Astrophysics V Observational Cosmology

Sheet 2: Assignments

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Exercise 1 : Colours

The magnitude m of an object with flux F is defined by : $m = -2.5 \times \log_{10}(F) +$ constant. When considering two filters centered at wavelengths $\lambda_1 < \lambda_2$, we say that an object O is redder than an object O' if $F_2/F_1 > F_2'/F_1'$.

Which is redder : $m_1 - m_2 = 0$ or $m_1 - m_2 = 1$?

Exercise 2 : Surface of the sky

In astronomy/cosmology, we measure angles on the sky. The use unit is : 1 deg = 60 arcmin = 3600 arcsec. What is the definition of a radian (angle) ? How many radians is one degree ?

What is the definition of a steradian (solid angle) ? How many steradians in one square degree ? (just square the degree-radian relation).

How many square degrees are there in the full sky ?

For the sake of comparision :

— The Orion constellation (currently visible in the evening sky) covers approximatively $35°$ by $15°$, its area is 594 deg^2 (the constellation boundaries are not rectangles in R.A. and Dec...)

— The apparent diameter of the Moon (and the Sun) is of about 30 arcminutes, corresponding to a surface of 0.25 deg^2.

— The field of view of the Wide Field Camera 3 (installed in 2009) aboard the Hubble Space Telescope covers 2.7 by 2.7 arcminutes, that is $2 \cdot 10^{-3}$ deg^2.

Exercise 3 : Pencil-beam vs. wide-field

Assume that the light of the object is measured in an aperture of n_{pix} pixels, and that the sky has a constant value with Poissonian noise.

a) demonstrate that, when the main source of noise is the sky background, the signal-to-noise ratio S/N scales as $\sqrt{t}$, the square root of the observation time.

b) Infer that the flux F of sources that are still just above a choosen S/N is proportional to $1/\sqrt{t}$.

For many observations, including HST imaging, the depth of an image is proportional to the square root of the time spent integrating. Suppose that the galaxy source counts are Euclidean, i.e. $N(F > F_{\text{thresh.}}) \propto F_{\text{thresh.}}^{-3/2}$. [Note this comes from the fact that in a Euclidean universe, the number of galaxies out to distance R would be $N \propto R^3$. The flux F goes like $F \propto R^{-2}$, so that $N \propto F^{-3/2}$].

a) Would you detect more galaxies in a pencil-beam survey or a wide-field survey, for a fixed amout of observing time ?

b) What source count slope would tip the balance in the opposite direction ?

Exercise 4 : The Tully-Fisher relation

It is known that spiral galaxy rotation curves flatten, i.e. that the circular velocity tends to a limit value V_{max} at large radii. This provides a constraint on the mass within a given radius R. The absolute magnitude $\mathcal{M}$ is defined as the observed magnitude the object would have when it is placed at a distance of 10 pc : $\mathcal{M} = -2.5 \times log_{10}(L/(4\pi[10pc]^2))$. The luminosity $L = 4\pi d^2 F$, where d is the distance of the galaxy, F is the flux from the galaxy. Ideally, the apparent magnitude is $m = -2.5 log_{10} F$. With assuming that all spiral galaxies have the same mass-to-luminosity ratio $M/L = \Upsilon$ and the same surface brightness $\mu = L/(\pi R^2)$,

a) Express the relation between the galaxy mass M and the circular velocity V_{max}.

b) Demonstrate the Tully-Fisher relation which expresses the absolute magnitude as function of the circular velocity as follows :

$$\mathcal{M} = -10 \log_{10} V_{max} + \text{cte}$$

This relation is in good agreement with observations. Indeed, we observe for Sa galaxies : $\mathcal{M}_B = -9.95 \log_{10} V_{max} + 3.15$, with V_{max} in km.s^{-1}. The NGC 2639 galaxy is a Sa galaxy : its maximal circular velocity is 324 km.s^{-1} and its apparent magnitude is $B = 11.5$ mag.

a) What is the distance to that galaxy ? and what is its luminosity L_B (expressed in $L_{B,\odot}$), given that $\mathcal{M}_{B,\odot} = 5.47$?

b) Express G in km^2 kpc $M_\odot^{-1}$ s^{-2} ($G = 6.67 \cdot 10^{-11}$ m^3 kg^{-1} s^{-2}, 1pc=3.26 light-year, $M_\odot = 2 \cdot 10^{30}$ kg).

c) What is the mass enclosed in a $R = 30$ kpc radius ?

d) Then deduce the M/L_B ratio.