

# Lecture 8: Diffusion in the cell

Goal: Role of Brownian motion in living systems.  
Compute the time to travel a distance, model diffusion  
in gradient.

- Brownian motion
- Concentration fields and diffusive dynamics

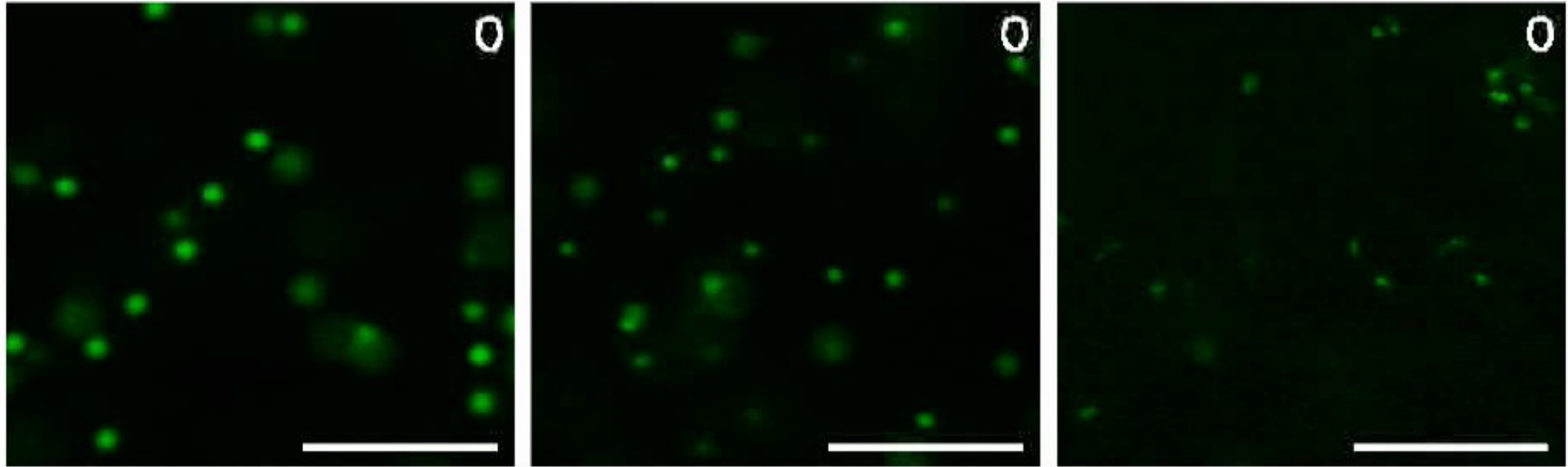
PBOC Chapter 13.1, 13.2.1-13.2.3

# Diffusion in the cell

*Active vs. passive transport*

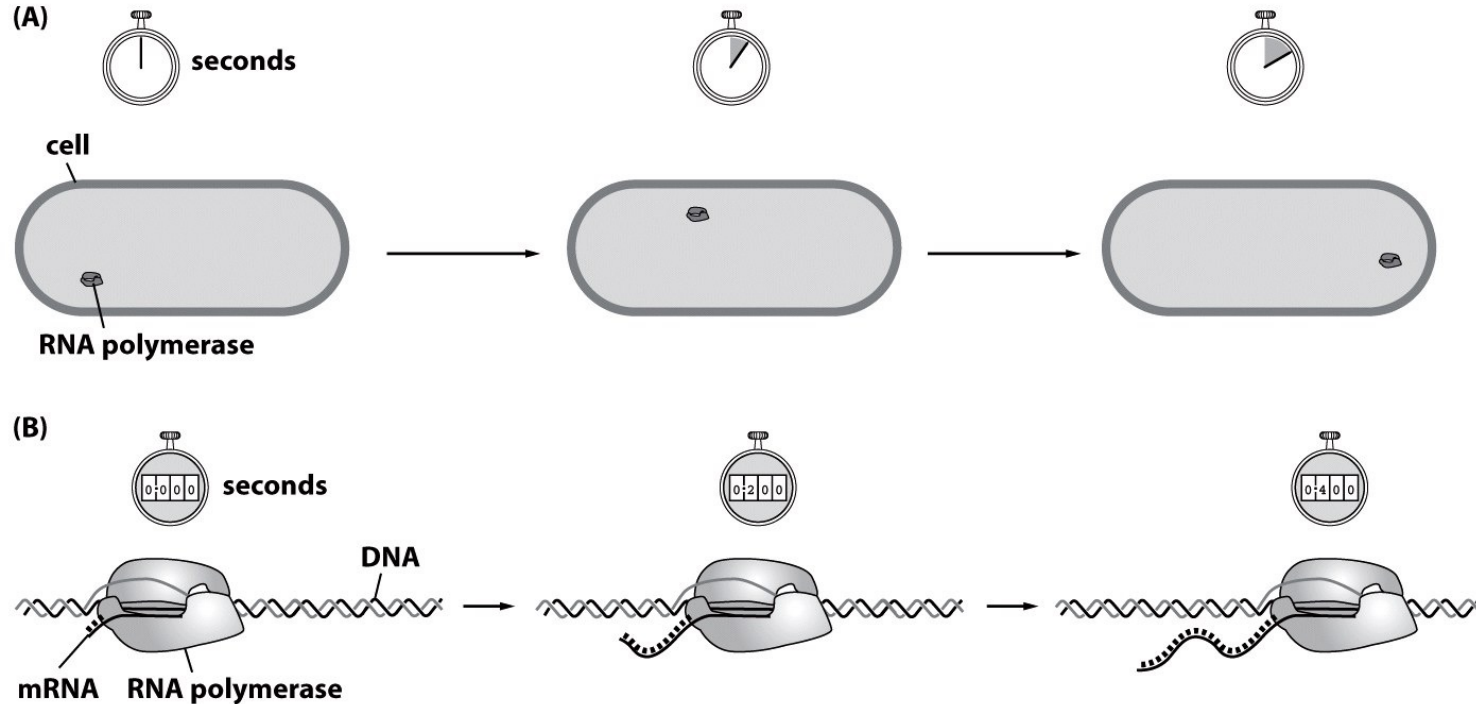
