Chapter 1

All is not well with Classical Mechanics

1.1 Introduction

Throughout the history of physics, there have been several moments when our understanding of nature fundamentally changed. Accepting new ideas is always challenging, especially when they are radically different from existing, widely accepted concepts. A notable example is gravity; it was initially described as a force in Newtonian mechanics, but later redefined by Einstein as a geometric property of four-dimensional spacetime. Quantum physics is another such revolutionary example.

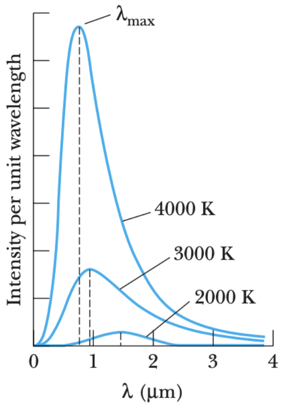
Documenting the entire history of quantum mechanics would take countless pages. Fortunately, there are many excellent references for those interested in how our understanding in this topic has evolved over time. We propose some at the end of this chapter. In these notes, we will explore some key milestones of quantum mechanics and examine their implications. These include the correct description of the black body radiation, the photoelectric effect, the Rutherford model for the atom, the Bohr model for the atom, and the concept of matter waves. [also SG]

This brief overview will offer insight into the need for a new theory that accurately explains many phenomena that remain inexplicable within the framework of classical physics.

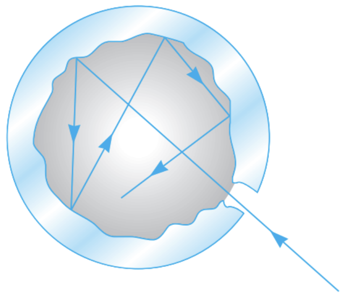
Disclaimer: This chapter shares the same title as the third chapter of *Principles of Quantum Mechanics by Ramamurti Shankar (1980)*. We believe this title properly reflects the content of this chapter and emphasizes the necessity for a new theory.

1.2 The Black-body radiation

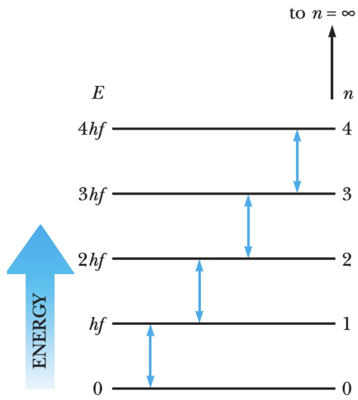
The problem of calculating the intensity of radiation from a heated cavity is regarded as the beginning of quantum physics. It is a matter of classical statistical mechanics. The solution to this problem was provided in 1900 by Max Planck, a strong proponent of classical thermodynamics. We will soon explore his approach. This is known as the blackbody problem.



(a) Emission from a glowing solid.



(b) A cavity representation of a blackbody.



(c) Allowed energy levels based on Planck's hypothesis.

Figure 1.1: The blackbody problem.

The physical scenario is as follows: we consider a hot, glowing solid at a specific temperature T, and our objective is to predict the radiation intensity at a given wavelength λ. Experiments from previous years have shown that glowing solids exhibit a continuous emission spectrum, as illustrated in Figure 1.1a. Notably, the amount of emitted radiation increases rapidly with temperature.

We can visualize this system as a cavity with a small opening, similar to the one shown in Figure 1.1b. Light enters the cavity through this opening, undergoes multiple reflections, and is eventually absorbed entirely. This serves as a nice illustration of a blackbody, which is defined as an object that absorbs all incident radiation and therefore appears black.

Planck originally conceived his formula for the spectral energy density $u(f, T)$ for black body radiation based on experimental observations.[1]. This formula reads

$$u(f,T) = \frac{8\pi h f^3}{c^3} \frac{1}{e^{hf/k_B T} - 1}, \tag{1.2.2}$$

where h is the Planck constant equal to $6.626 \cdot 10^{-34} Js$ and k_B is the Boltzmann constant equal to $1.380 \cdot 10^{-23} J/K$.

