Today's agenda:

Electric flux.

You must be able to calculate the electric flux through a surface.

Gauss' Law.

You must be able to use Gauss' Law to calculate the electric field of a high-symmetry charge distribution.

Gauss' Law, other examples.

You must be able to use Gauss' Law to calculate the electric field of a high-symmetry charge distribution.

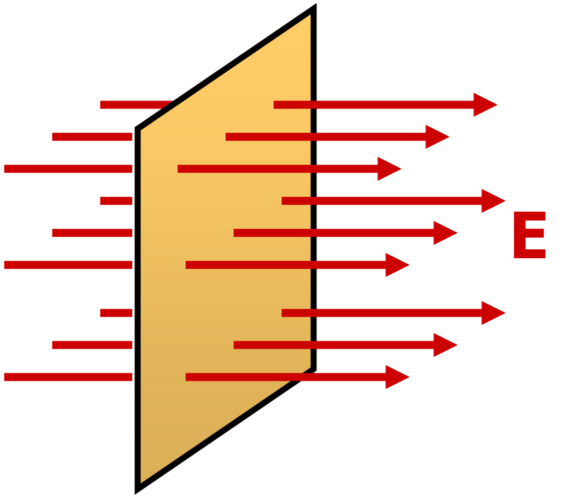
Conductors in electrostatic equilibrium.

You must be able to use Gauss' law to draw conclusions about the behavior of charged particles on, and electric fields in, conductors in electrostatic equilibrium.

Electric Flux

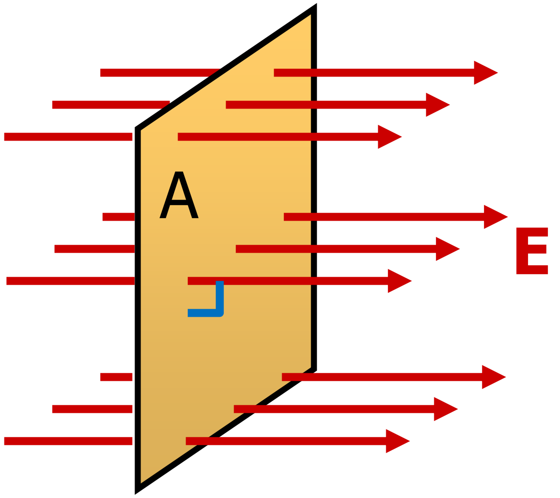
We have used electric field lines to visualize electric fields and indicate their strength.

We are now going to count* the number of electric field lines passing through a surface and use this count to determine the electric field.



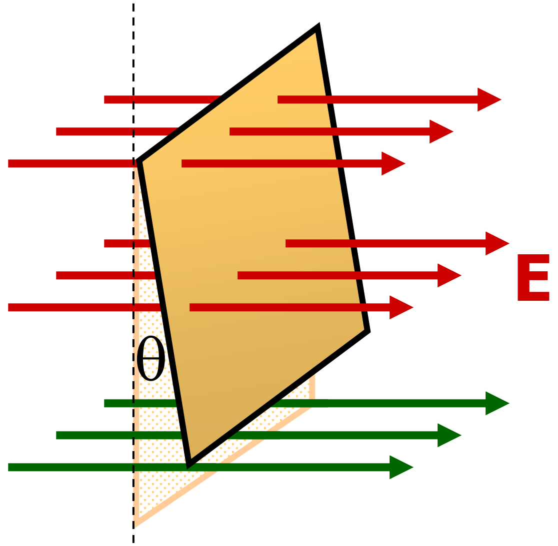
The **electric flux** passing through a surface is the number of electric field lines that pass through it.

Because electric field lines are drawn arbitrarily, we **quantify** electric flux like this: $\Phi_E = EA$,



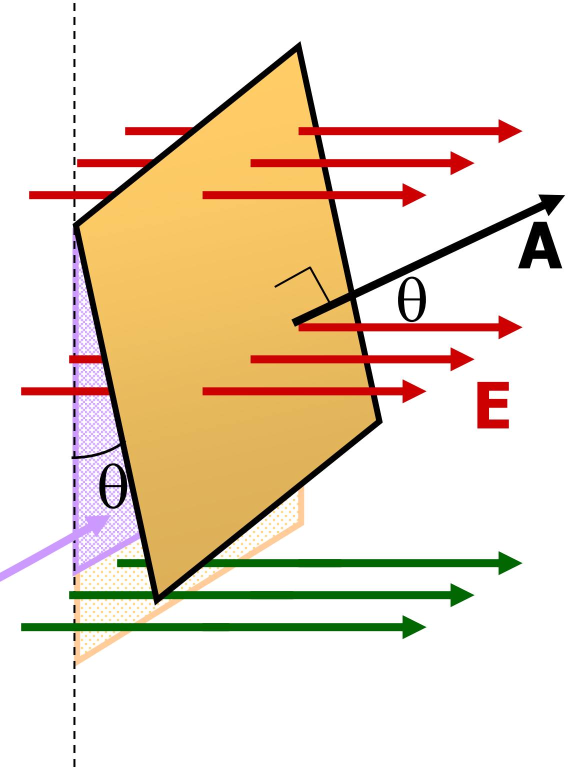
…except that…

If the surface is tilted, fewer lines cut the surface.



Later we'll learn about magnetic flux, which is why I will use the subscript E on electric flux.

We define $\vec{A}$ to be a vector having a magnitude equal to the area of the surface, in a direction normal to the surface.



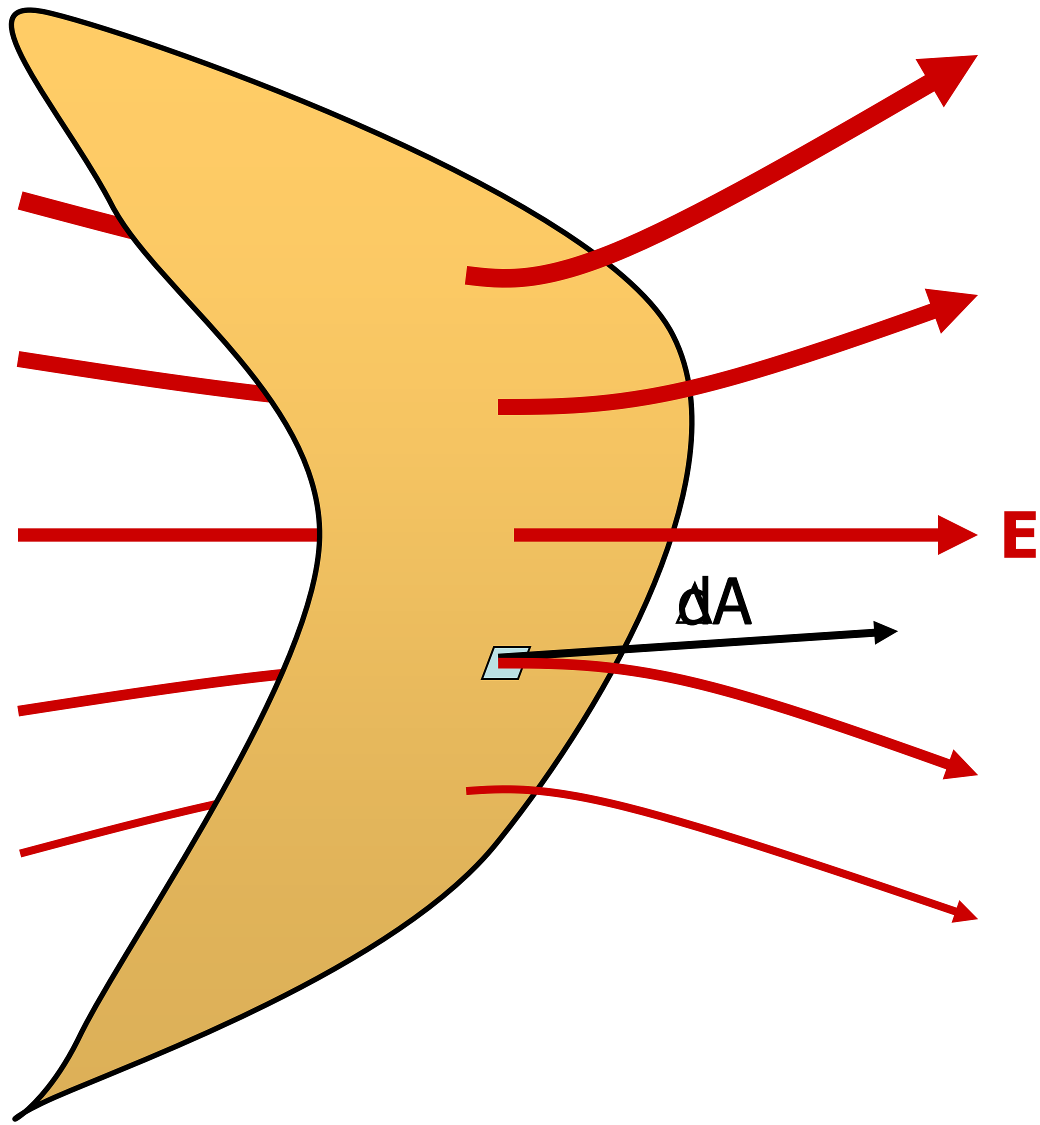
The "amount of surface" perpendicular to the electric field is $A\cos\theta$.

Therefore, the amount of surface area effectively "cut through" by the electric field is $A\cos\theta$.

$$A_{Effective} = A\cos\theta \quad \text{so} \quad \Phi_E = EA_{Effective} = EA\cos\theta.$$

$$\Phi_E = \vec{E} \cdot \vec{A}$$

If the electric field is not uniform, or the surface is not flat…



divide the surface into infinitesimal surface elements and add the flux through each…

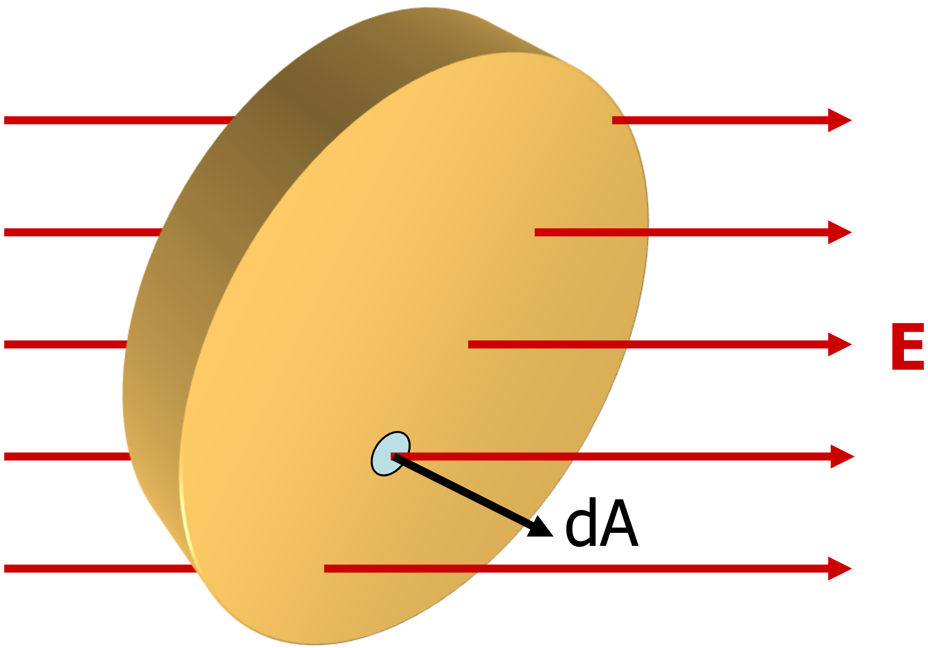
$$\Phi_E = \lim_{\Delta A_i \to 0} \sum_i \vec{E}_i \cdot \Delta\vec{A}_i$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

a surface integral,
therefore, a double integral $\iint$

Remember, the direction of dA is normal to the surface.

If the surface is closed (completely encloses a volume)…



…we count* lines going out as positive and lines going in as negative…

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

a surface integral, therefore, a double integral $\oiint$

For a closed surface, dA is normal to the surface and always points away from the inside.

The circle just reminds you to integrate over a closed surface.

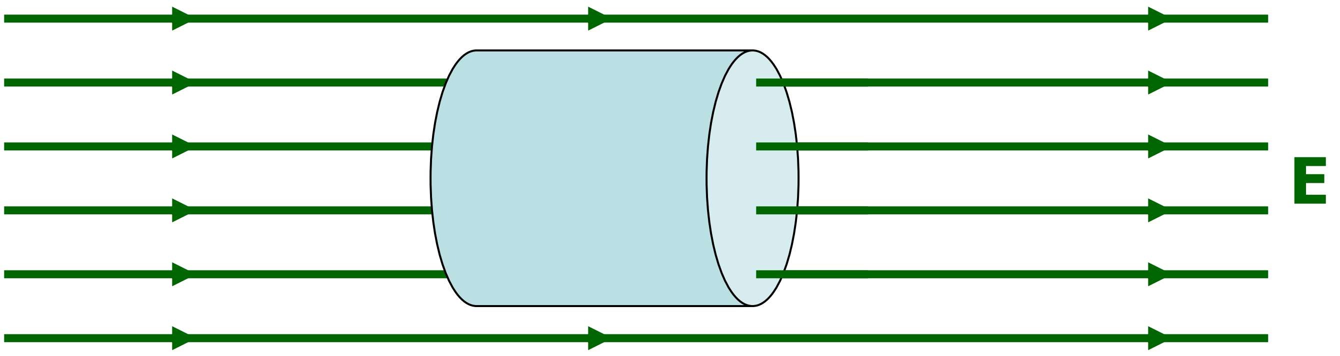
Question: you gave me five different equations for electric flux. Which one do I need to use?

Answer: use the simplest (easiest!) one that works.

$\Phi_E = EA$	Flat surface, $\vec{E} \parallel \vec{A}$, $\vec{E}$ constant over surface. Easy!
$\Phi_E = EA\cos\theta$	Flat surface, $\vec{E}$ not $\parallel \vec{A}$, $\vec{E}$ constant over surface.
$\Phi_E = \vec{E}\cdot\vec{A}$	Flat surface, $\vec{E}$ not $\parallel \vec{A}$, $\vec{E}$ constant over surface.
$\Phi_E = \int \vec{E}\cdot d\vec{A}$	Surface not flat, $\vec{E}$ not uniform. Avoid, if possible. This is the definition of electric flux, so it is on your equation sheet.
$\Phi_E = \oint \vec{E}\cdot d\vec{A}$	Closed surface. The circle on the integral just reminds you to integrate over a closed surface.

If the surface is closed, you may be able to "break it up" into simple segments and still use $\Phi_E=\vec{E}\cdot\vec{A}$ for each segment.

Electric Flux Example: Calculate the electric flux through a cylinder with its axis parallel to the electric field direction.



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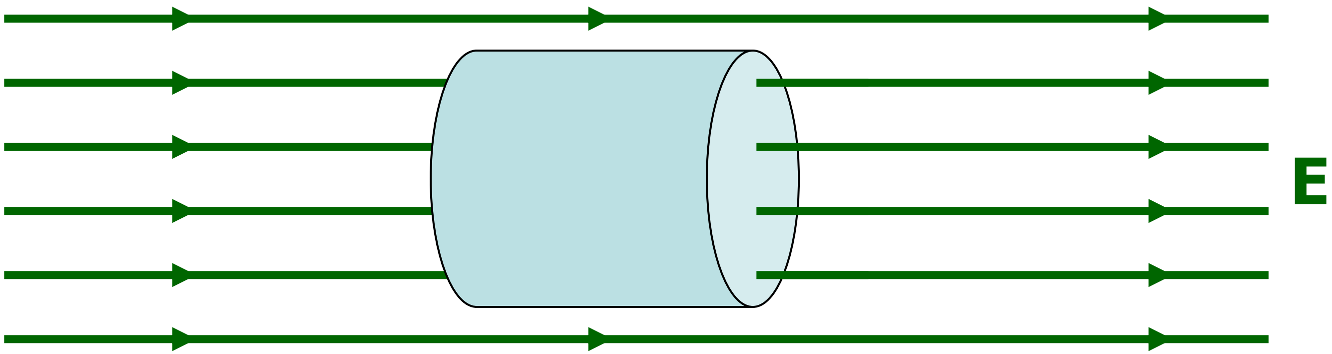
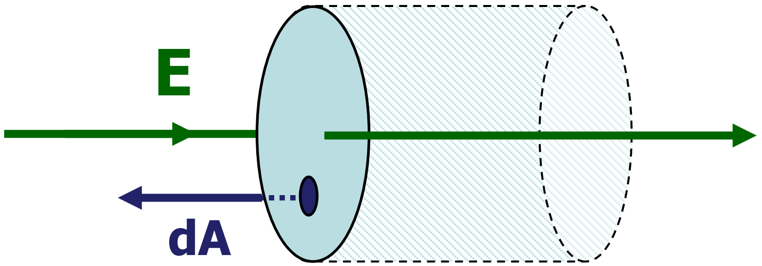
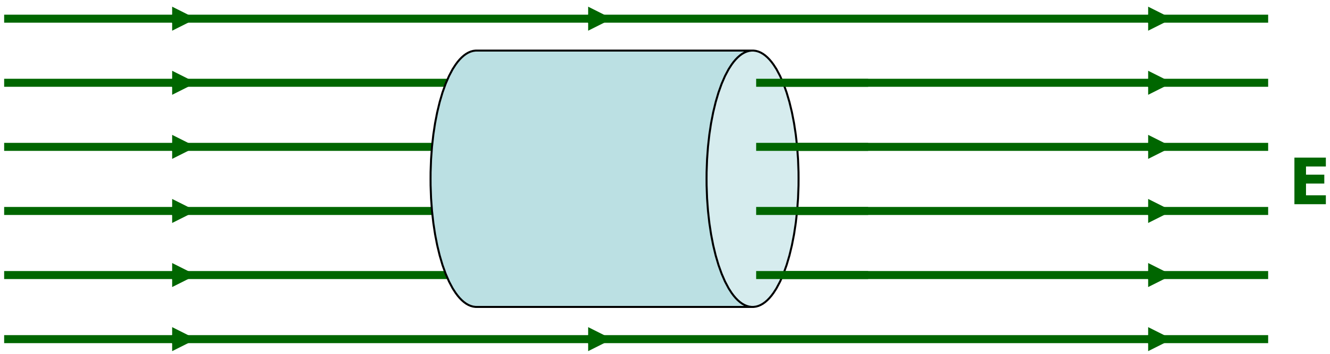


I see three parts to the cylinder:

The left end cap.

