

Lecture 10: Genomes

Goal: Model to obtain insights into how the same DNA sequence can result in a diversity of gene expression profiles

<https://youtu.be/nQQJNlJbzd8?si=1QLyWGcKHZ0pvAi1>

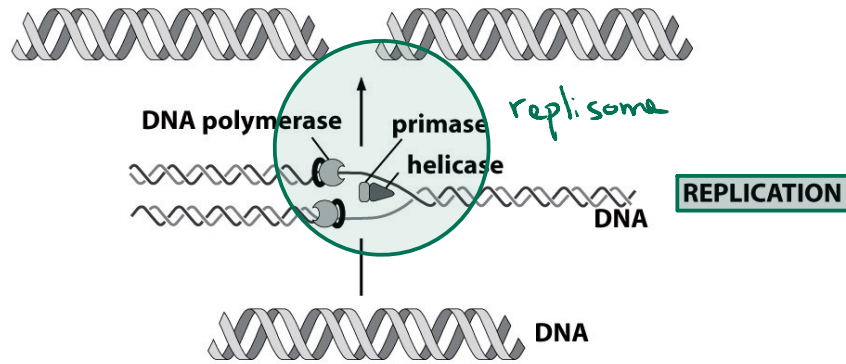
- Central dogma of molecular biology
- Genome regulation
- Genetic networks

PBOC Chapter 3.2.1, 6.1.2 (refresh Lecture 06), 19.2 (except 19.2.5)

Central dogma of molecular biology

Recall (Lecture 1, 6)