After publishing his formula, Planck was eager to derive it through rigorous theoretical justification, which he achieved shortly thereafter. He was convinced that blackbody

[1] Planck originally published his formula as

$$u(\lambda,T) = \frac{C_1}{\lambda^5} \frac{1}{e^{C_2/\lambda T} - 1}, \tag{1.2.1}$$

where $C_1 = 8\pi ch$ and $C_2 = hc/k_B$. He obtained the numerical values of h and k_B by comparing to the experimental data. Planck himself told upon receiving the Nobel prize that *"... even if the radiation formula is perfectly correct, it would after all have been only an interpolation formula found by luck guess-work..."*. Fortunately, he eventually succeeded in providing a proper physical explanation.

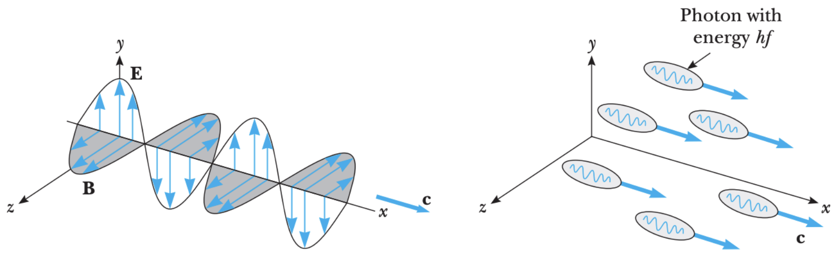


Figure 1.2: Left: Classical picture of light as an electromagnetic wave. Right: Light itself is composed of irreducible finite amounts or quanta of energy, called photons.

radiation was produced by resonators comprising the walls of the glowing cavity, all vibrating at different frequencies. According to Maxwell, each oscillator should emit radiation at a frequency corresponding to its vibration frequency. Furthermore, an oscillator of a given frequency could, in principle, possess any energy value and vary its amplitude continuously. Here, however, Planck introduced an alternative description. He proposed that the total energy of an oscillator with frequency f could only take integer multiples of the quantity hf:

$$E_n = nhf, \quad n = 1, 2, 3, \ldots, \tag{1.2.3}$$

where h is the Planck constant. The resulting energy spectrum will take the form of Figure 1.1c. The radiation emitted by this resonator corresponds to the energy change as it transitions to a lower-energy state, i.e., $\Delta E = hf$. By employing principles from statistical mechanics, Planck successfully derived his renowned formula. Max Planck proved that energy is quantized, meaning it is emitted or absorbed in discrete packets called "quanta," laying the foundation for quantum mechanics, and he is called the father of quantum mechanics because of this groundbreaking work.

1.3 The Photoelectric Effect

Another revolutionary discovery came from one of the brightest minds in the history of physics, Albert Einstien, for whom no further introduction is needed, in 1905. This year found Einstein very productive, and among his great works he formulated the theory of light quanta, to explain the photoelectric effect. This challenges the classical picture of light as an electromagnetic wave (see left picture in Figure 1.2), which was predicted by Maxwell and experimentally confirmed by Heinrich Hertz. Based on the ideas of Planck, he conjectured that light itself is composed of irreducible finite amounts or quanta of energy, called photons. Each quantum contains energy hf, much more like the right picture of Figure 1.2.

The photoelectric effect describes the emission of electrons (photoelectrons) from a metal excited by shining light on the surface. In 1902, Philip Lenard studied this phenomenon and discovered that the emitted electrons exhibited a range of velocities. Notably, he found that the maximum kinetic energy of the photoelectrons K_{max} was independent of the intensity of the incident light. He also observed that K_{max} increased