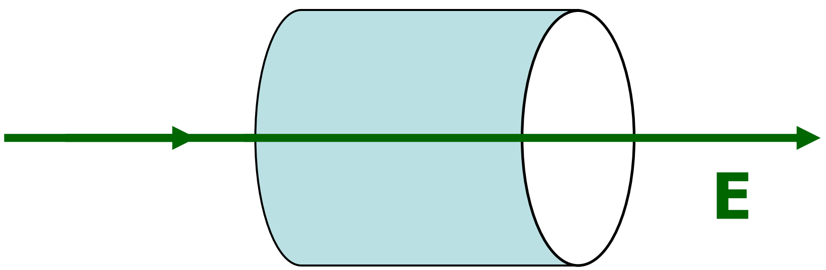


Electric Flux Example: Calculate the electric flux through a cylinder with its axis parallel to the electric field direction.

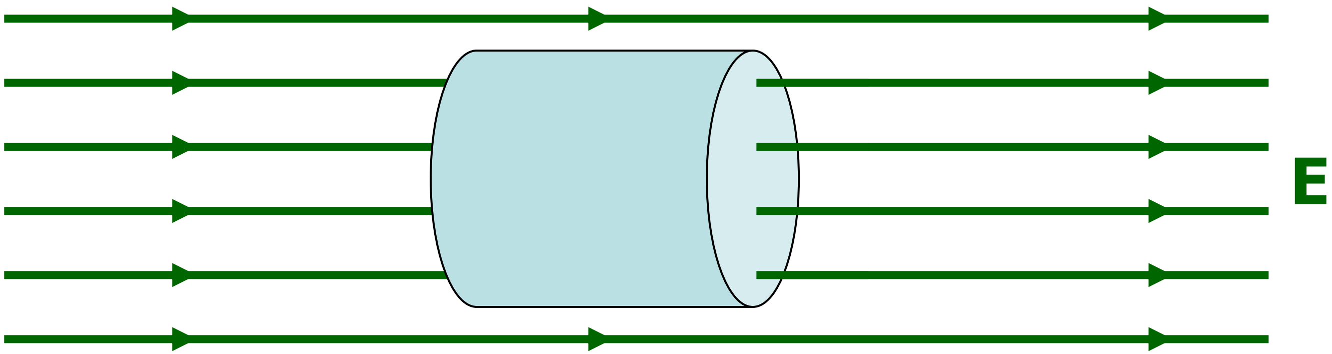


I see three parts to the cylinder:

The tube.

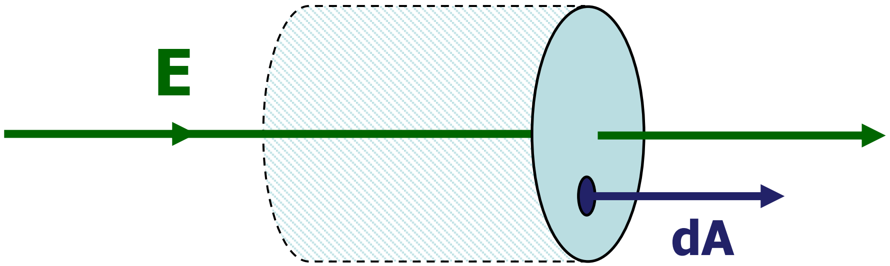


Electric Flux Example: Calculate the electric flux through a cylinder with its axis parallel to the electric field direction.



I see three parts to the cylinder:

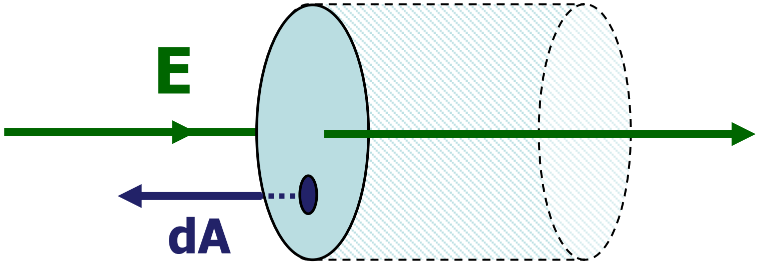
The right end cap.



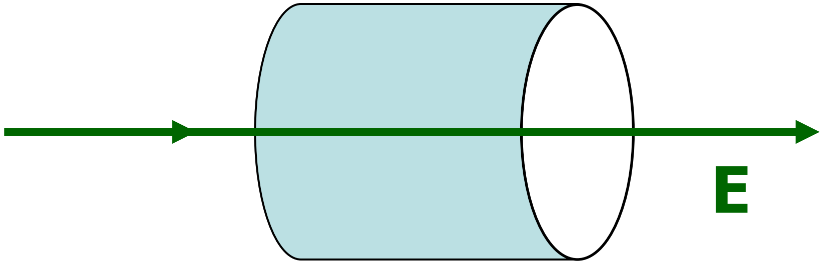
Let's separately calculate the contribution of each part to the flux, then add to get the total flux.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \int_{left} \vec{E} \cdot d\vec{A}_{left} + \int_{tube} \vec{E} \cdot d\vec{A}_{tube} + \int_{right} \vec{E} \cdot d\vec{A}_{right}$$

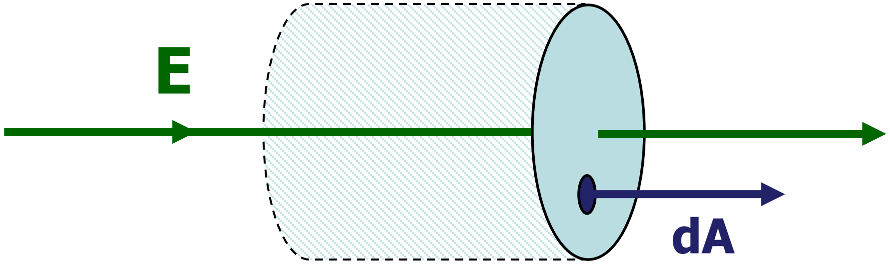
The left end cap.



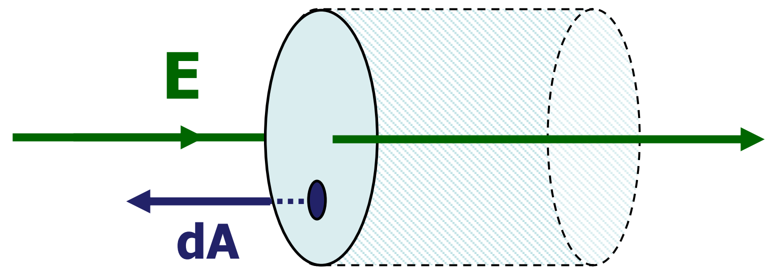
The tube.



The right end cap.



The left end cap.

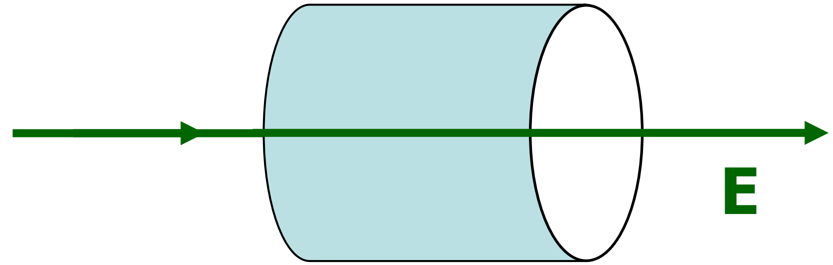


Every $\overrightarrow{dA}$ on the left end cap is antiparallel to $\vec{E}$. The angle between the two vectors is 180°

$$\int_{left} \vec{E} \cdot d\vec{A}_{left} = \int_{left} E\, dA_{left} \cos 180° = \int_{left} -E\, dA_{left}$$

$\vec{E}$ is uniform, so $\displaystyle\int_{left} -E\, dA_{left} = -E \int_{left} dA_{left} = -EA_{left}$

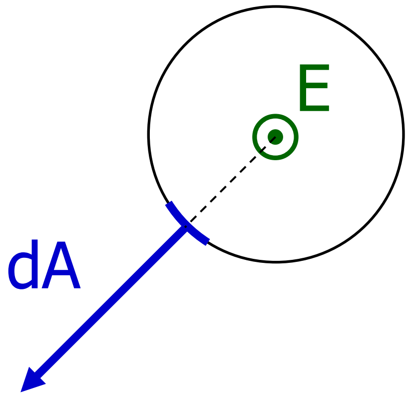
The tube.



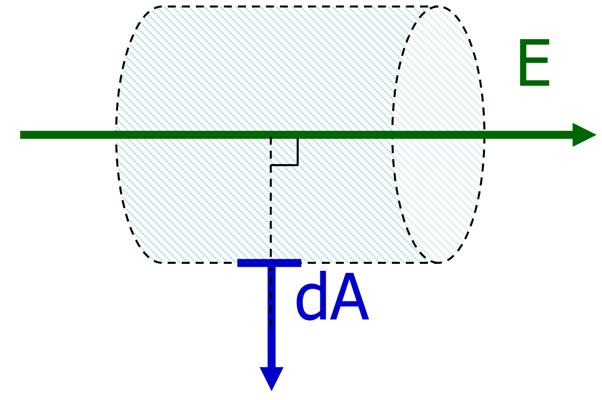
Let's look down the axis of the tube.

$\vec{E}$ is pointing at you.

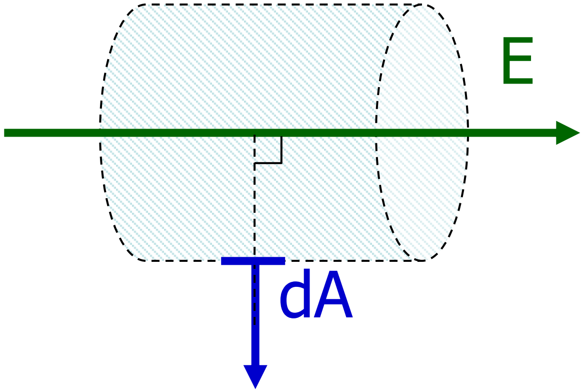
Every $\overrightarrow{dA}$ is radial (perpendicular to the tube surface).



The angle between $\vec{E}$ and $\overrightarrow{dA}$ is 90°.



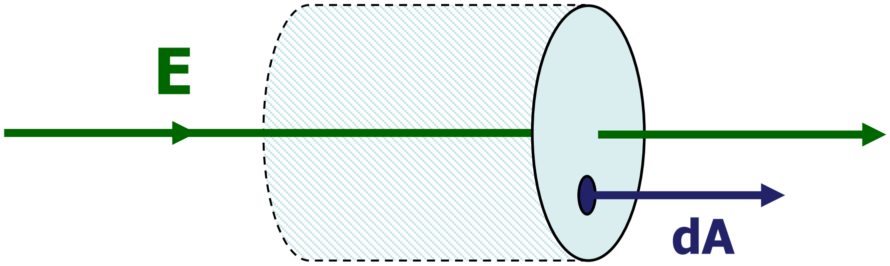
The angle between $\vec{E}$ and $\overrightarrow{dA}$ is 90°.



$$\int_{tube} \vec{E} \cdot d\vec{A}_{tube} = \int_{tube} E \, dA_{tube} \cos 90° = \int_{tube} 0 \, dA_{tube} = 0$$

The tube contributes nothing to the flux!

The right end cap.



Every $\overrightarrow{dA}$ on the right end cap is parallel to $\vec{E}$. The angle between the two vectors is 0°

$$\int_{right} \vec{E} \cdot d\vec{A}_{right} = \int_{right} E\, dA_{right} \cos 0\,° = \int_{right} E\, dA_{right}$$

$\vec{E}$ is uniform, so $\int_{right} E\, dA_{right} = E \int_{right} dA_{right} = EA_{right}$

The net (total) flux

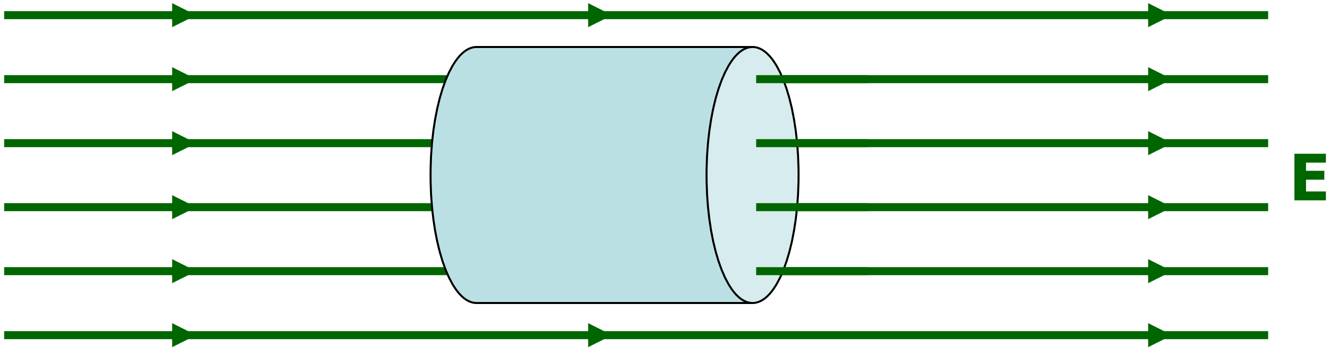
$$\Phi_E = \int_{left} \vec{E} \cdot d\vec{A}_{left} + \int_{tube} \vec{E} \cdot d\vec{A}_{tube} + \int_{right} \vec{E} \cdot d\vec{A}_{right}$$

$$\Phi_E = -EA_{left} + 0 + EA_{right} = 0$$

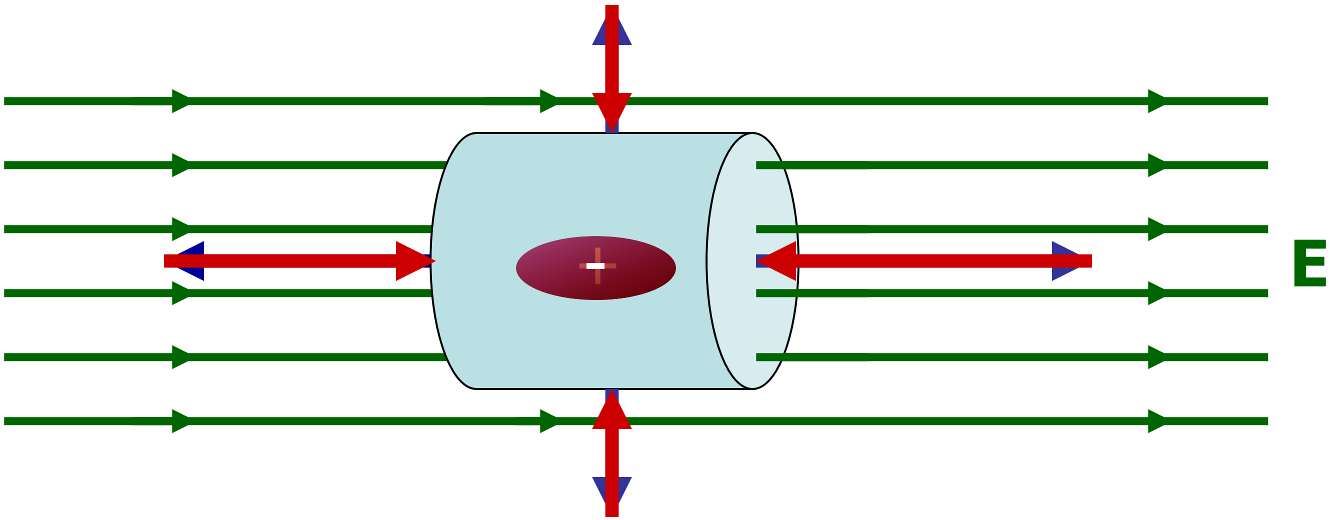
Assuming a right circular cylinder.*

The flux is zero!
Every electric field line that goes in also goes out.

*We will see in a bit that we don’t have to make this assumption.



The net electric flux through a closed cylindrical surface is zero.



If there were a + charge inside the cylinder, there would be more lines going out than in.

If there were a - charge inside the cylinder, there would be more lines going in than out…

…which leads us to…

Today’s agenda:

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You must be able to calculate the electric flux through a surface.

Gauss’ Law.

You must be able to use Gauss’ Law to calculate the electric field of a high-symmetry charge distribution.

Gauss’ Law, other examples.

You must be able to use Gauss’ Law to calculate the electric field of a high-symmetry charge distribution.

Conductors in electrostatic equilibrium.

You must be able to use Gauss’ law to draw conclusions about the behavior of charged particles on, and electric fields in, conductors in electrostatic equilibrium.

Gauss' Law

Mathematically*, we express the idea of the last two slides as

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

Gauss' Law

Always true, not always useful.

We will find that Gauss' law gives a simple way to calculate electric fields for charge distributions that exhibit a high degree of symmetry…

…and we will save more complex charge distributions for advanced classes.

Example: use Gauss' Law to calculate the electric field from an isolated point charge q.