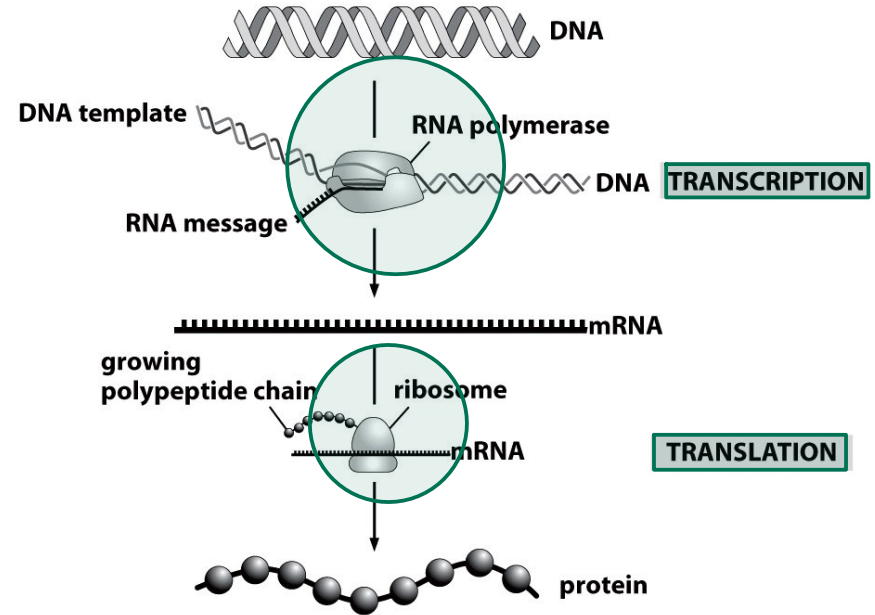
Processes of the central dogma



DNA → DNA

replisome: DNA polymerase

+ primase + helicase



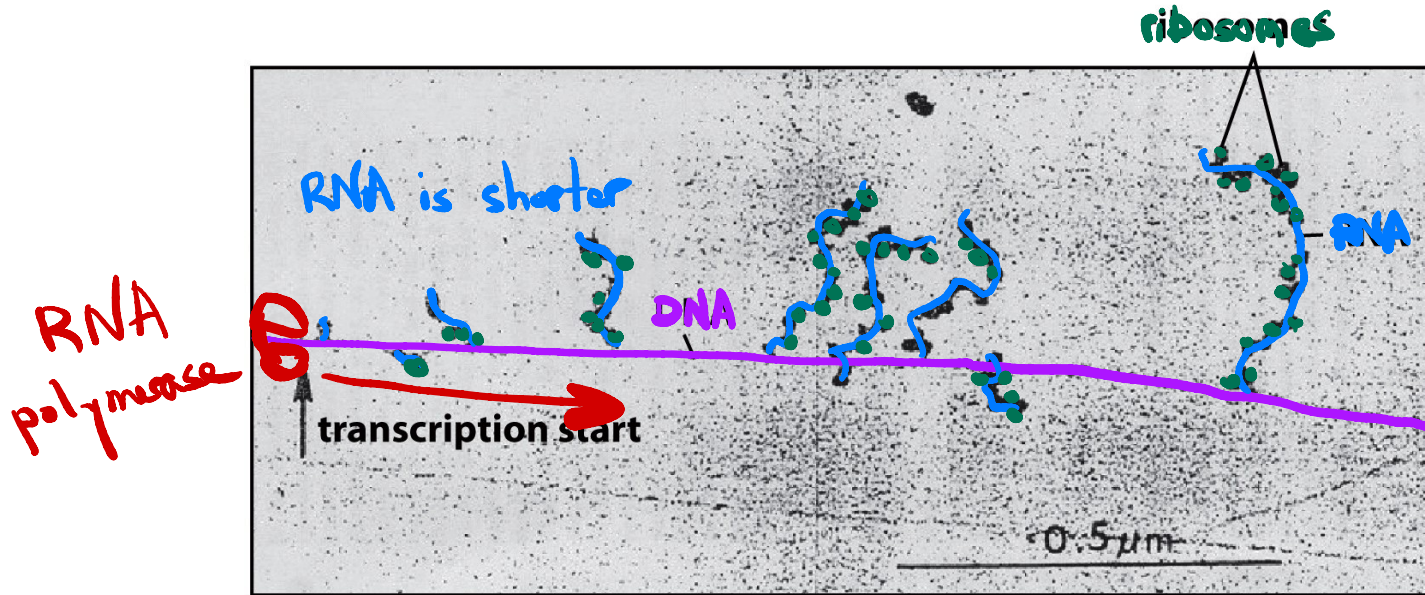
DNA → mRNA → protein

<https://youtu.be/TNkWgcFPHqw?si=IGHHDFNOwNdEYNT7>

<https://youtu.be/gG7uCskUOrA?si=c6mmpHIrsmIUtaF>

Central dogma of molecular biology

Processes of the central dogma



In bacteria, ribosomes can bind to RNA even before it has finished being transcribed

Central dogma of molecular biology

Processes of the central dogma

What are the rates of processes involved in the central dogma?

DNA replication
Given replication rate per replisome of 250-1000 bp/s, and *E. coli* genome size of 5×10^6 bp, what range of times should it take to replicate the genome?

transcription
Given an average protein molecular weight of 46 kDa, an average amino acid molecular weight of 110 Da, and a transcription rate of 40 nucleotides/s, how long should it take on average to make an mRNA transcript?

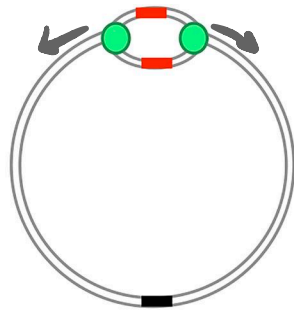
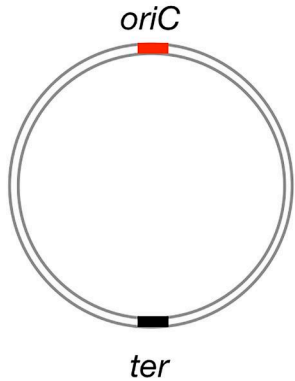
translation
Given an *E. coli* cell cycle time of 3,000 seconds, and the facts that the number of proteins must double during this time, and that there are ~20,000 ribosomes per cell, what is the rate of translation per ribosome (units aa/second)? If needed, look back at Lecture 2 where we estimated the number of proteins per cell.

