

Lecture 10: Genomes

Goal: Model to obtain insights into how the same DNA sequence can result in a diversity of gene expression profiles

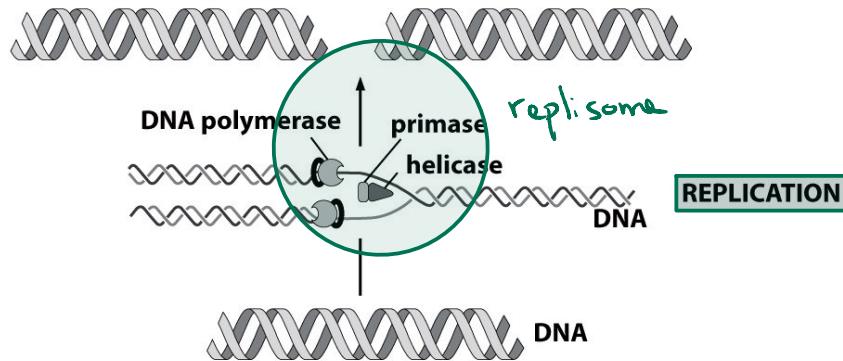
<https://youtu.be/nQQJNlJbzd8?si=1QLyWGcKHZ0pvAi1>

- Central dogma of molecular biology
- Genome regulation
- Genetic networks

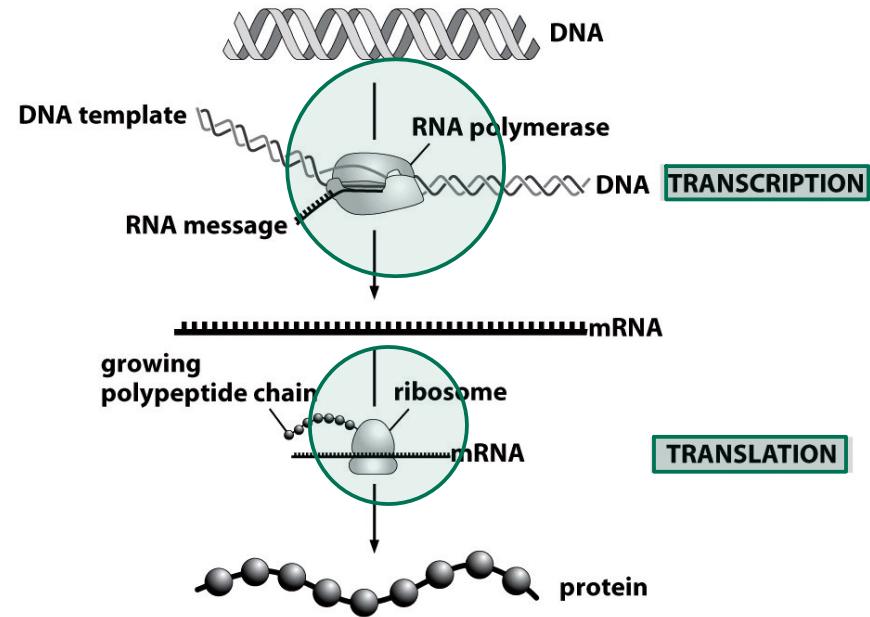
PBOC Chapter 3.2.1, 6.1.2 (refresh Lecture 06), 19.2 (except 19.2.5)

Central dogma of molecular biology

Recall (Lecture 1, 6) *Processes of the central dogma*



$\text{DNA} \rightarrow \text{DNA}$
replisome: DNA polymerase
+ primase + helicase



