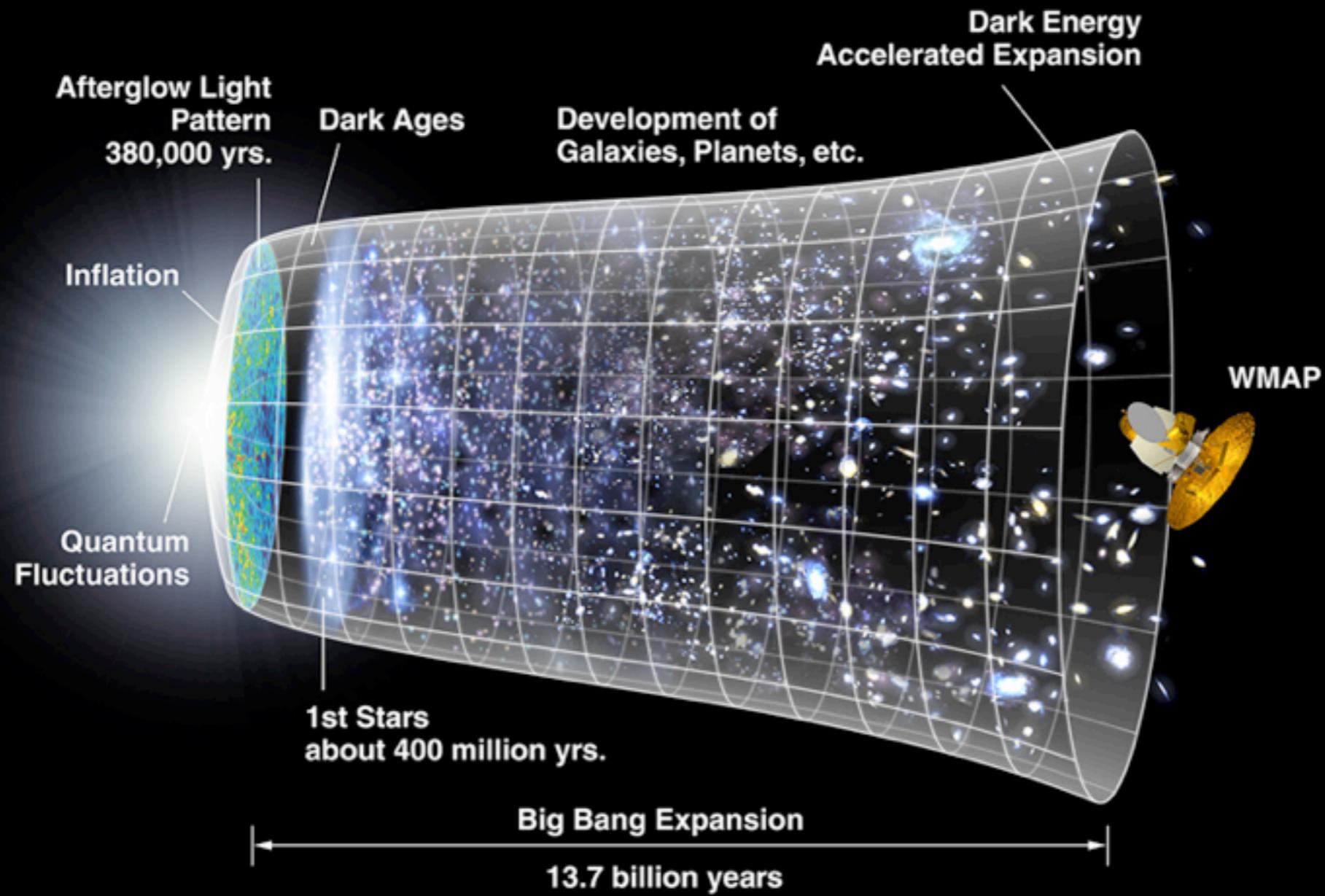
Planck



PLANCK

Most recent measurement of the anisotropy with WMAP & Planck

WMAP, launched in 2001.
Planck, launched in 2009.



NASA/WMAP Science Team

The History of the Universe

inflation, CMB, Dark Ages, First Galaxies, Galaxy evolution, Dark Energy

Some Useful Numbers

- c = speed of light in vacuum (2.998×10^{10} cm sec $^{-1}$)
- G = gravitational constant (6.67×10^{-8} cm 3 g $^{-1}$ sec $^{-2}$)
- $h\text{-bar} = h/2\pi$: Planck constant (1.05×10^{-27} erg sec)
- L_\odot = Solar Luminosity (4×10^{33} erg sec $^{-1}$)
- M_\odot = Solar mass (2×10^{33} g)
- H_0 : Hubble constant, $H_0 = 100 h$ kms $^{-1}$ Mpc $^{-1}$, $h \sim 0.7$
- 1 Jy = 1 Jansky = 10^{-23} erg cm $^{-2}$ sec $^{-1}$ Hz $^{-1}$,
- $1 \text{ keV} = 1.16 \times 10^7$ K,

Some Useful Distance Scale

- Earth-Moon: 384,400 km = ~ 1.3 light-second
- 1 Astronomical Unit (AU) [Earth-Sun] = $1.496 \cdot 10^{13}$ cm
= 499 light-second.
- 1 parsec (pc) = $3.086 \cdot 10^{18}$ cm = $206'265$ AU =
3,261566 light-year.
- 1 Megaparsec (Mpc) = $3.086 \cdot 10^{24}$ cm.
- Size of a galaxy cluster 1-5 Mpc.
- Size of a Super-cluster 10-30 Mpc.
- Size of Voids ≈ 50 Mpc.
- Width of Filaments ≈ 5 Mpc.
- Baryonic Acoustic scale: 150 Mpc
- Homogeneity scale >100 Mpc
- Radius of the Visible Universe ~ 14 Gpc

Some Useful Mass Scale

- M_\odot = Solar mass (2×10^{33} g)
- Mass of Galaxies $10^6 - 10^{13} M_\odot$.
- Mass of Cluster of Galaxies $\approx 10^{14} - 10^{15} M_\odot$.
- Density of the Universe: $\rho_0 = 1.9 \times 10^{-29} \Omega h^2 \text{ g cm}^{-3}$
 $= 1.05 \times 10^4 \Omega h^2 \text{ eV cm}^{-3}$.
- Mass of the visible Universe: $1.46 \times 10^{56} \text{ g} = 7.3 \times 10^{22} M_\odot$

Wavelength Domain

- Radio: $\lambda = 30\text{cm}-10\text{m}$,
- Microwave: $\lambda = 1\text{ mm} - 30\text{ cm}$,
- Sub-millimeter : $\lambda = 0.2\text{ mm} - 1\text{ mm}$,
- Infrared : $\lambda = 0.8\text{ }\mu\text{m} - 0.1\text{ mm}$
- Visible : $\lambda = 0.3-0.8\text{ }\mu\text{m}$
- UV: $\lambda = 0.01-0.3\text{ }\mu\text{m}$ ($100\text{ \AA} - 3000\text{ \AA}$)
- X-rays: $\lambda = 0.01\text{ \AA} - 100\text{ \AA}$
- γ -rays : $\lambda = 0.0001\text{ \AA} - 0.01\text{ \AA}$