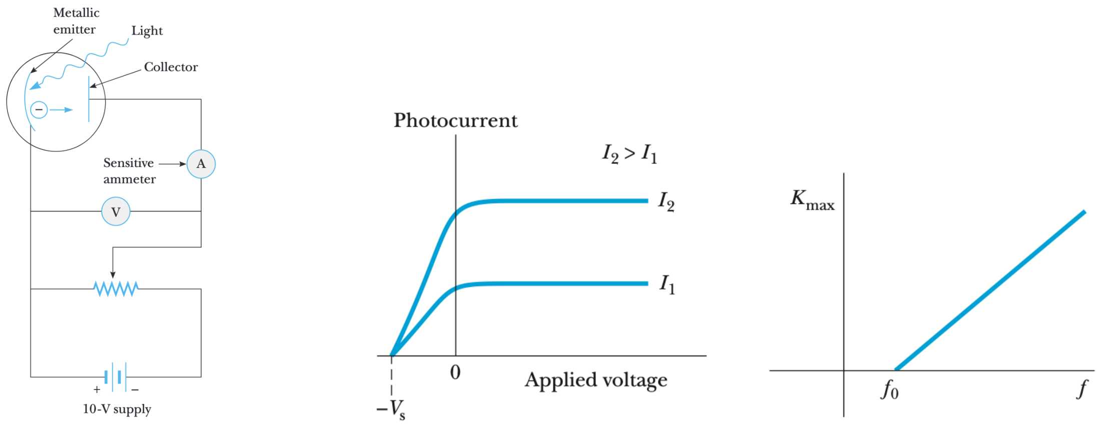


Figure 1.3: Properties of the photoelectric effect. Left figure: Experimental setup. Middle figure: A plot of the photocurrent against applied voltage. It is evident that the maximum kinetic energy is independent of the light intensity I of fixed frequency. Right figure: The linear dependence of K_{max} on the light frequency.

with the frequency of light, and that there existed a threshold frequency below which no photoelectrons were emitted. Additional experiments revealed that there was no time lag between the onset of illumination and the start of the photocurrent. These observations are illustrated in Figure 1.3.

Einstein studied this effect, and he imposed the quantization of light into photons. He obtain that the maximum kinetic energy follows

$$K_{max} = hf - \phi \,, \tag{1.3.1}$$

where ϕ is the work function of the metal, and corresponds to the minimum energy an electron must have to be bound to the metal. The threshold frequency depends on the material of the metal, through the work function ϕ. For frequencies below $f_0 = \phi/h$, no photocurrent is produced, as seen in experiments.

Exercise 1.1 The photoelectric effect suggests that light of frequency f can be regarded as consisting of photons of energy $E = hf$.

1. What are the energy and frequency of a photon in the visible spectrum ($400 - 700nm$)?
2. The microwave in our kitchen operates at roughly $2.5GHz$ at a max power of $300W$. How many photons per second can it emit?
3. How many such microwave photons does it take to warm a $200ml$ glass of water by 10^oC? (the heat capacity of water is approximately $4J/gK$ and the density is $1g/ml$).
4. How many photons per second can a low-power laser ($10mW$ at $633nm$) emit?
5. How many photons per second can a cell phone ($0.25W$ at $850MHz$) emit?

Einstein introduced the concept of the quantum of light in 1905, and a year later he showed that this carries momentum E/c along the direction of its motion. Despite this, he did not explore this topic further, including the scattering between photons and other particles. The study of photon-particle collisions was later developed by Peter Debye and Arthur Holly Compton (1923). They independently demonstrated that x-ray photon scattering from electrons could be explained by treating photons as particles with energy hf and momentum hf/c, conserving relativistic energy and momentum. This work completed the particle picture of light by confirming that photons both carry momentum and scatter like particles. The treatment of photon-particle scattering requires a relativistic approach, thus this problem will be addressed at a later stage.

1.4 Classical Atoms Cannot Exist

One of the biggest failures of Classical Mechanics regards the basic components that forms all matter, atoms. If atoms were obeying the laws of Classical mechanics, then they would be unstable, and as a consequence, all matter would also be unstable.

The history behind this development involves Rutherford. Following the experimental discovery of the atomic nucleus in 1911, conducted within his research group, Rutherford proposed a model to explain the properties of the atom. Drawing inspiration from the orbital motion of planets around the Sun, he envisioned the atom as consisting of electrons orbiting a positively charged massive center, the nucleus. However, it was soon recognized that this model, when considered within the framework of classical physics, suffered from two fundamental problems:

- Atoms are unstable
- Atoms radiate energy over a continuous range of frequencies

Where do these deficiencies come from? The instability of atoms arises by applying the classical electromagnetic (EM) theory to Rutherford's model. The EM theory tells us that accelerating charges radiate energy[2]. In the Rutherford model, the electron orbits around the nucleus, thus is accelerating and as a result radiates (loses) energy. The loss of energy will result in the decrease of the radius of the encircling orbit, and eventually the electron collapses onto the nucleus! If this were truly how things worked, we wouldn't be here pursuing our physics majors! Clearly, this picture is not consistent.

What about the second deficiency? The frequency of the radiated energy is the same as the orbiting frequency. As the electron orbit shrinks, its orbiting frequency increases continuously. Thus, the spectrum of the radiation emitted by the electron should be continuous. This conclusion completely disagrees with the experiment, in which atoms radiate energy in discrete frequencies, instead of a continuous frequency spectrum.