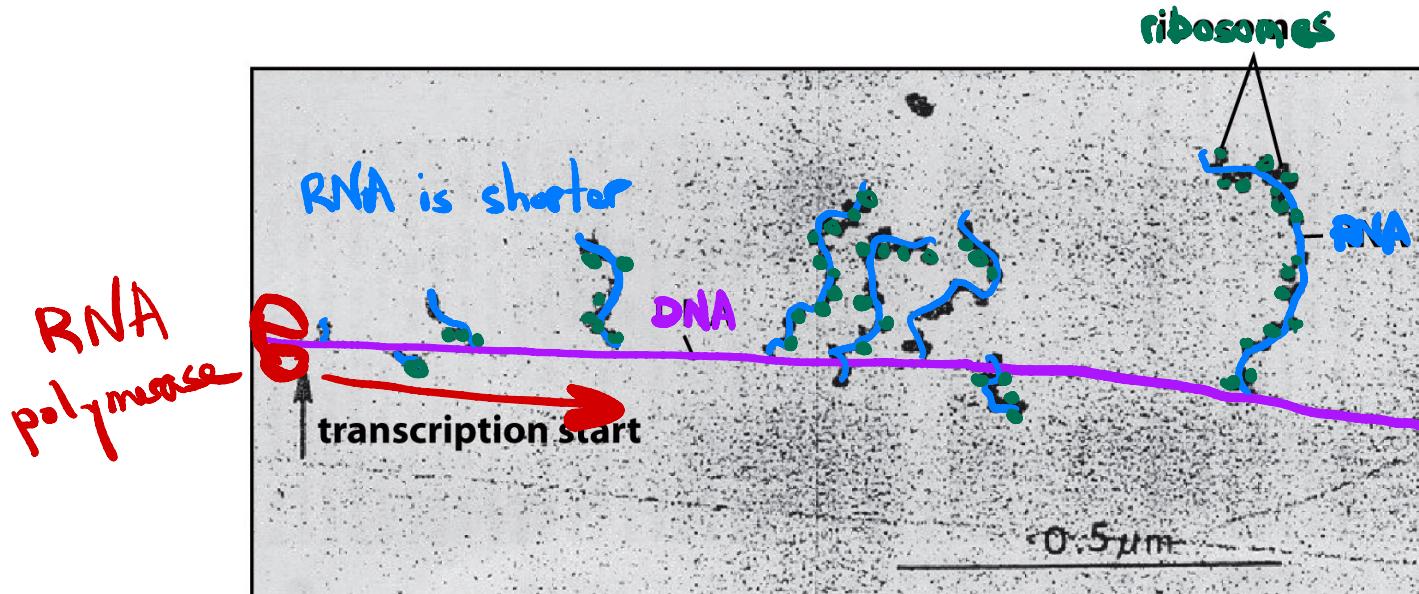
$\text{DNA} \rightarrow \text{mRNA} \rightarrow \text{protein}$

<https://youtu.be/gG7uCskUOrA?si=c6mmpHITsrmlUtaF>

<https://youtu.be/TNKWgcFPHqw?si=IGHHDFNOwNdEYNT7>

Central dogma of molecular biology

Processes of the central dogma



In bacteria, ribosomes can bind to RNA even before it has finished being transcribed

Central dogma of molecular biology

Processes of the central dogma

What are the rates of processes involved in the central dogma?

DNA replication Given replication rate per replisome of 250-1000 bp/s, and *E. coli* genome size of 5×10^6 bp, what range of times should it take to replicate the genome?

transcription Given an average protein molecular weight of 46 kDa, an average amino acid molecular weight of 110 Da, and a transcription rate of 40 nucleotides/s, how long should it take on average to make an mRNA transcript?

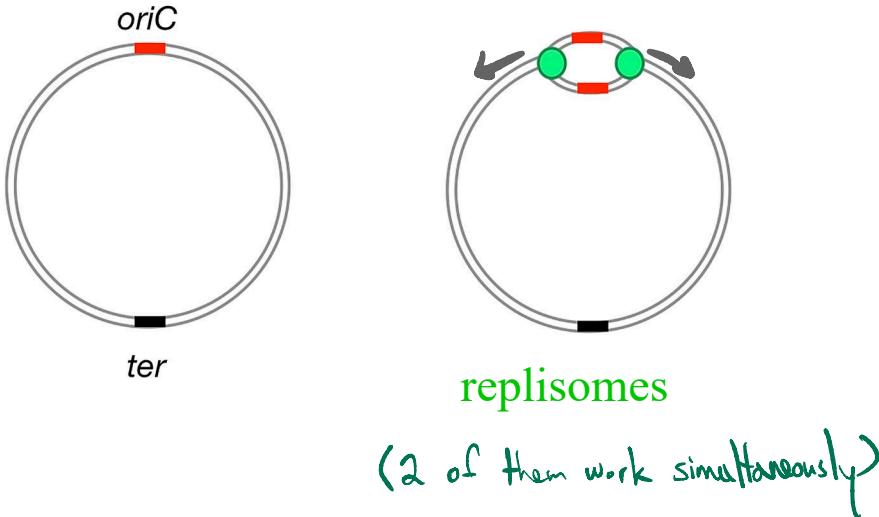
translation Given an *E. coli* cell cycle time of 3,000 seconds, and the facts that the number of proteins must double during this time, and that there are ~20,000 ribosomes per cell, what is the rate of translation per ribosome (units aa/second)? If needed, look back at Lecture 2 where we estimated the number of proteins per cell.

