Astrophysics V   Observational Cosmology

Sheet 6: Assignments

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Exercise 1 : Cosmological constant vs. dark energy

What is the difference between a cosmological constant and dark energy ?

Exercise 2 : Critical Density

a) Express the critical density as a function of $H(z)$.

b) Given $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, what is the value of today's critical density in kg/m^3 ?

c) in hydrogen atoms per m^3 ?

d) in $10^{11} \times \mathrm{M_\odot\,Mpc^{-3}}$?

Exercise 3 : Flatness problem

a) Write the first Friedmann equation,

$$H^2(z) = (\dot{a}/a)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \tag{1}$$

using the reduced densities Ω_r, Ω_m, Ω_k, and Ω_Λ.

b) Express $(H/H_0)^2$ as a function of the reduced densities at t_0 and the scale factor $a(t)$.

c) Using the first Friedmann equation, express how a and H depend on time t for a radiation-dominated Universe ; and for a matter-dominated Universe.

d) Express Ω_k as a function of a and H, how does it evolve with time for a radiation-dominated Universe ? and for a matter-dominated Universe ? The current observations securely constrain $|\Omega_k| < 0.1$. Assume we have been in a matter dominated universe since the early universe, what's the Ω_k at time $t \sim 1$s (the order of magnitude is enough) ? Is it normal ?

Exercise 4 : Redshift of matter-radiation equality

Taking into account neutrinos, the present-day total radiation energy density is $\Omega_{\mathrm{r},0} \approx 4.2\,h^{-2} \cdot 10^{-5}$, where the energy-density of radiation is defined by the usual expression :

$$\Omega_r = \frac{8\pi G\rho_r}{3H^2}. \tag{2}$$

From this result, show that the redshift of matter-radiation equality z_{eq}, when the energy densities of matter and radiation were comparable, is given by

$$1 + z_{\mathrm{eq}} \approx 23800\,\Omega_{\mathrm{m},0} h^2.$$

(which corresponds to ∼50,000 yr)