[2]For a reference on this, you can check any standard book on electromagnetic theory such as: Griffiths D. Introduction to Electrodynamics (1981), Jackson J. D. Classical Electrodynamics (1962).

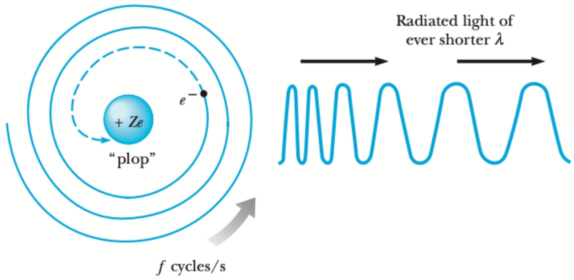


Figure 1.4: The collapse of the classical atom.

These findings led Niels Bohr and others to the first official formulation of a new theory called quantum mechanics (informally known as the "old quantum theory"). It was largely empirical, and attempted to fix both issues.

Exercise 1.2 In this exercise, we will compute the typical time for the collapse of the atom using classical concepts. We know that a non-relativistic, accelerating electron radiates energy at a rate given by the Larmor formula:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{e^2 a^2}{6\pi\varepsilon_0 c^3} \tag{1.4.1}$$

where e is the electron charge, c is the speed of light, a is the magnitude of the electrons acceleration and ε_0 is the permittivity in free space. Because of this, the classical atom may have a stability problem. We want to understand how big this effect is. The electron potential energy in the presence of the photon is

$$U = -\frac{e^2}{4\pi\varepsilon_0 r}\,. \tag{1.4.2}$$

(a) Show that for a non-relativistic ($v \ll c$) electron the energy ΔE lost per revolution is small compared to the electron's kinetic energy K. Hence, it is possible to regard the orbit as circular at any instant, even though the electron eventually spirals into the proton. You have to show that

$$\frac{|\Delta E|}{K} = \frac{8\pi}{3}\left(\frac{v}{c}\right)^3, \tag{1.4.3}$$

and since v/c is very small, thus the energy lost per revolution is small compared to the kinetic energy.

(b) Show that the time required for the electron to reach a final radius r_f starting from r_i is

$$\Delta t = \frac{\pi\varepsilon_0 m^2 c^3}{q^4}(r_i^3 - r_f^3)\,. \tag{1.4.4}$$

(c) A good estimate for the size of the hydrogen atom is $0.05nm$, and a good estimate for the size of the nucleus is $1fm$. Calculate how long it would take for the electron to spiral towards the nucleus.

(d) As the electron approaches the proton, what happens to its energy? Is there a minimum value for the energy the electron can have?

Remark: If you compute the ratio v/c from r_i to r_f, you will realize that the non-relativistic approach breaks down before the electron reaches the proton. Hence, relativistic corrections become important in latter case.

1.5 The Bohr Theory

One of the most remarkable discoveries was made by Bohr, who developed a simple atomic model based on a few key assumptions. Rutherford noted that *"Bohr's original quantum theory of spectra was one of the most revolutionary"*, while Einstein simply referred to it as *"...one of the greatest discoveries"*, when he heard about Bohr's calculation for the hydrogen atom. Thus, we will dedicate a section to this topic, and you will be guided to solve it yourselves.

The motivation behind Bohr's work originates from the discrete line spectrum emitted by a low-pressure gas subjected to an electric discharge. Until then, the continuous spectra of glowing solids and liquids had been successfully explained by Planck, as previously discussed. However, this new discovery required an entirely new framework of physics to describe it. Figure 1.5a illustrates typical discrete spectra for a selection of elements.

In the following, we present the assumptions of the Bohr model.

1. The electron moves around the proton in a circular orbit under the influence of the Coulomb force.

2. Only certain of the circular orbits are allowed.

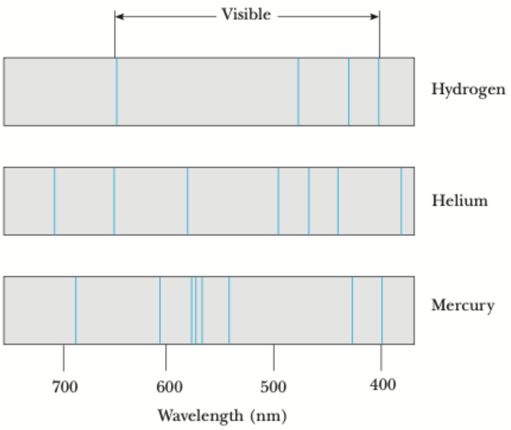
3. Radiation is emitted by the atom when the electron "jumps" from a more energetic state to a less energetic one. This jump cannot be described classically.