Central dogma of molecular biology

Processes of the central dogma: Replication

Given replication rate per replisome of 250-1000 bp/s, and *E. coli* genome size of 5×10^6 bp, what range of times should it take to replicate the genome?

bacteria contain a single circular genome



$$\frac{5 \times 10^6 \text{ bp}}{2 \cdot 250 \text{ bp/s}} = 10^4 \text{ s}$$

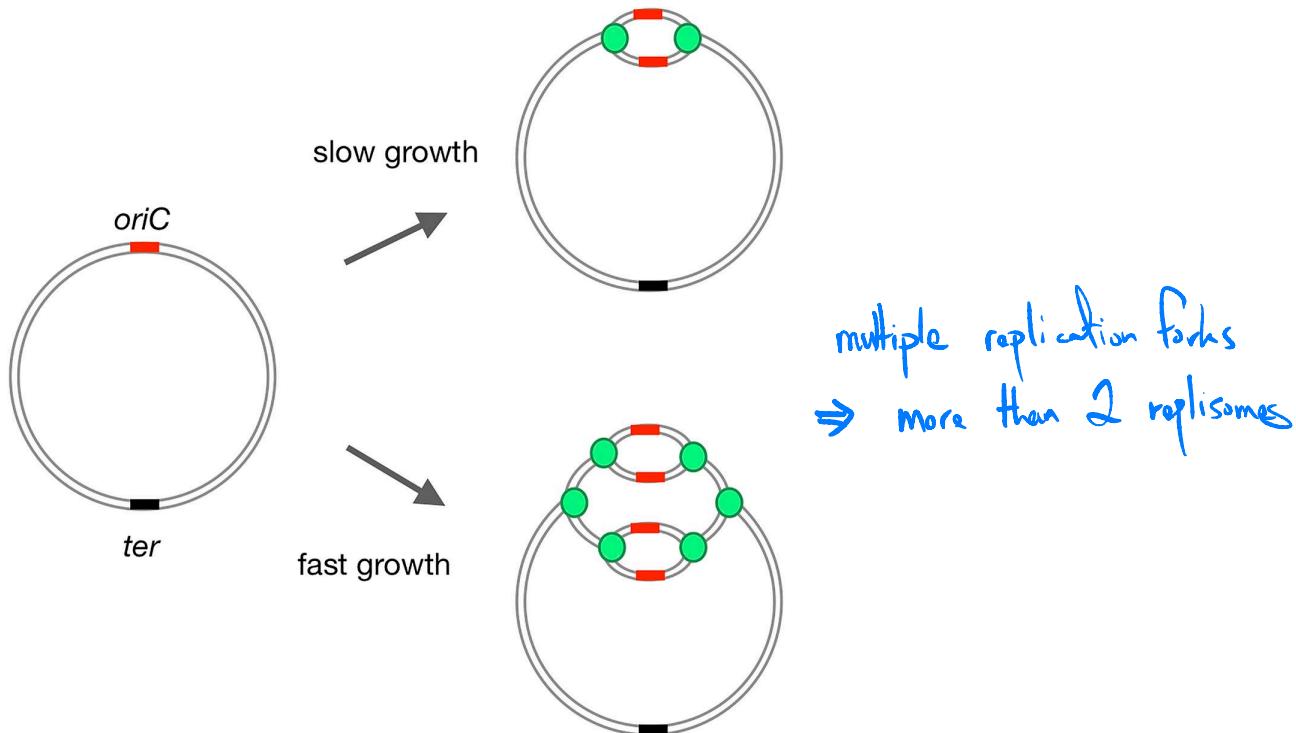
$$\frac{5 \times 10^6 \text{ bp}}{2 \cdot 1000 \text{ bp/s}} = 2.5 \times 10^3 \text{ s}$$

one to a few hours.

Central dogma of molecular biology

Processes of the central dogma: Replication

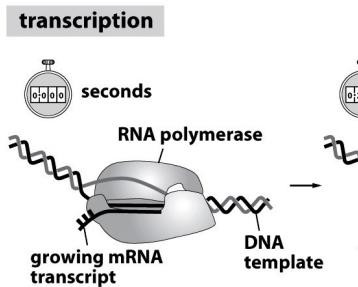
E. coli can divide in much less than 3000 seconds, in fact, as little as 1000 seconds



Central dogma of molecular biology

Processes of the central dogma: Transcription

Given an average protein molecular weight of 46 kDa, an average amino acid molecular weight of 110 Da, and a transcription rate of 40 nucleotides/s, how long should it take on average to make an mRNA transcript?