Central dogma of molecular biology

Processes of the central dogma: Replication

Given replication rate per replisome of 250-1000 bp/s, and *E. coli* genome size of 5×10^6 bp, what range of times should it take to replicate the genome?

bacteria contain a single circular genome



replisomes

(2 of them work simultaneously)

$$\frac{5 \times 10^6 \text{ bp}}{2 \cdot 250 \text{ bp/s}} = 10^4 \text{ s}$$

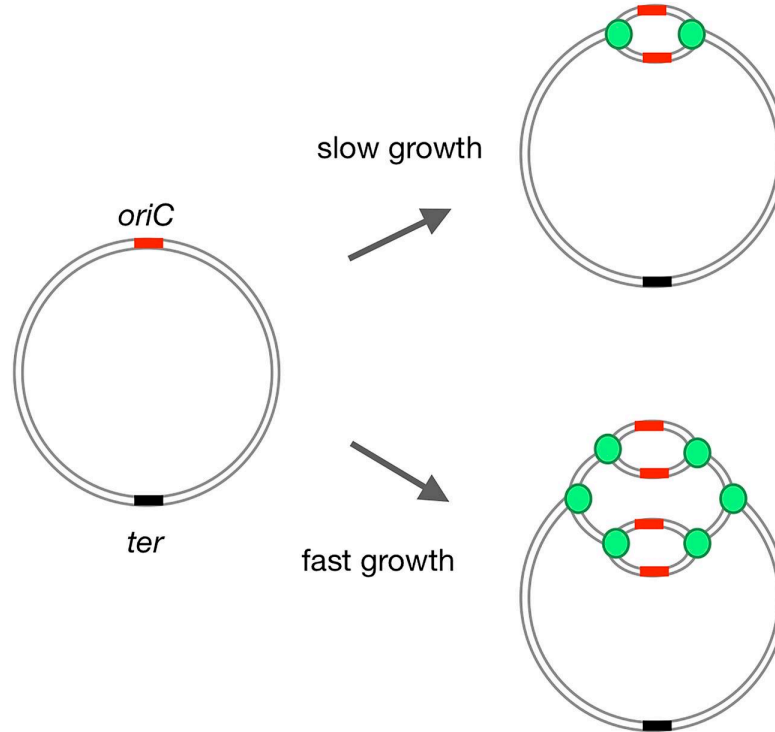
$$\frac{5 \times 10^6 \text{ bp}}{2 \cdot 1000 \text{ bp/s}} = 2.5 \times 10^3 \text{ s}$$

one to a few hours.

Central dogma of molecular biology

Processes of the central dogma: Replication

E. coli can divide
in much less than
3000 seconds, in
fact, as little as
1000 seconds



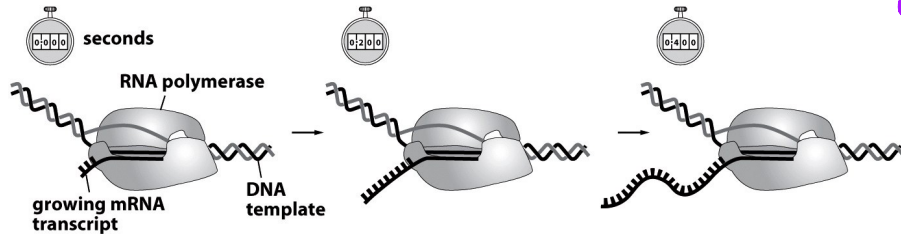
multiple replication forks
⇒ more than 2 replisomes

Central dogma of molecular biology

Processes of the central dogma: Transcription

Given an average protein molecular weight of 46 kDa, an average amino acid molecular weight of 110 Da, and a transcription rate of 40 nucleotides/s, how long should it take on average to make an mRNA transcript?

transcription



average protein: $\frac{4.6 \times 10^4 \text{ Da/protein}}{110 \text{ Da/aa}} = 420 \text{ aa/protein}$

Each aa is encoded by three nucleotides (a "codon")

$$\left(\frac{1260 \text{ nucleotides}}{\text{protein}} \right) / \left(40 \text{ nucleotides/s} \right)$$