$$E_i - E_f = hf \tag{1.5.1}$$

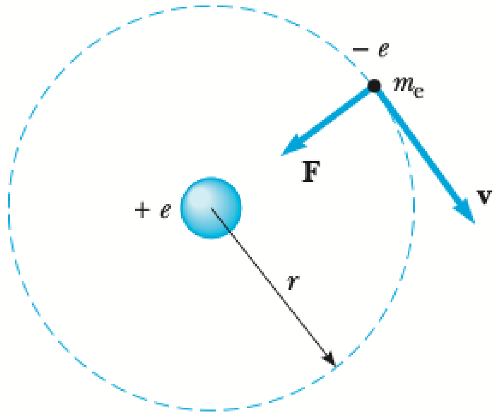
   where E_i, E_f are the energies of the initial and final states respectively.

4. The size of the allowed electron orbits is determined by an additional condition, namely the allowed orbits are those ones for which the electrons orbital angular momentum is an intergral multiple of the reduced Planck constant,

$$L = m_e v r = n\hbar, \quad n = 1, 2, 3, ... \tag{1.5.2}$$



(a) Emission line spectra of a few representative elements.



(b) Bohr representation of the Hydrogen atom. Electrons encircle the nucleus following circular trajectories of constant energy.

Figure 1.5: The motivation behind Bohr's theory.

where m_e is the electronic mass, v is the speed of the electron along a circular orbit, and r is the distance between the electron and the nucleus (which is the radius of the circular orbit). This picture be schematically represented in Figure 1.5b.

Using these four assumptions, Bohr was able to calculate the allowed energy levels and emission wavelengths of the Hydrogen atom.

Exercise 1.3 In the following, we will use the assumptions of the Bohr model to find the allowed energies for the Hydrogen atom.

(a) Given that the electrical potential energy of the system is $U = qV = -ke^2/r$ where k is the Coulomb constant, write down the total energy of the atom.

(b) Apply Newton's second law and obtain an expression for the kinetic energy.

(c) Express the total energy of the system in terms of k, e, r. Comment on the sign and its physical meaning.

(d) Use your result from Question (b) and the angular momentum condition 1.5.2, show that the radii of the allowed electron orbits are:

$$r_n = \frac{n^2\hbar^2}{m_e k e^2} = n^2 a_0, \quad n = 1, 2, 3, \dots, \tag{1.5.3}$$

where $a_0 = 0.0529nm$ is the Bohr radius.

(e) Using your result from Question (d), show that the allowed energies are

$$E_n = -\frac{\alpha}{n^2}, \quad n = 1, 2, 3, \dots. \tag{1.5.4}$$

Find an expression for constant α and compute its numerical value.

(f) Compute the first few energy levels, eg $n = 1, 2, 3$, where $n = 1$ corresponds to the ground state, $n = 2$ to the excited state and so on.

Exercise 1.4 In this exercise, we will look at the emission spectrum of the Hydrogen atom within the Bohr theory.

1. Given your main result from the previous exercise (that of quantized energy levels), derive an expression for the wavelength of the radiation emitted when an electron "jumps" from an initial state n_i to a final state n_f. Show that this expression takes the form of:
$$\frac{1}{\lambda} = \frac{ke^2}{2a_0hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right). \tag{1.5.5}$$

2. Consider now transitions from an initial state n_i to the final state $n_f = 2$. This series of radiation lines is called the Balmer series[3]. Compute the wavelengths for $n_i = 3, 4, 5$. Do they match with the experimental observations, as seen in Figure 1.5a?

The truth is that the description that Bohr gave is false. However, the remarkable aspect of this simple model is that it predicts the precise energy levels of the allowed states. By Chapter 10, we will have acquired the necessary tools to derive the correct energy levels within the framework of quantum mechanics.