average protein: $\frac{4.6 \times 10^4}{110}$ Da/protein = 420 aa/protein

Each aa is encoded by three nucleotides (a "codon")

$$\left(1260 \text{ nucleotides/protein} \right) / \left(40 \text{ nucleotides/s} \right)$$

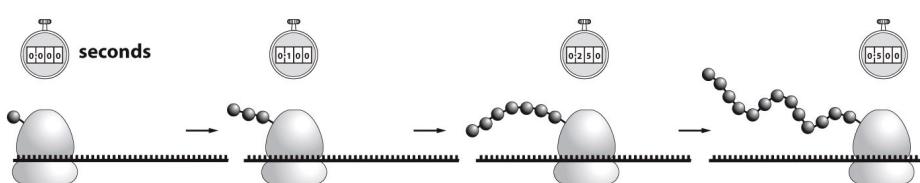
$$= 31 \text{ seconds}$$

Central dogma of molecular biology

Processes of the central dogma: Translation

Given an E. coli cell cycle time of 3,000 seconds, and the facts that the number of proteins must double during this time, and that there are ~20,000 ribosomes per cell, what is the rate of translation per ribosome (units aa/second)? If needed, look back at Lecture 2 where we estimated the number of proteins per cell. 3×10^6 proteins/cell

protein synthesis



$$\frac{3 \times 10^6 \text{ proteins}}{2 \times 10^4 \text{ ribosomes}} = 150 \frac{\text{proteins}}{\text{ribosome}} \text{ (per cell cycle)}$$

$$150 \frac{\text{proteins}}{\text{ribosome}} \cdot 1260 \frac{\text{nucleotides}}{\text{protein}}$$

$$/ 3000 \text{ s} \approx 63$$

$$\frac{\text{nucleotides}}{\text{ribosome} \cdot \text{s}} \approx 21 \frac{\text{aa}}{\text{ribosome} \cdot \text{s}}$$

(last question)
or 420 aa/protein

Around 20 s to make a protein
About same time as to make mRNA.

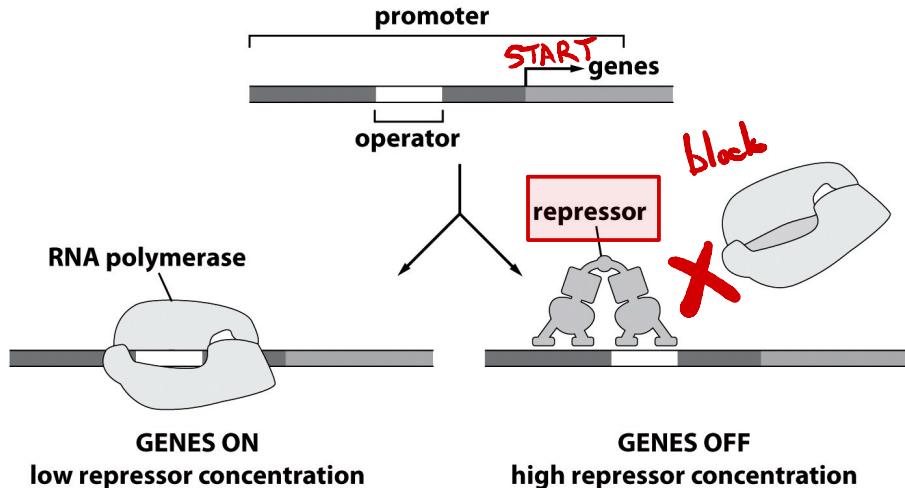
Genome regulation

Statistical mechanics of gene expression

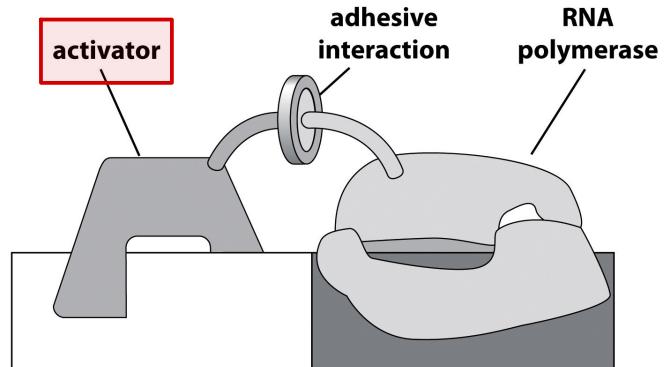
Review Lecture 6 annotated slides 13-20, along with PBoC 6.1.2