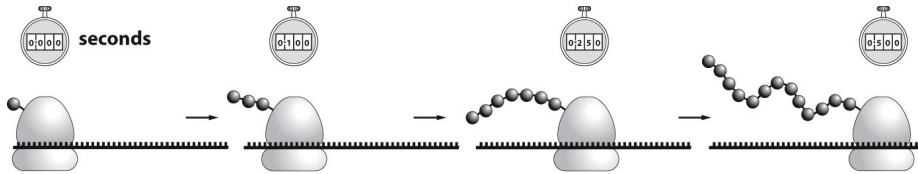
$$= 31 \text{ seconds}$$

Central dogma of molecular biology

Processes of the central dogma: Translation

Given an E. coli cell cycle time of 3,000 seconds, and the facts that the number of proteins must double during this time, and that there are ~20,000 ribosomes per cell, what is the rate of translation per ribosome (units aa/second)? If needed, look back at Lecture 2 where we estimated the number of proteins per cell. 3×10^6 proteins/cell

protein synthesis



$$\frac{3 \times 10^6}{2 \times 10^4} \frac{\text{proteins}}{\text{ribosome}} = 150 \frac{\text{proteins}}{\text{ribosome}} \text{ (per cell cycle)}$$

$$150 \frac{\text{proteins}}{\text{ribosome}} \cdot 1260 \frac{\text{nucleotides}}{\text{protein}} / 3000 \text{ s} = 63 \frac{\text{nucleotides}}{\text{ribosome} \cdot \text{s}} \approx 21 \frac{\text{aa}}{\text{ribosome} \cdot \text{s}}$$

(last question)
or 420 aa/protein

Around 20 s to make a protein
About same time as to make mRNA.

Genome regulation

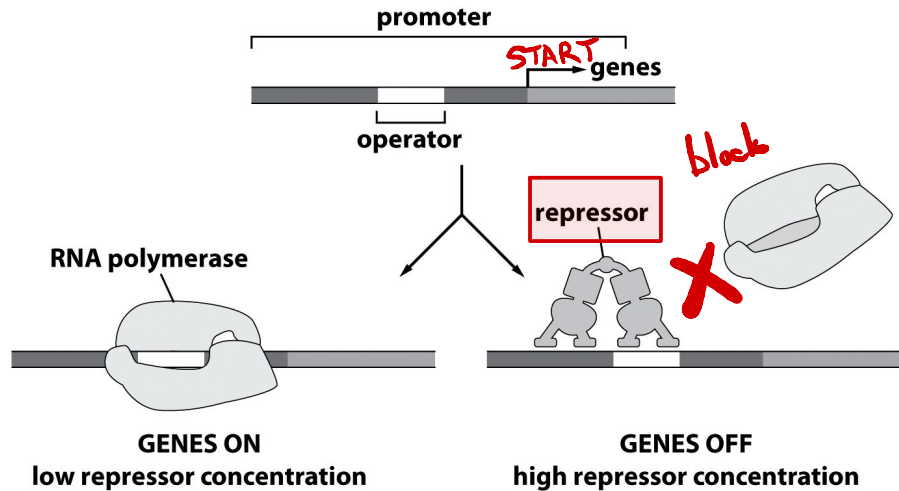
Statistical mechanics of gene expression

Review Lecture 6 annotated slides 13-20, along with PBoC 6.1.2

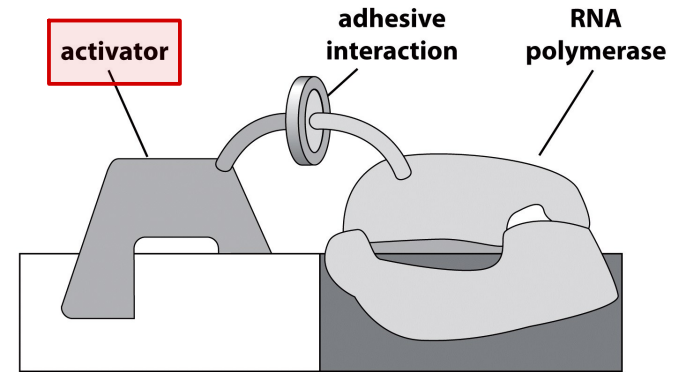
Genome regulation

Genetic networks: Molecules

negative regulation



positive regulation



Repressors and activators change the probability of RNA polymerase binding to the promoter of a gene

