

PART II

Dark Matter

Evidence for dark matter (DM)

Evidence for Dark Matter

A. From astrophysics

1. Galactic rotation curves
2. Clusters

B. From cosmology

3. Abundance of primordial elements
4. SN type Ia luminosity
5. CMB anisotropies

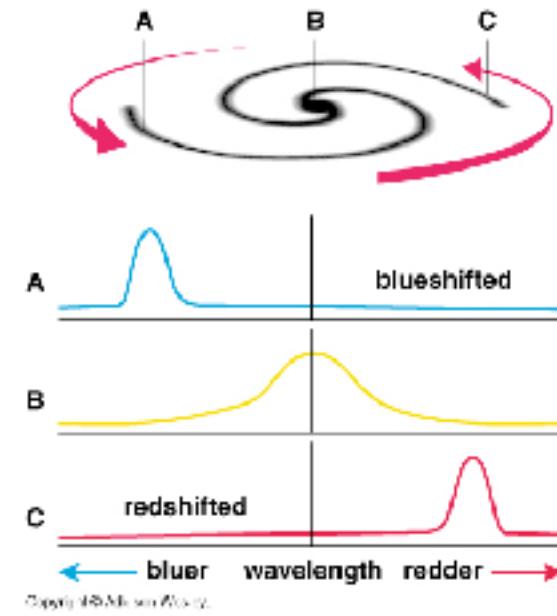
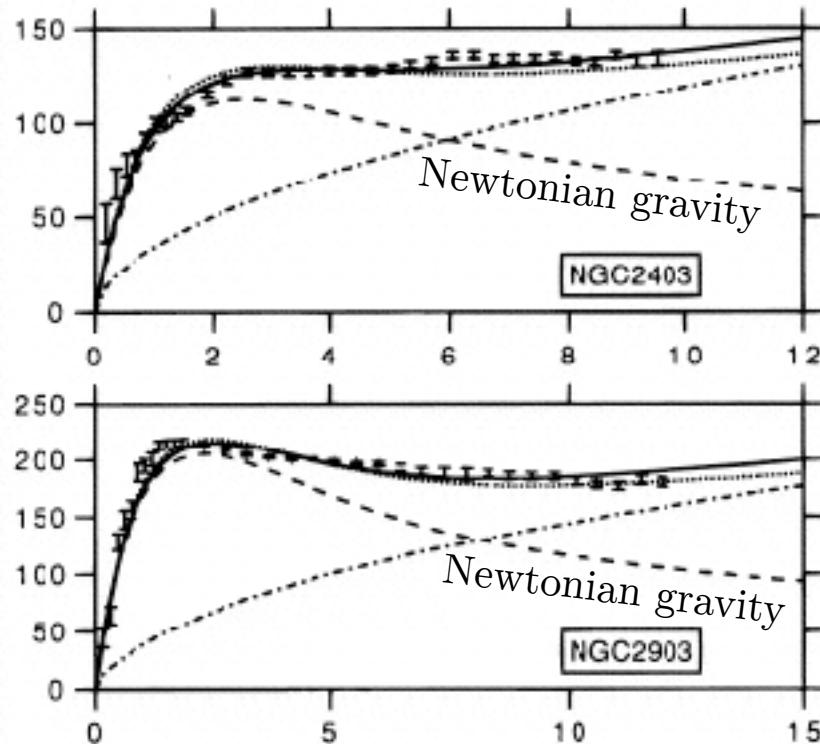
A.1 Galactic rotation curves

$$\frac{v^2}{r} = \frac{GM(r)}{r^2}$$

- If all matter in the bulb, expect

$$v \propto \frac{1}{\sqrt{r}}$$

- But we observe $v(r) = \text{constant}$

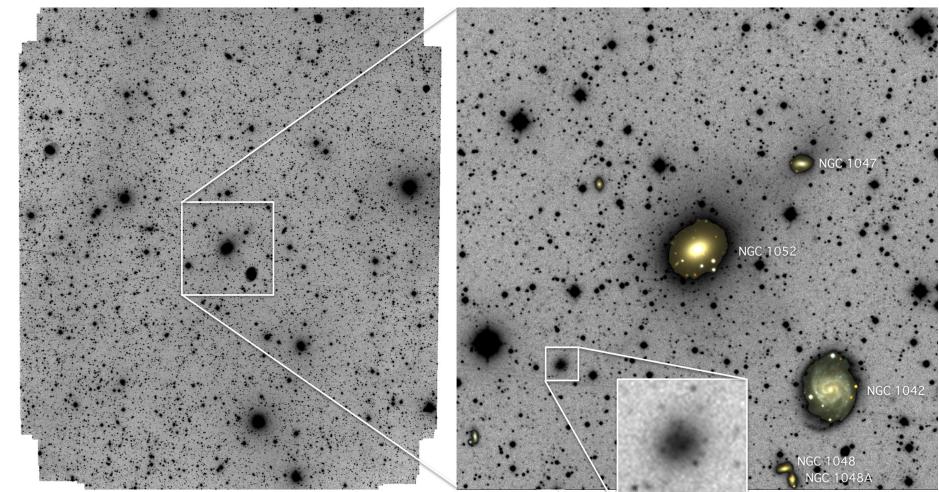


- Possible solutions:
 - modify gravity (not favoured)
 - add halo component of invisible (dark) matter (DM)
 - all our ignorance is transferred to DM!

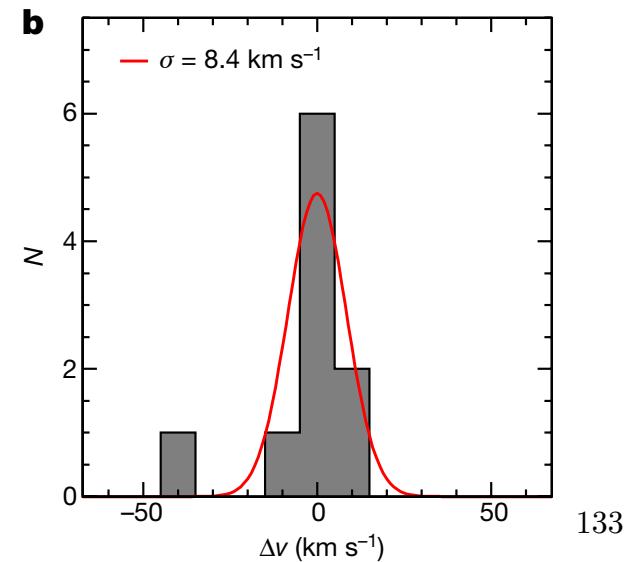
A.1 Observation of a galaxy without DM

- Observation of a galaxy without a Dark Matter component
⇒ inconsistent with “Modified Newtonian Dynamics” (MOND)...
⇒ ...but consistent with the existence of DM !
- NGC1052-DF2 ($M_{\text{star}} \simeq 2 \times 10^8 M_{\odot}$)
 - measure ratio of dynamical mass M_{dyn} over visible mass M_{star}
 - M_{dyn} measured from velocity dispersion σ of 10 globular clusters
 - measure dispersion $\sigma = 8.4 \text{ km s}^{-1}$, and deduce intrinsic dispersion
 $\sigma_{\text{intr}} = (3.2^{+5.5}) \text{ km s}^{-1} < 10.5 \text{ km s}^{-1}$ (@90 % C. L.)
 $\Rightarrow M_{\text{dyn}} < 3.4 \times 10^8 M_{\odot}$
 - with DM, expect $M_{\text{dyn}}/M_{\text{star}} \gg 1$ (≈ 400 !)
 - Result for NGC1052-DF2 : $M_{\text{dyn}}/M_{\text{star}} \leq 2$

⇒ this constitutes evidence for a galaxy without DM!



Extended Data Figure 1 | NGC1052-DF2 in the Dragonfly field. The full Dragonfly field, approximately 11 degree², centred on NGC 1052. The zoom-in shows the immediate surroundings of NGC 1052, with NGC1052-DF2 highlighted in the inset.



A.2 Clusters of galaxies (I)

1. Galactic motion within clusters incompatible with visible matter

⇒ same conclusion as for galaxy rotation curves



2. X-rays from hot gas in clusters

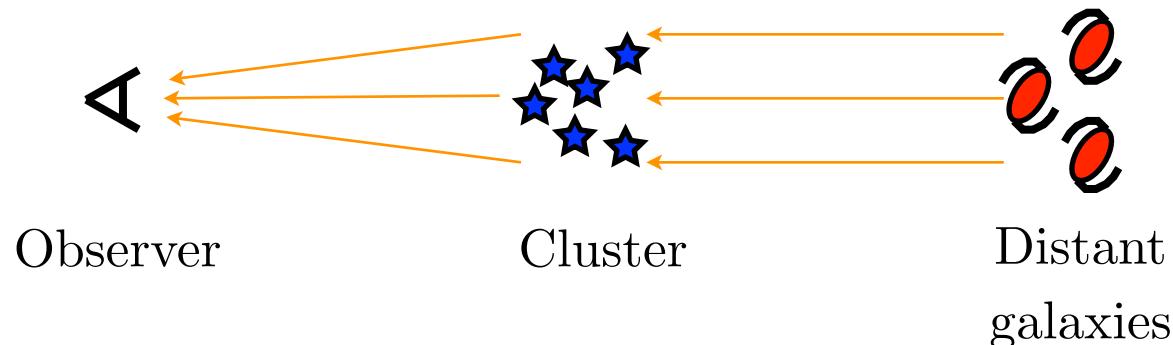
- the temperature T is related to the density ρ
 - measure T and determine the corresponding density ρ_T
 - compare with ρ_{visible}

⇒ Result: $\rho_{\text{visible}} \approx 10\% \rho_T$

Clusters of Galaxies (II)

3. Weak lensing (since 1990's)

- measure the mass distribution from average deformation of distant galaxies through gravitational lensing in closer cluster



- Remark: this is a statistical method
- reach same conclusion that the total mass in the cluster is significantly larger than the visible mass

Bullet Cluster

- Observe clusters that have collided:
 - galaxies in visible spectrum
 - hot gas in X-ray (magenta)
 - mass distribution from weak lensing (blue)

⇒ blue has only interacted gravitationally
⇒ strongest direct evidence for weakly interacting Dark Matter!