# Diffusion in the cell

*Particles in a fluid*



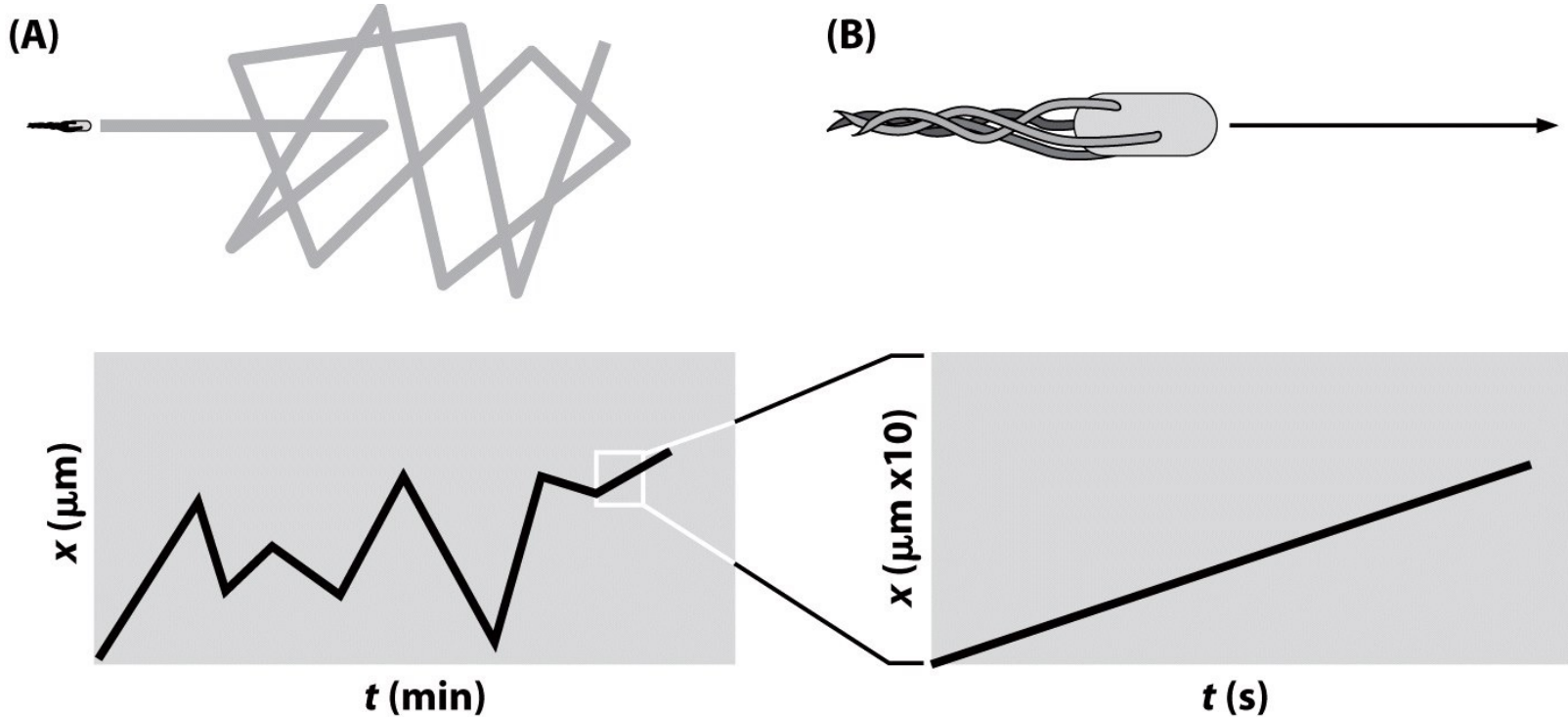
# Diffusion in the cell

## *Active vs. passive transport*



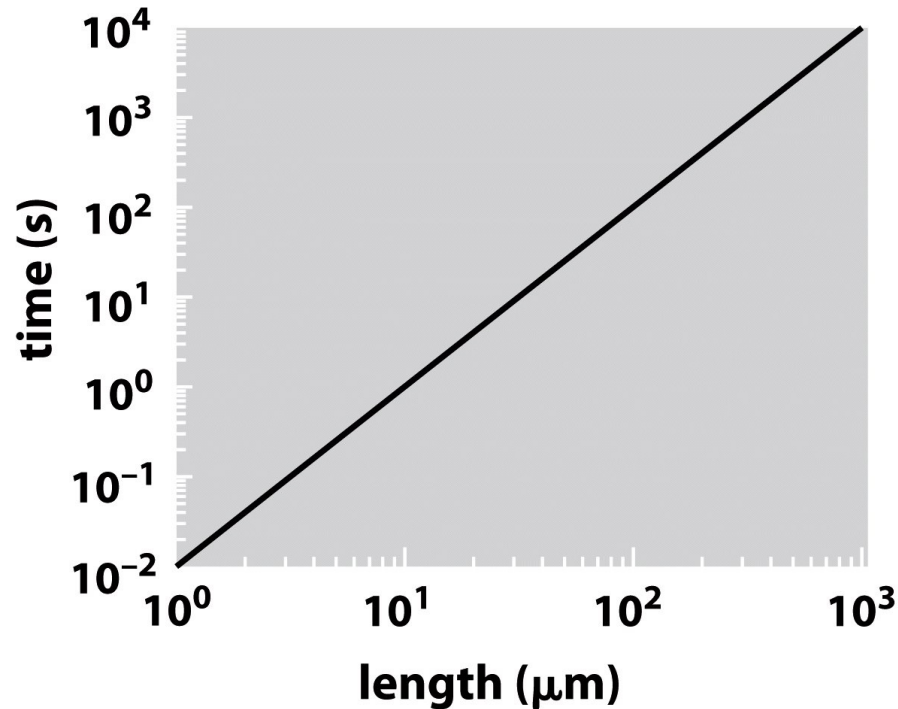
# Diffusion in the cell

## *Active vs. passive transport*



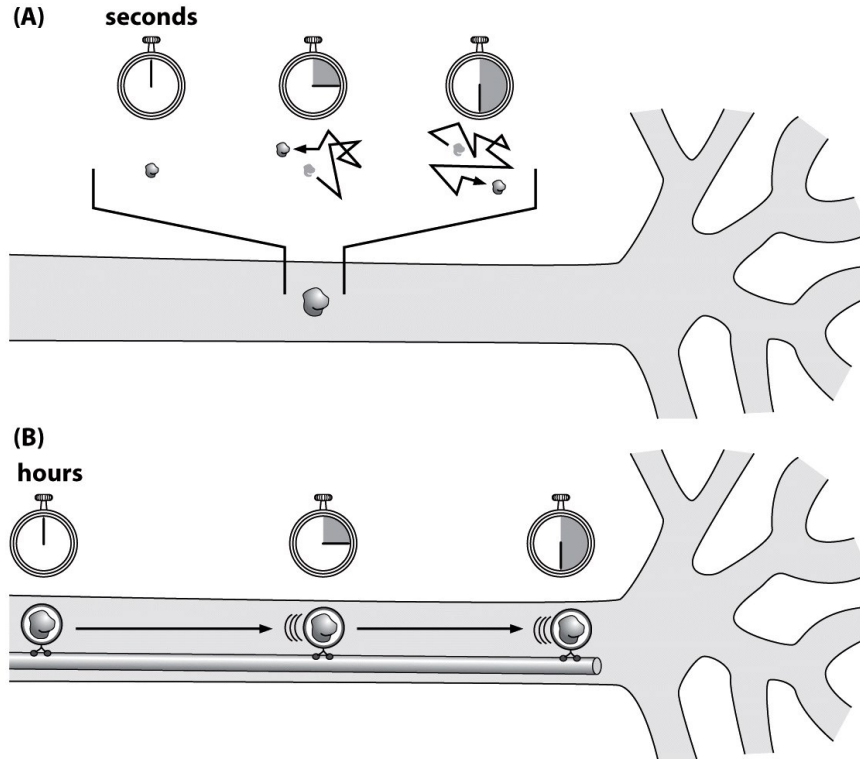
# Diffusion in the cell

*Time to diffuse biological distances*