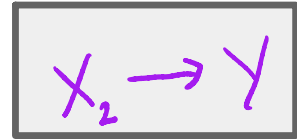
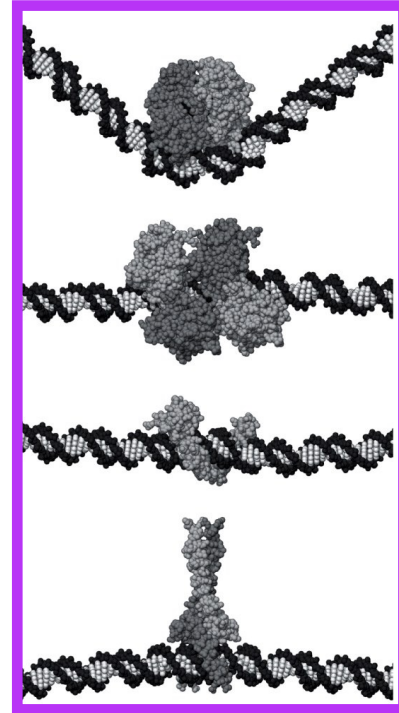
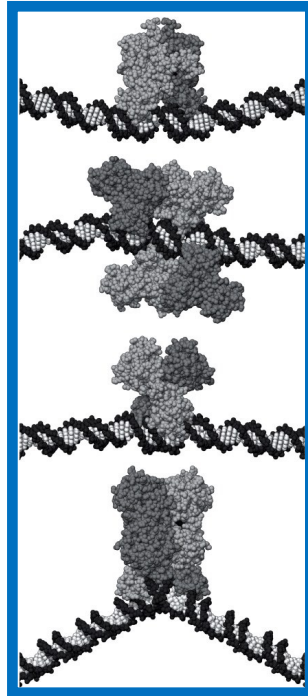
Genome regulation

Genetic networks: Molecules

Repressors and activators can both bind to DNA and deform it in the promoter region of a gene.



Repressors bind to the promoter site, to block RNA polymerase from binding

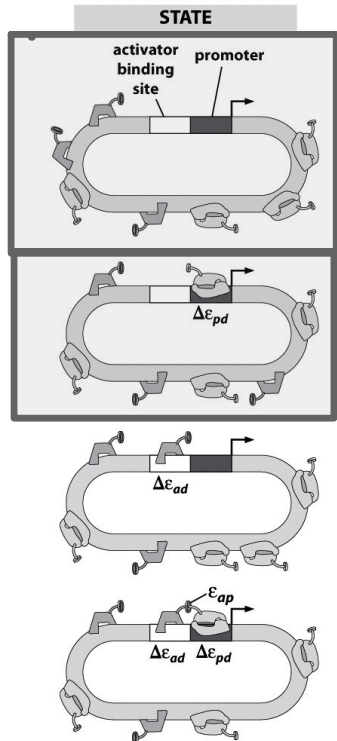


Activators bind upstream of the promoter site, to enhance RNA polymerase binding

Genome regulation

Genetic networks: Models (activation)

P # of polymerases
 A # of activators
 R # of repressors
 N_{NS} # of boxes



RENORMALIZED WEIGHT

1

$$\frac{P}{N_{NS}} e^{-\Delta\epsilon_{pd}/k_B T}$$

$$\frac{A}{N_{NS}} e^{-\Delta\epsilon_{ad}/k_B T}$$

$$\frac{P}{N_{NS}} \frac{A}{N_{NS}} e^{-(\Delta\epsilon_{pd} + \Delta\epsilon_{ad} + \epsilon_{ap})/k_B T}$$

renormalize: divide each weight by unoccupied state

activator
? promoter
unoccupied

promoter
occupied

activator
occupied

activator
? promoter
occupied

Energies:

$$\Delta\epsilon_{pd} = \epsilon_{pd}^S - \epsilon_{pd}^{NS}$$

$$\Delta\epsilon_{ad} = \epsilon_{ad}^S - \epsilon_{ad}^{NS}$$

$$\epsilon_{ap}$$

NS non-specific

pd polymerase-DNA

ad activator-DNA

ap activator-polymerase

Genome regulation

Genetic networks: Models (activation)

$$P_{bound} = \frac{\text{states with RNAP bound}}{\text{all states}}$$

The diagram illustrates four states of a bacterial cell, represented as an oval with internal components. The states are separated by plus signs. The first state shows an unbound cell. The second state shows a cell with RNAP bound to the promoter, labeled $\Delta\epsilon_{pd}$. The third state shows a cell with RNAP bound to the activator binding site, labeled $\Delta\epsilon_{ad}$. The fourth state shows a cell with RNAP bound to both sites, labeled $\Delta\epsilon_{ad}$, $\Delta\epsilon_{pd}$, and ϵ_{ap} .