1.6 Matter Waves and Double-Slit Experiments

Light, initially considered to obey wave mechanics, was ultimately found to be comprised of tiny quantized packets of energy called photons, as demonstrated by Einstein through the photoelectric effect, for which he was awarded the Nobel Prize in 1921.[4]. This conclusion led Louis de Broglie to conjecture that electrons (and any other matter particle), in addition to their particle nature, should also exhibit wavelike behavior. It was suggested that a particle with momentum p would produce an interference pattern corresponding to a wavenumber k in the double-slit experiment. The momentum and wavenumber are related by
$$\lambda = \frac{h}{p}, \tag{1.6.1}$$
where h is the Planck constant.

Exercise 1.5 In this simple exercise, we get a feeling of the de Broglie relations and how it sets the scale for quantum effects.

[3] Transitions from $n_i \to n_f = 1$ constitute the Lyman series and the transitions $n_i \to n_f = 3$ belongs to the Paschen series.

[4] Remarkably, this was the only Nobel Prize he received, despite having formulated two of the most successful physical theories, Special and General Relativity.

Since a wavelength can be associated with every moving particle, let us do this also for objects in everyday life. Calculate the de Broglie wavelength $\lambda = h/p$, with h being the Planck's constant and p the associated momentum, of each of the following examples:

1. a car with mass $2000kg$ traveling at a speed of $20m/s$
2. a $10g$ marble moving with a speed of $10cm/s$
3. a smoke particle of diameter $100nm$ and a mass of $1fg$ being jostled about by air molecules at room temperature, $T = 300K$ (assume that the particle has the same translational kinetic energy as the thermal average of the air molecules, $KE = \frac{3}{2}k_B T$ with k_B the Boltzmann constant).
4. a $^{8}7Rb$ atom that has been laser cooled to a temperature of $T = 100\mu K$ (again, assume $KE = \frac{3}{2}k_B T$)

We will now present a series of double-slit experiments using various physical entities. Ultimately, our focus will be on the behavior of electrons, for which we demonstrate its wave-like properties.

First, we consider two classical objects whose behavior is quite familiar to us. Let us begin with the case of bullets, as depicted in Figure 1.6(a). A gun fires randomly toward a wall with two slits. We associate a probability P_1 with bullets passing through slit 1, and similarly, P_2 for slit 2. When both slits are open, the total probability distribution is given by

$$P_{12} = P_1 + P_2\,. \tag{1.6.2}$$

In this simple example, the probabilities simply add up, and correspond to the formation of two distinct strips on the screen, as shown in Figure 1.6(a).

The next experiment, which is shown in Figure 1.6(b), involves water waves which are described by plane waves of the form $h_j = |h_j|e^{i\theta_j}$, where $j = 1, 2$[5]. The intensity is $I_1 = |h_1|^2$ when slit 2 is closed, and similarly $I_2 = |h_2|^2$ when slit 1 is closed. However, when both slits are open the intensity will be the sum of amplitudes of the two waves, i.e. $I_{12} = |h_1 + h_2|^2$. Expanding this expression gives

$$I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta\,, \tag{1.6.3}$$

where $\delta = \theta_1 - \theta_2$. The last term is the "interference term". In fact, Eqs. (1.6.2) and (1.6.3) differ precisely due to this term, which is responsible for the distinct interference pattern observed.

Our last experiment involves electrons, shown in Figure 1.6(c). Consider an electron gun and electrons are shot towards the slits. We are interested in answering the question "What is the relative probability of the electrons on the screen?". The result remarkably

[5] To be precise, we should have used spherical waves.

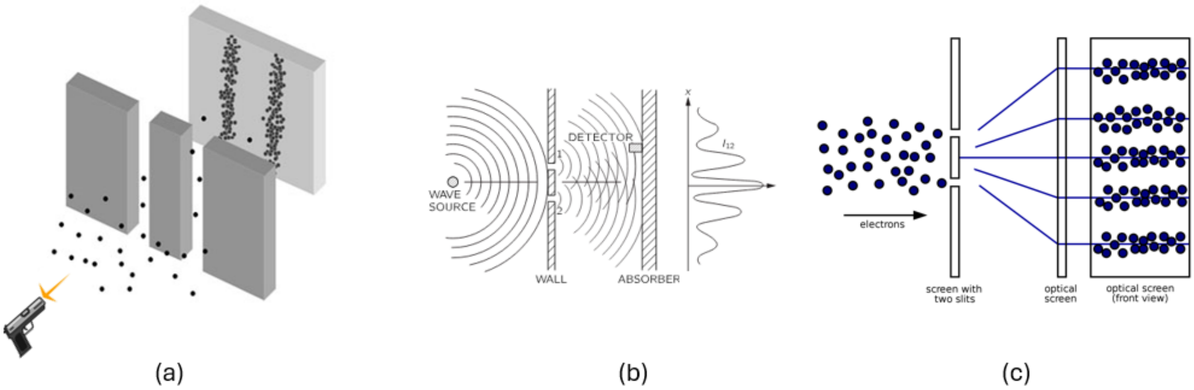


Figure 1.6: (a) Double slit experiment with bullets. (b) Double slit experiment with water waves (c) Double slit experiment with electrons.

