Astrophysics V   Observational Cosmology

Sheet 11: Assignments

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Exercise 1 : The BAO scale

The sound waves generated inside the baryon-photon plasma in the early Universe, left an imprint in the distribution of matter and radiation with a characteristic size called the sound horizon at radius r_s. This typical length can be computed as the comoving distance traveled by the acoustic waves from the Big Bang at $t = 0$ until the time of decoupling t_{dec}. The comoving integral for the sound horizon r_s at the time of decoupling, which depends only on the speed of sound $v_s(z)$ inside the baryon-photon plasma and the Hubble parameter $H(z)$, reads as :

$$r_s = \int_{z_{dec}}^{\infty} \frac{v_s(z)}{H(z)} dz \tag{1}$$

where the redshift of baryon decoupling $z_{dec} = 1060$.

Using the definition of the speed of sound with respect to the pressure p and density ρ of the medium : $v_s^2 = dp/d\rho$, find that its expression inside the baryon-photon plasma is given by :

$$v_s(z) = \frac{c}{\sqrt{3\left(1 + \frac{3\Omega_b}{4\Omega_\gamma(1+z)}\right)}} \tag{2}$$

The resolution of the comoving integral (1) gives a value of $r_s \simeq 100$ Mpc/h. This is why we refer to BAO as a "standard ruler", because it provides a length of known size as a function of cosmic time.

Exercise 2 : CMB radiation density

The Stefan–Boltzmann law implies that the energy density of photons radiated by a black body of temperature T is :

$$U = \frac{4}{c} \cdot \sigma T^4 \tag{3}$$

This is often written as $\rho_r c^2 = U = aT^4$, where $a \approx 7.56 \cdot 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$.

a) Calculate the CMB energy density of today.

b) Assuming $\rho_{\mathrm{crit},0} = \frac{3H_0^2}{8\pi G} \approx 1.88 \cdot 10^{-26}\ h^2$ kg m^{-3} at t_0, calculate $\Omega_{\mathrm{CMB},0} = \rho_{\mathrm{CMB},0}/\rho_{\mathrm{crit},0}$, the present day fractional contribution of the CMB radiation energy density to the total density. $h = 100 \times H_0$, you can leave it as a unit.

c) Give a rough estimate of the present day mean number density of CMB photons. The total energy density $U_\nu = n \cdot h\nu$ where n is the number density of photons. The typical energy of a photon in a black-body distribution of temperature T is roughly $3kT$, where $k = 1.38 \cdot 10^{-23}\ J \cdot K^{-1}$.

d) Calculate the present day baryon number density, provided that $\Omega_{b,0} = \rho_{\mathrm{b},0}/\rho_{\mathrm{crit},0} \approx 0.022h^{-2}$ and that $m_{\mathrm{proton}} \approx 1.7 \cdot 10^{-27}$ kg. Are baryons in the whole universe as compact as on the earth ?

Exercise 3 : Redshift of matter-radiation equality

Taking into account neutrinos, the present-day total radiation energy density is $\Omega_{\mathrm{r},0} \approx 4.2\,h^{-2} \cdot 10^{-5}$, where the energy-density of radiation is defined by the usual expression :

$$\Omega_r = \frac{8\pi G\rho_r}{3H^2}. \tag{4}$$

What is the redshift of matter-radiation equality (when they have equal energy densities) ? How does this compare to the approximate redshift of recombination for the CMB ? Recall that the CMB temperature today is $T = 2.7K$ and that recombination occurred at $\sim 4000K$.

Exercise 4 : Inflation

Explain the flatness and horizon problems and why inflation solves these. Explain how inflation can explain structure formation in the Universe.