PBOC
19.1-19.6

$$= \frac{1}{1 + \left[\frac{N_{ns}}{PF_{reg}^A(A)} \right] e^{\beta A \epsilon_{pd}}}$$

like changing $[P]$

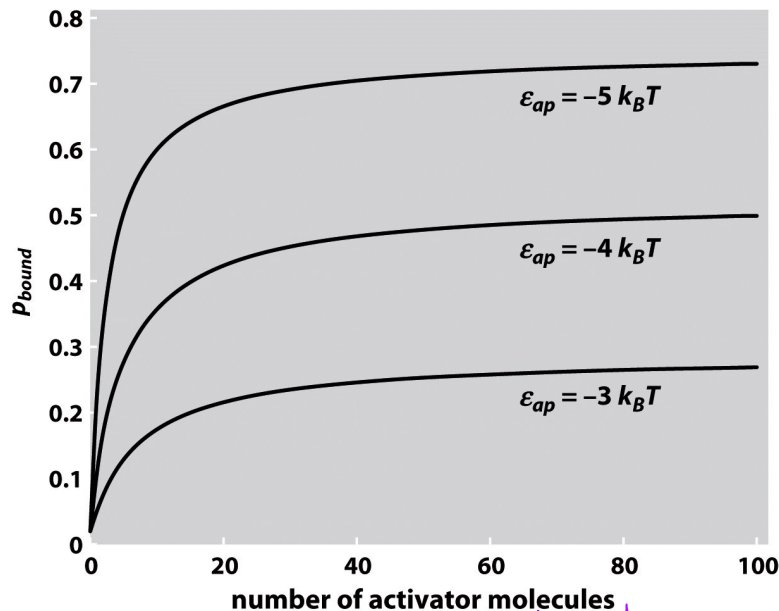
where

$$F_{reg}^A(A) = \frac{1 + \frac{A}{N_{ns}} e^{-\beta \Delta\epsilon_{ad}} - \beta \epsilon_{ap}}{1 + \frac{A}{N_{ns}} e^{-\beta \Delta\epsilon_{ad}}}$$

takes on values > 1 for $\epsilon_{ap} < 0$

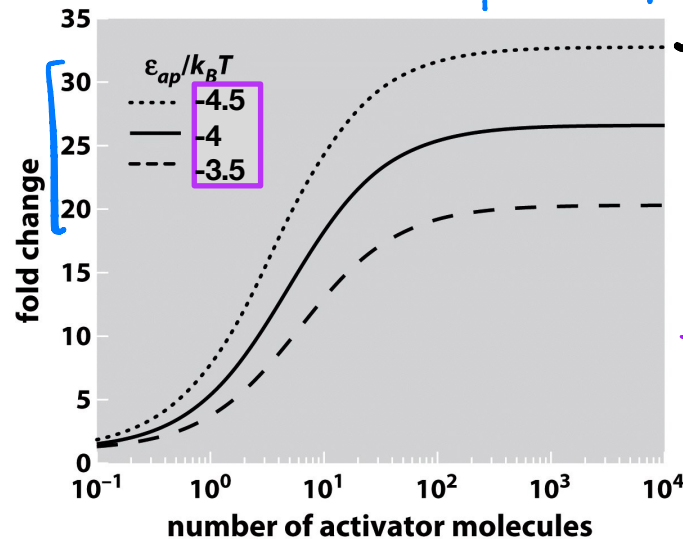
Genome regulation

Genetic networks: Models (activation)



p_{bound} increases with $|\epsilon_{ap}|$ and $[A]$ concentration of activator molecules