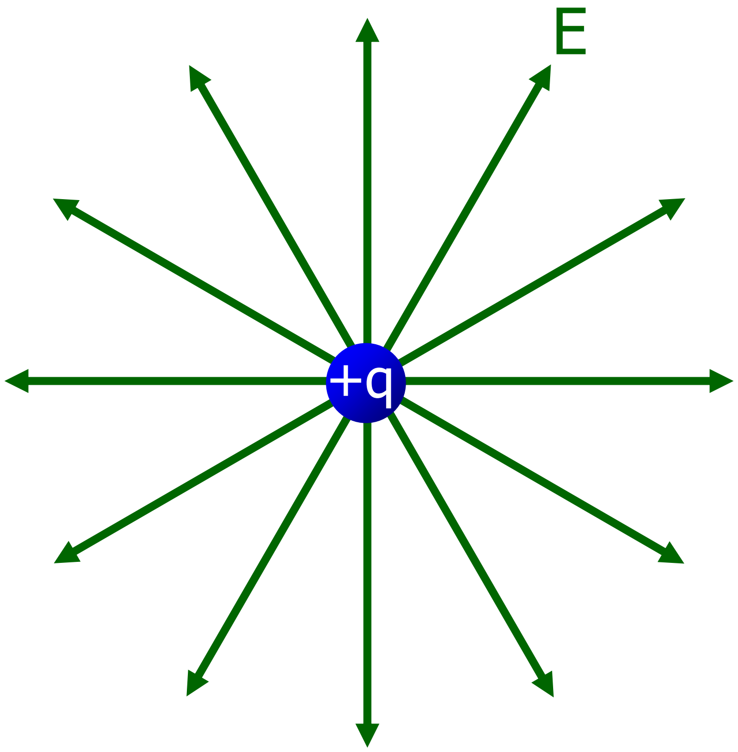
To be worked at the blackboard in lecture…

Example: use Gauss' Law to calculate the electric field from an isolated point charge q.

Let's assume the point charge is +.

The electric field everywhere points away from the charge.

If you go any distance r away from +q, the electric field is always directed "out" and has the same magnitude as the electric field at any other r.

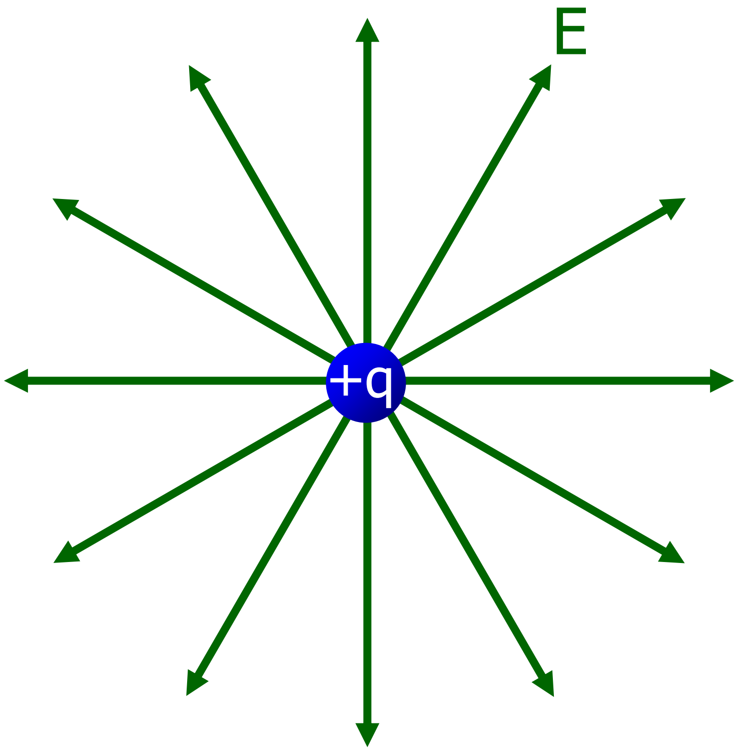


What is the symmetry of the electric field?

Spherical !

Example: use Gauss' Law to calculate the electric field from an isolated point charge q.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$



To apply Gauss' Law, we really want to pick a surface for which we can easily evaluate $\vec{E} \cdot d\vec{A}$.

That means we want $\vec{E}$ to everywhere be either parallel or perpendicular to the surface.

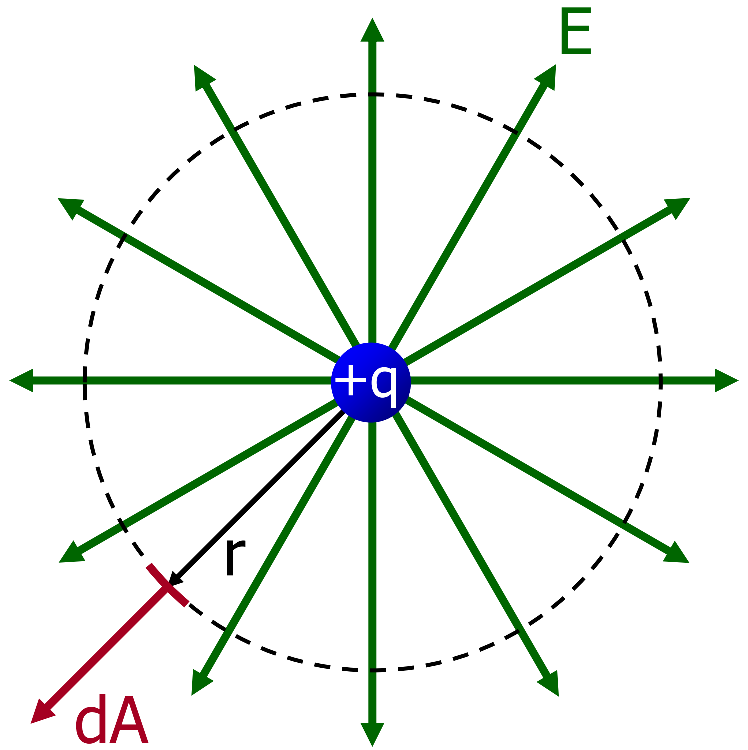
Let's see, for what kind of surface would this spherically-symmetric electric field always be parallel or perpendicular?

"a sphere"

Example: use Gauss' Law to calculate the electric field from an isolated point charge q.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

So let's draw a Gaussian sphere of radius r, enclosing and centered on +q. "Centered on" makes it easy to evaluate $\vec{E} \cdot d\vec{A}$.

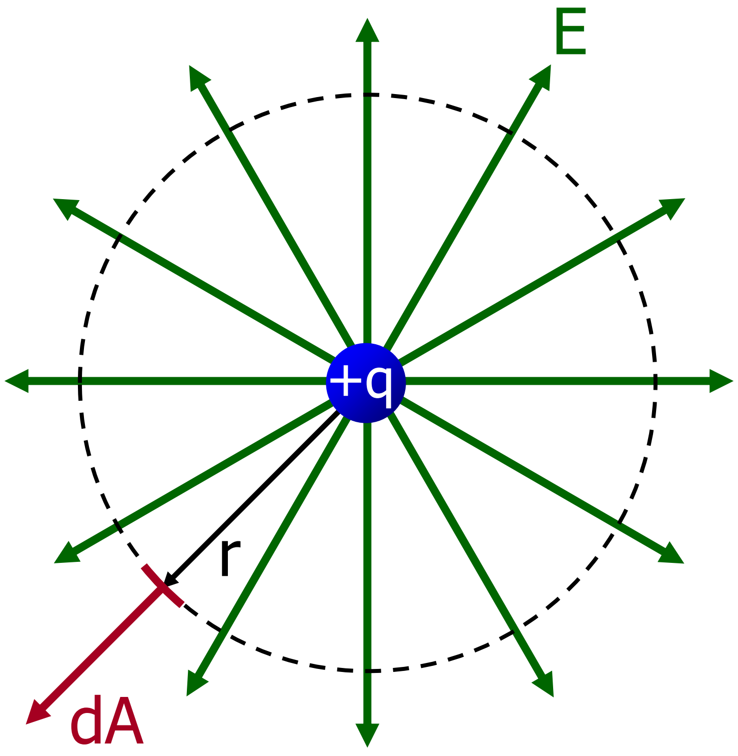


Everywhere on the sphere, $\vec{E}$ and $d\vec{A}$ are parallel and E is constant so

$$\oint \vec{E} \cdot d\vec{A} = \oint E\, dA = E \oint dA = E\, A_{sphere} = E\, 4\pi r^2$$

Example: use Gauss' Law to calculate the electric field from an isolated point charge q.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$



The charge enclosed by my Gaussian sphere is q, so

$$\oint \vec{E} \cdot d\vec{A} = E\ 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$E\ 4\pi r^2 = \frac{q}{\varepsilon_o}$$

$$E = \frac{q}{4\pi\varepsilon_o r^2}$$

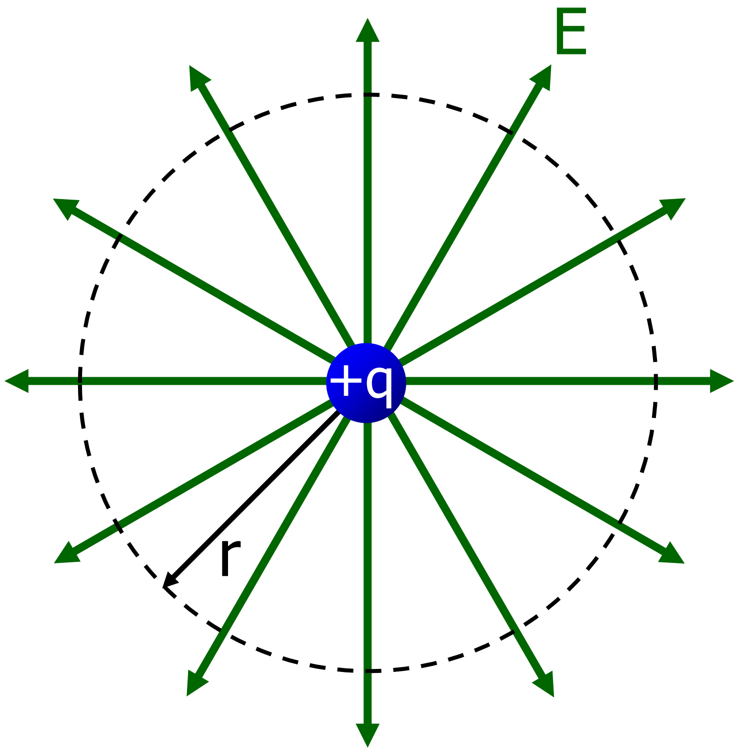
The direction of $\vec{E}$ is shown in the diagram.
Or you can say $\vec{E}$ is "radially out."

Example: use Gauss' Law to calculate the electric field from an isolated point charge q.

$$E = \frac{q}{4\pi\varepsilon_o r^2}, \text{ away from } +q$$

"But wait," you say, "the parameter r does not appear in the problem statement, so it can't appear in the answer.*"

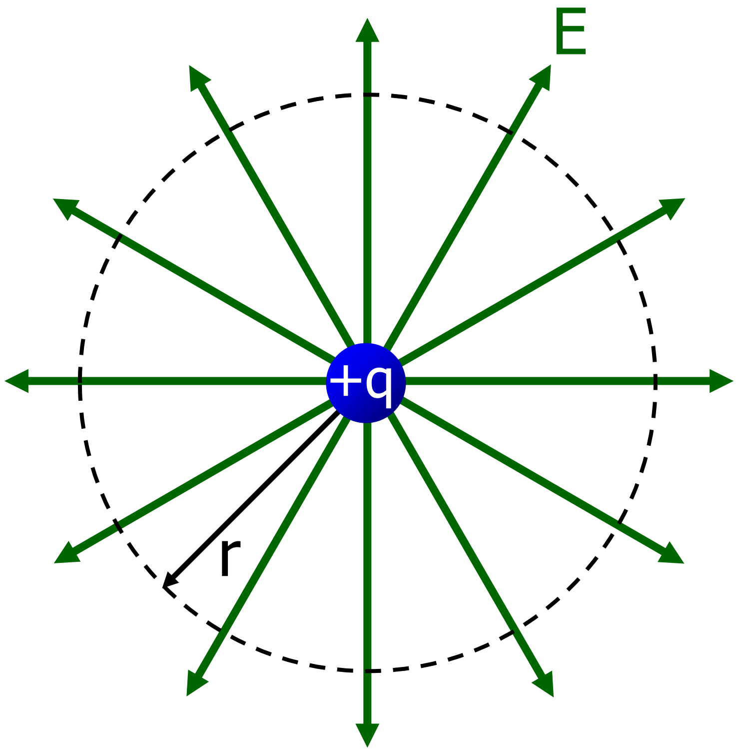
Wrong! The problem statement implies you should calculate $\vec{E}$ as a function of r.



*r does not appear to be a "system parameter."

Example: use Gauss' Law to calculate the electric field from an isolated point charge q.

$$E = \frac{q}{4\pi\varepsilon_o r^2}, \text{ away from } +q$$



"But wait," you say, "we already know the equation for the electric field of a point charge. We haven't learned anything new. It was a lot of work for nothing."

Wrong! You have learned how to apply Gauss' Law. You might find this technique useful on a future test.

You could use a cube instead of a sphere for your Gaussian surface. The flux would be the same, so the electric field would be the same. But I don't recommend that because the flux would be more difficult to calculate.

Strategy for Solving Gauss' Law Problems

- Select a Gaussian surface with symmetry that "matches" the charge distribution.

  Use symmetry to determine the direction of $\vec{E}$ on the Gaussian surface.

  You want $\vec{E}$ to be constant in magnitude and everywhere perpendicular to the surface, so that $\vec{E} \cdot d\vec{A} = E\,dA$…

  … or else everywhere parallel to the surface so that $\vec{E} \cdot d\vec{A} = 0$.

- Evaluate the surface integral (electric flux).

- Determine the charge inside the Gaussian surface.

- Solve for $\vec{E}$.

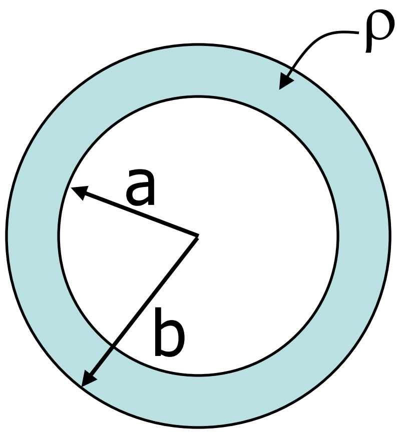
  Don't forget that to completely specify a vector, your answer must contain information about its direction.

Example: calculate the electric field for $0<r<\infty$ for an insulating spherical shell of inner radius a, outer radius b, and with a uniform volume charge density ρ spread throughout shell.

Note: if a conductor is in electrostatic equilibrium, any excess charge must lie on its surface (we will study this in more detail next time), so for the charge to be uniformly distributed throughout the volume, the object must be an insulator.

Example: calculate the electric field for $0<r<\infty$ for an insulating spherical shell of inner radius a, outer radius b, and with a uniform volume charge density ρ spread throughout shell.

Before I can choose a Gaussian surface, I need to have a clear picture of the charge distribution.



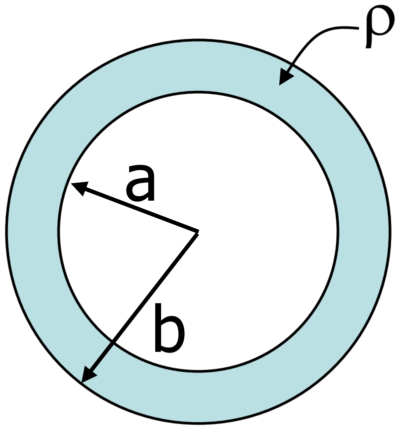
Draw a spherical shell. Drawn is a 2D slice through the center of the 3D sphere.

A uniform volume charge density ρ is distributed throughout the shell. Let's indicate this in the diagram.

The inner radius is 'a' and the outer radius is 'b'.

Example: calculate the electric field for $0<r<\infty$ for an insulating spherical shell of inner radius a, outer radius b, and with a uniform volume charge density ρ spread throughout shell.

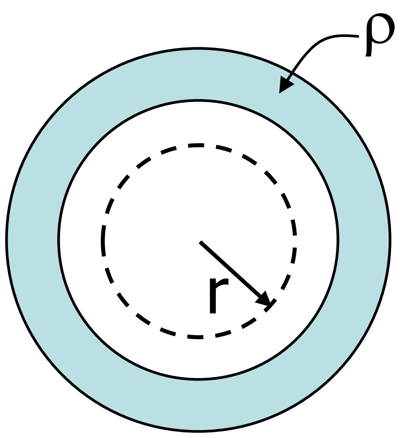
I see three different regions: $0<r<a$, $a<r<b$, and $r>b$. We should do each region separately.



What is the symmetry of the charge distribution?

Example (First Part): calculate the electric field for 0<r<a.

For 0<r<a, draw a Gaussian sphere of radius r<a, centered on the center of the shell.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

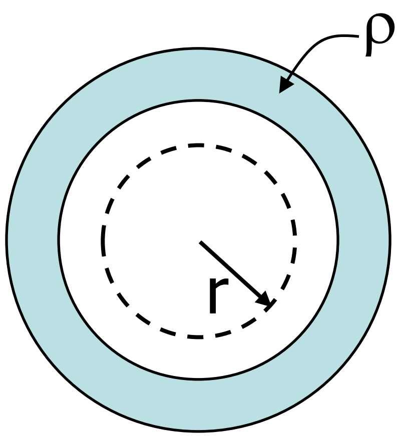
How much charge is enclosed by the sphere? $q_{enclosed}=0$.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

Unless $\vec{E}$ is some kind of pathological function, the only way for the integral to be zero is if $\vec{E} = 0$.

Example (First Part): calculate the electric field for 0<r<a.