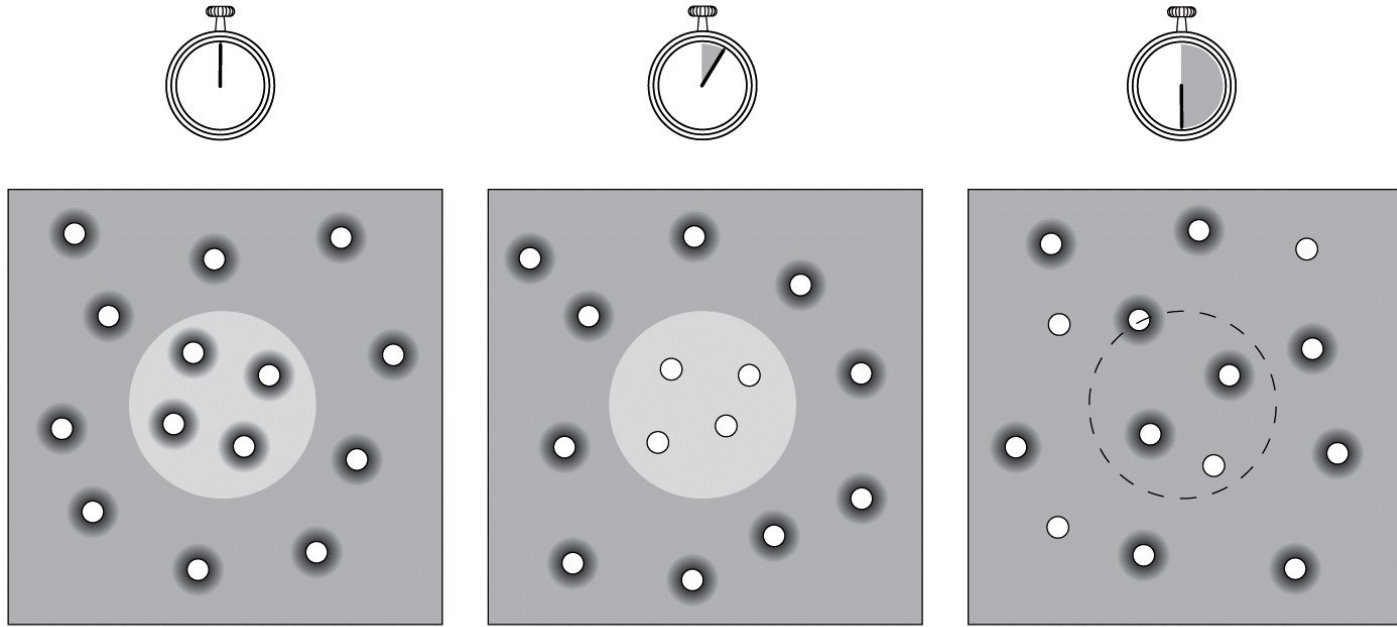
# Diffusion in the cell

## *Time to diffuse biological distances*



# Diffusion in the cell

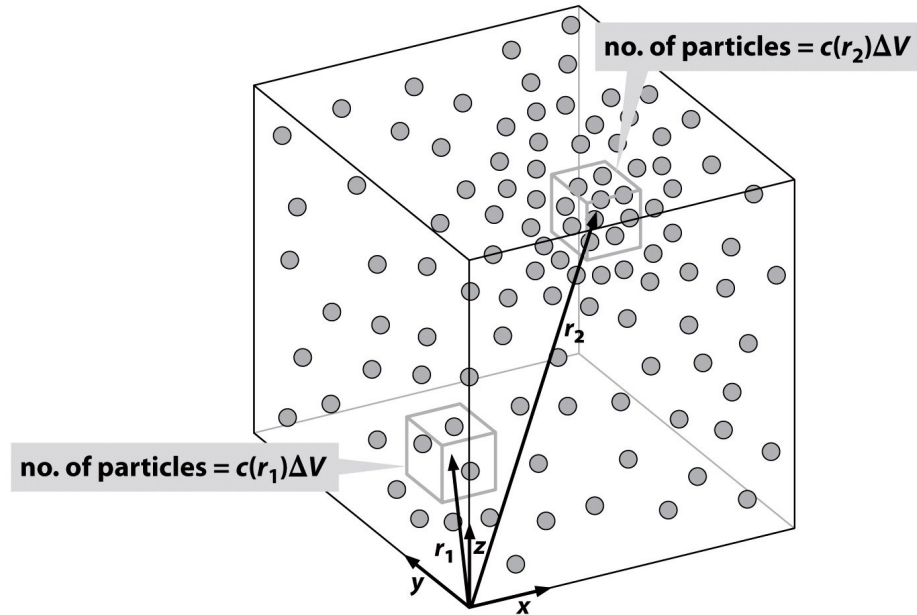
*Time to diffuse biological distances*





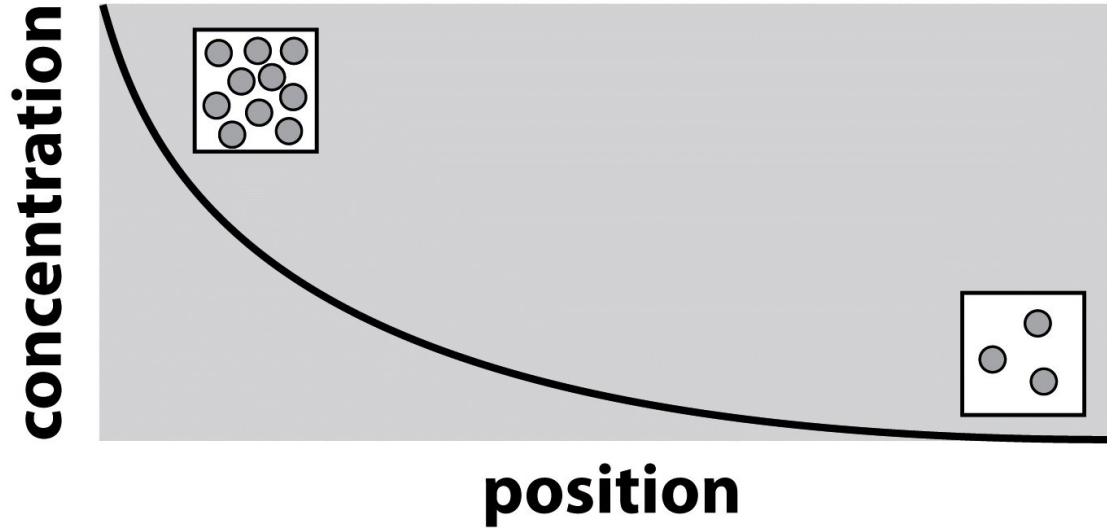
# Diffusion in the cell

## *Concentration fields and diffusion*



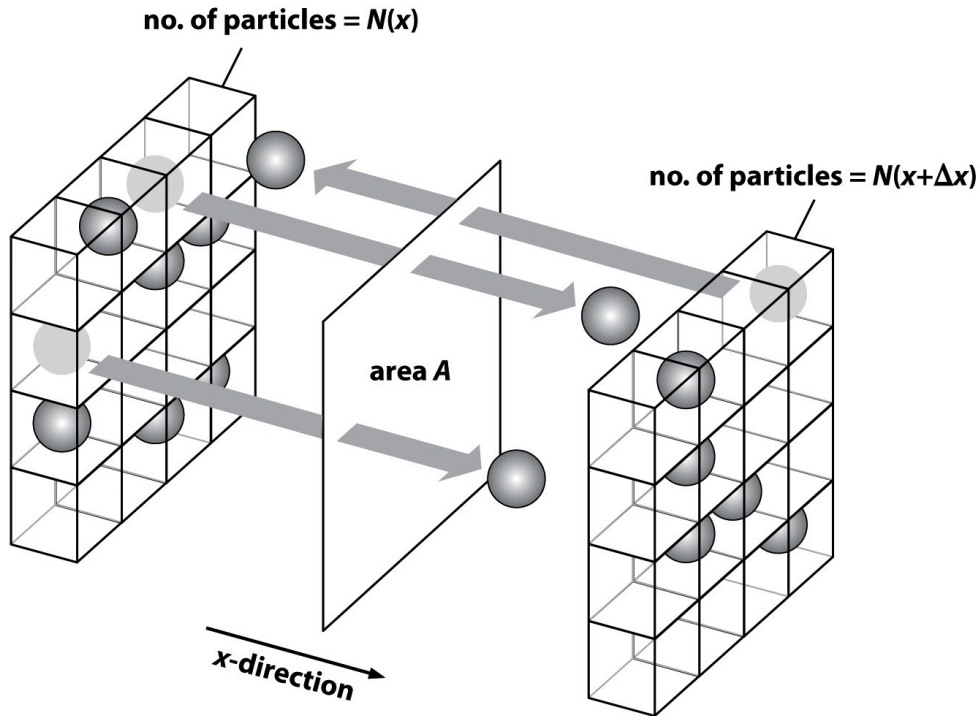