Interlude: (short) introduction to cosmology

Cosmology

- The Universe is described by the Robertson-Walker metric

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

- t = cosmological time measured by free-falling observer (comoving)

- k is the curvature:

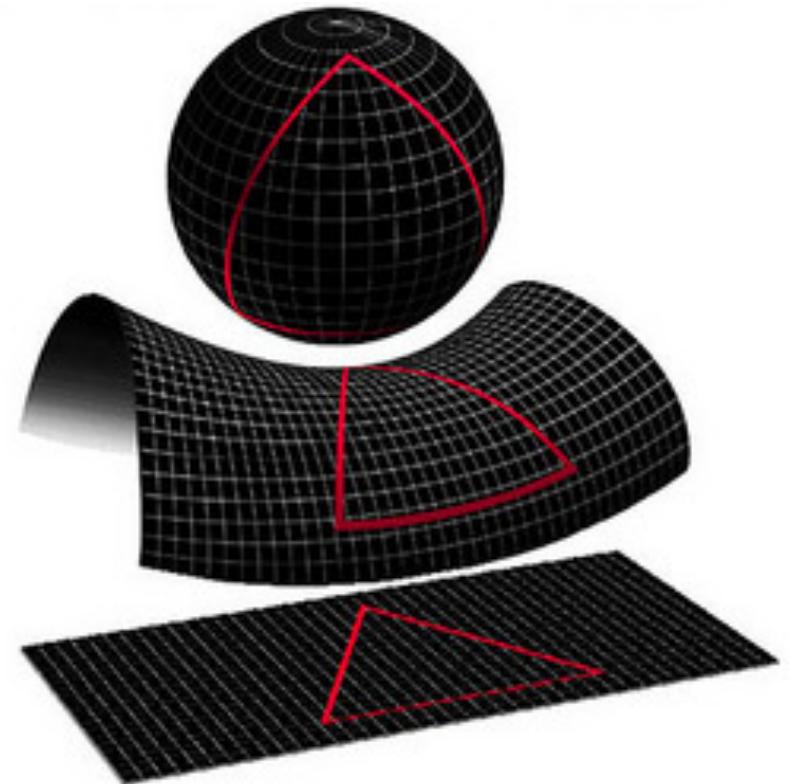
- $k = +1$ \Rightarrow closed Universe

- $k = -1$ \Rightarrow open Universe

- $k = 0$ \Rightarrow flat Universe

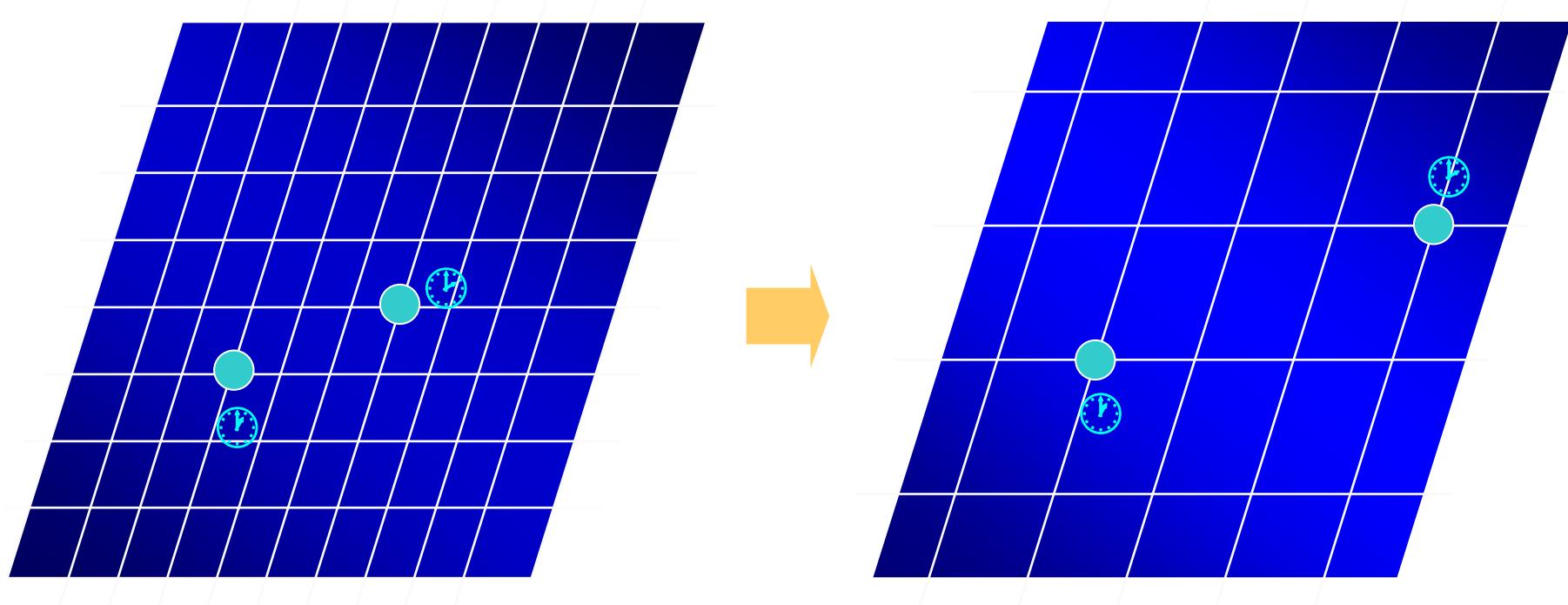
- $a(t)$ drives the expansion

\Rightarrow What is the form of $a(t)$?



Comoving coordinates

- Free-falling objects are said to be *comoving*
 - they measure the cosmological time
 - but their relative distance changes with $a(t)$
 - $a(t)$ depends on the content of the Universe, and is therefore the parameter that allows to identify the various components of the Universe



Cosmology dynamics

- R-W metric + Einstein's equation (relating curvature and energy density ρ) + adiabatic expansion

⇒ Friedmann equation

$$H^2 = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}$$

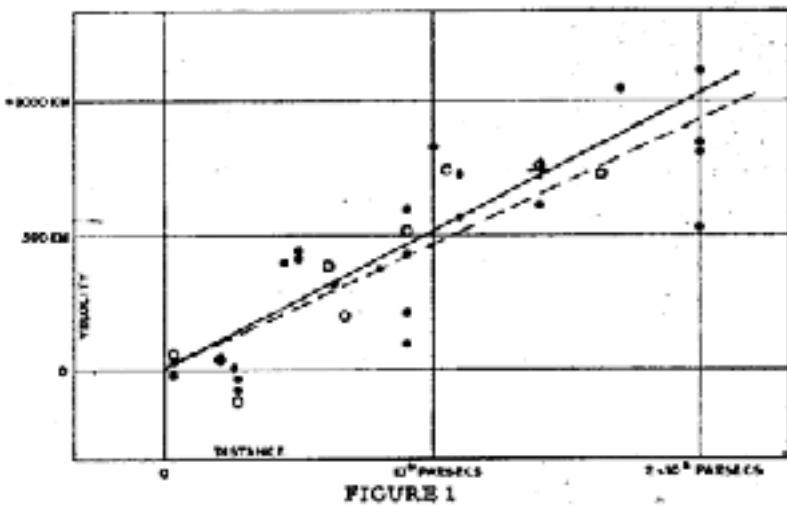
- G : gravitational constant
- H : the Hubble parameter
- ρ : energy (mass) density
- Energy conservation equation,
relating density ρ and pressure p