So for 0<r<a, $\vec{E} = 0.$

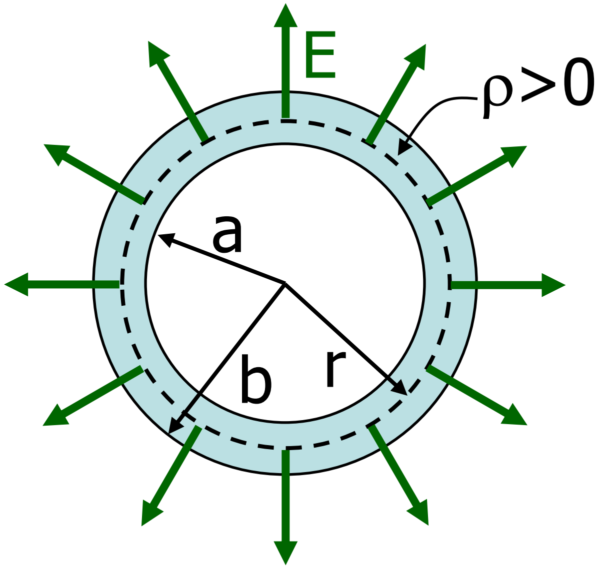


"But wait," you say, "there is a whole bunch of charge nearby. How can the electric field possibly be zero anywhere?"

Answer: Pick any charge on the shell. Assume a positive charge so you can draw an electric field line. Draw an electric field line from the charge out to infinity. The line never goes into the sphere (and if it did, it would go out anyway, because there is no - charge to "land on"). It contributes nothing to the flux.

Example (Second Part): calculate the electric field for a<r<b.

For a<r<b, draw a Gaussian sphere of radius a<r<b, centered at the center of the shell.



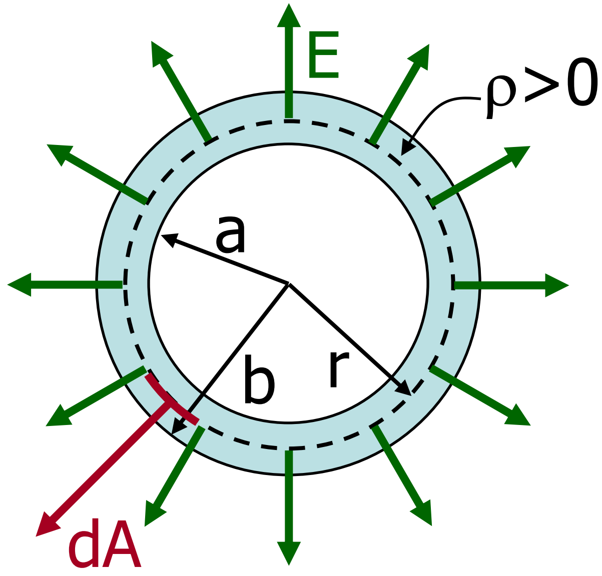
Let's assume ρ is positive, so that I have a direction to draw the electric field. A negative ρ just reverses the direction of the electric field.

The charge distribution is spherically symmetric (you see the "same" thing at any given r). Therefore you must see the same electric field at any given r, so the electric field is also spherically symmetric.

A spherically symmetric electric field is everywhere radial and has the same magnitude at any given r. For ρ>0, $\vec{E}$ is "out."

Example (Second Part): calculate the electric field for a<r<b.

Everywhere on the sphere, $\vec{E}$ and $d\vec{A}$ are parallel and E is constant so



$$\oint \vec{E} \cdot d\vec{A} = \oint E\, dA = E \oint dA$$

$$\oint \vec{E} \cdot d\vec{A} = E\, A_{sphere} = E\, 4\pi r^2$$

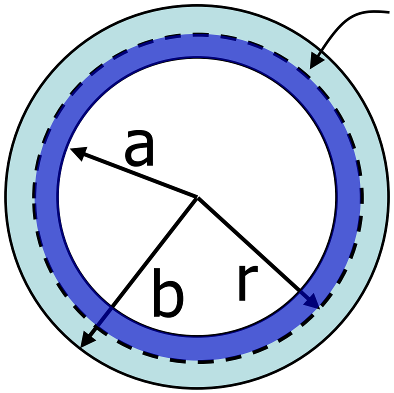
That was easy so far, wasn't it.

The hard part is finding $q_{enclosed}$. We have to determine the charge inside the dashed-line Gaussian sphere.

The volume charge density is ρ. The amount of charge in a volume V is simply ρV.

Example (Second Part): calculate the electric field for a<r<b.

The charge enclosed by the dashed-line Gaussian sphere is the total charge on a spherical shell of inner radius 'a' and outer radius 'r'.



$$q_{\text{enclosed}} = q_{\text{shell of inner radius a and outer radius r}}$$

$$q_{\text{enclosed}} = \rho V_{\text{shell of inner radius a and outer radius r}}$$

$$q_{\text{enclosed}} = \rho \left(V_{\text{sphere of radius r}} - V_{\text{sphere of radius a}}\right)$$

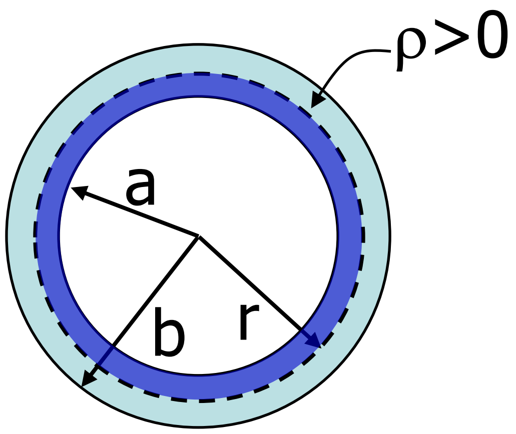
$$q_{\text{enclosed}} = \rho \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right) = \frac{4}{3}\pi\rho(r^3 - a^3)$$

Notice it doesn't matter how large b is. It's only the charge inside r that counts.

Example (Second Part): calculate the electric field for a<r<b.

Finishing…

$$E\ 4\pi r^2 = \frac{\frac{4}{3}\pi\rho(r^3 - a^3)}{\varepsilon_0}$$



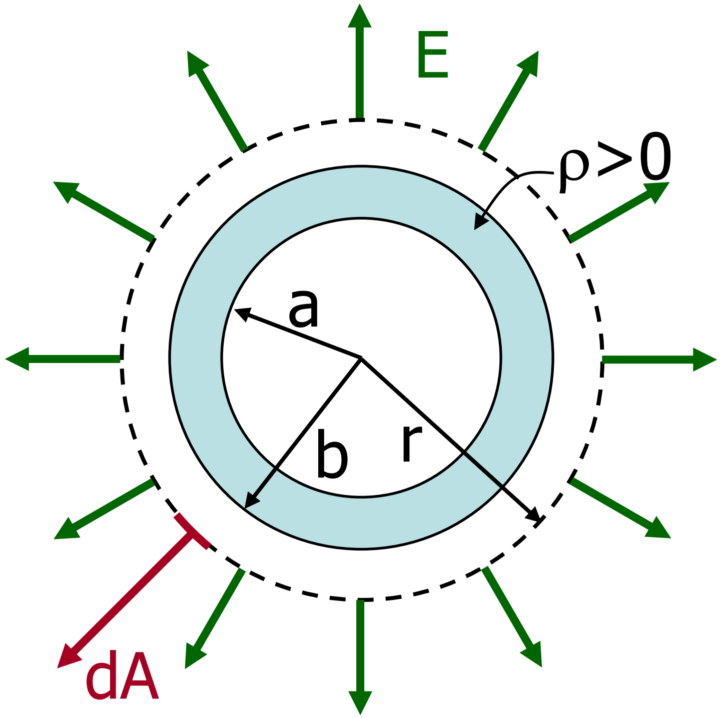
$$E = \frac{\rho}{3}\frac{(r^3 - a^3)}{\varepsilon_0 r^2}$$

Above is the magnitude E, the direction is radially out.

We still need to calculate the electric field for r>b.

Example (Third Part): calculate the electric field for r>b.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$



$$E\ 4\pi r^2 = \frac{\rho V_{\text{shell of inner radius a and outer radius b}}}{\varepsilon_o}$$

$$E = \frac{\rho \frac{4}{3}\pi(b^3 - a^3)}{4\pi r^2\ \varepsilon_o}$$

This is just $Q/4\pi\varepsilon_0 r^2$, like a point charge.

$$\vec{E} = \frac{\rho(b^3 - a^3)}{3\varepsilon_o r^2}, \text{radially out}$$

Summary: electric field for $0<r<\infty$ for an insulating spherical shell of inner radius a, outer radius b, and with a uniform volume charge density ρ spread throughout shell.

0<r<a $\quad \vec{E} = 0$

a<r<b $\quad \vec{E} = \dfrac{\rho(r^3 - a^3)}{3\varepsilon_0 r^2}$, radially out

b>r $\quad \vec{E} = \dfrac{\rho(b^3 - a^3)}{3\varepsilon_o r^2}$, radially out

<u>Something to note</u>: $\vec{E}$ is continuous at both r=a and r=b. This is true because in this problem we are dealing with continuous volumetric charge distributions. This would not be the case in presence of a surface charge distribution.

Today’s agenda:

Electric flux.

You must be able to calculate the electric flux through a surface.

Gauss’ Law.

You must be able to use Gauss’ Law to calculate the electric field of a high-symmetry charge distribution.

Gauss’ Law, other examples.

You must be able to use Gauss’ Law to calculate the electric field of a high-symmetry charge distribution.

Conductors in electrostatic equilibrium.

You must be able to use Gauss’ law to draw conclusions about the behavior of charged particles on, and electric fields in, conductors in electrostatic equilibrium.

Gauss' Law

Last time we learned that

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

Gauss' Law

Always true, not always easy to apply.

and used Gauss' Law to calculate the electric field for spherically-symmetric charge distributions

Let's calculate electric fields for charge distributions that are non-spherical but exhibit a high degree of symmetry, and then consider what Gauss' Law has to say about conductors in electrostatic equilibrium.

Example: calculate the electric field outside a long cylinder of finite radius R with a uniform volume charge density ρ spread throughout the volume of the cylinder.

To be worked at the blackboard in lecture.

"Long" cylinder with "finite" radius means neglect end effects; i.e., treat cylinder as if it were infinitely long.

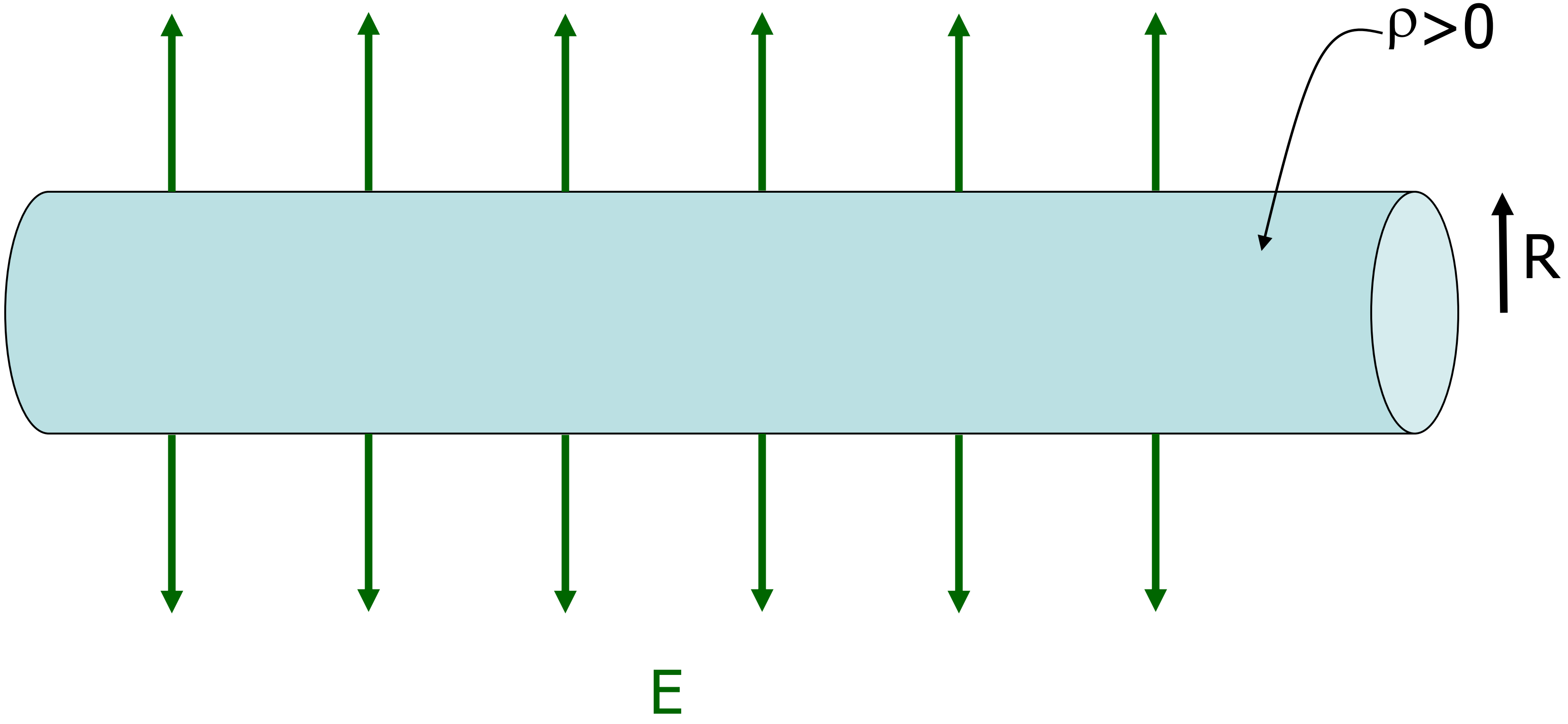
$$E = \frac{|\rho| R^2}{2\varepsilon_0 r}$$

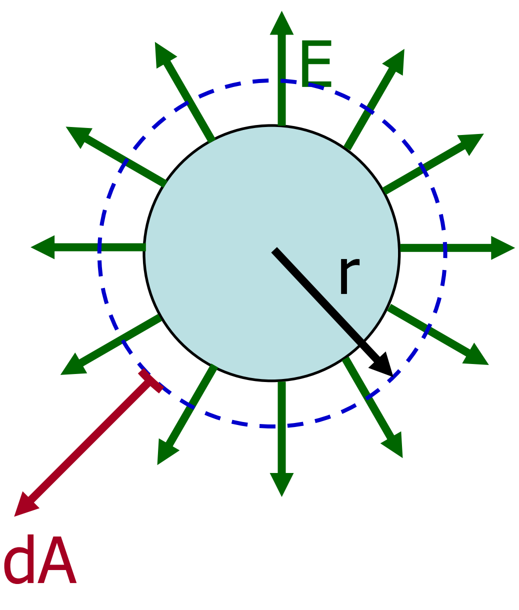
Example: calculate the electric field outside a long cylinder of finite radius R with a uniform volume charge density ρ spread throughout the volume of the cylinder.

Cylinder is looooooong. I'm just showing a bit of it here.

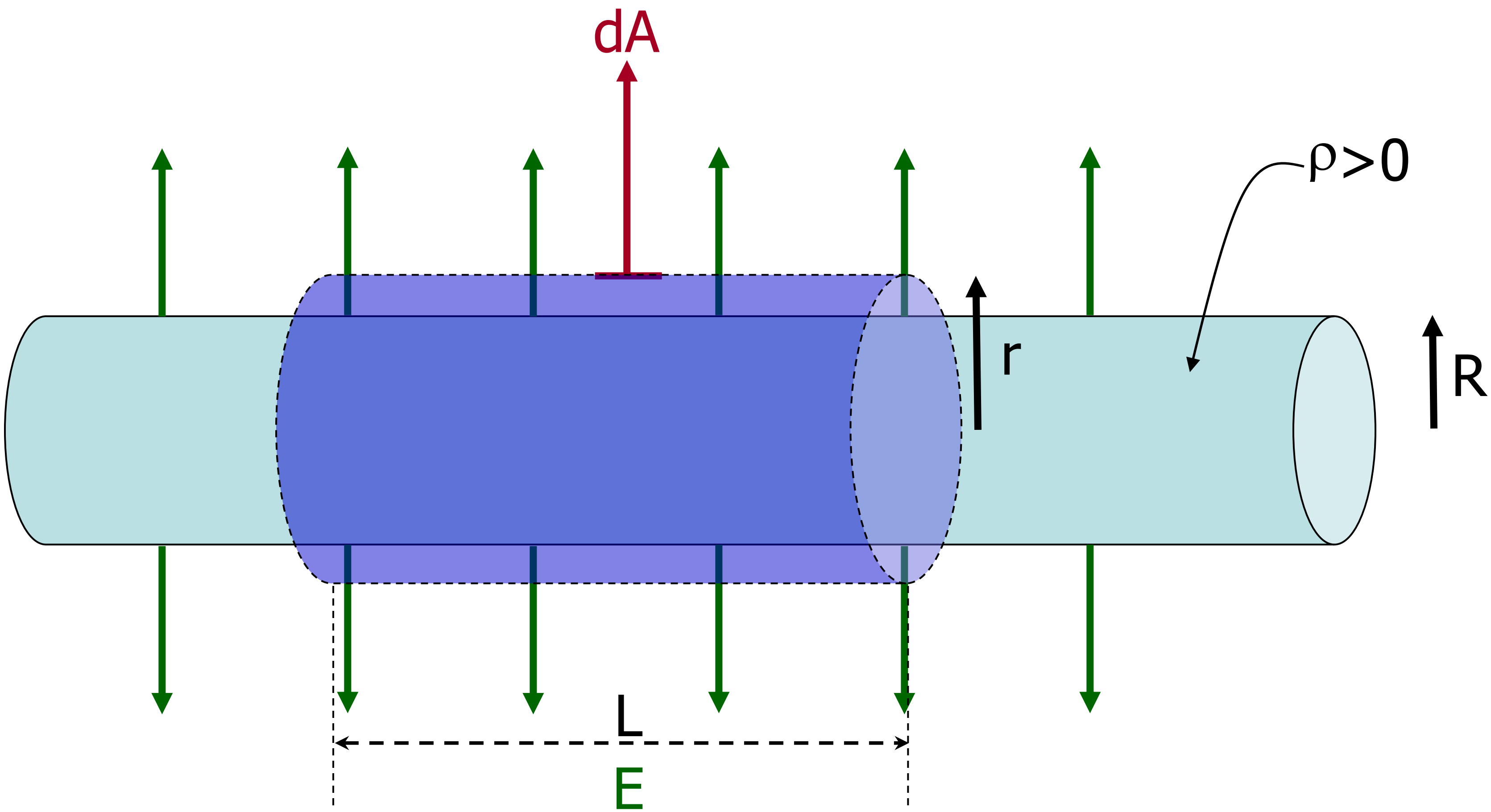
I don't even want to think of trying to use $dE=k|dq|/r^2$ for this.