Genome regulation

Genetic networks: Molecules negative regulation



positive regulation



Repressors and activators change the probability of RNA polymerase binding to the promoter of a gene

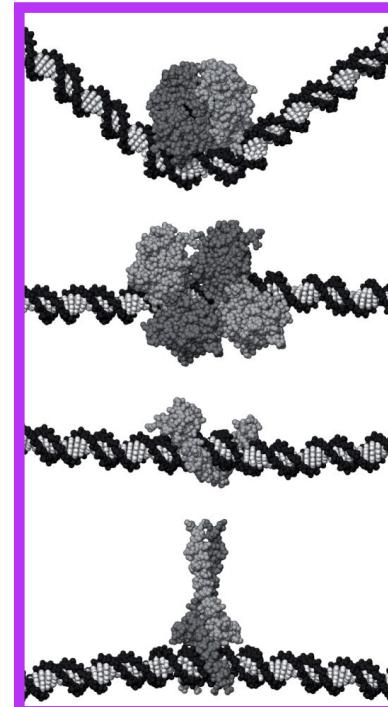
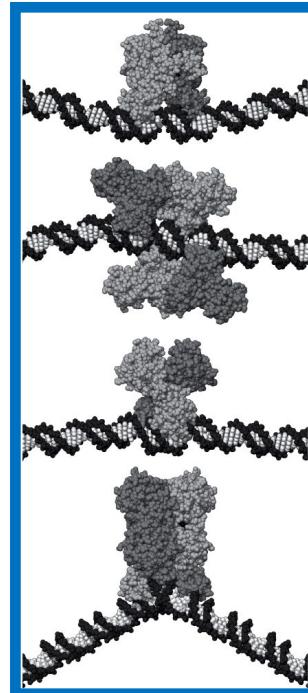
Genome regulation

Genetic networks: Molecules

Repressors and activators can both bind to DNA and deform it in the promoter region of a gene.

$$X_1 \rightarrow Y$$

Repressors bind to the promoter site, to block RNA polymerase from binding

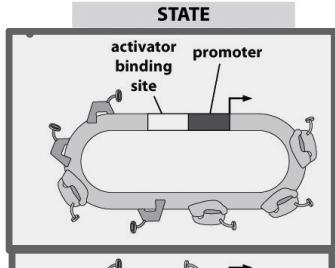


$$X_2 \rightarrow Y$$

Activators bind upstream of the promoter site, to enhance RNA polymerase binding

Genome regulation

Genetic networks: Models (activation)



RENORMALIZED WEIGHT

1

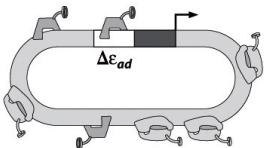
renormalize : divide each weight
by unoccupied state

activator
? promoter
unoccupied

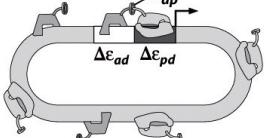
promoter
occupied

activator
occupied

$$\frac{P}{N_{NS}} e^{-\Delta \varepsilon_{pd}/k_B T}$$



$$\frac{A}{N_{NS}} e^{-\Delta \varepsilon_{ad}/k_B T}$$



$$\frac{P}{N_{NS}} \frac{A}{N_{NS}} e^{-(\Delta \varepsilon_{pd} + \Delta \varepsilon_{ad} + \varepsilon_{ap})/k_B T}$$

activator
is
promoter
occupied

P # of polymerases
A # of activators
R # of repressors
 N_{NS} # of boxes

NS non-specific

Energies:

$$\Delta \varepsilon_{pd} = \varepsilon_{pd}^s - \varepsilon_{pd}^{NS}$$

pd polymerase-DNA

$$\Delta \varepsilon_{ad} = \varepsilon_{ad}^s - \varepsilon_{ad}^{NS}$$

ad activator-DNA

ε_{ap}

ap activator-polymerase

Genome regulation

Genetic networks: Models (activation)

$$P_{\text{bound}} = \frac{\text{states with RNAP bound}}{\text{all states}} = \frac{\text{+}}{\text{+} + \text{+} + \text{+}}$$

..

