Problem set 2: Solutions

Problem 1

The NEMO-3 experiment observed 15 *candidates for neutrino-less double* β *decays, while* 18.0 ± 0.6 *background events are expected. Knowing that the experimental efficiency is* 4.7%*, and that the exposure is* 34.3 *kg·yr, compute the upper limit at 90% C.L. (*$\approx 1.3\sigma$*) for the half-life time of the* ^{100}Mo *(atomic mass* = 95.94 *g/mol)* $\beta\beta - 0\nu$ *decay. Compare the result with the published result :* $\tau_{1/2} > 1.1 \times 10^{24}$yr *at 90% C.L.*

Note: we can simplify the computation considering that the uncertainty for the Poisson distribution ($\sigma(N) = \sqrt{N}$*) is gaussian* $\Rightarrow 1.3\sigma = 1.3\sqrt{N}$.

Solution:

The half-life time $\tau_{1/2}$ is given by:

$$\tau_{1/2} = \ln 2 \frac{\varepsilon \cdot N_{\text{nucl}} \cdot \Delta t}{N_{\beta\beta}}$$

We have the following values for the parameters in the formula:

- $\varepsilon = 0.047$
- $N_{\text{nucl}}(^{100}\text{Mo}) = N_{Av} M/u$, with $u = 0.096\,\text{kg/mol}$. Therefore:

$$N_{\text{nucl}} \cdot \Delta t = N_{Av} \frac{34.3\,\text{kg} \cdot \text{yr}}{0.096\,\text{kg/mol}} = 215 \times 10^{24}\,\text{yr}$$

- $N_{\beta\beta} = N_{\text{obs}} - N_{\text{bkg}} = (15 - 18.0) \pm \sqrt{15 + 18.0 + 0.6^2} = -3.0 \pm 5.8$, where we have taken into account the Poissonian fluctuations of the measurement and the estimate for the background. A variation of 1.3σ gives $N_{\beta\beta} < 4.5(= -3.0 + 1.3 \times 5.8)$

Finally we obtain:

$$\tau_{1/2} > \ln 2\, \frac{0.047 \cdot 215 \times 10^{24}}{4.5} = 1.6 \times 10^{24}\,\text{yr at 90\% C.L.}$$

Problem 2

(a) *Determine the eigenvalues of the mass matrix of the see-saw Lagrangian, without neglecting the value of* m_L.

$$-\frac{1}{2}\begin{pmatrix}\bar{\nu}_L^c & \bar{\nu}_R\end{pmatrix}\begin{pmatrix}m_L & m_D \\ m_D & m_R\end{pmatrix}\begin{pmatrix}\nu_L \\ \nu_R^c\end{pmatrix}$$

(b) *Why can one neglect* m_L *in the mass hierarchy?*

Solution:

(a) The eigenvalues for the matrix are:

$$\lambda_\pm = \frac{1}{2}\left[m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right]$$

(b) We have the following hierarchy: $m_L \ll m_D \ll m_R$. The value of the eigenvalues $\lambda_\pm$ in the limit $m_L \to 0$ gives

$$\lambda_\pm \approx \frac{1}{2}\left[m_R \pm \sqrt{m_R^2 + 4m_D^2}\right],$$

which is the same result as obtained from considering $m_L = 0$ in the mass matrix. Therefore one can neglect m_L. However, m_D can not be neglected, although it is much smaller than m_R.

Problem 3

Considering the flavor basis for the neutral mesons ($|P^0\rangle$, $|\bar{P}^0\rangle$)*, the mass matrix can be written as:*

$$\begin{pmatrix} m & \Delta m \\ \Delta m & m \end{pmatrix}$$

(a) *Calculate the mass eigenvalues* M_H *and* M_L *and the corresponding eigenstates* $|P_H\rangle$ *("heavy") and* $|\bar{P}_L\rangle$ *("light").*

(b) *Discuss how this property of the neutral mesons can be detected by experiments ("mixing"). Consider for instance the creation of a neutral meson in a strong interaction, followed by its desintegration via the weak interaction. Draw the corresponding Feynman diagrams.*

Solution:

(a) The mass eigenstates are a superposition of the two flavor states (interaction eigenstates):

$$\lambda_\pm \equiv m_\pm = m \pm \Delta m$$

$$|P_H\rangle = \frac{1}{\sqrt{2}}\left(|P^0\rangle + |\bar{P}^0\rangle\right)$$

$$|P_L\rangle = \frac{1}{\sqrt{2}}\left(\left|P^0\right\rangle - \left|\bar{P}^0\right\rangle\right)$$

As a consequence, while a pure flavor state is produced in strong interactions, mixing with the opposite flavor state is possible during the space-time evolution of the mass eigenstate. In the situation where $\Delta m = 0$, the mass eigenstates become $\left|P^0\right\rangle$ and $\left|\bar{P}^0\right\rangle$ and no mixing can be present.

(b) A possible example is the creation of the pure neutral kaon K^0, in the strong desintegration of the K^{*+} ($K^{*+} \to K^0\pi^+$), or of the $\bar{K}^0$ in the desintegration of K^{*-} (see Fig. 1). The evolution of the neutral kaon allows the mixing process (see Fig. 2), and the component K^0 ($\bar{K}^0$) can be detected by its semi-leptonic desintegration $K^0 \to \pi^- e^+ \nu$ ($\bar{K}^0 \to \pi^+ e^- \bar{\nu}$), revealing its flavor at the time of the decay (see Fig. 3).

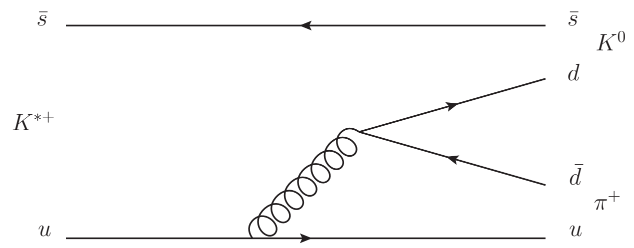

Figure 1: Feynman diagram of the disintegration of a K^{*+} leading to the creation of a neutral kaon.

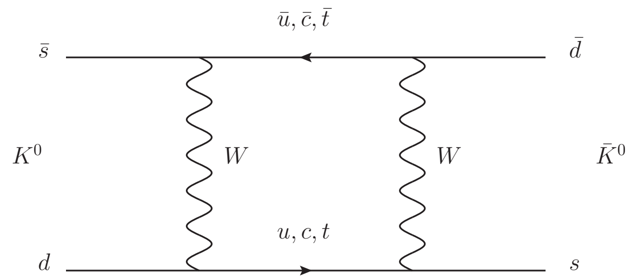

Figure 2: Feynman diagram of the neutral kaon mixing.

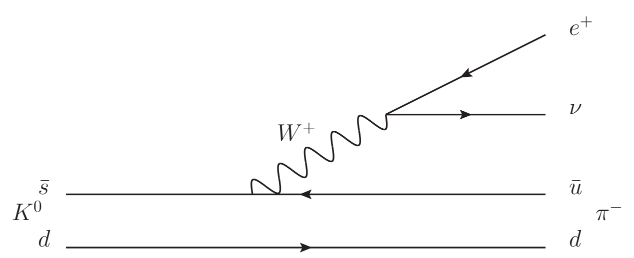

Figure 3: Feynman diagram of the semi-leptonic decay of a neutral kaon.