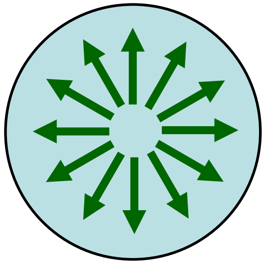
Example: calculate the electric field outside a long cylinder of finite radius R with a uniform volume charge density ρ spread throughout the volume of the cylinder.



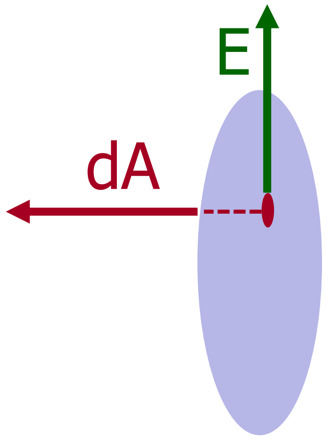


Looking down the axis of the cylinder.

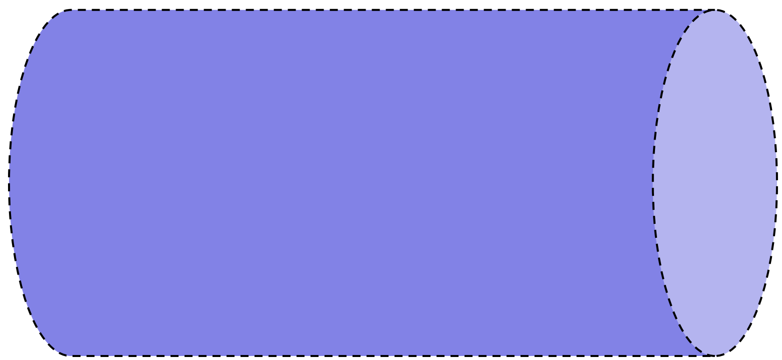

dA
ρ>0
r
R
L
E



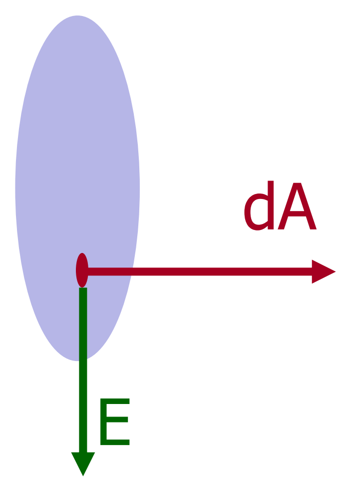
Inside the charged cylinder, by symmetry $\vec{E}$ must be radial.

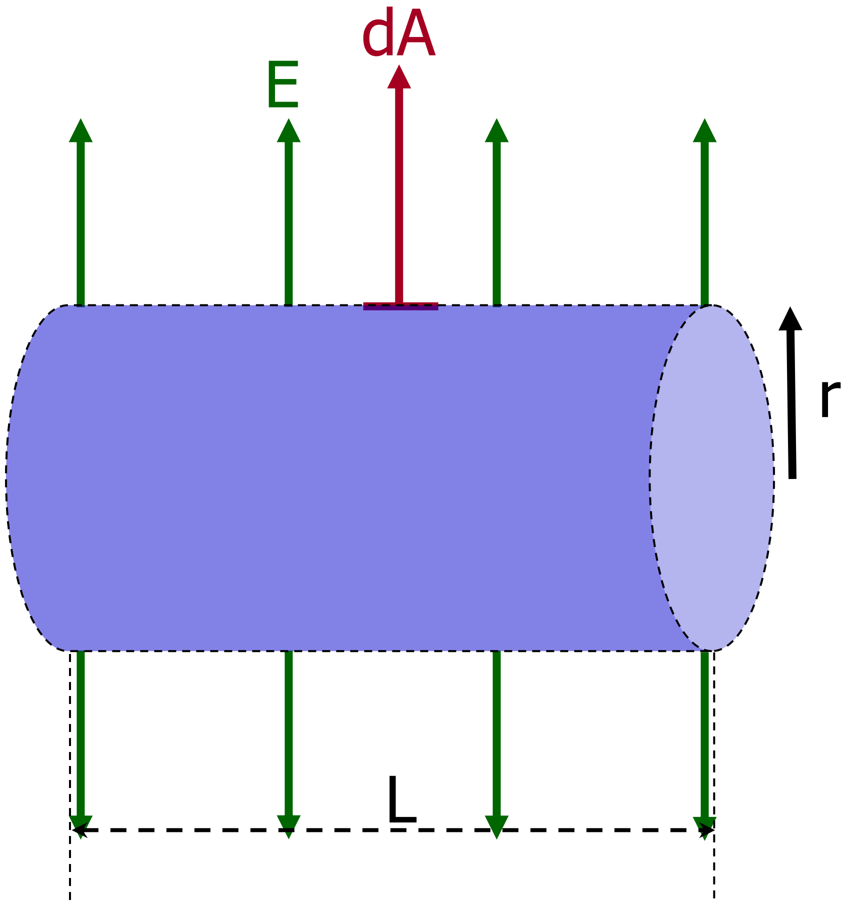


$$\vec{E} \cdot d\vec{A} = 0 \quad \text{because } \vec{E} \perp d\vec{A}$$



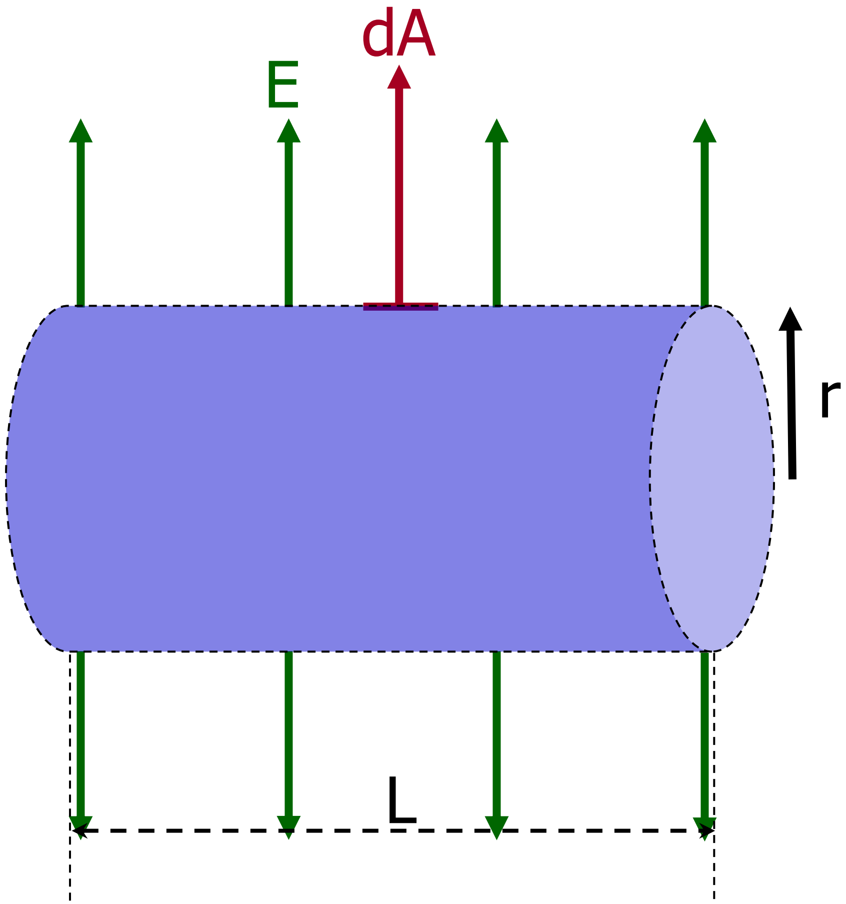
$$\vec{E} \cdot d\vec{A} = 0 \quad \text{because } \vec{E} \perp d\vec{A}$$





$$\vec{E} \cdot d\vec{A} = E\,\mathrm{dA} \text{ because } \vec{\mathrm{E}} \parallel \mathrm{d}\overrightarrow{\mathrm{A}}$$

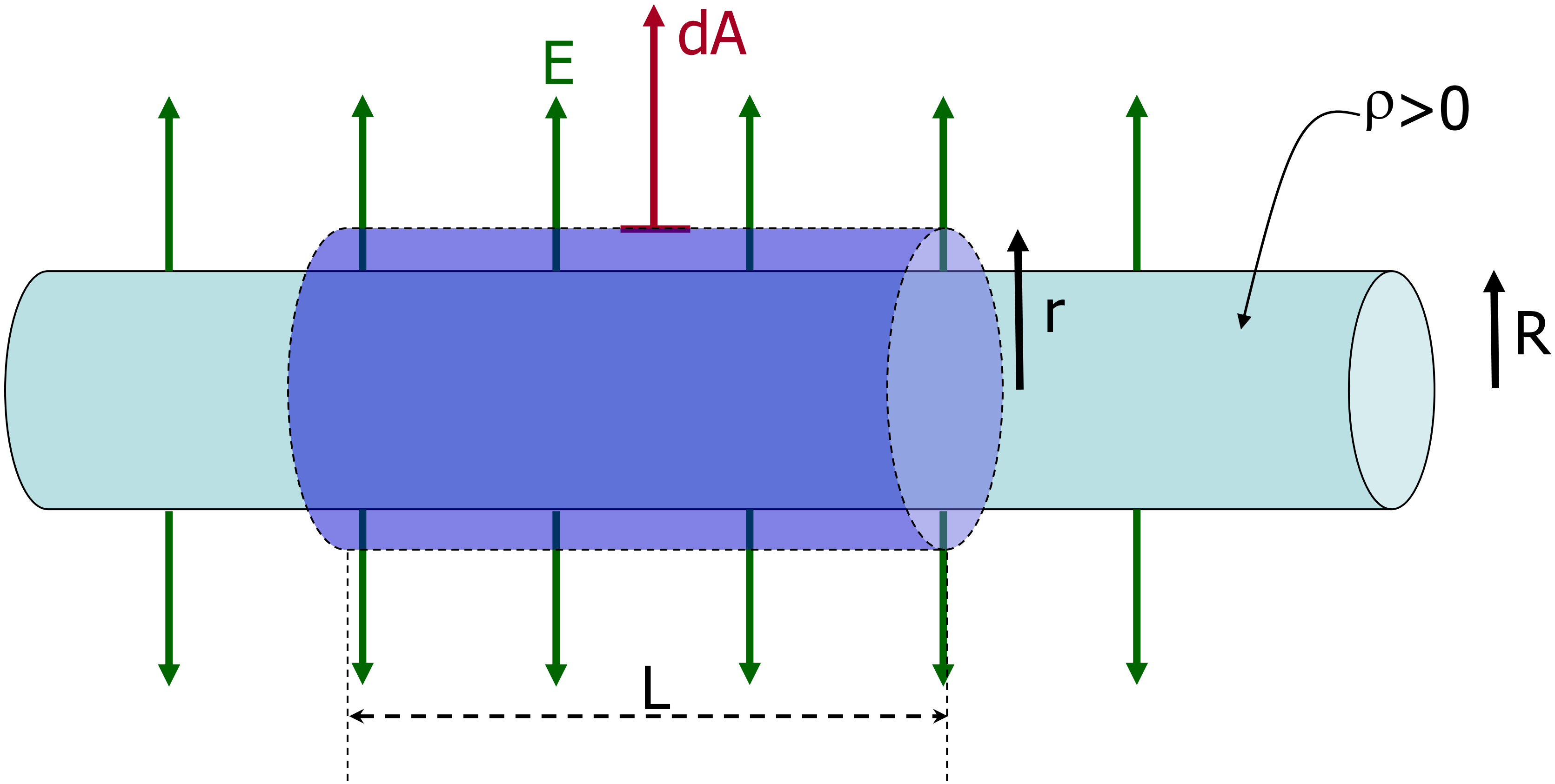
Also $\left|\vec{\mathrm{E}}\right| = \mathrm{E}$ must be constant at any given r.



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \int_{tube} \vec{E} \cdot d\vec{A} = \int_{tube} E\, dA = E \int_{tube} dA$$

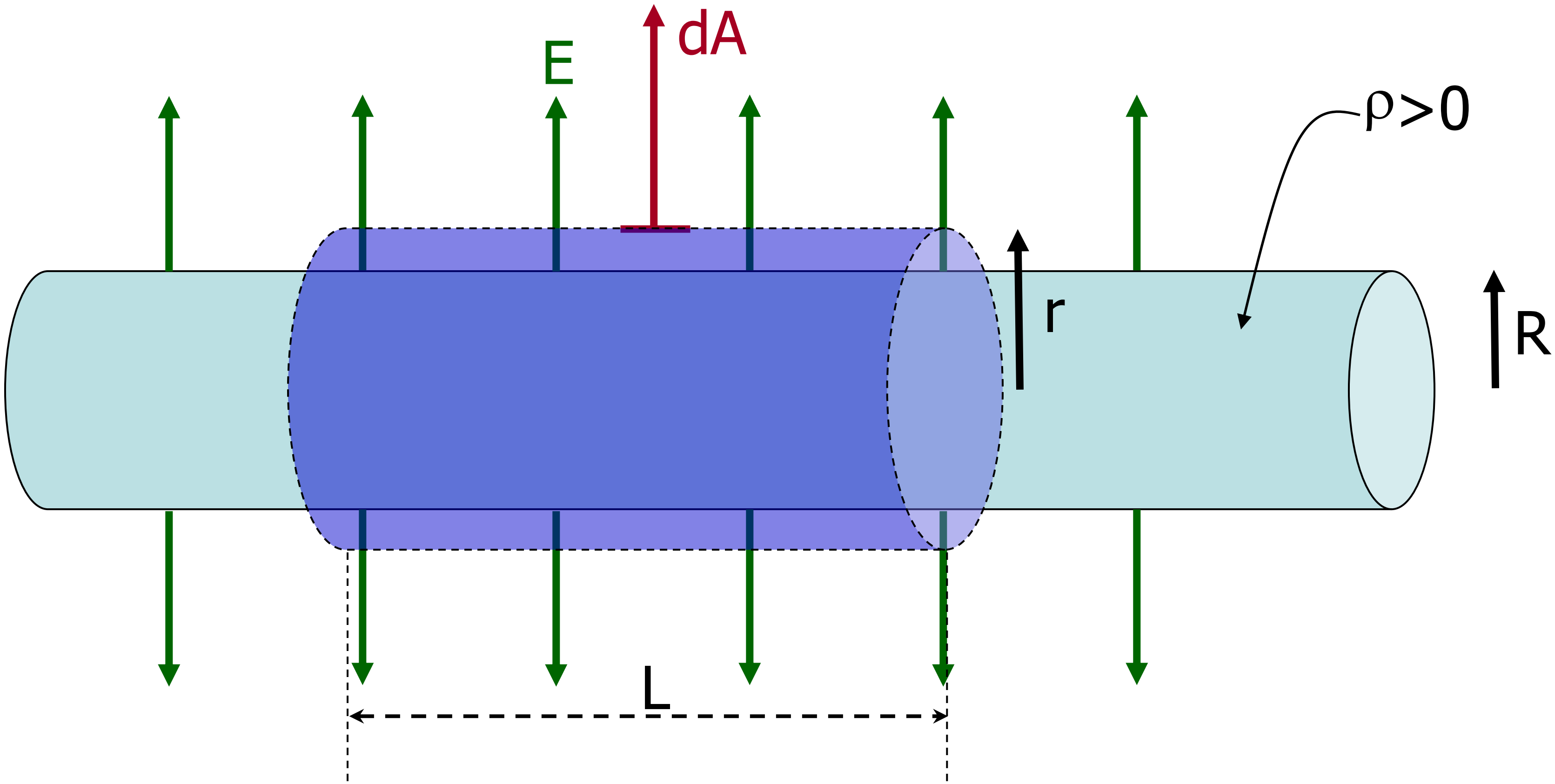
$$= \mathrm{E}\left(\text{circumference of Gaussian cylinder}\right)\left(\text{length of GC}\right)$$