PBOC 19.1-19.6

$$= \frac{1 + \left[N_{\text{NS}} \cancel{P F_{\text{reg}}^A(A)} \right] e^{\beta \Delta \varepsilon_{\text{pd}}}}{1 + \frac{A}{N_{\text{NS}}} e^{-\beta \Delta \varepsilon_{\text{ad}}}}$$

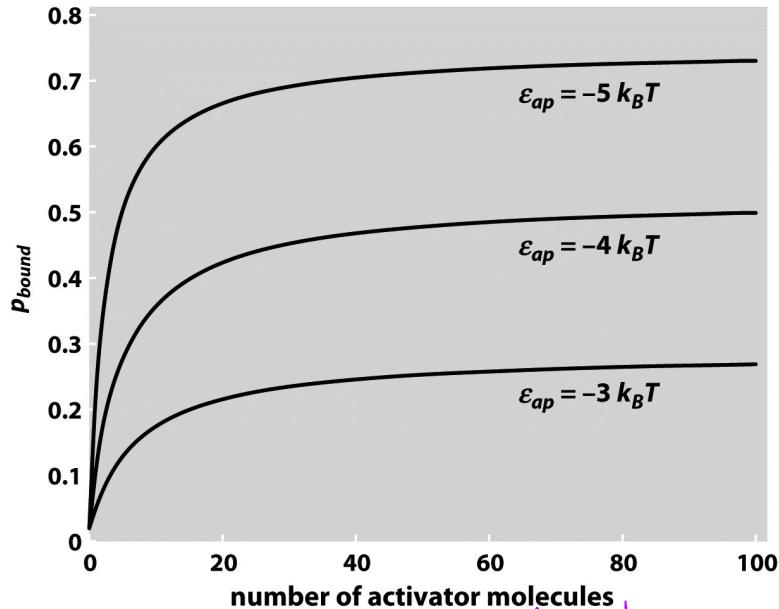
where $F_{\text{reg}}^A(A) = \frac{1 + \frac{A}{N_{\text{NS}}} e^{-\beta \Delta \varepsilon_{\text{ad}}}}{1 + \frac{A}{N_{\text{NS}}} e^{-\beta \Delta \varepsilon_{\text{pd}}}}$

like changing $[P]$

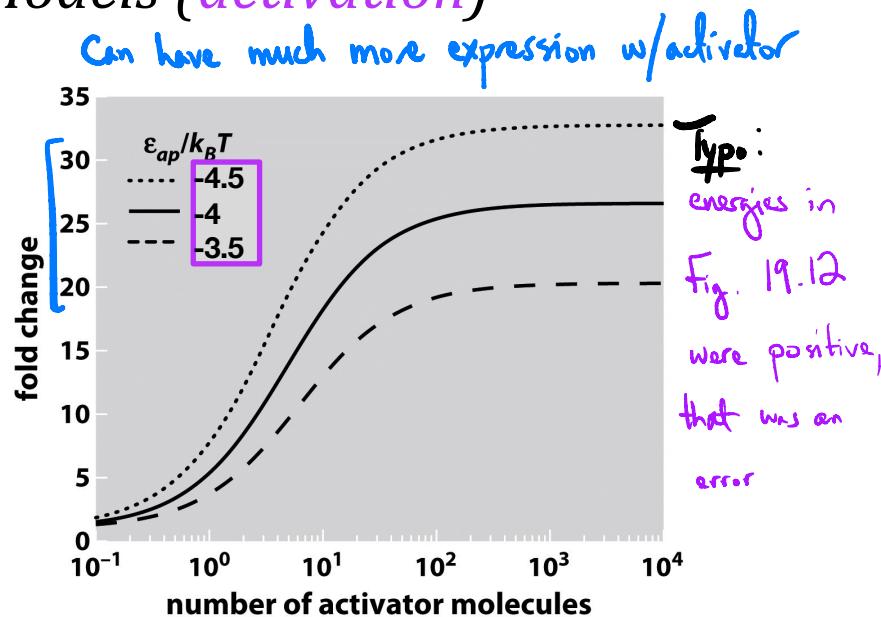
takes on values > 1 for $\varepsilon_{\text{ap}} < 0$

Genome regulation

Genetic networks: Models (*activation*)



P_{bound} increases with $|\epsilon_{ap}|$ and $[A]$ concentration of activator molecules



normalized to display fold-change $P_{\text{bound}}(A)/P_{\text{bound}}(A=0)$

$$= \frac{1 + [N_{\text{ns}}/P] e^{\beta \Delta \epsilon_{\text{p}2}}}{1 + [N_{\text{ns}}/P F_{\text{frag}}] e^{\beta \Delta \epsilon_{\text{p}2}}} \approx \frac{[N_{\text{ns}}/P] e^{\beta \Delta \epsilon_{\text{p}2}}}{[N_{\text{ns}}/P F_{\text{frag}}] e^{\beta \Delta \epsilon_{\text{p}2}}} = F_{\text{reg}}$$

