Problem set 12

Problem 1

Using the definitions of transverse mass m_{T} and of the rapidity y, demonstrate that:

$$p_z = m_{\mathrm{T}} \sinh y$$

$$E = m_{\mathrm{T}} \cosh y$$

Use this result to show that $\tanh y = \frac{p_z}{E}$

Problem 2

The goal of this exercise is to show that the energy density in central collisions (i.e. for which the impact parameter $b = 0$, and therefore all nucleons participate in the collision) of nuclei of atomic mass A is given by:

$$\epsilon \approx \frac{A^{1/3}}{2\pi} \frac{dN}{dy} \frac{m_{\mathrm{T}}}{\tau}\,, \tag{1}$$

where dN/dy is the average number of particles (mostly pions) produced per unit rapidity y and per nucleon-nucleon collision, m_{T} is the transverse mass of these particles, and τ is the proper time of the particles produced in the collision.

To obtain the energy density, proceed as follows:

(a) Under the assumption that the colliding nuclei have a radius of $r \approx 1.4A^{1/3}$, show that the energy density in a cylindrical volume with radius equal to the nucleus radius can be written as $\epsilon \approx \frac{A^{1/3}}{2\pi} \frac{dN}{dy} \frac{dy}{dz} E_\pi$.

(b) Use the definition of the rapidity to find that $z = t \tanh y$, where z and t are the distance measured along the beam axis and the time of flight measured in the laboratory rest frame.

(c) Using $z = t \tanh y$, show that $t = \tau \cosh y$, where the proper time τ (Lorentz invariant) of a particle is given by $\tau^2 = t^2 - z^2$.

(d) Finally, deduce the dz/dy dependence as a function of τ and y, and obtain the expression for the energy density given in the introduction above.