$$= \mathrm{E}\left(2\pi \mathrm{r}\right)\left(\mathrm{L}\right)$$



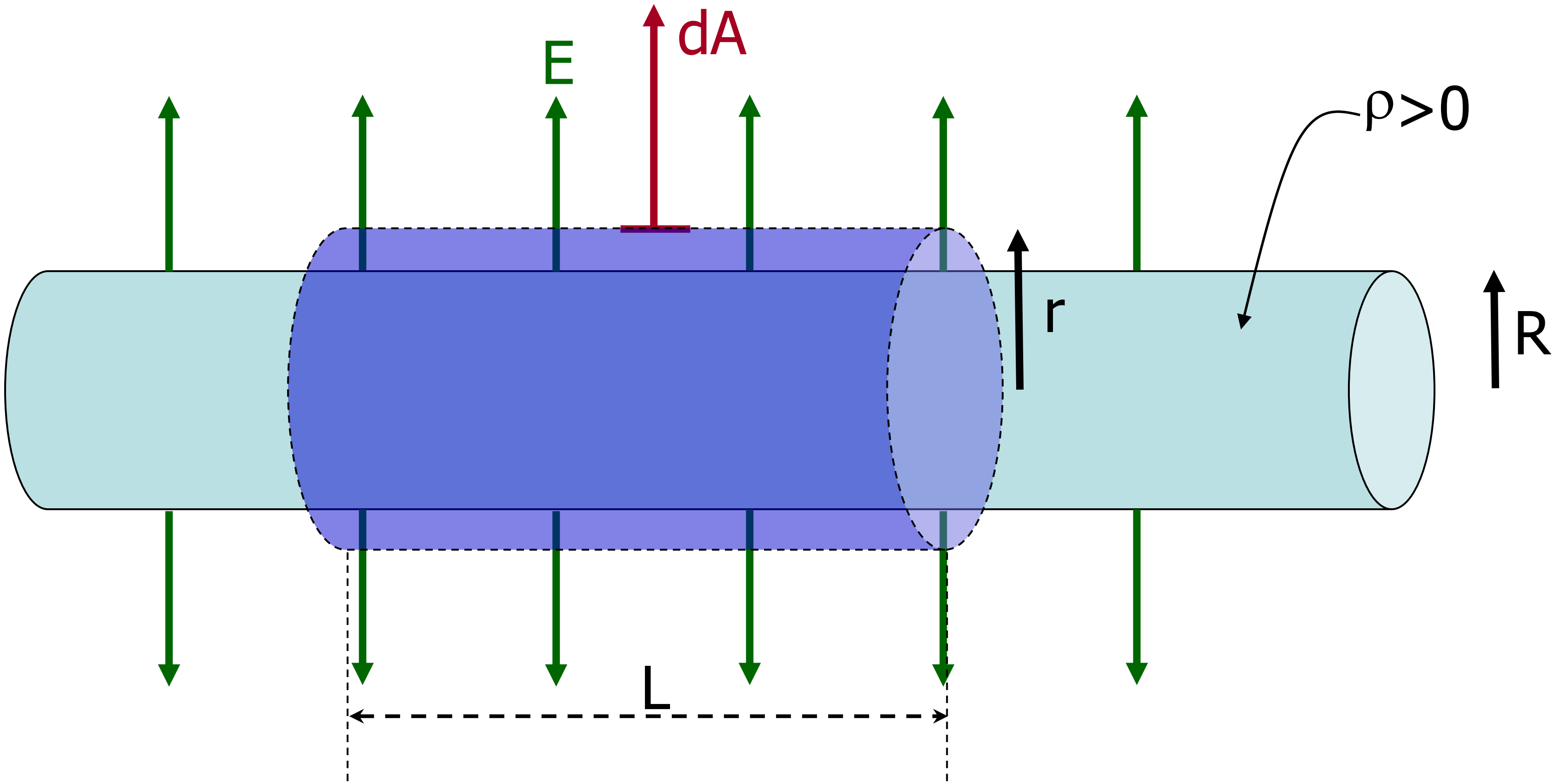
$$\oint \vec{E} \cdot d\vec{A} = E\ 2\pi r\ L = \frac{q_{enclosed}}{\varepsilon_o}$$

$$E\ 2\pi r\ L = \frac{\rho V_{enclosed}}{\varepsilon_o} = \frac{\rho(A_{base})(length)}{\varepsilon_o} = \frac{\rho(\pi R^2)(L)}{\varepsilon_o}$$



$$E\ 2\pi r\ L = \frac{\rho(\pi R^2)(L)}{\varepsilon_o}$$

$$E = \frac{\rho\pi R^2}{2\pi\varepsilon_o r} = \frac{\rho R^2}{2\varepsilon_o r}$$



For positive ρ: $\vec{E} = \dfrac{\rho R^2}{2\varepsilon_o r}$, radially out

In general: $E = \dfrac{|\rho| R^2}{2\varepsilon_o r}$

Why does this vary as 1/r instead of 1/r^2?

For a solid cylinder…

Charge per volume is $\rho = \dfrac{Q}{\pi R^2 L}$

Charge per length is $\lambda = \dfrac{Q}{L}$

So $\rho = \dfrac{Q}{\pi R^2 L} = \dfrac{1}{\pi R^2}\dfrac{Q}{L} = \dfrac{\lambda}{\pi R^2}$

And $E = \dfrac{|\rho| R^2}{2\varepsilon_o r} = \dfrac{\left|\dfrac{\lambda}{\pi R^2}\right| R^2}{2\varepsilon_o r} = \dfrac{|\lambda|}{2\pi\varepsilon_o r}$

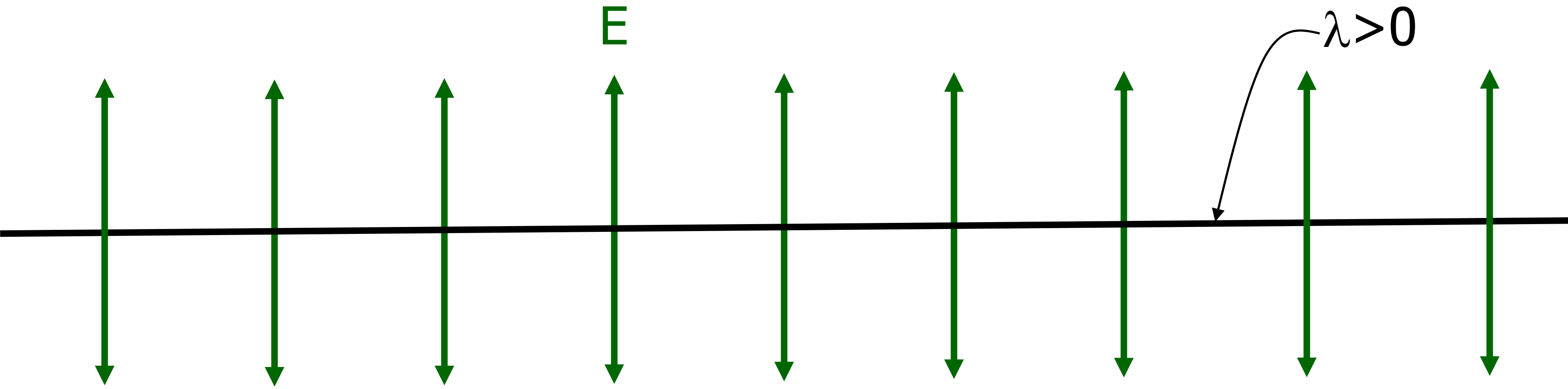
Example: use Gauss' Law to calculate the electric field due to a long line of charge, with linear charge density λ.

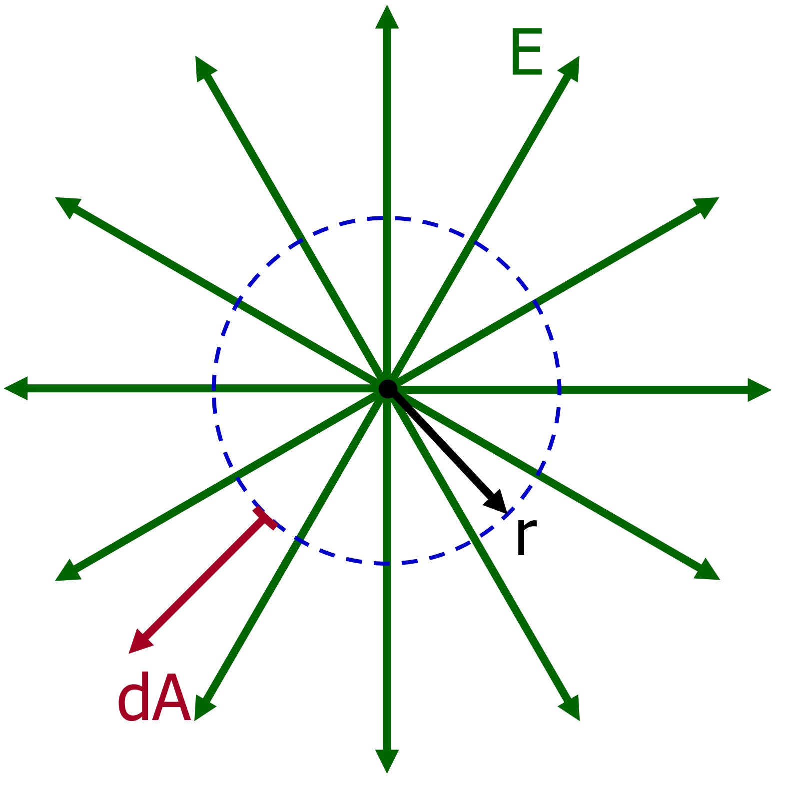
This is easy using Gauss' Law (remember what a pain it was in the previous chapter).

$$E = \frac{|\lambda|}{2\pi\varepsilon_0 r}$$

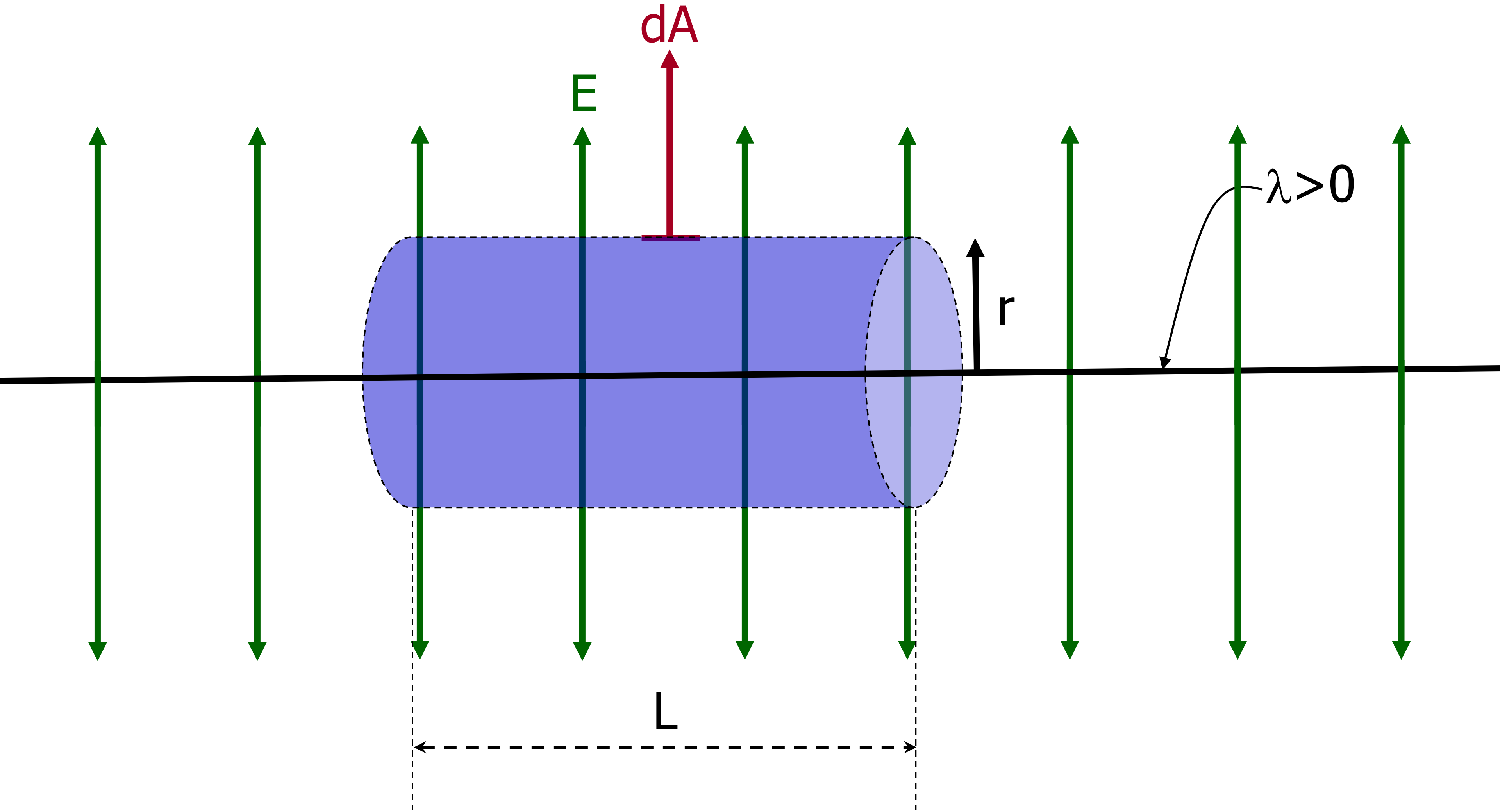
Example: use Gauss' Law to calculate the electric field due to a long line of charge, with linear charge density λ.

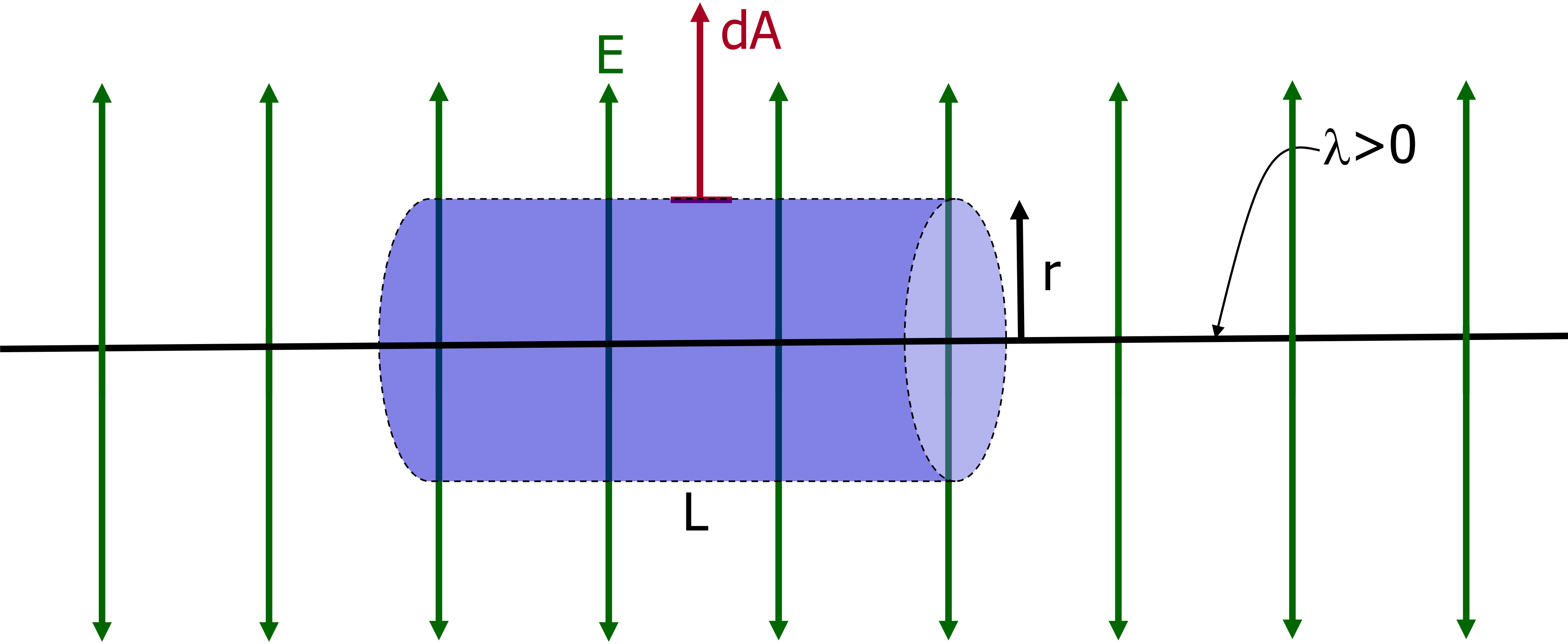
Line is looooooong.





