PHYS-302 - Physics of Biological Systems

Langevin equation

Consider the Langevin equation

$$\frac{dx(t)}{dt} = -\frac{dV(x)}{dx} + \eta(t) \tag{1}$$

with noise function

$$\langle \eta \rangle = 0 \tag{2}$$

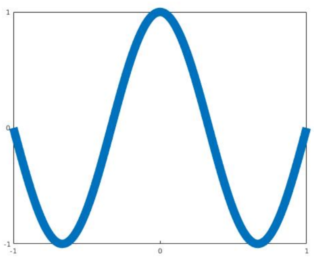
$$\langle \eta(t)\eta(t') \rangle = 2\delta(t - t') \tag{3}$$

and with a double-well potential

$$V(x) = A\cos\left(\frac{3\pi}{2}x\right) \quad \text{for } -1 \le x \le 1 \tag{4}$$

$$V(x) = \infty \quad \text{otherwise,} \tag{5}$$

depicted here:



Problem 1: Explore the steady states of the system computationally

Set $A = 1$. Discretize time and space. Simulate the particle's position as a function of time. Histogram the position and plot it together with the formula for the steady-state probability distribution derived in class.

Problem 2: Explore transition times

Vary the values of $A = 0.1, 0.2, ..., 10$. Start the particle at the bottom of the left well. Measure the time it takes for the particle to move to the right of the wall which is between the two wells. Repeat this simulation until you get an average time for hopping over the wall. Plot this average time versus A.

Hand in your two plots on the first page of your homework set and the code on the subsequent sheets.