is not that of Eq. (1.6.2) but that of Eq. (1.6.3)! Electrons behave like waves!

The series of experiments we described in this section demonstrates the strangeness behind the behaviour of electrons. In fact, this will be the case not just for electrons, but also other particles of matter. Even more remarkably, the double slit experiment has been shown with large organic molecules[6]. The question on whether an electron is a wave or a particle cannot be addressed with a single answer. They have a dual personality, behaving either like particles or waves, depending on the kind of experiment performed on them. Many feel that the elder Bragg's remark, originally made about light, is a more satisfying answer: *"Electrons behave like waves on Mondays, Wednesdays, and Fridays, like particles on Tuesdays, Thursdays, and Saturdays, and like nothing at all on Sundays."*

1.7 The Stern-Gerlach Experiment

The Stern-Gerlach (SG) experiment[7] displays yet another phenomenon that Classical Mechanics fails to explain. In this section, we will describe it phenomenologically. Once we establish the postulates of Quantum Mechanics in the next chapter, we will be able to provide a correct interpretation of the results.

The goal of the experiment was to measure the magnetic moment of a silver atom. In order to understand what factors can contribute to the magnetic moment, let us concentrate for a moment on a simpler example. Consider the classical model of the hydrogen atom:

[6] Gerlich, S., Eibenberger, S., Tomandl, M. et al. Quantum interference of large organic molecules. Nat Commun 2, 263 (2011). https://doi.org/10.1038/ncomms1263

[7] The original paper is in German: *Gerlach, W., & Stern, O. Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld. The European Physical Journal A, 9(1), 349–352 (1922).* There is however an English translation written by particle physicist Martin Bauer: *Bauer, M. The Stern-Gerlach Experiment, Translation of: 'Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld', arXiv:2301.11343v1 (2023).*

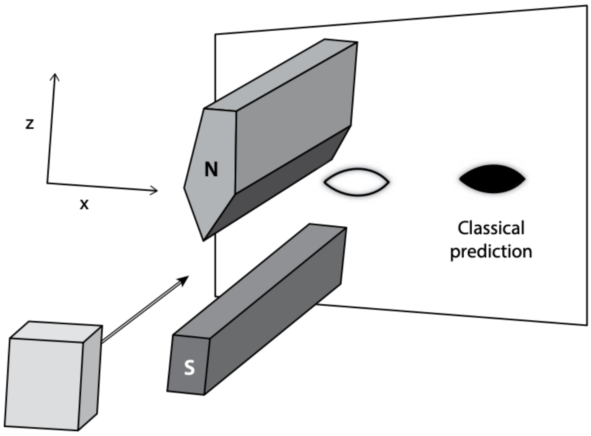


Figure 1.7: The Stern-Gerlach configuration.

an electron orbiting a proton of opposite charge with angular momentum $\vec{L}$. Since the electron mass m is much smaller than the proton mass, the magnetic moment of this system is simply given by

$$\vec{\mu}_L = -\frac{e}{2m}\vec{L}\,. \tag{1.7.1}$$

Now, we can imagine that we send the hydrogen atom through a magnetic field $\vec{B}$. Since the atom has no net electric charge, the interaction energy between the magnetic field and the magnetic moment is simply given by the classical formula $U = -\vec{\mu} \cdot \vec{B}$. We assume for simplicity that the magnetic field points along the z-direction. Thus, the magnitude of the force $\vec{F} = -\vec{\nabla}U$ acting on the atom is

$$F = \frac{\partial}{\partial z}\vec{\mu} \cdot \vec{B} \approx \mu_z \frac{\partial B_z}{\partial z}\,. \tag{1.7.2}$$