# Diffusion in the cell

*Concentration fields and diffusion*



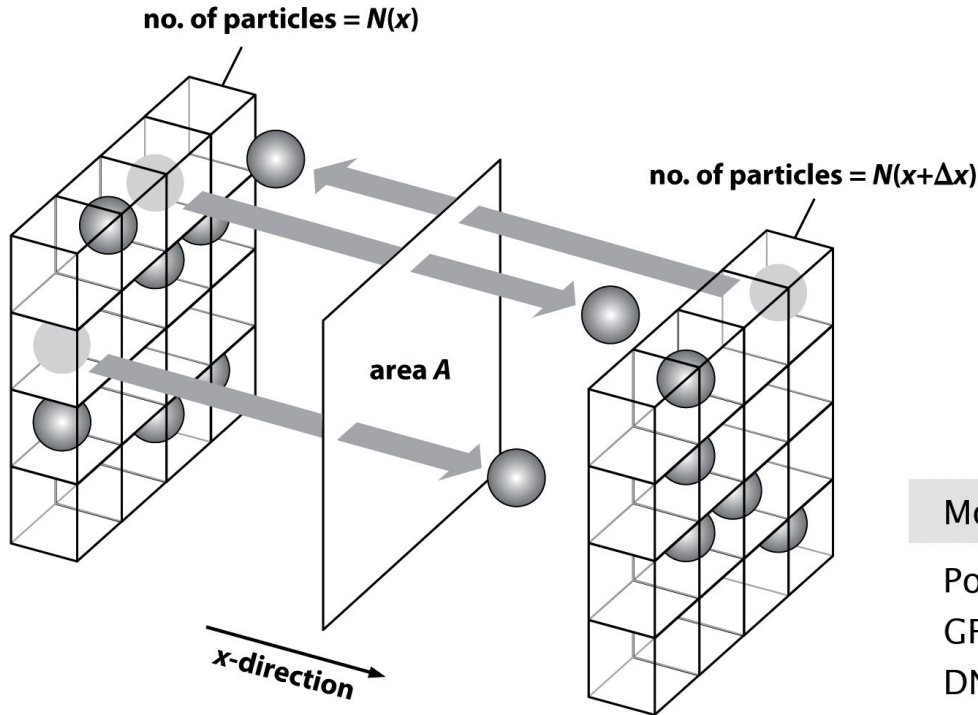
# Diffusion in the cell

## *Concentration fields and diffusion: Fick's Law*



# Diffusion in the cell

## *Concentration fields and diffusion: Fick's Law*



Molecule

Diffusion coefficient

Potassium ion in water

$\approx 2000 \mu\text{m}^2/\text{s}$

GFP in *E.coli*

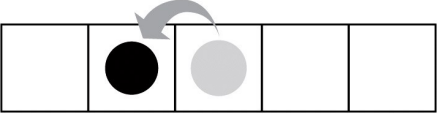