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

Expansion: Hubble parameter

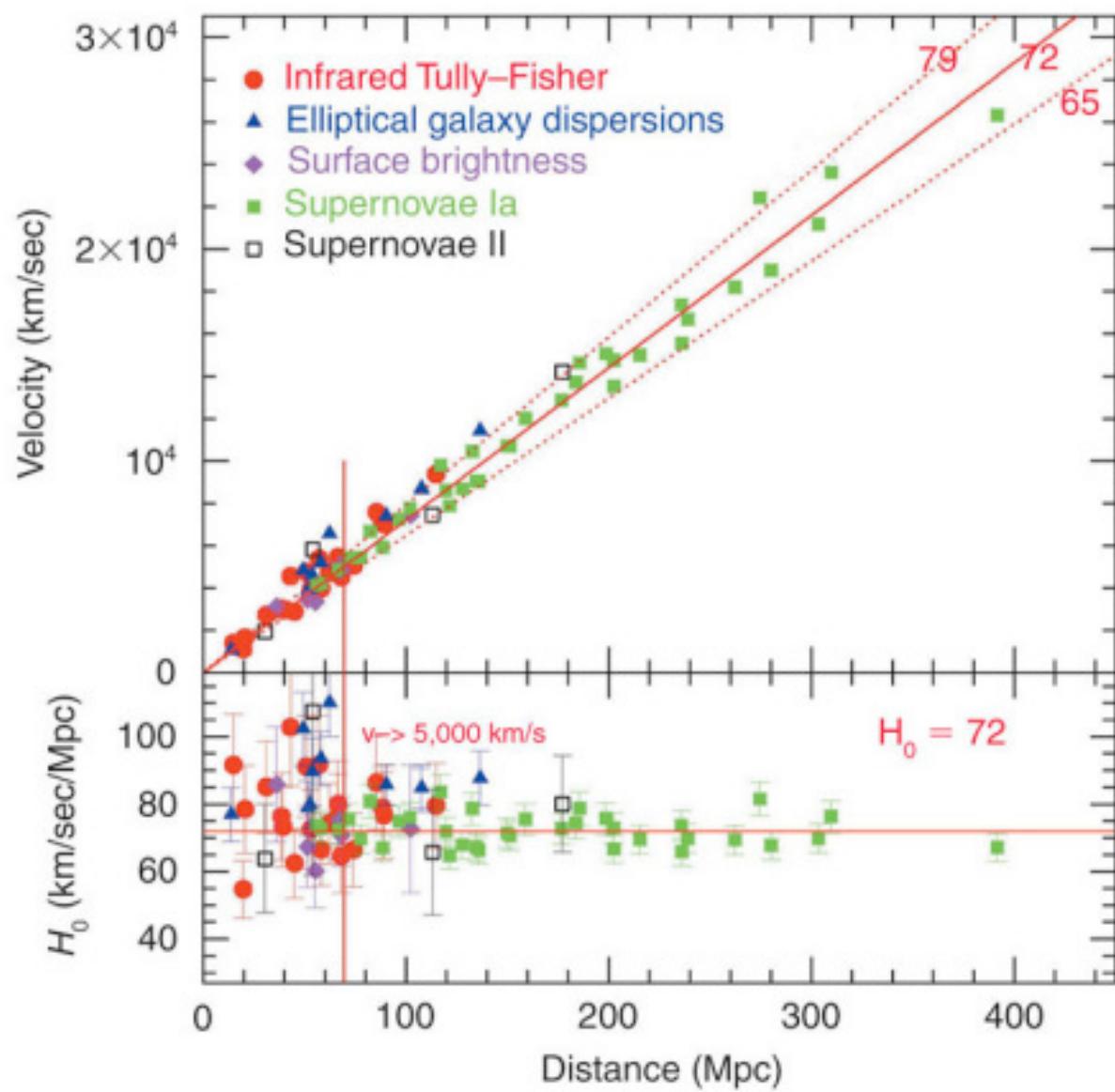
- Current (t_0) expansion rate H_0 $v = \dot{a}(t_0)r = H_0ar = H_0D$
- $H_0 = 100 \times h$ [km/s/Mpc]
with $h = 0.674 \pm 0.005$

Original Hubble results, up to 2Mpc



- Define redshift z :

$$z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{emitted}}} - 1$$



Critical density

- The curvature of the Universe could be closed, flat, or open
- The present (t_0) critical density for having a flat Universe ($k = 0$):

$$\rho_c^0 = \frac{3H_0^2}{8\pi G} \quad \Rightarrow \quad \rho_c^0 = 10.6 \text{ keVcm}^{-3} \times h^2$$

- Contribution Ω_A of a species A, expressed as a fraction of the critical density ρ_c^0 :

$$\Omega_A = \frac{\rho_A}{\rho_c^0}$$

- Sum of all contributions: $\Omega_0 = \sum_i \Omega_i$

$$\Omega_0 = 1 \Leftrightarrow k = 0$$

$$\Omega_0 < 1 \Leftrightarrow k = -1$$

$$\Omega_0 > 1 \Leftrightarrow k = +1$$

- Content of the Universe : radiation (R), matter (M), and possibly vacuum energy (Λ)
$$\Omega_0 = \Omega_R + \Omega_M + \Omega_\Lambda$$
 - Radiation contains photons and neutrinos (relativistic)
 - Matter can contain baryonic and “cold” non-baryonic matter (non-relativistic)
 - Dark energy

Radiation density

- Radiation (R): photons and neutrinos

- relativistic \Rightarrow not in comoving rest frame

$$H^2 = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^3}$$

- pressure is proportional to density:

$$p_R = \rho_R / 3 \Rightarrow \rho_R \propto a^{-4}$$

- Ω_γ calculated from measured CMB temperature ($T_0 = 2.725 \text{ K}$)

$$\Omega_\gamma = \left(\frac{\pi^2}{15} T_0^4 \right) \left(\frac{8\pi G}{3H_0^2} \right)$$

- neutrino density related to photon density as

$$\Omega_\nu = \Omega_\gamma \times 3 \times \frac{7}{8} \times \left(\frac{4}{11} \right)^{4/3}$$

- Total:

$$\Omega_R = \Omega_\gamma + \Omega_\nu \approx 8 \times 10^{-5}$$

Cosmological neutrinos

- Cosmological Microwave Background (CMB)

$$\Rightarrow \sum m_i \simeq 1 \text{ eV}/c^2$$

$$\Rightarrow \text{one } \nu \text{ mass of at least } \sqrt{\Delta m_{23}^2} = \sqrt{2.1 \times 10^{-3}} \simeq 0.05 \text{ eV}/c^2$$

$$\Rightarrow \text{for three neutrinos: } 0.05 \text{ eV}/c^2 < \text{mass of heaviest } \nu_i < 0.1 \text{ eV}/c^2$$

- Neutrino contribution to the cosmological evolution

- if neutrinos have very **small mass** \Rightarrow they are relativistic \Rightarrow they are not trapped in a gravitational wells \Rightarrow they don't contribute to the gravitational potentiel of clusters
 - if neutrinos have **larger mass** \Rightarrow they become non-relativistic \Rightarrow they can be trapped by structures smaller than the Universe (e.g clusters)

\Rightarrow the larger the ν mass, the earlier they become non-relativistic as the Universe cools down \Rightarrow their effect can be observed on small clusters

April 2024:

New result from DESI (BAO)

$\sum m_i < 0.113 \text{ eV} @ 95\% \text{ C.L.}$

[arXiv:2404.03002](https://arxiv.org/abs/2404.03002)

Matter density

- Matter (M): baryonic and non-baryonic

$$H^2 = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^3}$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

$$\Omega_M = \Omega_b + \Omega_{DM}$$

- cold (\equiv non-relativistic) matter in the comoving frame $\Rightarrow p \rightarrow 0$
- $p_M = 0 \Rightarrow \rho_M \propto a^{-3}$
- therefore:
 - we know how ρ_M varies with time
 - but we need to measure the value of Ω_M (Ω_b and Ω_{DM})

Vacuum energy

- Cosmological constant Λ first introduced by Einstein to make the Universe static