In addition to the orbital angular momentum, electrons and protons have an additional degree of freedom, an intrinsic angular momentum or spin S. In classical terms, we could think of this degree of freedom as a rotation around their proper axis. The spin angular momentum also induces a magnetic dipole moment

$$\vec{\mu}_S = g\frac{-e}{2m}\vec{S}\,, \tag{1.7.3}$$

where $g \simeq 2$ (for the electron) is known as the gyromagnetic ratio, and this magnetic moment couples to the magnetic field as before. In general, we expect that the total magnetic moment is the sum of the spin and orbital angular momenta. In their experiment, Stern and Gerlach used silver atoms passing through an inhomogeneous magnetic field. Silver is made up of a nucleus and 47 electrons, where 46 out of the 47 electrons can be visualized as forming a spherically symmetrical electron cloud with no net angular momentum (both spin and orbital components). The only contribution to the total angular momentum of the atom is due to the intrinsic spin of the 47th electron. The force in Eq. (1.7.2) shows that the SG apparatus is an effective way to measure the z-component of $\vec{\mu}$. Atoms with $S_z < 0$ will experience a downward force while those with $S_z > 0$ will experience an upward force. By measuring how many atoms emerge at which vertical position we therefore have an indirect measurement of S_z. The atoms

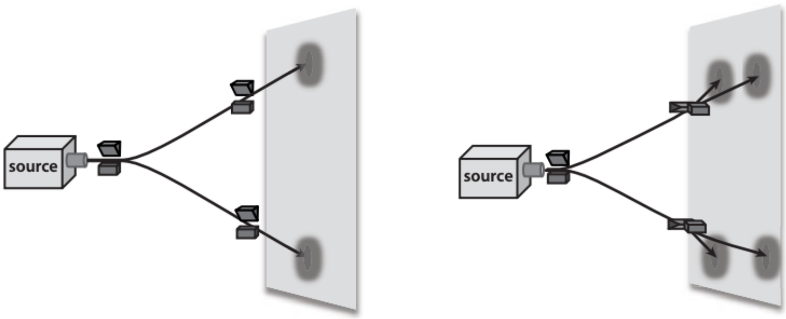


Figure 1.8: A sketch of a sequential Stern-Gerlach experiment. In both figures the first magnet after the source creates a magnetic field along z. Left figure: We place a second set of magnets oriented along z after the initial one, in the path of each deflected beam. Right figure: We place a second set of magnets oriented along x in the path of each deflected beam.

entering the apparatus are randomly oriented (because they come from a high-temperature source), thus we could expect that on average each atom will have a random value of $-|\mu| < \mu_z < |\mu|$. We would therefore expect that at the exit of the SG apparatus, a vertical continuous spot would appear like the one on the right hand side of the screen in Figure (1.7). However, it is experimentally observed that only two isolated spots appear on the screen like the ones on the left hand side of the screen in Figure (1.7). This implies that S_z can take only two possible values and experimentally, it is found that these two values are

$$S_z^{(+)} = +\frac{\hbar}{2}, \quad S_z^{(-)} = -\frac{\hbar}{2}\,. \tag{1.7.4}$$

The fact that the z component of the intrinsic spin of the electron can take only two values cannot be explained by classical mechanics. This is a striking example of quantization, where microscopic objects subjected to quantum mechanics often exhibit discrete values upon measurement, rather than the continuous range predicted by classical theory, as seen in this case.

We take this experiment a step further by introducing an additional SG apparatus. This will be a thought (Gedanken) experiment, which demonstrates additional features that we will need to explain with the full quantum theory. Let the first SG apparatus be oriented in the z-direction, splitting the beam into an upper and lower branch. We then place a second apparatus, also oriented along the z-direction as shown in the left figure in Figure 1.8. The whole upper beam is deflected upwards and the whole lower beam is deflected downwards, as seen in Figure 1.8. It appears that the first SG apparatus has fixed the direction of deflection of each particle.

We now rotate and orient the second SG apparatus along the x direction (see the right figure of Figure 1.8). In this case, each beam splits in two. So preparing a beam with a known z-orientation results in complete uncertainty about the x direction. If we test the output of the x SG apparatus with yet another z one, we find that the beam is once again split in two! In the next Chapter, we will acquire all the tools and concepts to explain this behavior.

1.8 References

This chapter was inspired by several references including:

1. Sakurai J.J. et al. Modern Quantum Mechanics (2020) : Chapter 1 briefly touches upon these topics, mainly the Stern-Gerlach experiment.

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