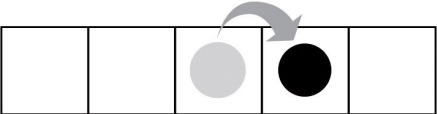
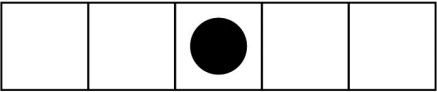
$\approx 7 \mu\text{m}^2/\text{s}$

DNA in yeast

$5 \times 10^{-4} \mu\text{m}^2/\text{s}$

# Diffusion in the cell

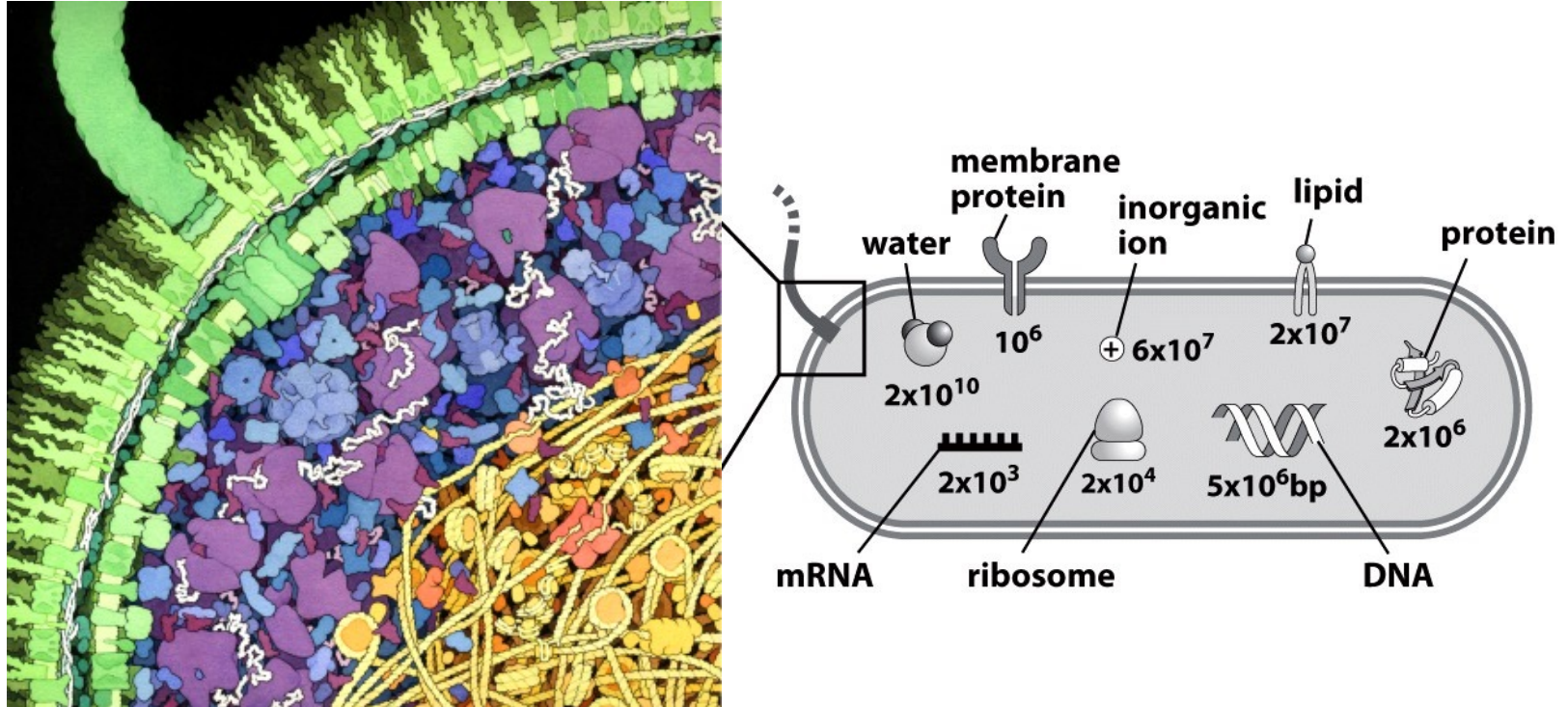
## *Summing over microtrajectories*

TRAJECTORY	WEIGHT
	$k\Delta t$
	$k\Delta t$
	$1-2k\Delta t$

# An ode to E. coli

Previously:

