Problem set 9

Problem 1

We consider a dark matter detector that measures the recoil energy of a nucleus A of mass M_A, initially at rest, after an elastic collision with a dark matter particle χ, of mass M_χ. The detector has a detection threshold corresponding to a minimal kinetic energy of the nucleus A after the collision.

We make the following assumptions:

- the elastic collisions are central (head-on collision), such that the initial and final momenta are all parallel;
- the typical velocity of the χ particles in our region of the galaxy is approximately similar to the velocity of the sun in the galaxy ($\approx 240\,\mathrm{km/s}$);
- the mass per unit volume, $\rho = M_\chi \cdot n$, of the χ particles is a constant, where n is the number of particles per unit of volume.

(a) What is the expression for the initial momentum of the dark matter particle? (hint: $\beta = \frac{v}{c} = \frac{p}{E}$ and $\gamma = (1-\beta^2)^{-1/2} = \frac{E}{M}$). Show that in the limit $\beta \ll 1$ we have $p_\chi \simeq \beta M_\chi$. Justify the approximation $\beta \ll 1$.

(b) Using energy and momentum conservation, show that the momentum p'_A of the nucleus A after the collision is

$$p'_A \approx 2\beta\gamma \frac{M_A M_\chi}{M_A + M_\chi},$$

where we used the fact that $p_i \ll M_i$, $i = \chi, A$. Infer from this result that the probability for observing a collision decreases for small masses M_χ, and consequently that the sensitivity of the detector is low for $M_\chi \ll M_A$.

(c) Find an expression for the number of collisions $N_{\text{collisions}}$ observed during a time Δt, as a function of the cross section and of the density of dark matter particles. Show that for a constant mass density of dark matter, the number of collisions decreases when M_χ increases.

Problem 2

The superCDMS detector observes 11 candidate collisions for dark matter particles χ, while a background of 6 ± 1 events is expected. The exposure time T is 577 kg-days for germanium detectors with molar mass $72.64\,\mathrm{g/mol}$. The mean density of dark matter in the region of the solar system is measured to be $\rho = (0.39 \pm 0.03)\,[\mathrm{GeV/c^2}]/\mathrm{cm}^3$, and the velocity of the solar system in the galaxy is $240\,\mathrm{km/s}$.

Determine the cross section of the dark matter particle χ in Germanium, σ_χ, for $M_\chi = 10\,\mathrm{GeV/c^2}$.