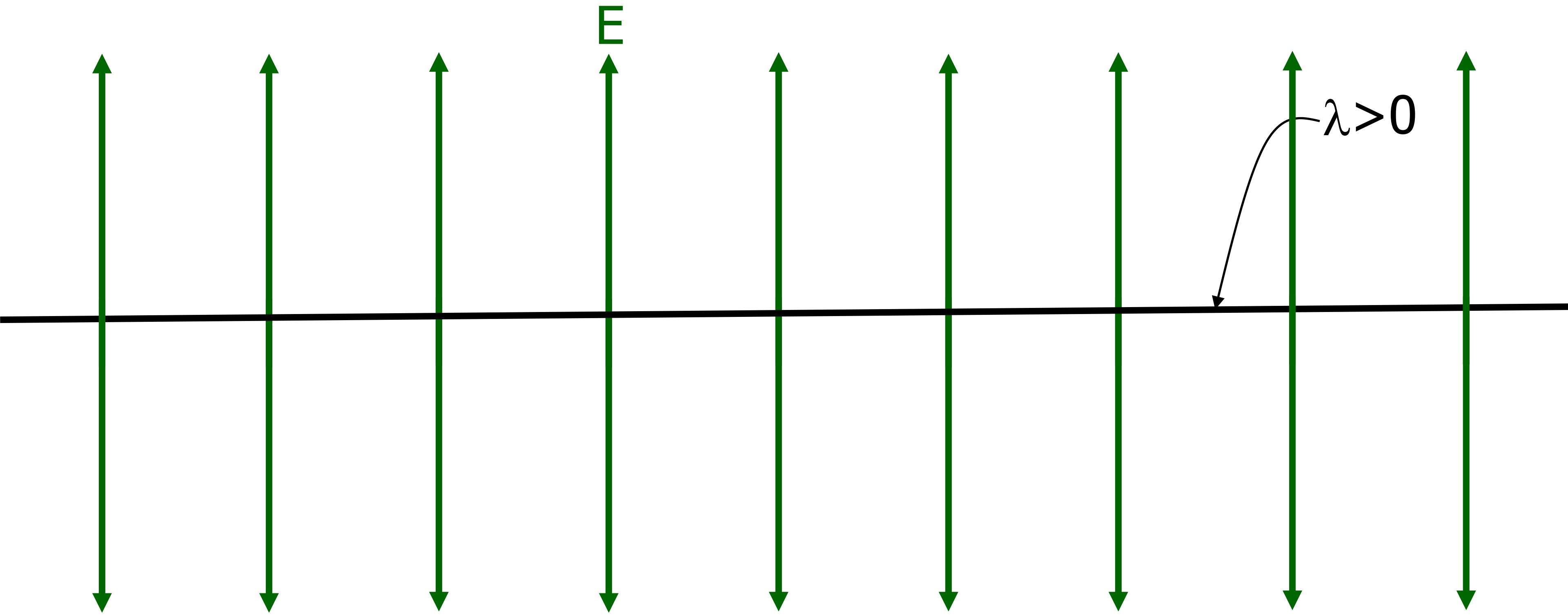
Looking down the line.


dA
E
λ>0
r
L



$$\oint \vec{E} \cdot d\vec{A} = E(2\pi\, r)\ (L) = \frac{q_{enclosed}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



For positive λ: $\vec{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}$, radially out

In general: $E = \dfrac{|\lambda|}{2\pi\varepsilon_0 r}$

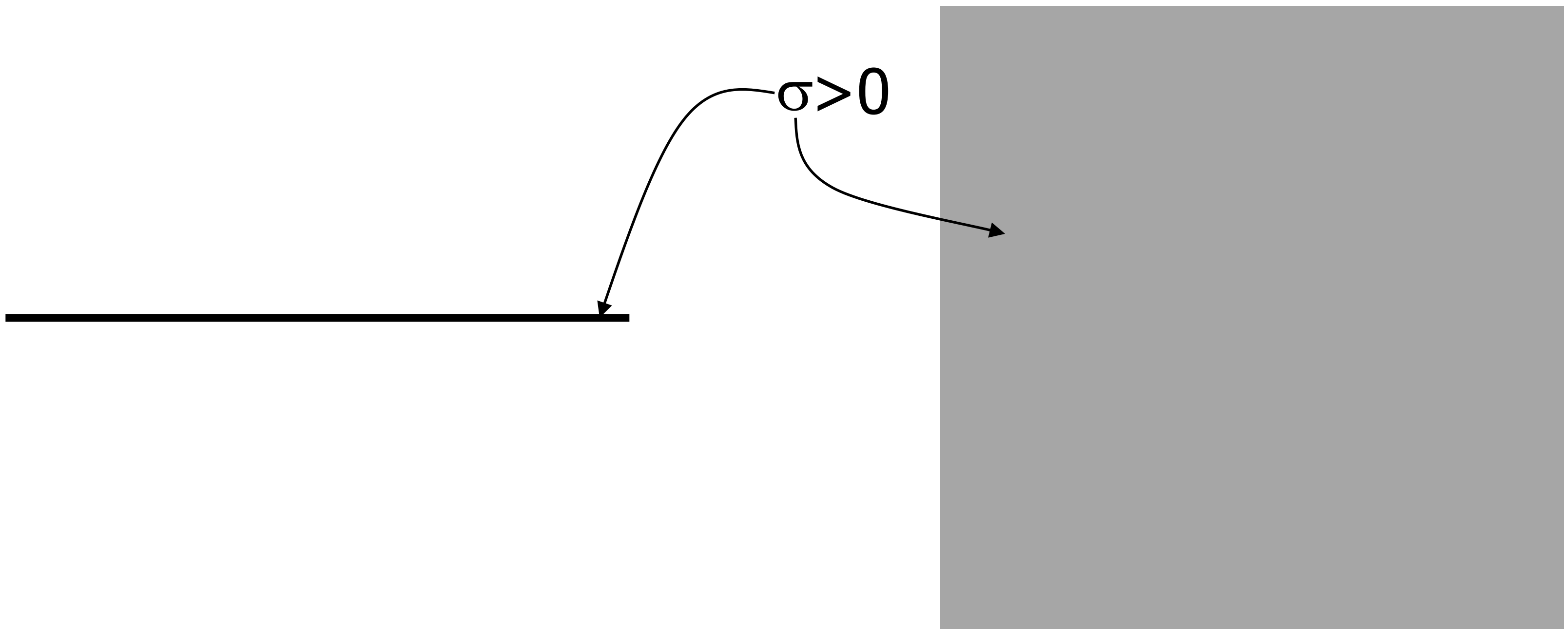
Same as outside a solid cylinder!

Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ.

This is easy using Gauss' Law (remember what a pain it was in the previous chapter).

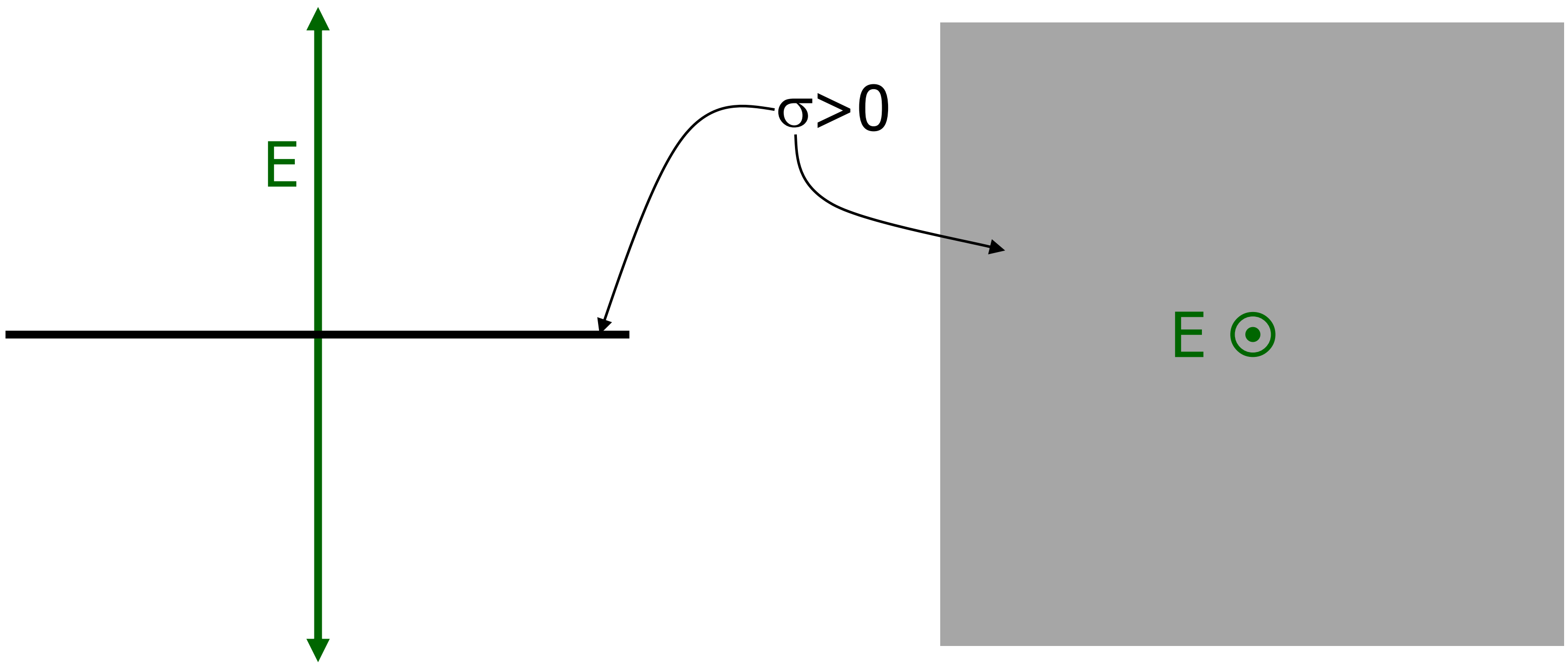
$$E_{sheet} = \frac{|\sigma|}{2\varepsilon_0}.$$

Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ.

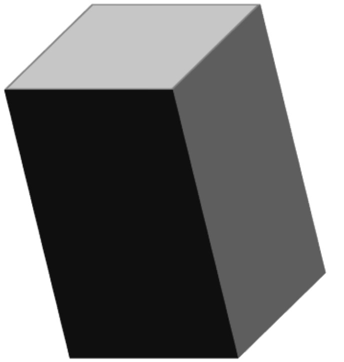


Two views of sheet of charge; side view looking edge on, and top view looking down. Sheet extends infinitely far in two dimensions.

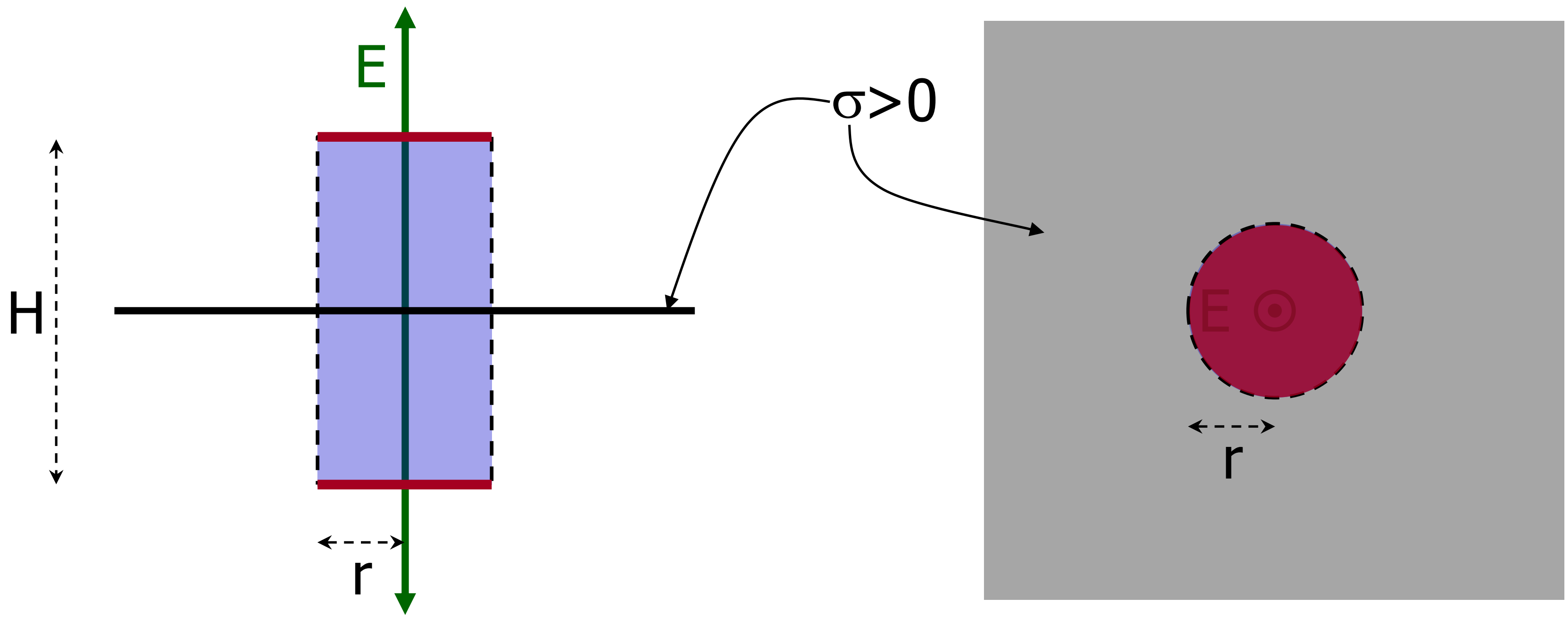
Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ.



For this electric field symmetry, we usually use a "pillbox" (cylinder shape) for our Gaussian surface. In the views above, it will look like a rectangle and a circle. You could also use a rectangular box.



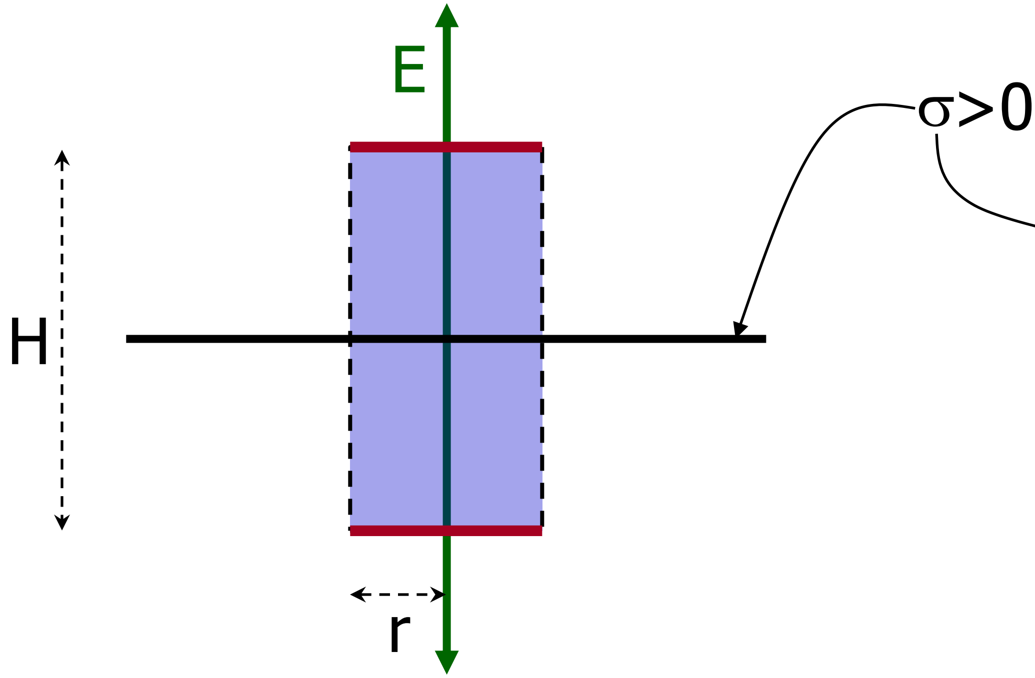
Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ.

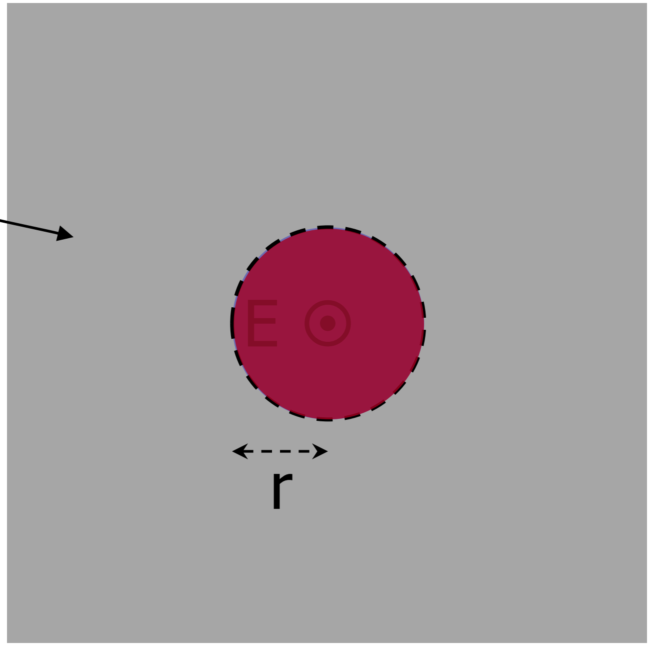


$$\oint \vec{E} \cdot d\vec{A} = 2[E(\pi r^2)] = \frac{q_{enclosed}}{\varepsilon_0}$$

$$q_{enclosed} = \sigma(\pi r^2)$$

Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ.

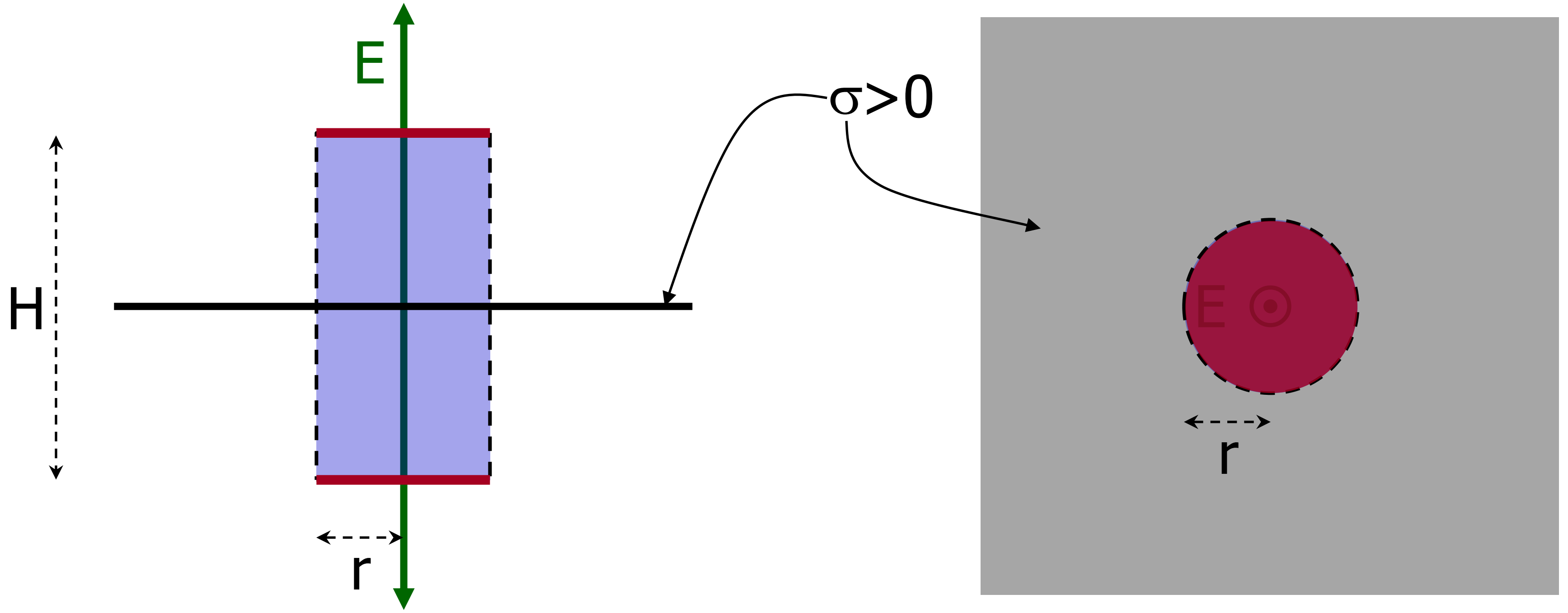




$$2[E(\pi\, r^2)\,] = \frac{\sigma(\pi\, r^2)}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Example: use Gauss' Law to calculate the electric field due to an infinite sheet of charge, with surface charge density σ.



