Problem set 11

Problem 1

Show that the latent heat for the transition of a pions gas into a plasma of quarks and gluons is equal to $4B$, where B is the energy density for the "bag" creation in the bag model which describes quark confinement into hadrons.

Problem 2

An ideal gas of fermions (bosons) in thermal equilibrium obeys Fermi-Dirac statistics (Bose-Einstein statistics): for each degree of freedom, the average number of particles that occupy a state of energy E is

$$\frac{1}{\exp\left(\frac{E}{k_B T}\right)+1} \quad \text{for fermions and} \tag{1}$$

$$\frac{1}{\exp\left(\frac{E}{k_B T}\right)-1} \quad \text{for bosons.} \tag{2}$$

Calculate the energy density per degree of freedom for these ideal gases. Note the following identities:

$$\int_0^\infty \frac{x^3}{e^x-1}\mathrm{d}x = \Gamma(4)\mathrm{Li}_4(1) \tag{3}$$

$$\int_0^\infty \frac{x^3}{e^x+1}\mathrm{d}x = -\Gamma(4)\mathrm{Li}_4(-1) \tag{4}$$

where $\Gamma(n) = (N)$ is the Gamma function and $\mathrm{Li}_N(x)$ is the polylogarithm. The function $\mathrm{Li}_N(x)$ is related to the Riemann zeta function, $\zeta(N)$, and to the Dirichlet eta function, $\eta(N) = (1-2^{1-N})\zeta(N)$, as

$$\mathrm{Li}_N(1) = \zeta(N)\,, \tag{5}$$

$$\mathrm{Li}_N(-1) = -\eta(N) = -(1-2^{1-N})\zeta(N)\,. \tag{6}$$

And the Riemann zeta function (with $\Gamma(N) = (N-1)!$):

$$\zeta(N) = \frac{1}{\Gamma(N)}\int_0^\infty \frac{u^{N-1}}{e^u-1}\mathrm{d}u \tag{7}$$

$$\Rightarrow \zeta(4) = \frac{1}{6}\int_0^\infty \frac{u^3}{e^u-1}\mathrm{d}u = \frac{\pi^4}{90}\,. \tag{8}$$