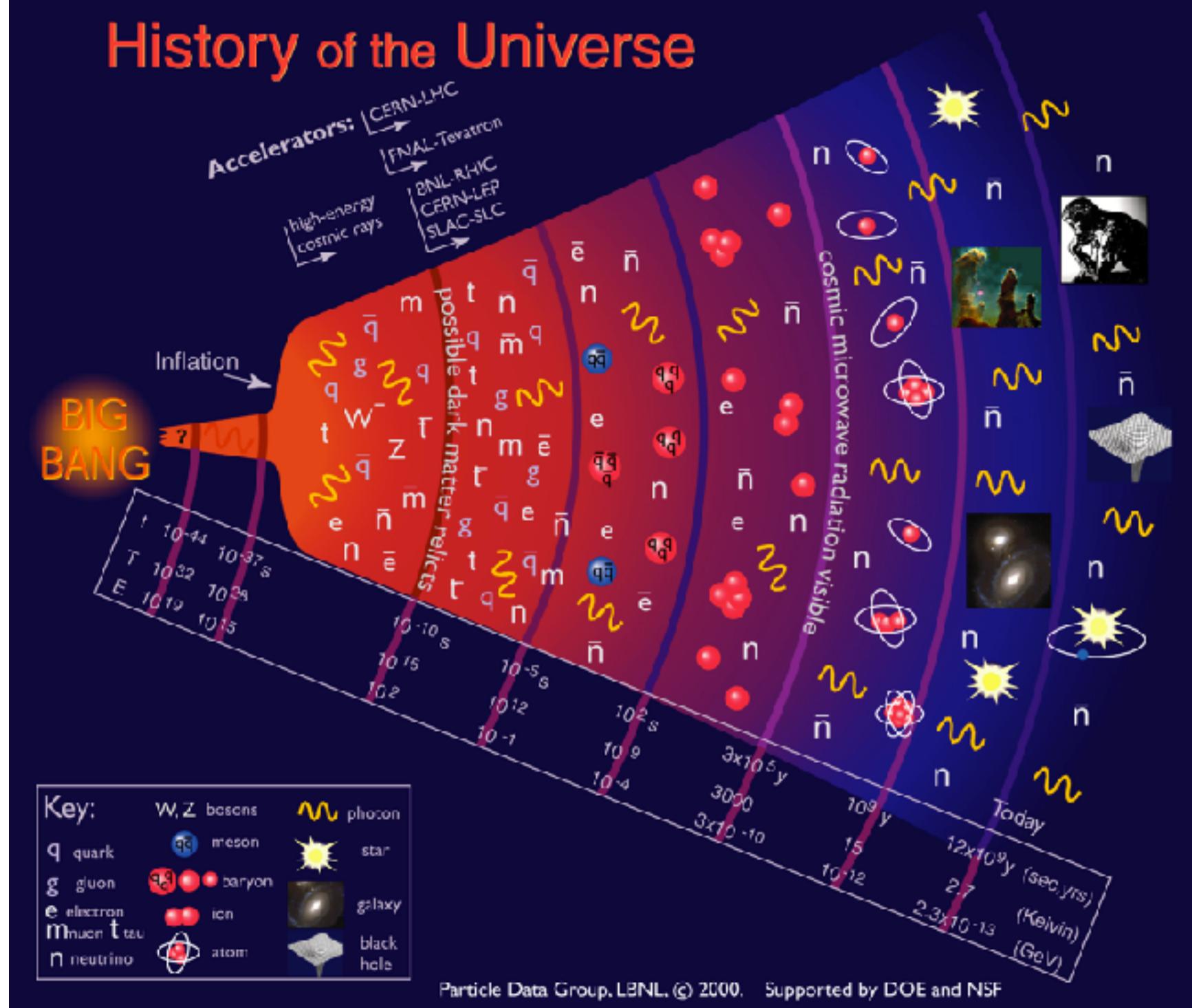
$$H^2 = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

- In modern cosmology, Λ is interpreted as vacuum energy (Dark Energy), and contributes to the energy density of the Universe as Ω_Λ
- Dark Energy suffers no dilution
 $\Rightarrow \rho_\Lambda = \text{cst} \Rightarrow p_\Lambda = -\rho_\Lambda \Rightarrow$ negative pressure!
- Like for matter, we a priori don't know Ω_Λ but we know its impact on the evolution of the Universe $\Rightarrow \Omega_\Lambda$ can be measured from the observation of the expansion

$$H^2 = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$$

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

History of the Universe



CP violation and cosmology

- When the temperature cools down below $T = 0.5 \text{ MeV}$, $\gamma\gamma \rightarrow e^+e^-$ cannot occur, and e^+e^- pairs annihilate
- Therefore, all matter and anti-matter should have converted into radiation
- **Then, why do we observe matter and no anti-matter?**
- Three conditions (Sakharov, 1967)
 1. Baryon number violation
 2. C- and CP- symmetry violation
 3. non-thermal equilibrium
- Baryogenesis? Leptogenesis?

Evidence for Dark Matter

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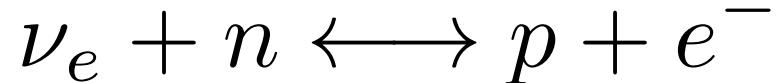
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B.1 Abundance of elements

- Baryon nucleosynthesis:

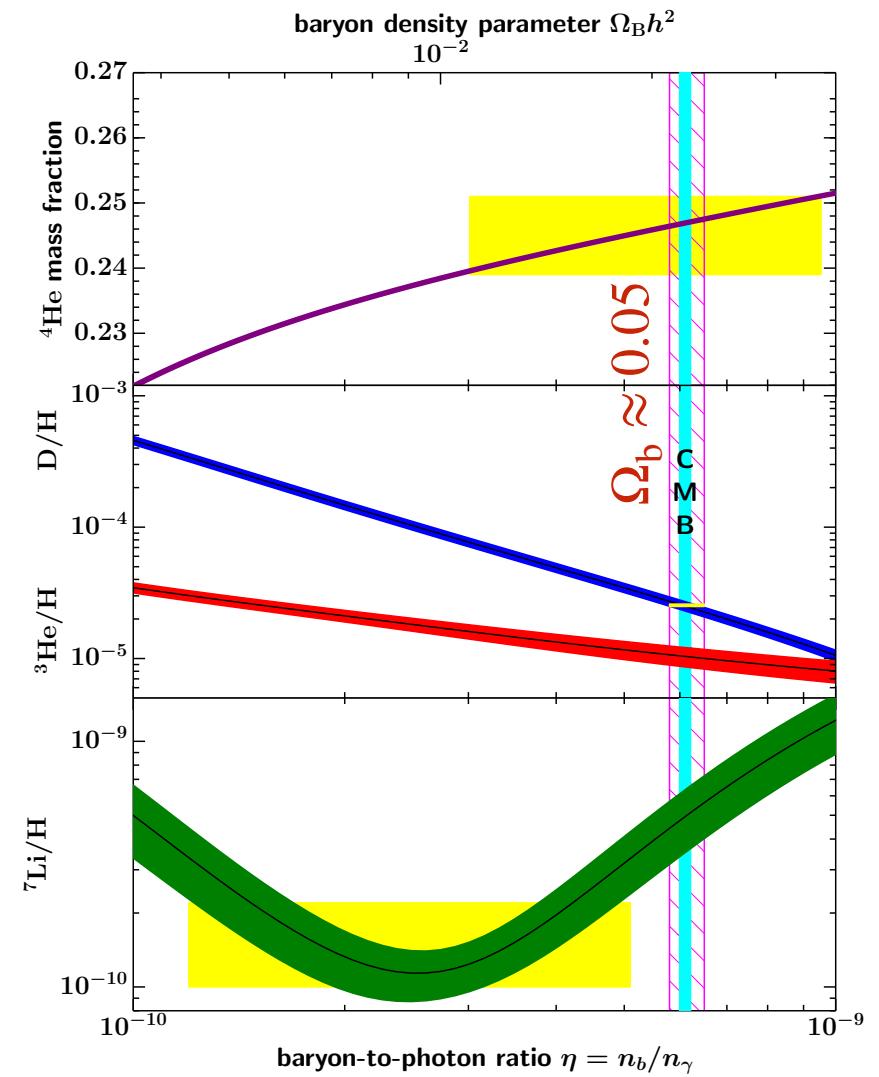
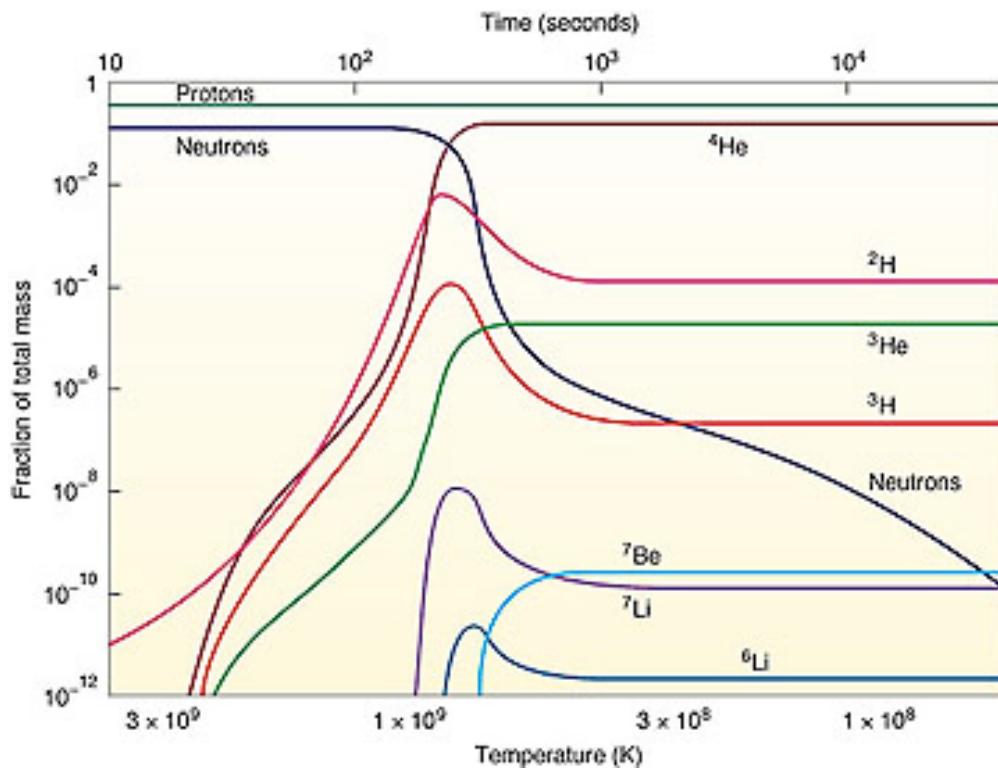


$$\frac{N_p}{N_n} \propto e^{\frac{m_n - m_p}{T}} \quad m_n - m_p = 1.29 \text{ MeV}$$