Can have much more expression w/ activator



Typo: energies in Fig. 19.12 were positive, that was an error

normalized to display fold-change $p_{\text{bound}}(A)/p_{\text{bound}}(A=0)$

$$= \frac{1 + [N_{\text{ns}}/P] e^{\beta \Delta \epsilon_{p2}}}{1 + [N_{\text{ns}}/PF_{\text{reg}}] e^{\beta \Delta \epsilon_{p2}}} \approx \frac{[N_{\text{ns}}/P] e^{\beta \Delta \epsilon_{p2}}}{[N_{\text{ns}}/PF_{\text{reg}}] e^{\beta \Delta \epsilon_{p2}}} = F_{\text{reg}}$$

Genome regulation

Genetic networks: Models (*activation*)

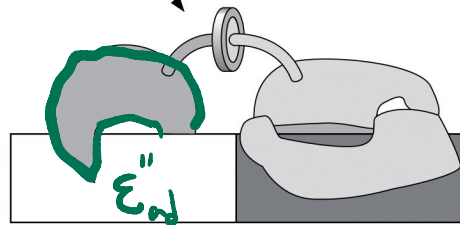
How do we know?

mix-and-match activator elements

(A)



*piece of activator
that interacts w/ RNAp*



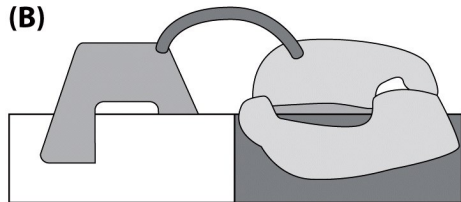
change $\Delta\epsilon_{ad}$

Experiment:

measure p_{bound}

for (A) ? (B)

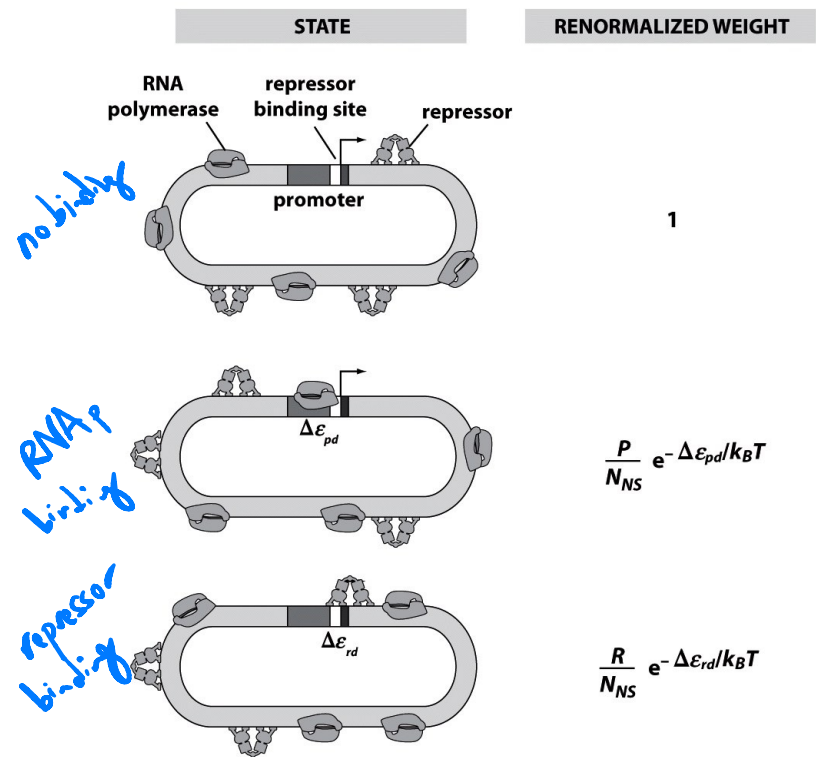
(B)



$\epsilon_{ap} \rightarrow \infty$, two states (1 : 4)

Genome regulation

Genetic networks: Models (repression)



$$P_{\text{bound}} = \frac{1}{1 + \left[\underbrace{N_{NS} \frac{P}{N_{NS}} F_{\text{reg}}^R(R)}_{\text{like changing } [P]} \right] e^{\beta \Delta\epsilon_{pd}}}$$




