Astrophysics V Observational Cosmology

Sheet 13: Assignments

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Exercise 1 : Friedmann Equations

a) From the Friedmann Equations

$$\begin{cases} \ddot{R}(t) & = -\frac{4\pi \mathrm{G}}{3}\left(\rho + \frac{3p}{\mathrm{c}^2}\right) R(t) + \frac{1}{3}\Lambda\, \mathrm{c}^2\, R(t) \\ \dot{R}^2(t) + k\,\mathrm{c}^2 & = \frac{8\pi \mathrm{G}}{3}\rho\, R^2(t) + \frac{1}{3}\Lambda\, \mathrm{c}^2\, R^2(t) \end{cases} \tag{1}$$

find the following expression for the density ρ and the pressure p :

$$\begin{cases} \frac{8\pi \mathrm{G}}{\mathrm{c}^2}\, p = -\frac{k\mathrm{c}^2}{R^2} - \frac{\dot{R}^2}{R^2} - 2\frac{\ddot{R}}{R} + \Lambda\, \mathrm{c}^2 \\ \frac{8\pi \mathrm{G}}{3}\, \rho = \frac{k\mathrm{c}^2}{R^2} + \frac{\dot{R}^2}{R^2} - \frac{1}{3}\Lambda\, \mathrm{c}^2 \end{cases} \tag{2}$$

b) Using the set of Equations 2, show that the density can be related to the pressure by :

$$\frac{d}{dt}\left(\rho c^2 R^3\right) = -p\frac{dR^3}{dt} \tag{3}$$

c) From Equation 3 show that for matter and radiation respectively, the density evolves as $\rho_M(t) \propto R(t)^{-3}$ and $\rho_{\mathrm{rad}}(t) \propto R(t)^{-4}$.

d) From **c)** recover the thermal law for radiation, i.e. that

$$R \cdot T = \text{constant} \tag{4}$$