*Molecular census*



# Diffusion in the cell

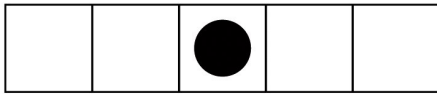
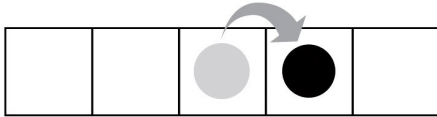
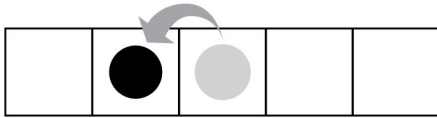
## *Diffusion in crowded environments*

What is the diffusion coefficient associated with “crowded” random walk?  
Assume fraction of occupied lattice sites  $\phi$

# Diffusion in the cell

*Summing over microtrajectories: Crowding (14.3.2)*

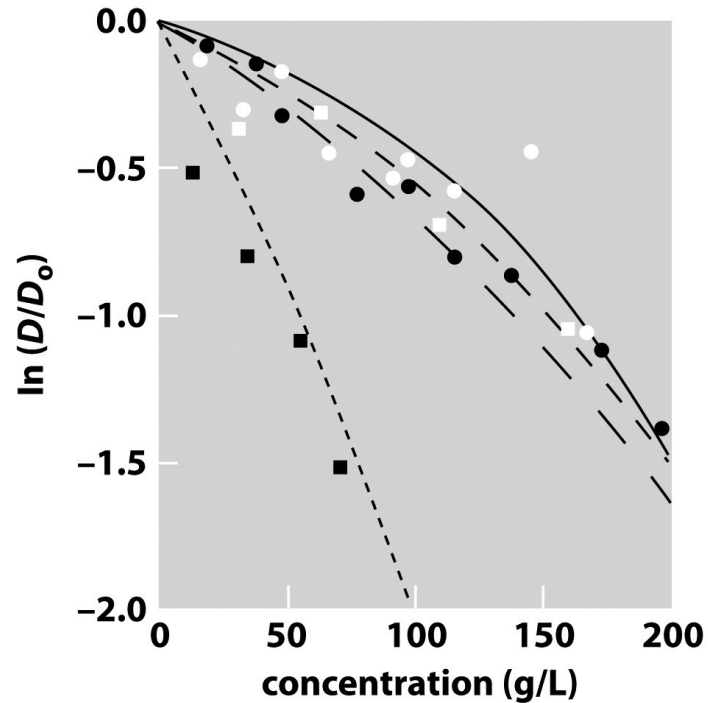
**TRAJECTORY**





# Transport in cellular systems

## *Diffusion in crowded environments*

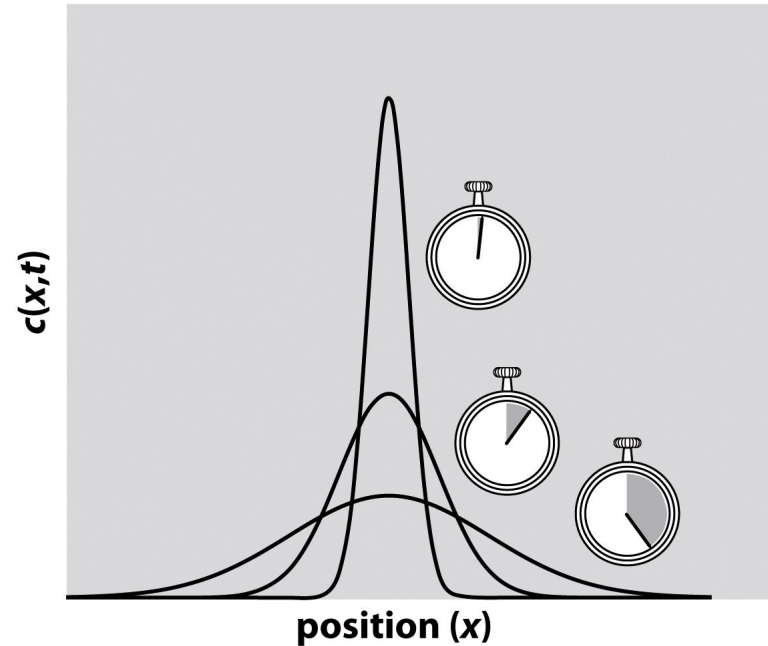


tracer diffusion in protein solution

Molecular species	$M_W$ [Da]	$r$
■ ribonuclease	12,400	2.3
□ ovalbumin	43,500	1.5
● BSA	70,000	1.3
○ aldolase	150,000	1