- when the temperature $T \gg \Delta m \equiv m_n - m_p \Rightarrow N_n = N_p$
- when the temperature $T \simeq \Delta m \Rightarrow N_n \approx N_p$ (freeze-out)
- when $T < 0.1 \text{ MeV}$, no more photo-dissociation is possible $(D + \gamma \leftrightarrow p + n)$, and the nucleosynthesis can begin
 \Rightarrow production of deuterium (D), Helium (He), Lithium (Li), Beryllium (Be)
- abundances can be predicted based on the Cosmological standard model, and compared to observations

Predicted abundances

- Observed relative abundances are consistent with predictions!
- Observed baryonic density $\Omega_b \approx 0.05$, consistent with CMB measurements (see later)



B.2 SuperNovae type Ia

- Measure luminosity distance d_L versus redshift of SN type Ia
 - from known intrinsic luminosity, apparent luminosity, and redshift, one can write

$$d_L = a(t_0)(1+z)f_k \left(\int_0^z \frac{dz}{a(t_0)H_0} [\Omega_M(1+z)^3 + \Omega_\Lambda]^{-\frac{1}{2}} \right)$$

$$f_k(x) = \sin x \quad \text{if } k = +1$$

$$f_k(x) = x \quad \text{if } k = 0$$

$$f_k(x) = \sinh x \quad \text{if } k = -1$$

- Results:

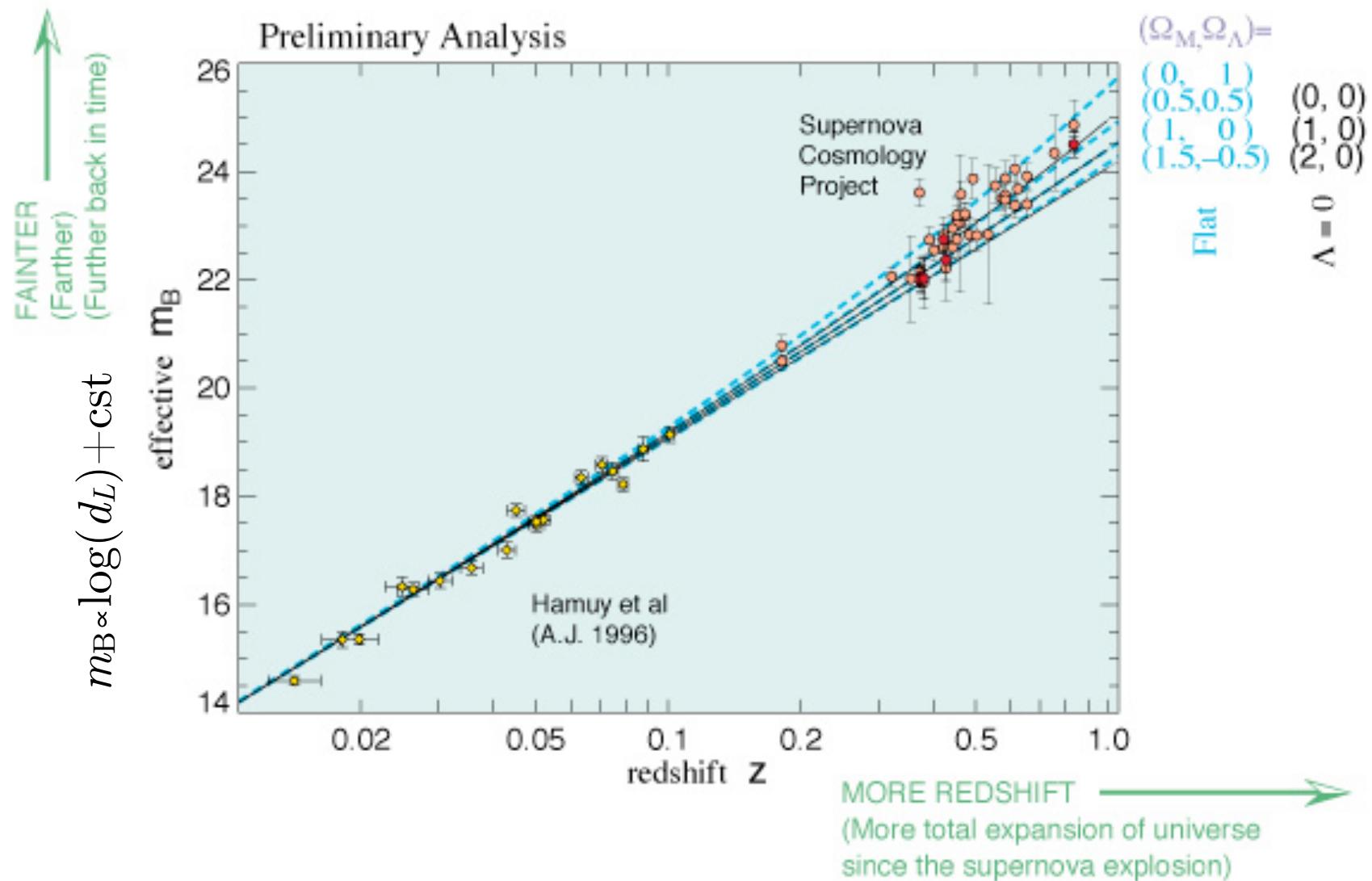
$$\Omega_M - \Omega_\Lambda = -0.49 \pm 0.12$$

$$\Omega_M + \Omega_\Lambda = +1.11 \pm 0.52$$

⇒ d_L is mostly sensitive to $\Omega_\Lambda - \Omega_M$

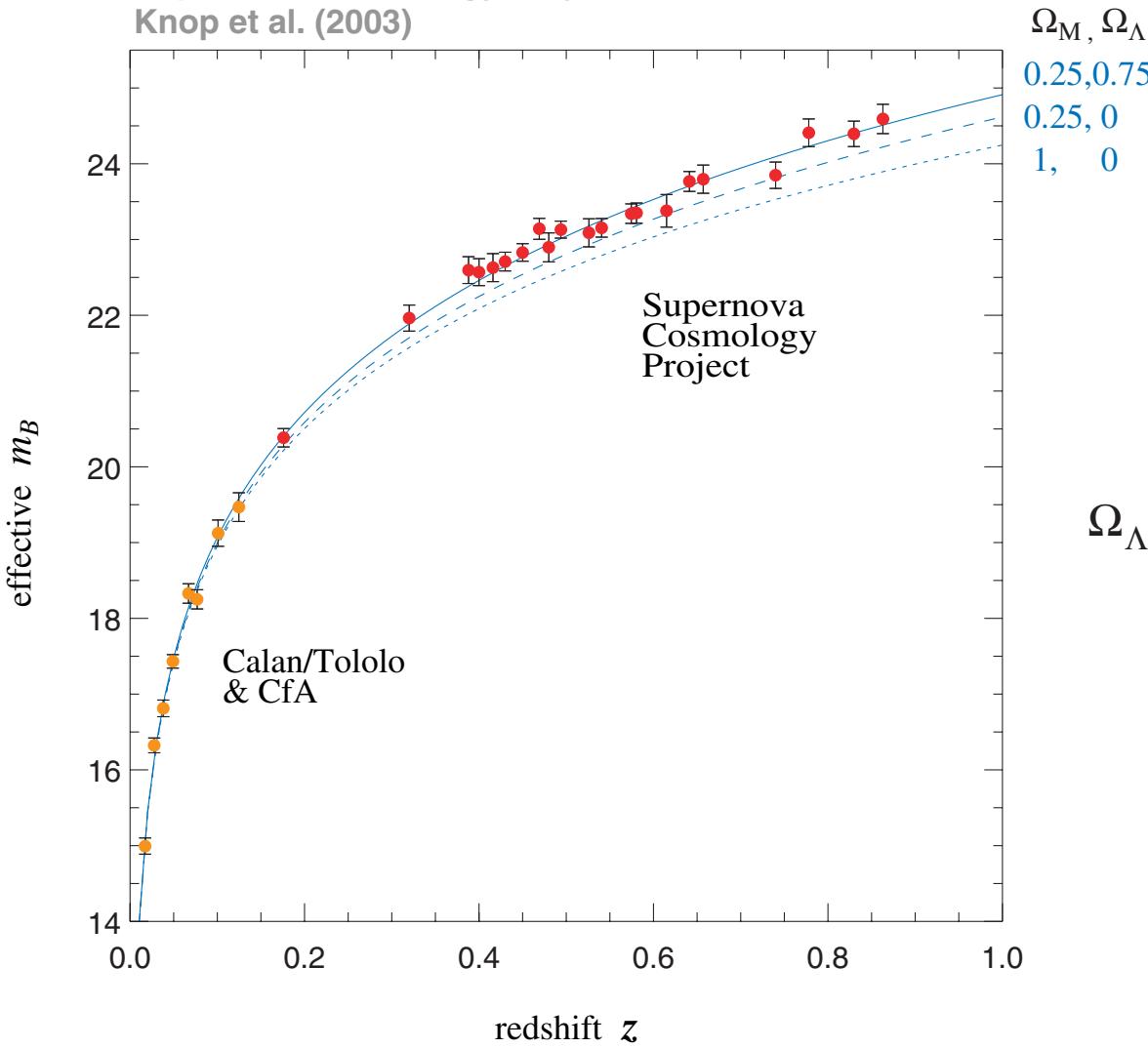
Supernova Cosmology Project (1998)

- Initial results from 1998 compatible with non-zero cosmological constant!