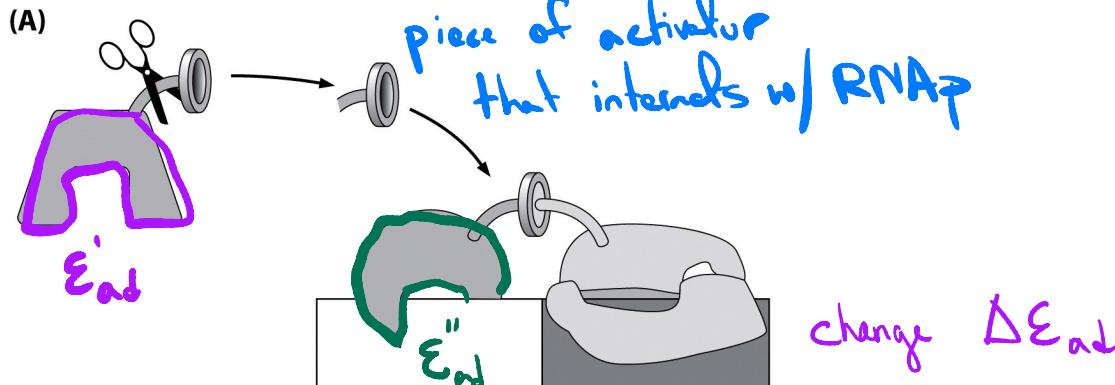
Hypo:
energies in
Fig. 19.12
were positive,
that was an
error

Genome regulation

How do we know?

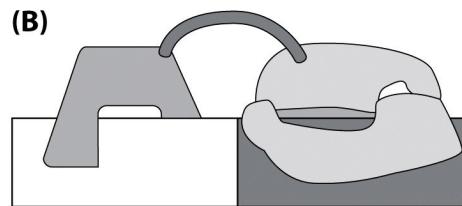
Genetic networks: Models (activation)

mix-and-match activator elements



Experiment:
measure μ_{ad}
for (A) : (B)

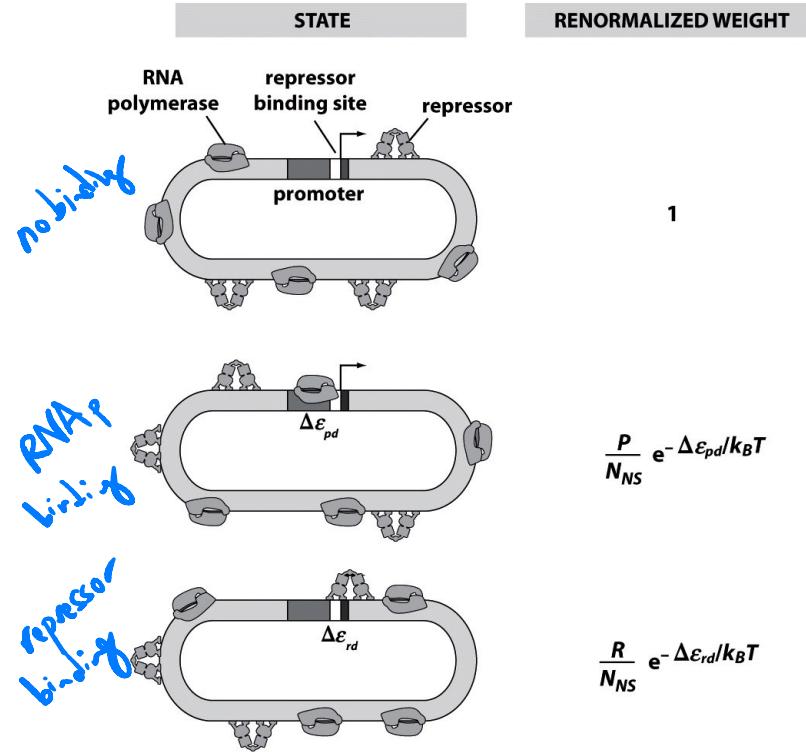
change $\Delta\epsilon_{ad}$



$\epsilon_{ad} \rightarrow \infty$, two states (1:4)

Genome regulation

Genetic networks: Models (repression)