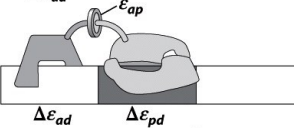
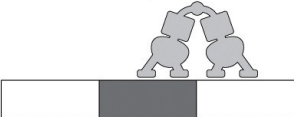
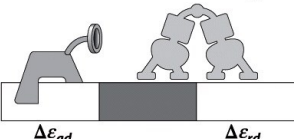
where

$$F_{\text{reg}}^R(R) = \frac{1}{1 + \frac{R}{N_{NS}} e^{-\Delta\epsilon_{rd}}}$$

takes on values < 1

Genome regulation

Genetic networks: Models (activation + repression)

STATE	RENORMALIZED WEIGHT
	1
	$\frac{P}{N_{NS}} e^{-\Delta\epsilon_{pd}/k_B T}$
	$\frac{A}{N_{NS}} e^{-\Delta\epsilon_{ad}/k_B T}$
	$\frac{P}{N_{NS}} \frac{A}{N_{NS}} e^{-(\Delta\epsilon_{pd} + \Delta\epsilon_{ad} + \epsilon_{ap})/k_B T}$
	$\frac{R}{N_{NS}} e^{-\Delta\epsilon_{rd}/k_B T}$
	$\frac{A}{N_{NS}} \frac{R}{N_{NS}} e^{-(\Delta\epsilon_{pd} + \Delta\epsilon_{rd})/k_B T}$

PB.6 19.2.4

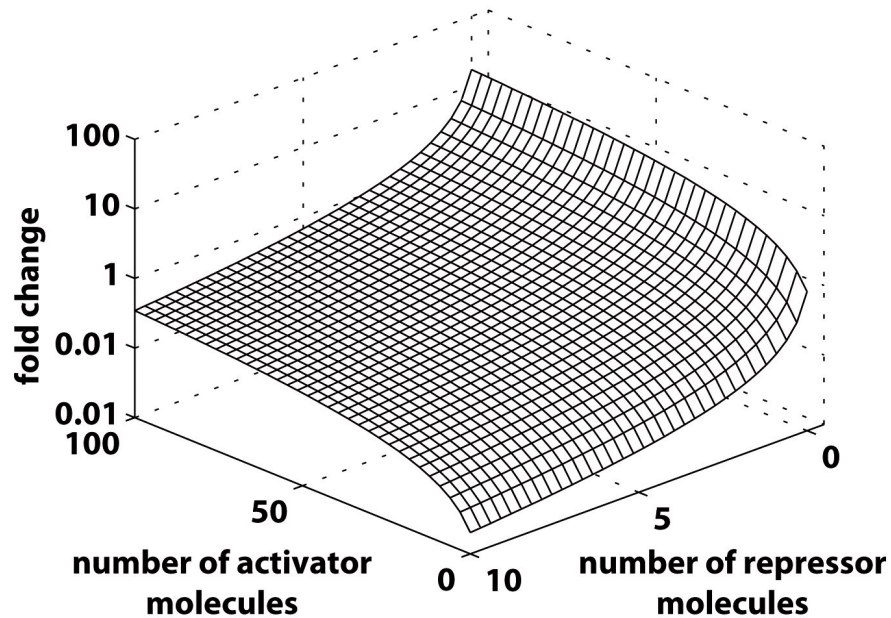
$$P_{\text{bound}} = \frac{1}{1 + \left[\frac{N_{NS}}{P F_{\text{reg}}^{\text{AR}}(A, R)} \right] e^{\beta \Delta \epsilon_{pd}}}$$

like changing [P]

Note: $F_{\text{reg}}^A(A) \neq F_{\text{reg}}^R(R) \neq F_{\text{reg}}^{\text{AR}}(A, R)$

Genome regulation

Genetic networks: Models (activation + repression)



Lecture 10: Genomes

Summary:

- Processes related to the "central dogma of molecular biology" include transcription, translation, and replication.
- The rates of such processes are governed by enzymatic activity, and can limit achievable rates of cell division.
- Thermodynamic models of gene expression are based on estimating the probability of RNA polymerase to bind to a gene promoter.
- Activators and repressors modify the probability of binding in distinct ways, and their effects can be generalized through a regulation factor, F_{reg} .

Lecture 11: Genomes

Goal: Model interactions between gene regulatory elements,
discuss common motifs in genetic circuits

Watch Prof. Uri Alon video before class

Answer guiding questions

In-class small group discussions

Reading : Alon Network Motifs