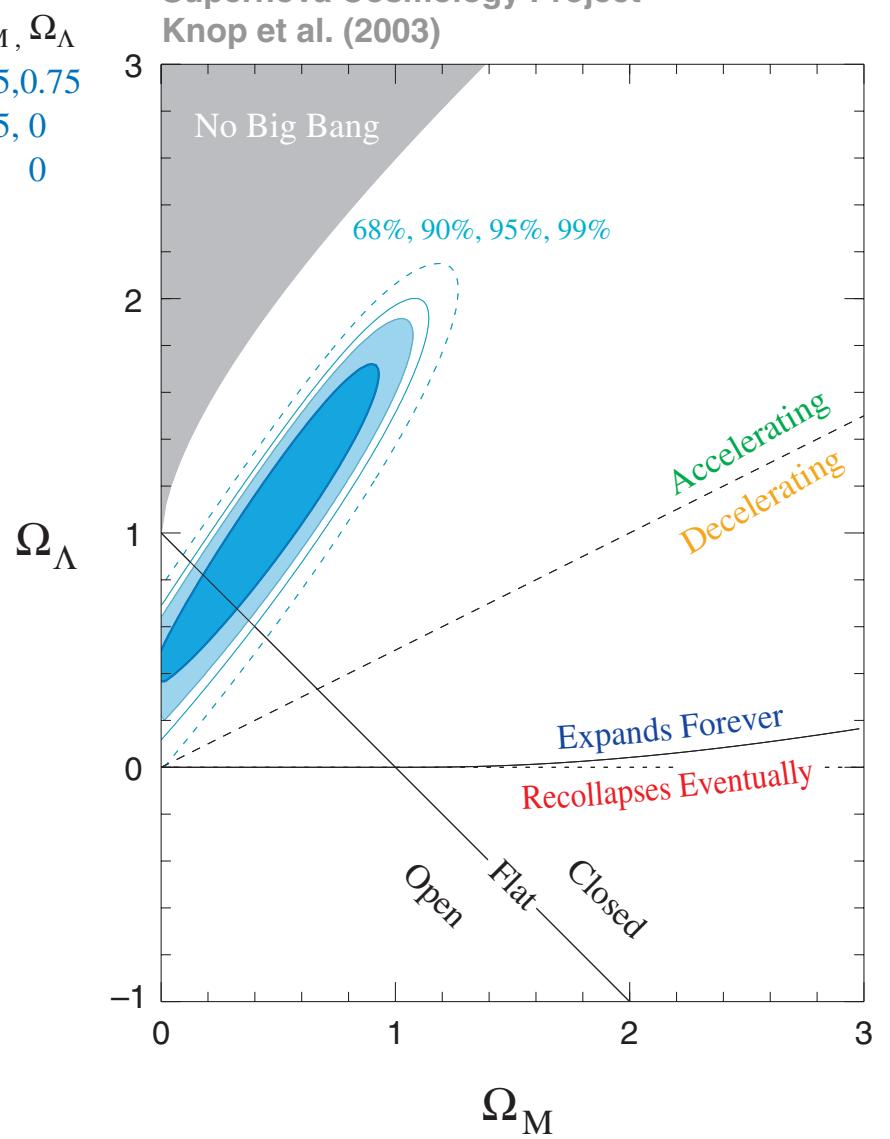


Supernova Cosmology Project (2003)

Supernova Cosmology Project
Knop et al. (2003)

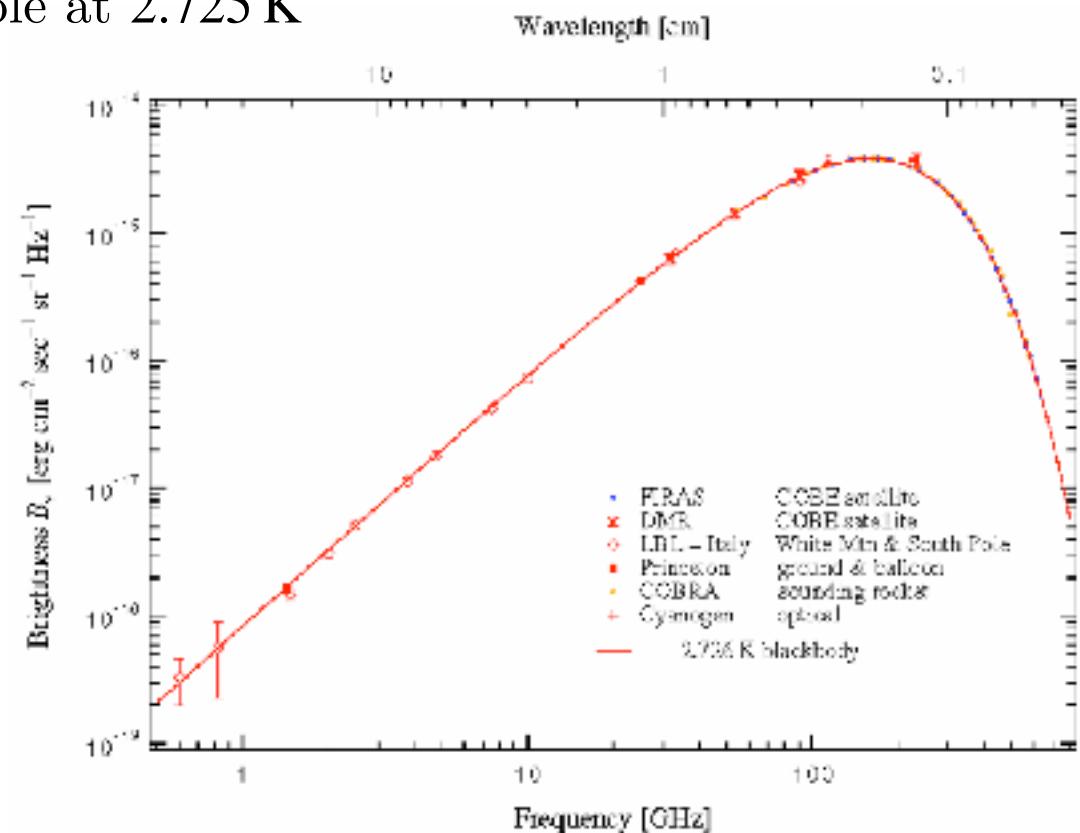


Supernova Cosmology Project
Knop et al. (2003)



B.3 Cosmic Microwave Background

- Cosmic Microwave Background (CMB)
 - decoupling of photons from matter at $T \simeq 3000 \text{ K}$
 $\Leftrightarrow z = 1100 \Leftrightarrow t = 380000 \text{ yr}$
 \Rightarrow picture of the Universe at this time
 - thermal equilibrium before decoupling
 \Rightarrow black body, now observable at 2.725 K



- almost perfect black body,
but...

