Astrophysics V   Observational Cosmology

Sheet 8: Assignments

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Exercise 1 : Newtonian lensing

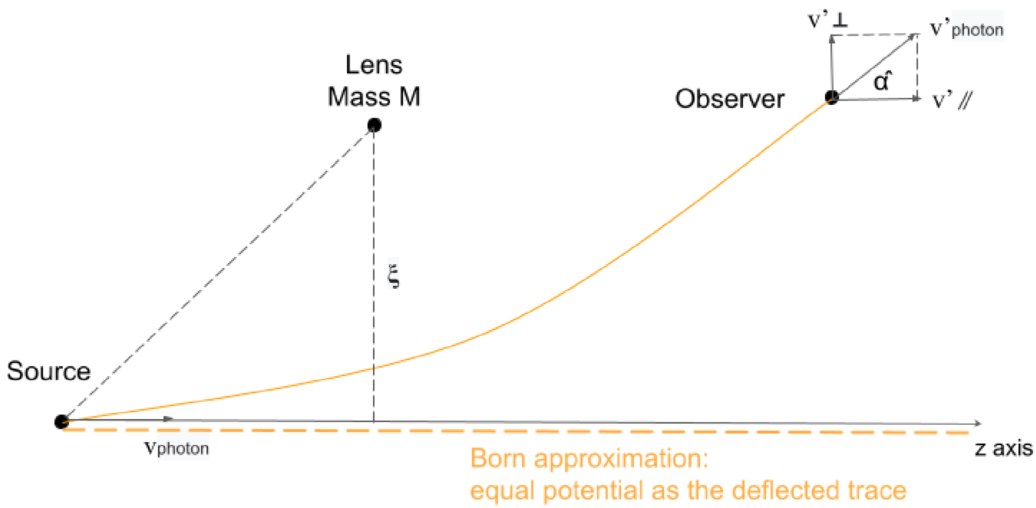


FIGURE 1 – An exagerated geometry of a gravitational lensing system.

a) For a gravitational lens system as depicted by figure 1, and assuming Newtonian gravity, what is the deflection angle $\hat{\alpha}$ for a point mass M and impact parameter ξ ?

Hints : Assume that the deflection angle is small and use the Born approximation (the gravitational potential along the true deflected path can be approximated as the potential along the undeflected path) to express the acceleration that contribute to the velocity change. For integral assistant, you can use `https://www.integral-calculator.com/` and the integral in this exercise is definite from -∞ to +∞.

b) Compare your answer to the result from General Relativity :

$$\hat{\alpha} = \frac{4GM}{c^2\,\xi} \qquad (1)$$

Exercise 2 : The gravitational lens

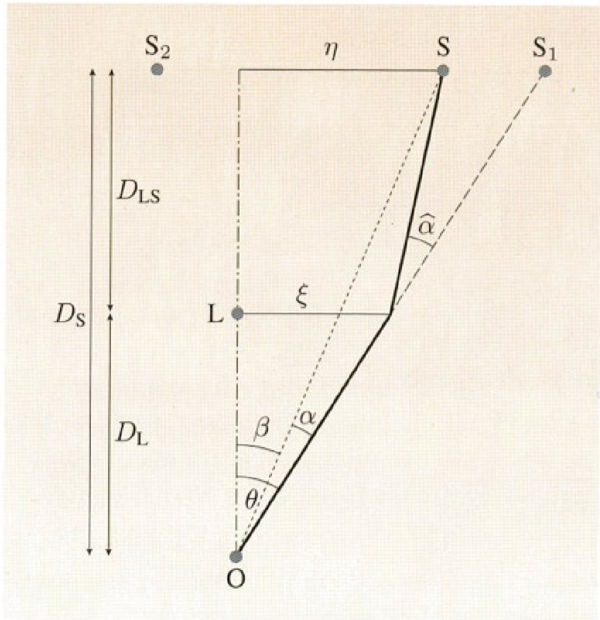


Figure 7.7 The geometry of gravitational lensing of a source S by a lens L, with the angles that are discussed in the text labelled. The apparent positions of the source seen from the Earth are S_1 and S_2. (Light rays for S_2 are not shown, for clarity.) The impact parameter is given the symbol ξ. D_{LS} is the distance to the source as seen from the lens. Note that the distances D_{L}, D_{S} and D_{LS} are all *angular diameter distances*, so $D_{\mathrm{S}} \neq D_{\mathrm{LS}} + D_{\mathrm{L}}$.

