Problem set 4

Problem 1

In the two-neutrino system (ν_e and ν_μ), show that the oscillation probability $\nu_e \to \nu_\mu$ is:

$$\begin{aligned} P_{\text{osc}} = |\langle \nu_\mu | \nu(t) \rangle|^2 &= 4 \sin^2\theta \cos^2\theta \sin^2 \frac{(E_1 - E_2)t}{2} \\ &= \sin^2 2\theta \sin^2 \frac{\Delta m^2 \, t}{4E}. \end{aligned}$$

Problem 2

The mean energy loss of a muon in a medium can be expressed as:

$$\left\langle \frac{dE}{dX} \right\rangle = -(a + bE).$$

The parameter a takes into account the energy loss due to ionization and excitation effects, while parameter b describes Bremsstrahlung, e^+e^- pair production and photo-nuclear reactions. We assume a and b to be constant.
Derive the expression for the mean free path of a muon of energy E_μ in rock. We consider that the particle is stopped for an energy below a certain threshold E_{min}.
For the computation, use: $a \equiv 2\text{MeV}\,\text{cm}^2/\text{g}$, $b \equiv 4 \times 10^{-6}\text{cm}^2/\text{g}$ (for rock as medium), $E_{\text{min}} \equiv 1$ GeV, $E_\mu = 100$ TeV, and $\rho_{\text{rock}} = 3000\,\text{kg/m}^3$.