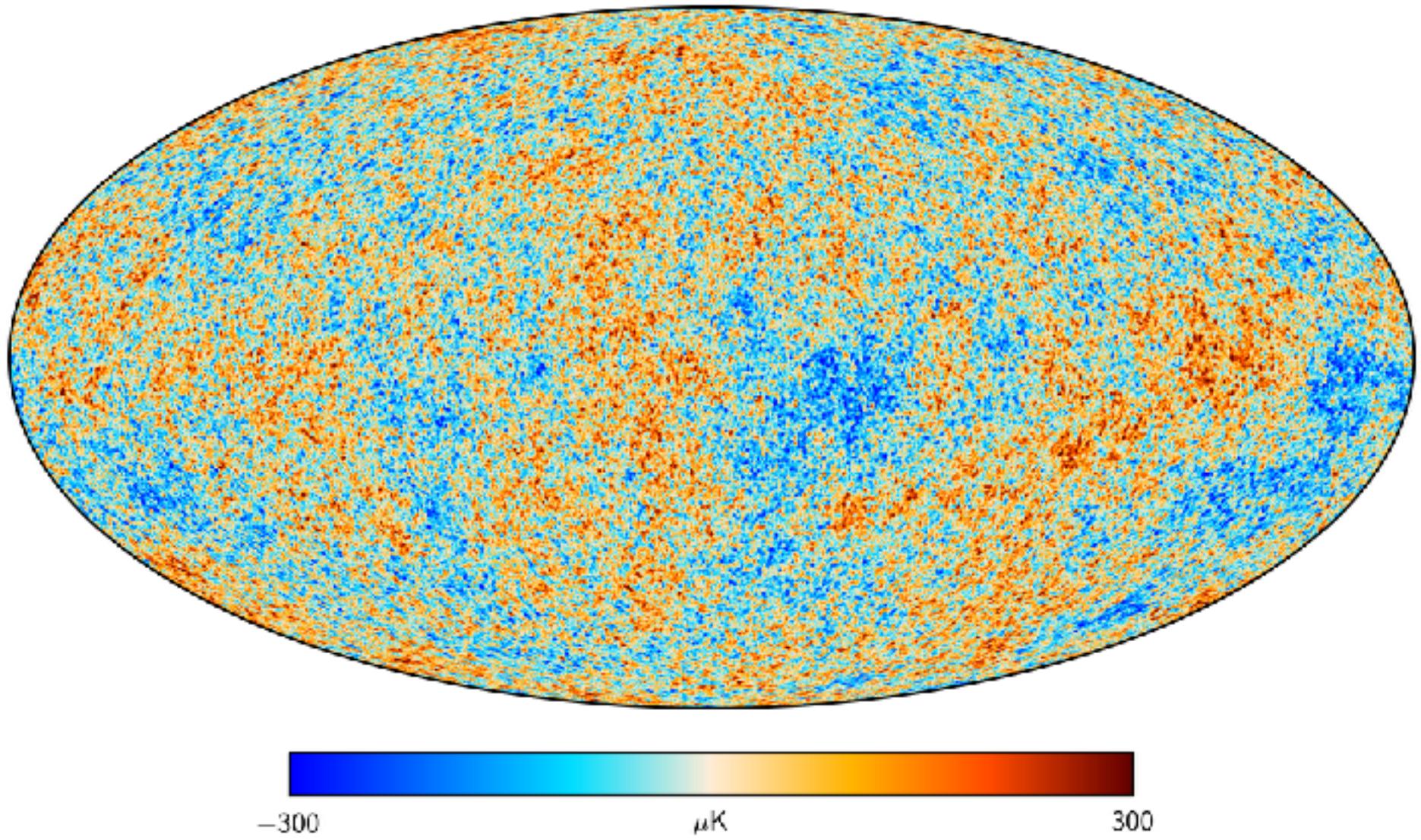
CMB anisotropies

- Anisotropies expected at level of 10^{-5} to explain large scale structures
 - observation by COBE (1994) :
$$\frac{\Delta T}{T} \propto 10^{-5}$$

⇒ consistent with theoretical considerations
- Source and nature of the fluctuations :
 - γ /nucleon/electron fluid under antagonist forces:
 - gravity and quantum pressure ⇒ density waves (sound)
 - limited velocity ⇒ effective horizon ⇒ sets the effective size of fluctuations at the time of decoupling
 - the size of the fluctuations depends on velocity, which depends on pressure, and pressure is different for Λ and matter

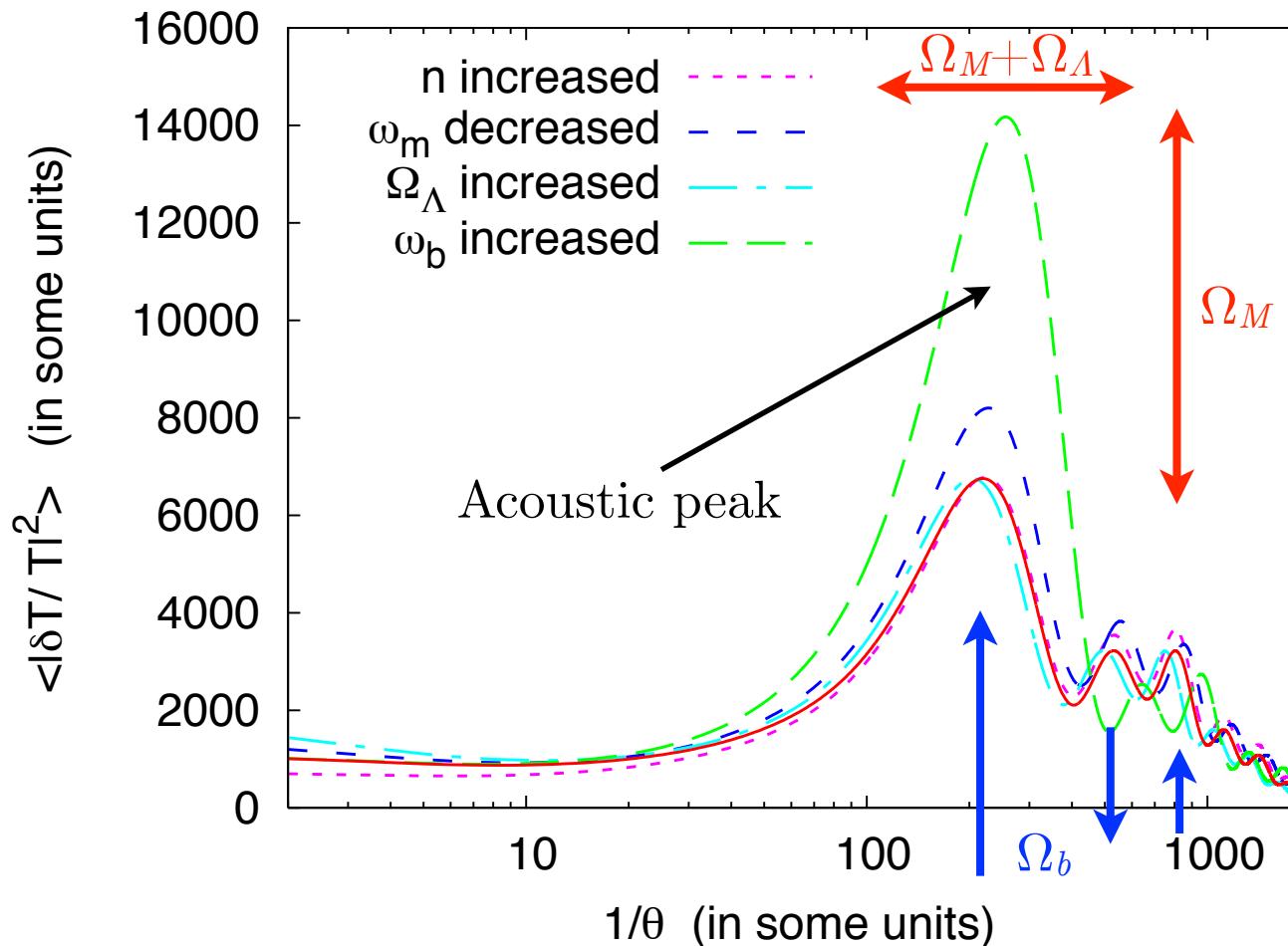
⇒ Ω_Λ and Ω_M can be determined from the map of anisotropies!

CMB anisotropies: Planck (2015)



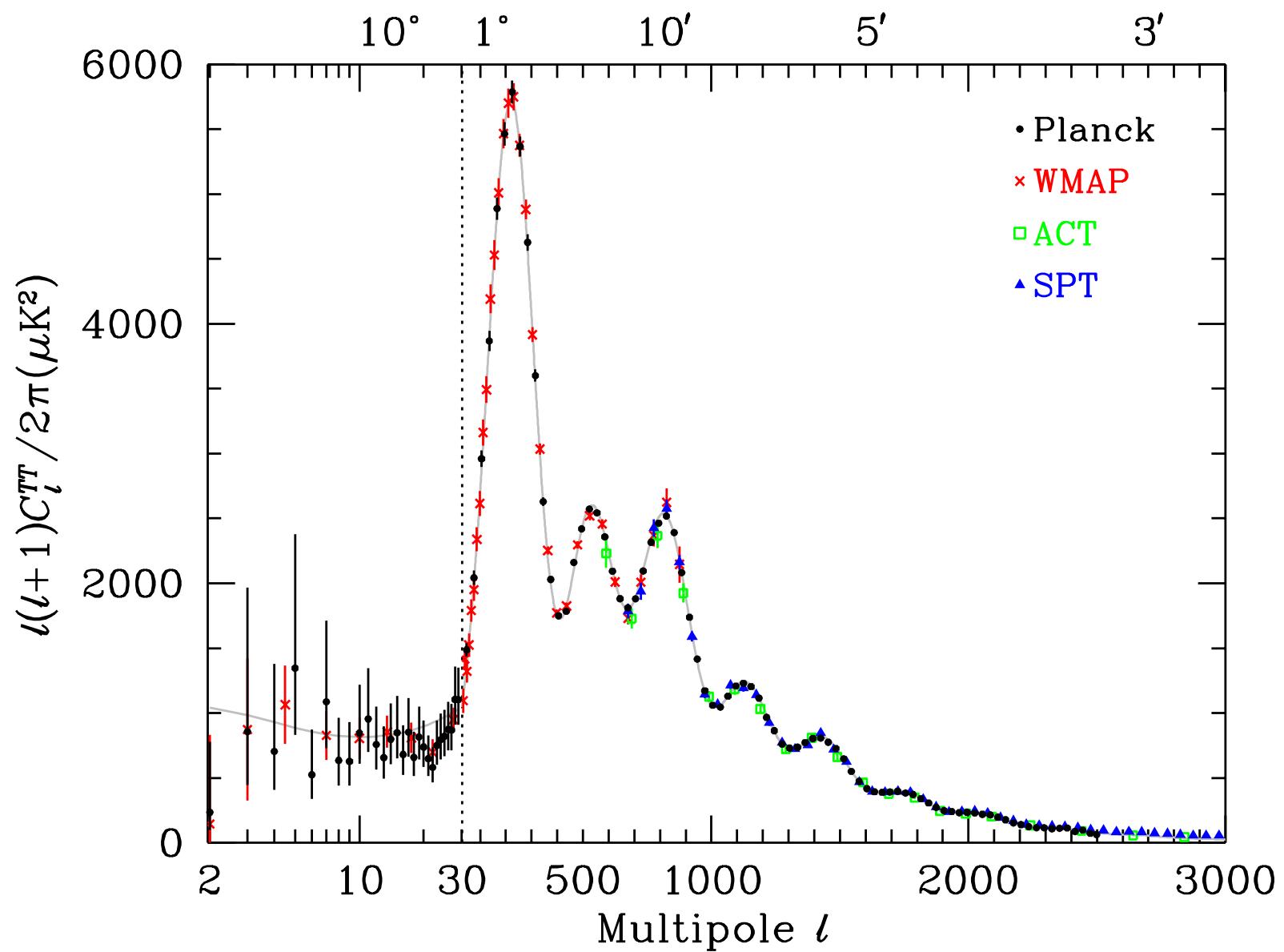
CMB power spectrum

- Fourier modes of angular CMB fluctuations
 - sensitive to several parameters
 - $\Omega_M + \Omega_\Lambda$, Ω_M , Ω_b , (and many more)