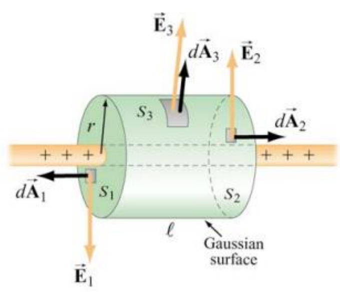
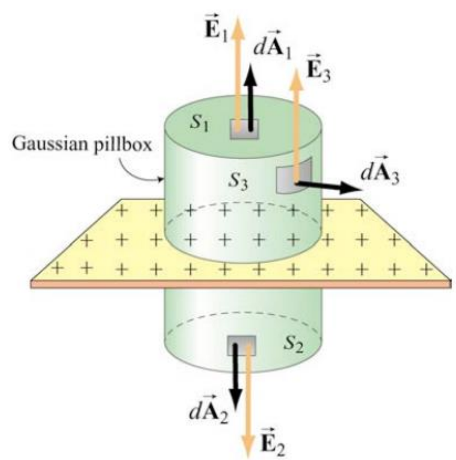
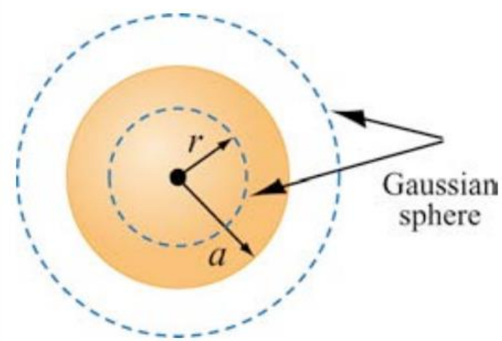
For positive σ: $\vec{E} = \frac{\sigma}{2\varepsilon_0}$, away from the sheet

In general: $E = \frac{|\sigma|}{2\varepsilon_0}$

That sure was easier than the derivation starting from the expression of the electric field produced by an infinitesimal charge dq

Gauss’ Law works well for three kinds of symmetry:

<u>Charge Symmetry</u>	<u>Gaussian Surface</u>
spherical	concentric sphere
cylindrical	coaxial cylinder
planar	pillbox

Choose Gaussian surface	 Coaxial cylinder	 Gaussian pillbox	 Concentric sphere

Today’s agenda:

Electric flux.

You must be able to calculate the electric flux through a surface.

Gauss’ Law.

You must be able to use Gauss’ Law to calculate the electric field of a high-symmetry charge distribution.

Gauss’ Law, other examples.

You must be able to use Gauss’ Law to calculate the electric field of a high-symmetry charge distribution.

Conductors in electrostatic equilibrium.

You must be able to use Gauss’ law to draw conclusions about the behavior of charged particles on, and electric fields in, conductors in electrostatic equilibrium.

Conductors in Electrostatic Equilibrium

Electrostatic equilibrium means there is no **net** motion of tne charges inside the conductor.

The electric field inside the conductor must be zero.

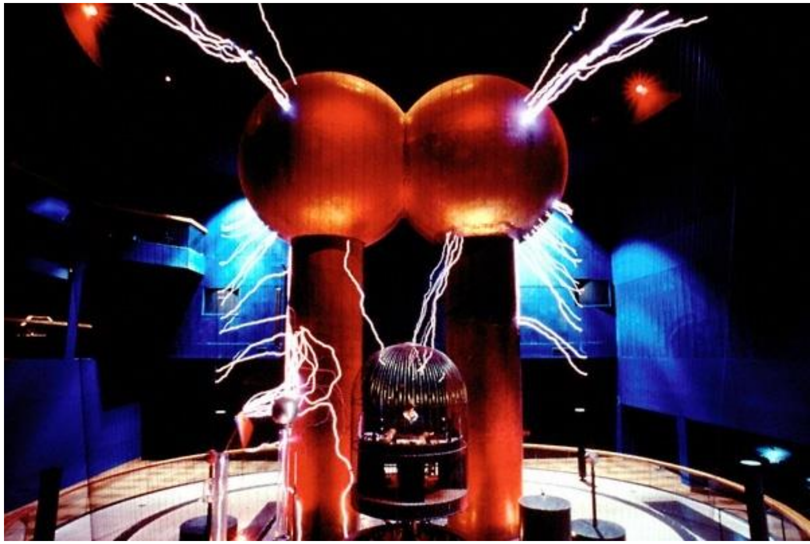
If this were not the case, charges would accelerate.

Any excess charge must reside on the outside surface of the conductor.

Apply Gauss' law to a Gaussian surface just inside the conductor surface. The electric field is zero, so the net charge inside the Gaussian surface is zero. Any excess charge must go outside the Gaussian surface, and on the conductor surface.

The electric field just outside a charged conductor must be perpendicular to the conductor's surface.

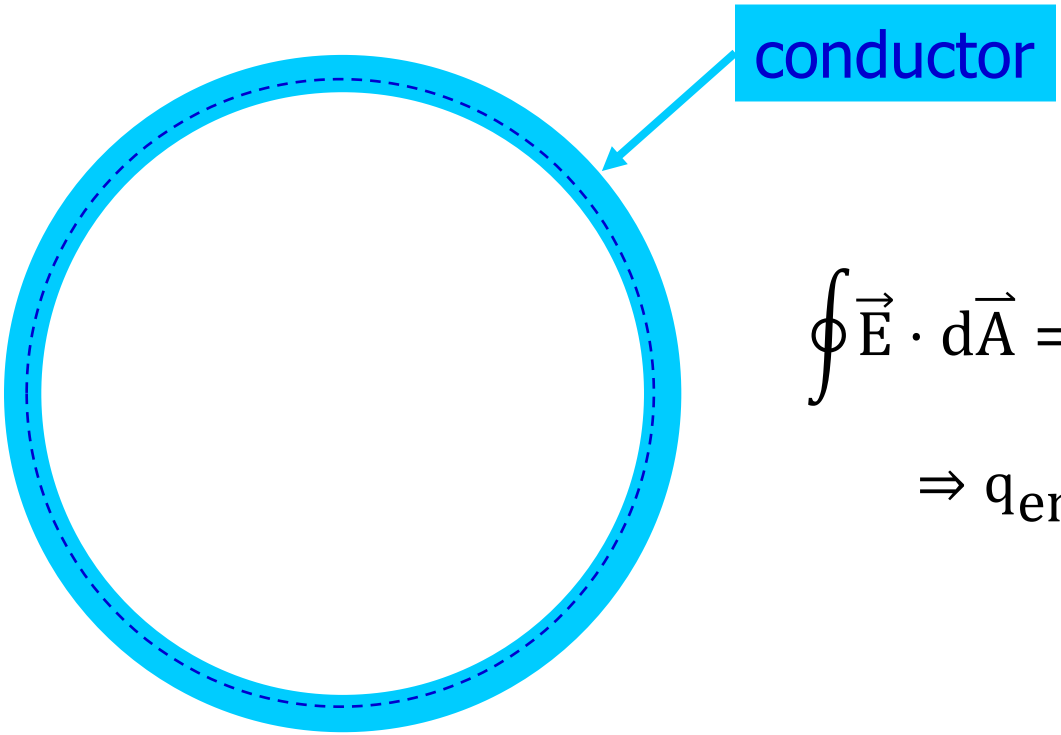
Otherwise, the component of the electric field parallel to the surface would cause charges to accelerate.



The magnitude of the electric field just outside a charged conductor is equal to $|\sigma|/\varepsilon_0$, where $|\sigma|$ is the magnitude of the local surface charge density.

A simple application Gauss' Law. Different from infinite sheet of charge because E is zero inside the conductor.

If there is an empty nonconducting cavity inside a conductor, Gauss' Law tells us there is no net charge on the interior surface of the conductor.

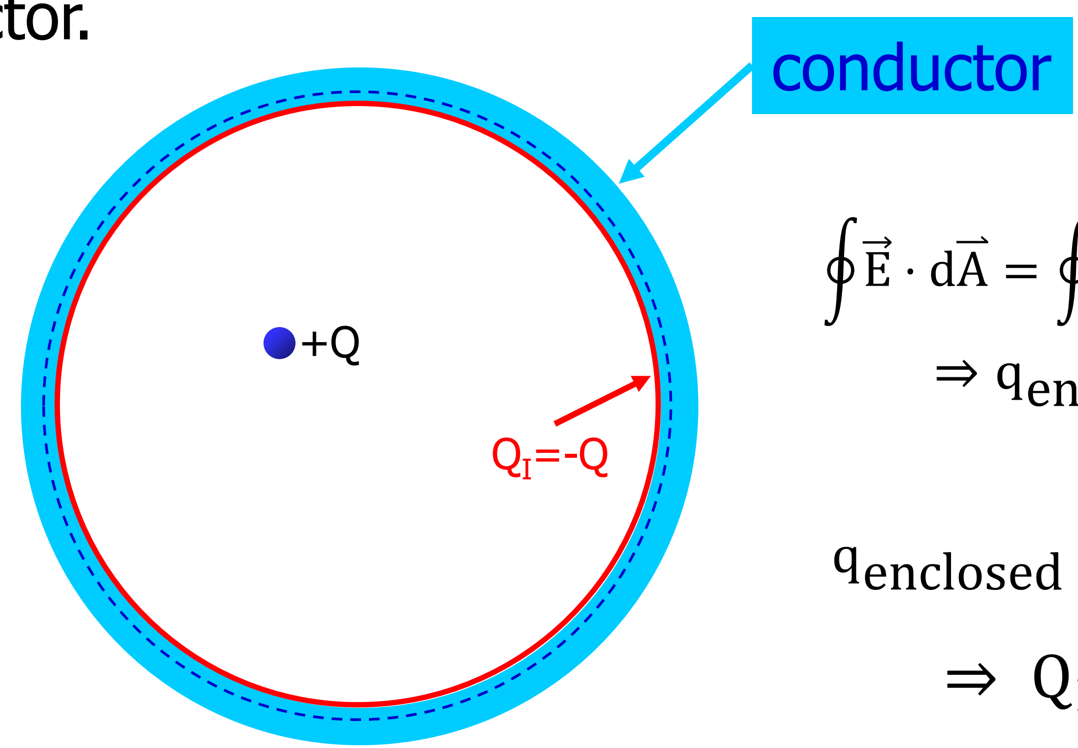


$$\oint \vec{E} \cdot d\vec{A} = \oint 0 \cdot dA = 0$$

$$\Rightarrow q_{enclosed} = 0$$

Construct a Gaussian surface that includes the inner surface of the conductor. The electric field at the Gaussian surface is zero, so no electric flux passes through the Gaussian surface. Gauss' Law says the charge inside must be zero. **Any excess charge must lie on the outer surface!** The conductor does not have to be symmetric, as shown.

If there is a nonconducting cavity inside a conductor, with a charge inside the cavity, Gauss' Law tells us there is an equal and opposite induced charge on the interior surface of the conductor.



$$\oint \vec{E} \cdot d\vec{A} = \oint 0 \cdot dA = 0$$

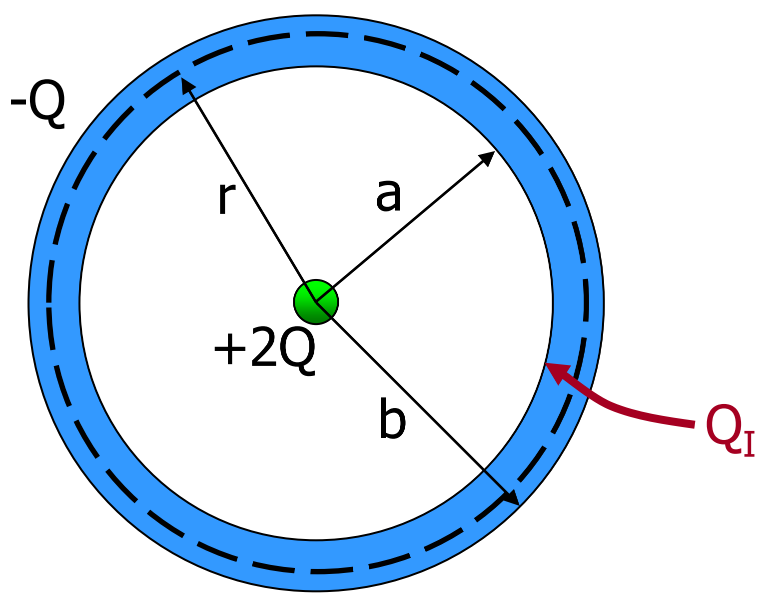
$$\Rightarrow q_{enclosed} = 0$$

$$q_{enclosed} = 0 = +Q + Q_I$$

$$\Rightarrow Q_I = -Q$$

Construct a Gaussian surface that includes the inner surface of the conductor. The electric field at the Gaussian surface is zero, so no electric flux passes through the Gaussian surface. Gauss' Law says the charge inside must be zero. There must be a –Q on the inner surface. **If the net charge on the conductor is not –Q, any additional charge must lie on the outer surface!** The conductor does not have to be symmetric.

Example: a conducting spherical shell of inner radius a and outer radius b with a net charge -Q is centered on point charge +2Q. Use Gauss's law to show that there is a charge of -2Q on the inner surface of the shell, and a charge of +Q on the outer surface of the shell.



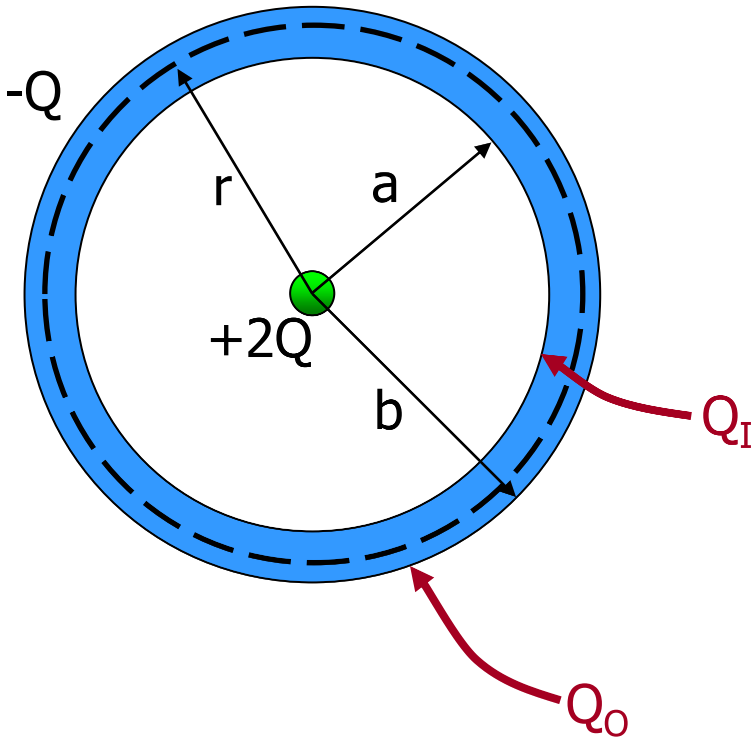
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

E=0 inside the conductor!

Let r be infinitesimally greater than a.

$$0 = \frac{q_{enclosed}}{\varepsilon_0} = \frac{Q_I + 2Q}{\varepsilon_0} \Rightarrow Q_I = -2Q$$

Example: a conducting spherical shell of inner radius a and outer radius b with a net charge -Q is centered on point charge +2Q. Use Gauss's law to show that there is a charge of -2Q on the inner surface of the shell, and a charge of +Q on the outer surface of the shell.



$$Q_I = -2Q$$

From Gauss' Law we know that excess* charge on a conductor lies on surfaces.

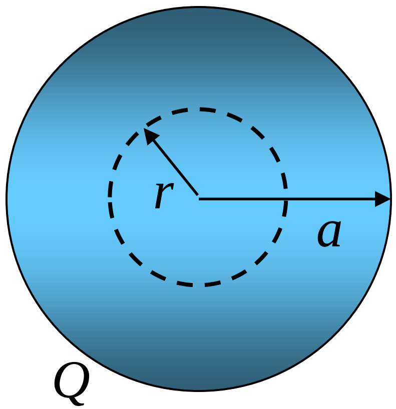
Electric charge is conserved:

$$Q_{shell} = -Q = Q_I + Q_O = -2Q + Q_O$$

$$-Q = -2Q + Q_O \Rightarrow Q_O = +Q$$

*excess=beyond that required for electrical neutrality

Example: an insulating sphere of radius a has a uniform charge density ρ and a total positive charge Q. Calculate the electric field at a point inside the sphere.

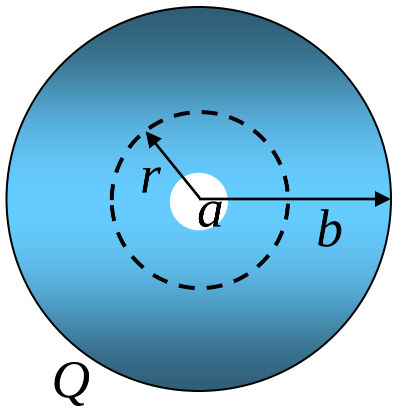


$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho V_{enclosed}}{\varepsilon_o}$$

$$E\left(4\pi r^2\right) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_o}$$

This object in this example is not a conductor.

Example: an insulating spherical shell of inner radius a and outer radius b has a uniform charge density ρ. Calculate the electric field at a point inside the sphere.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$q_{enclosed} = \rho V_{enclosed} = \rho \left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3 \right)$$

Calculate the electric field at a point outside the sphere.

$$q_{enclosed} = \rho V_{enclosed} = \rho \left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3 \right)$$