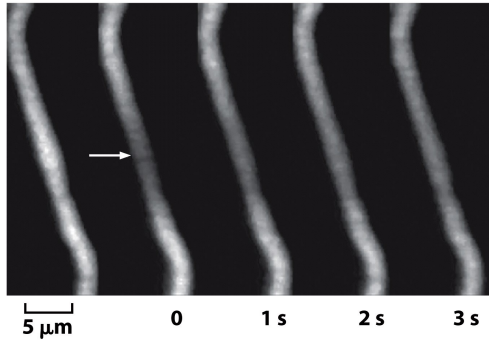
# Diffusion in the cell

*Solutions to the diffusion equation*



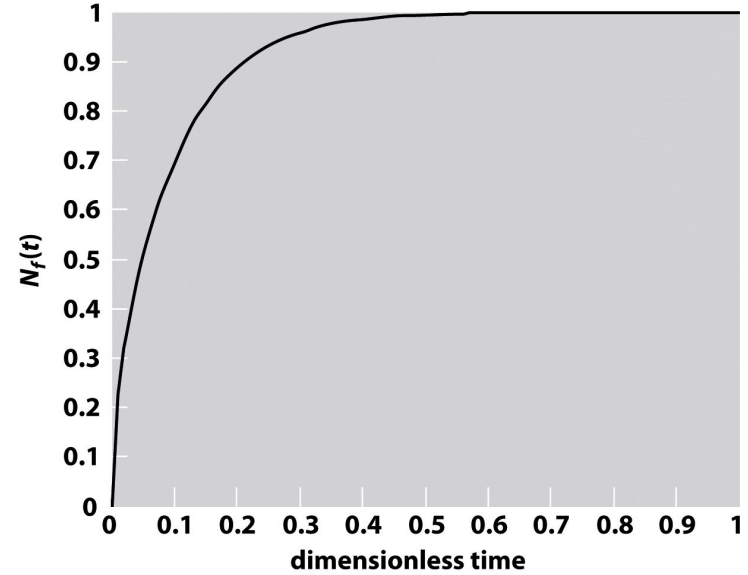
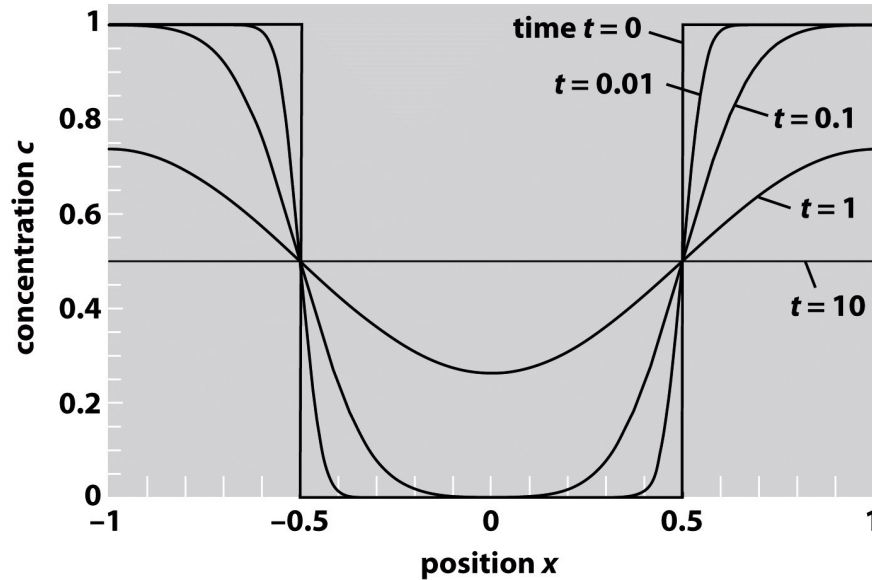
# Diffusion in the cell

## *Solutions to the diffusion equation*



# Diffusion in the cell

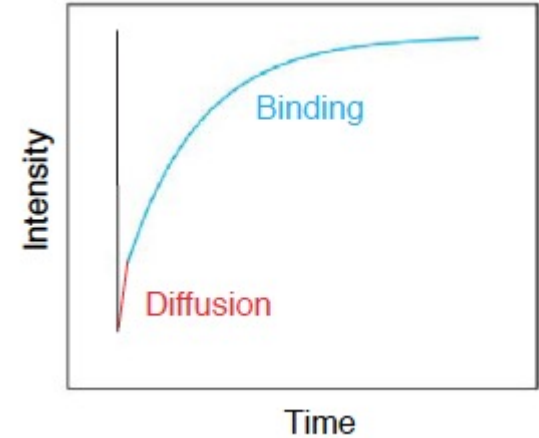
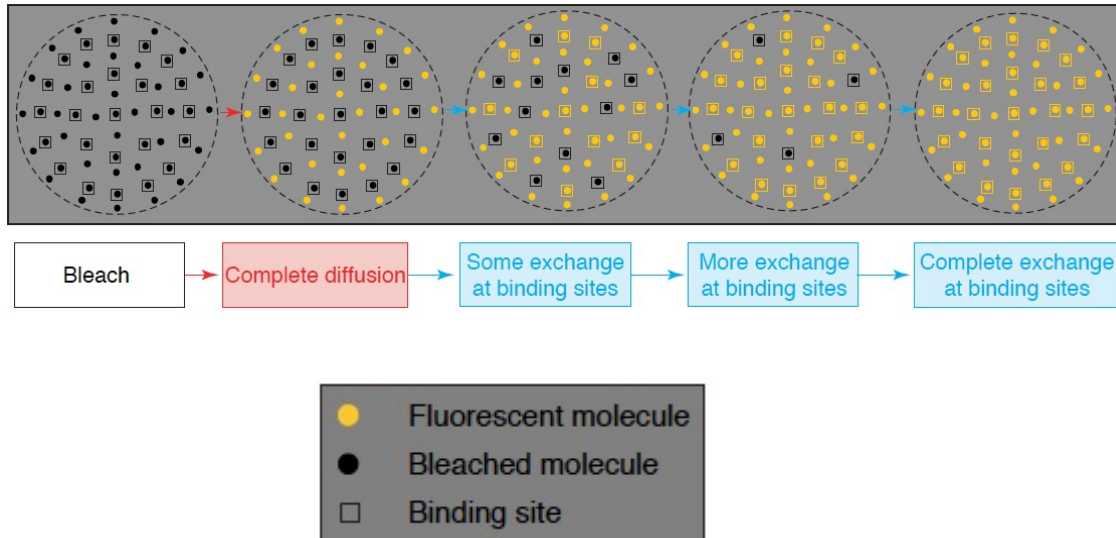
## *Solutions to the diffusion equation*