Simulation

CMB power spectrum: combined results



source: PDG 2023

Cosmological parameters

Parameter	Value	source
h	0.674 ± 0.005	SN, CMB, Clusters
Age of the Universe	13.797 ± 0.023 Gyr	
Ω_M	$(31.5 \pm 0.7)\%$	SN, CMB, Clusters
Ω_b	$(4.93 \pm 0.06)\%$	Baryon nucl., CMB
$\Omega_{DM} = \Omega_M - \Omega_b$	$(26.5 \pm 0.7)\%$	
Ω_Λ	$(68.5 \pm 0.7)\%$	SN, CMB
Ω_γ	$(5.38 \pm 0.15) \times 10^{-5}$	CMB temperature
Ω_ν	$< 0.3\%$	
$\Omega_{\text{tot}} = \sum \Omega_i$	1.011 ± 0.006	(SN, CMB)

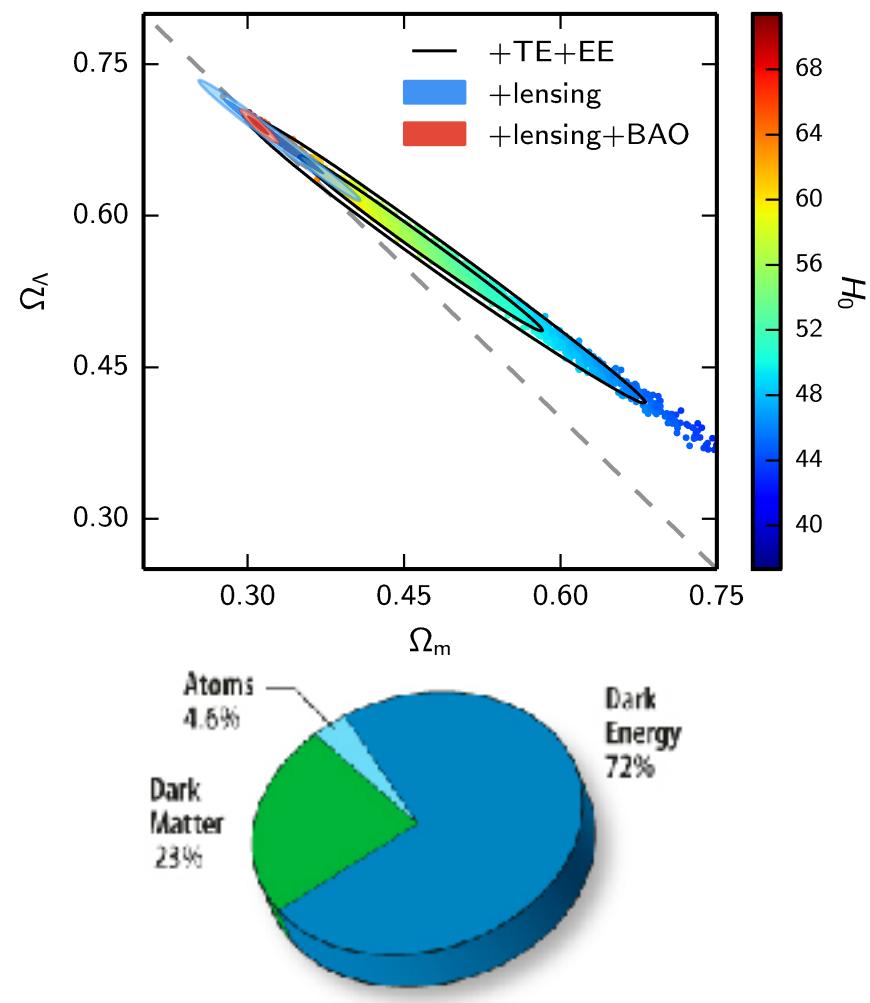
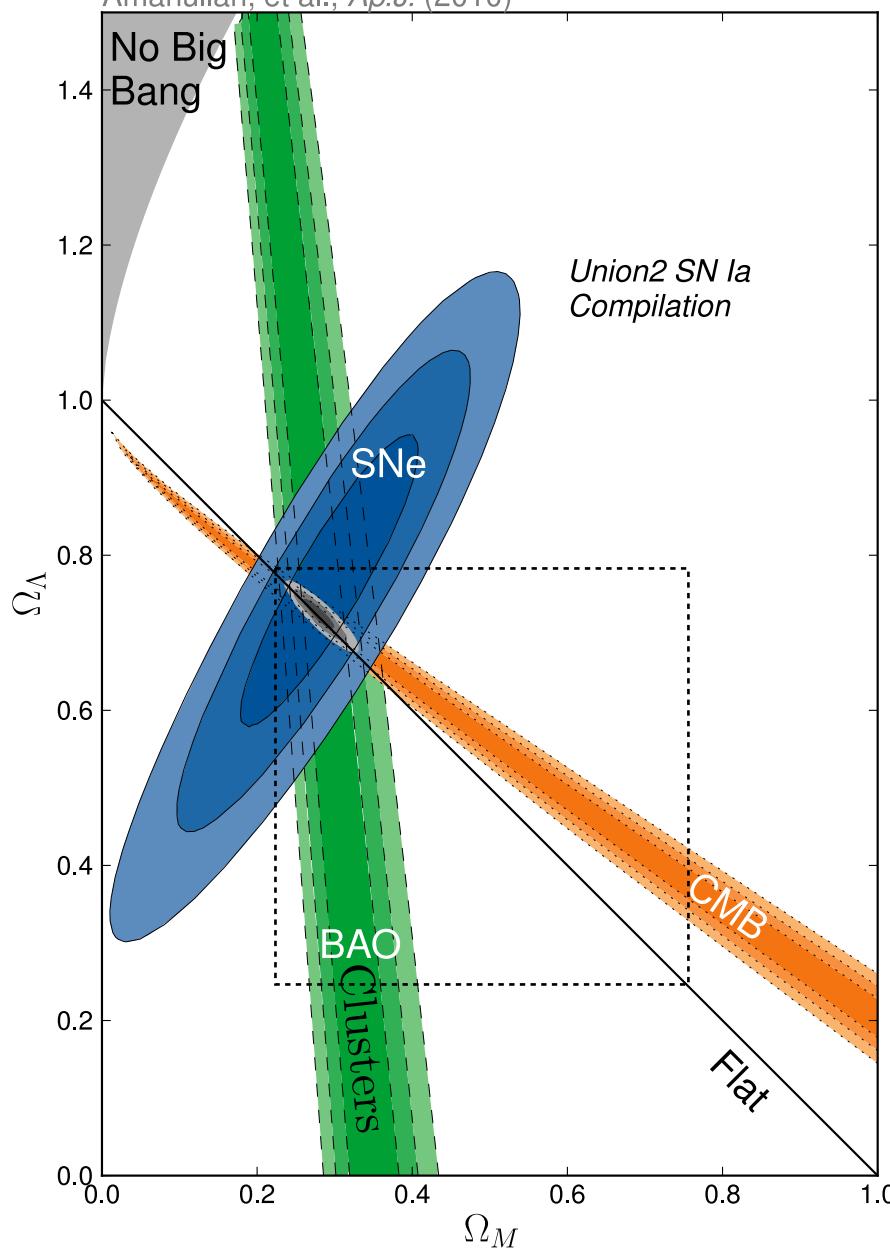
source: PDG 2021

Parameters are over-constrained

⇒ allows check of the consistency of the model

Cosmological parameters

Supernova Cosmology Project
Amanullah, et al., *Ap.J.* (2010)



Questions:

- What is Dark Matter?
- What is Dark Energy?

Dark Matter properties (summary)

- Dark Matter “particles” must be:

1. massive and non-relativistic (cold DM)

- constrained by large scale structures
- constrained by CMB power spectrum

2. non-baryonic

- constrained by nucleosynthesis, which fixes the baryonic contribution
- constrained by CMB power spectrum

3. stable on cosmological time scales

- because its effects are observable today through its gravitational effects

4. weakly interacting

- because not visible other than through gravitational effects