FIGURE 2 – From Serjeant, p220. Geometry of a gravitational lensing system.

a) Figure 2 is taken from Serjeant's textbook (fig 7.7 p. 220). We assume θ, β, α, $\hat{\alpha} \ll 1$. Using this figure, show that :

$$\beta = \theta - \frac{D_{\mathrm{LS}}}{D_{\mathrm{S}}}\hat{\alpha} \tag{2}$$

where $\hat{\alpha}$ is the deflection angle. Remember that the distances indicated in this figure are angular diameter distances, by construction.

b) The deflection angle of a light ray falling on a point mass M with an impact parameter ξ is :

$$\hat{\alpha} = \frac{4GM}{\mathrm{c}^2\,\xi} \tag{3}$$

Let's assume that our lens is indeed a point mass, or at least that it is much smaller than its Einstein radius θ_{E}, where

$$\theta_{\mathrm{E}} = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}}{D_S D_L}}. \tag{4}$$

For a given source position β, show that the images will be located at :

$$\theta_{\pm} = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_{\mathrm{E}}^2}\right) \quad \text{where} \quad \theta_{\mathrm{E}} = \sqrt{\frac{D_{\mathrm{LS}}}{D_{\mathrm{L}}D_{\mathrm{S}}}\frac{4GM}{\mathrm{c}^2}} \tag{5}$$

One of the two solutions of $\theta_{\pm}$ is always negative. What does this mean ? Does this negative solution θ_{-} exist even if β is much larger than θ_{E} ? What is the "problematic" assumption ?

c) What is the separation between the two images ? How can we approximate this separation if β is significantly smaller than θ_{E} (i.e., if the source is relatively well aligned with the line of sight to the lens) ?

d) Consider the gravitationlly lensed quasar QO957+561 (see Figure 7.29 of Serjeant if interested). The two images are located at $\theta_+ = 5.35$" and $\theta_- = -0.80$". The redshift of the quasar and the lens are $z_{\mathrm{S}} = 1.41$ and $z_{\mathrm{L}} = 0.36$. If $\Omega_{\mathrm{m}} = 0.3$, $\Omega_\Lambda = 0.7$, $\mathrm{H}_0 = 72$ km s^{-1} Mpc^{-1} these redshifts translate into angular diameter distances of $D_{\mathrm{S}} = 1693$ Mpc, $D_{\mathrm{L}} = 1011$ Mpc, and $D_{\mathrm{LS}} = 1123$ Mpc. Estimate the mass M of the lens. Be careful about the unit of angular quantities.

Exercise 3 : Gravitational lenses - MACHOs

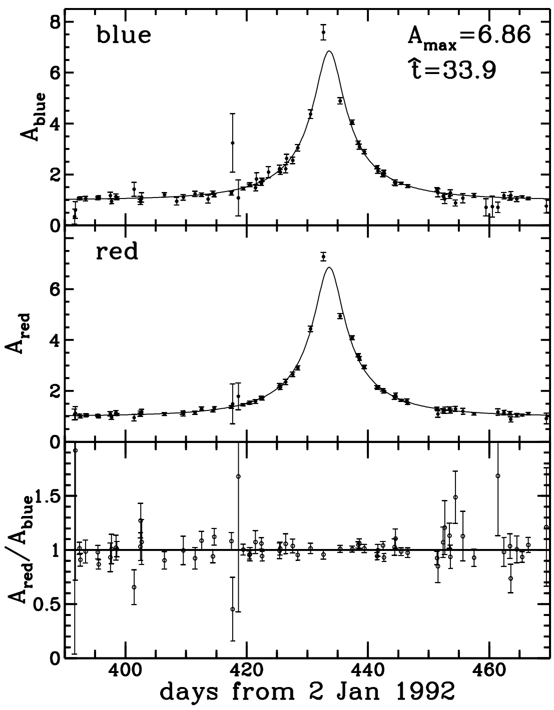
One hypothesis for the contents of our Galaxy suggests that a large fraction of mass could be in the form of MACHOs (massive compact halo objects). These objects could be, for instance, black holes or very faint dwarf stars.

Detection of these objects can be done using the gravitational lens effect. If a MACHO passes between the Earth and a distant star, the light of this star would be focused (the MACHO acting as a gravitational lens) and would lead to an increase in its apparent luminosity. In 1993, astronomers observed a temporary magnification of one star of the Large Magellanic Cloud (LMC), cf. Figure below.

Assume that the MACHO has a mass of 10 Jupiter masses, i.e. $1.899 \cdot 10^{28}$ kg, that it is halfway between the Earth and the LMC, and that it is moving perpendicular to the line of sight. How long would it take the MACHO to move an angle of $2\theta_E$ in front of the lensed star of the LMC ? The distance to the LMC is 50 kpc and the orbital speed of the MACHO is assumed to be 220 km s^{-1}. Is this duration compatible with observations ?

Exercise 4 : Magnification of a galaxy luminosity function (Serjeant Ex. 7.11)

Suppose that at a particular redshift the background galaxies have a luminosity function $\mathrm{d}\Phi/\mathrm{d}L \propto L^{-\alpha}$. For which values of α would lens magnification increase the number of detections among these background galaxies ? Don't forget that increasing the angular size of a distant region also decreases the comoving volume that's sampled, for a fixed observed area on the sky. Also assume $\alpha > 1$ due to finite total galaxies.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red
A_red
A_red/A_blue
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420
440
460
days from 2 Jan 1992