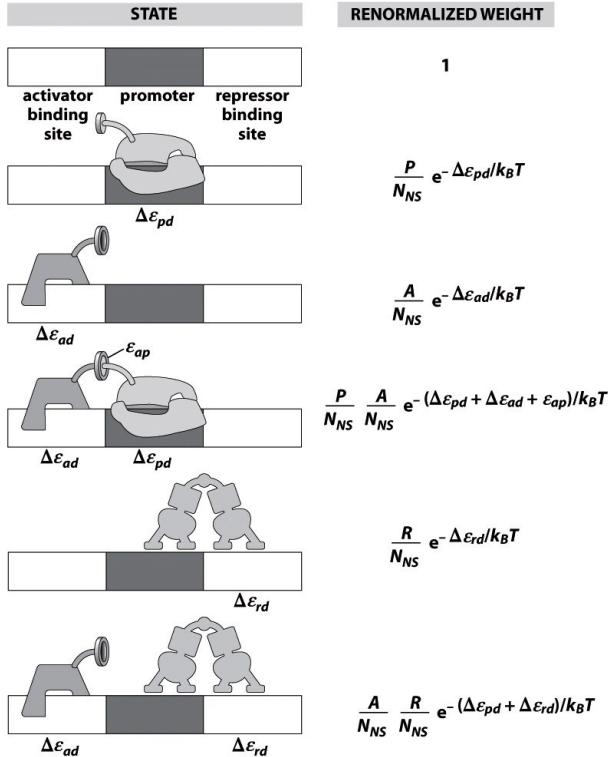
$$P_{\text{bind}} = \frac{1}{1 + \left[\frac{N_{\text{NS}}}{F_{\text{reg}}^R(R)} \right] e^{\beta \Delta \epsilon_{pd}}}$$

where $F_{\text{reg}}^R(R) = \frac{1}{1 + \frac{R}{N_{\text{NS}}} e^{-\Delta \epsilon_{rd}}}$

takes on values < 1

Genome regulation

Genetic networks: Models (activation + repression)



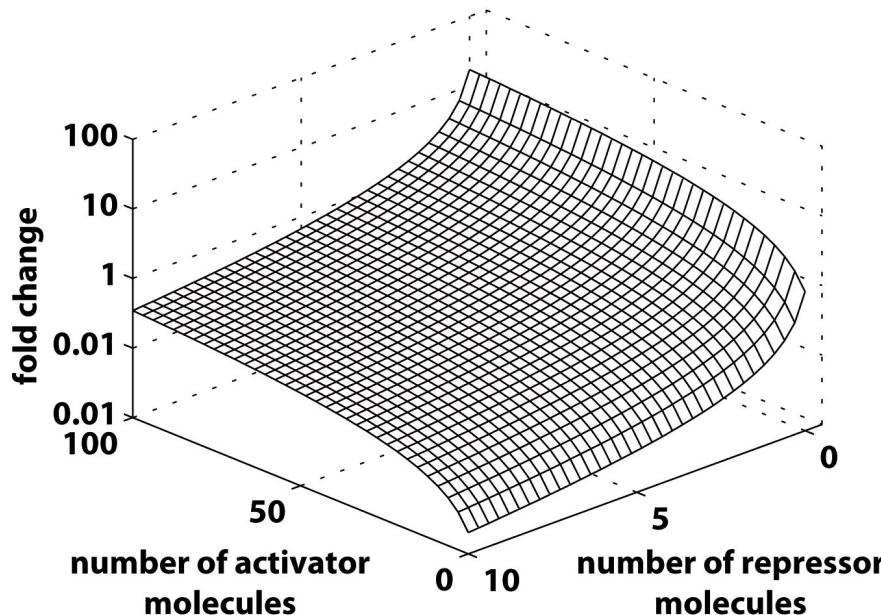
PB. C 19.2.4

$$P_{\text{bound}} = \frac{1}{1 + \left[\frac{N_{NS}}{P F_{\text{reg}}^{AR}} \right] e^{\beta \Delta \varepsilon_{pd}}}$$

Note: $F_{\text{reg}}^A(A) \neq F_{\text{reg}}^R(R) \stackrel{\text{like changing } [P]}{\neq} F_{\text{reg}}^{AR}(A, R)$

Genome regulation

Genetic networks: Models (activation + repression)



Lecture 10: Genomes

Summary:

- Processes related to the "central dogma of molecular biology" include transcription, translation, and replication.
- The rates of such processes are governed by enzymatic activity, and can limit achievable rates of cell division.
- Thermodynamic models of gene expression are based on estimating the probability of RNA polymerase to bind to a gene promoter.
- Activators and repressors modify the probability of binding in distinct ways, and their effects can be generalized through a regulation factor, F_{reg} .

Lecture 11: Genomes

Goal: Model interactions between gene regulatory elements, discuss common motifs in genetic circuits

Watch Prof. Uri Alon video before class

Answer guiding questions

In-class small group discussions

Reading : Alon Network Motifs