# Diffusion in the cell

## *Complexity: diffusion + binding*

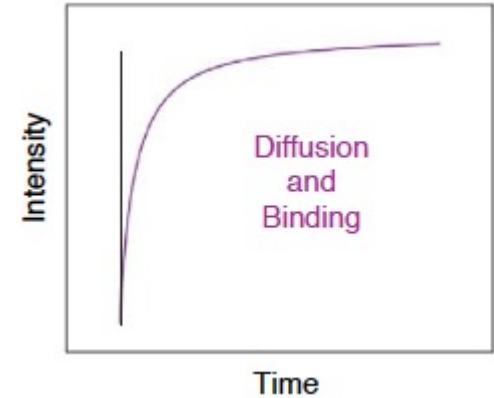
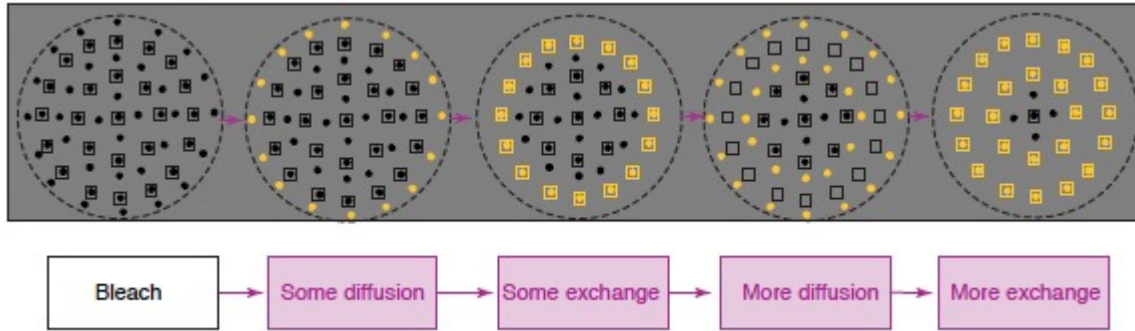
Limit case ( $t_{\text{diff}} \ll t_{\text{binding}}$ ): Two separable timescales



# Diffusion in the cell

*Complexity: diffusion + binding*

Case ( $t_{\text{diff}} \sim t_{\text{binding}}$ ): Mixing of dynamic modes



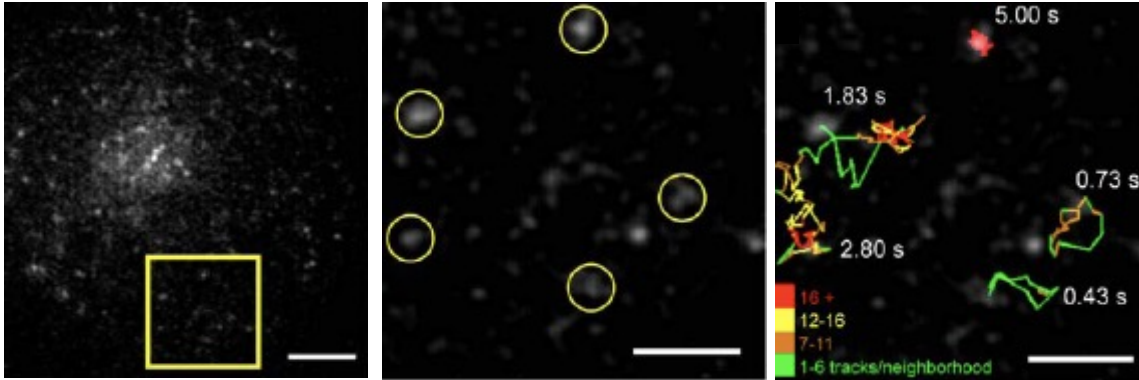
Anomalous diffusion:

$$\langle r^2(\tau) \rangle = 6D\tau$$

$$\langle r^2(\tau) \rangle = 6D\tau^\alpha = 6D(\tau)\tau$$

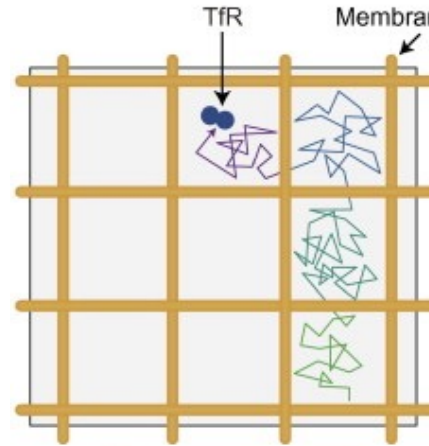
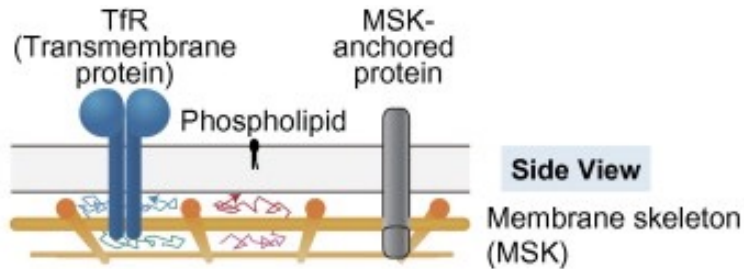
# Diffusion in the cell

## *Membrane proteins*

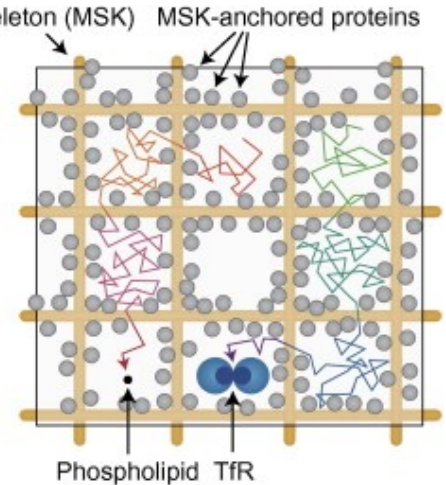


# Diffusion in the cell

## *Complexity: Caged motion*



Membrane-Skeleton “**Fence**”



Anchored-Protein “**Picket**”



# Lecture 8: Diffusion in the cell

Summary: