Today's agenda:

Capacitors and Capacitance.

You must be able to apply the equation C=Q/V.

Capacitors: parallel plate, cylindrical, spherical.

You must be able to calculate the capacitance of capacitors having these geometries, and you must be able to use the equation C=Q/V to calculate parameters of capacitors.

Circuits containing capacitors in series and parallel.

You must understand the differences between, and be able to calculate the "equivalent capacitance" of, capacitors connected in series and parallel.

Energy Storage in Capacitors.

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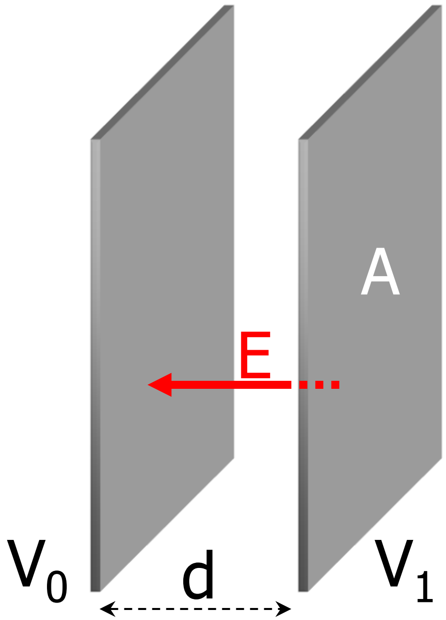
Dielectrics.

You must understand why dielectrics are used, and be able include dielectric constants in capacitor calculations.

Capacitors: the basics

What is a capacitor?

- device for **storing charge**
- simplest example: two parallel conducting plates separated by air

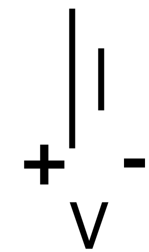


assortment of capacitors

Capacitors in circuits

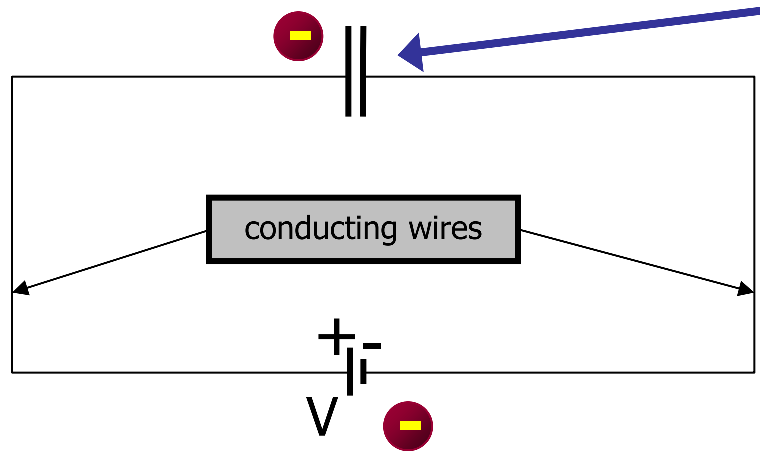
symbol for capacitor (think parallel plates)

symbol for battery, or external potential



battery voltage V is actually **potential difference** between the terminals

- when capacitor is connected to battery, charges flow onto the plates



Capacitor plates build up charges +Q and -Q

- when battery is disconnected, charge remains on plates

Capacitance

How much charge can a capacitor store?

Better question: How much charge can a capacitor store per voltage?

Capacitance: $$C = \frac{Q}{V}$$

V is really |ΔV|, the potential difference across the capacitor

capacitance C is a **device property**, it is always positive

unit of C: farad (F)
1 F is a large unit, most capacitors have values of C ranging from picofarads to microfarads (pF to μF).

micro $\Rightarrow 10^{-6}$, nano $\Rightarrow 10^{-9}$, pico $\Rightarrow 10^{-12}$ (Know for exam!)

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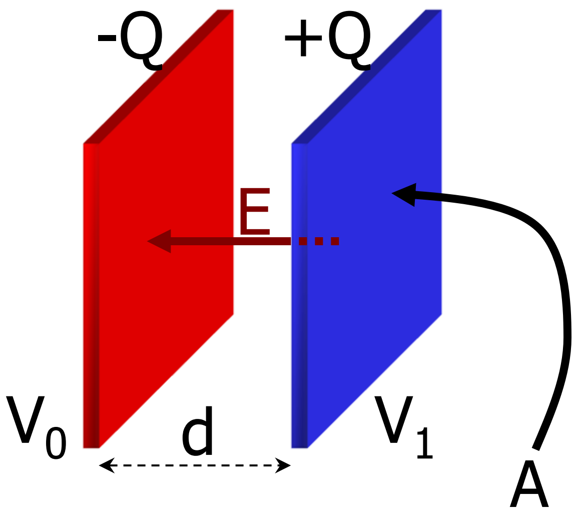
Capacitance of parallel plate capacitor

electric field between two parallel charged plates:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} .$$

Q is magnitude of charge on either plate.



potential difference:

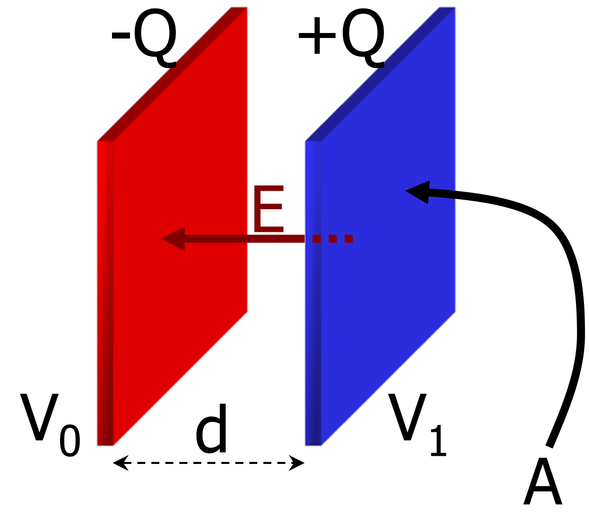
$$\Delta V = V_1 - V_0 = -\int_0^d \vec{E} \cdot d\vec{\ell} = E\int_0^d dx = Ed$$

capacitance:

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{\left(\frac{Q}{\varepsilon_0 A}\right) d} = \frac{\varepsilon_0 A}{d}$$

Parallel plate capacitance depends “only” on geometry.

$$C = \frac{\varepsilon_0 A}{d}$$



This expression is approximate, and must be modified if the plates are small, or separated by a medium other than a vacuum.

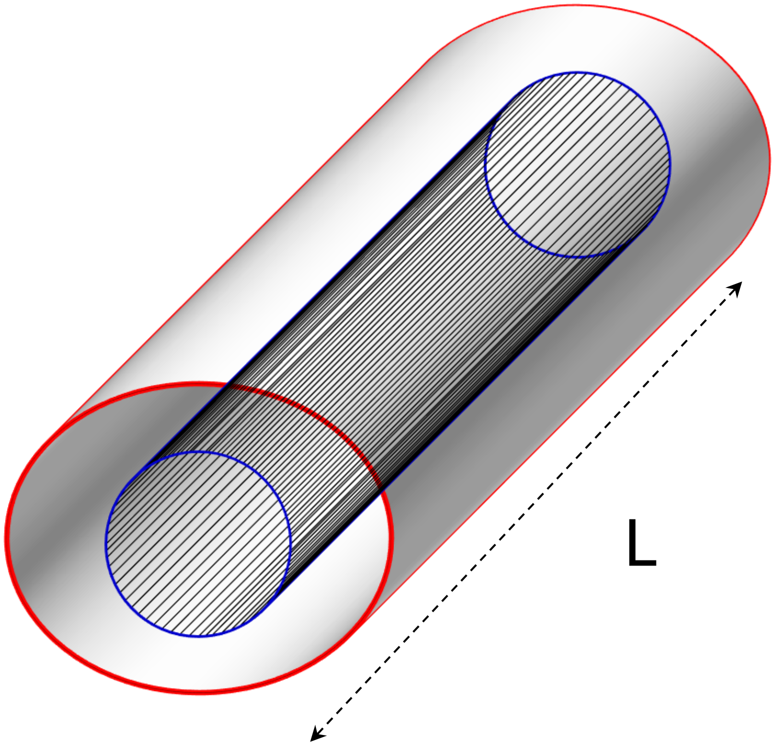
$$C = \frac{\kappa \varepsilon_0 A}{d}$$

Greek letter Kappa. (κ=1 for the vacuum).

κ is NOT the same as $k=9\times10^9$!

Capacitance of coaxial cylinder

- capacitors do not have to consist of parallel plates, other geometries are possible
- capacitor made of two coaxial cylinders:



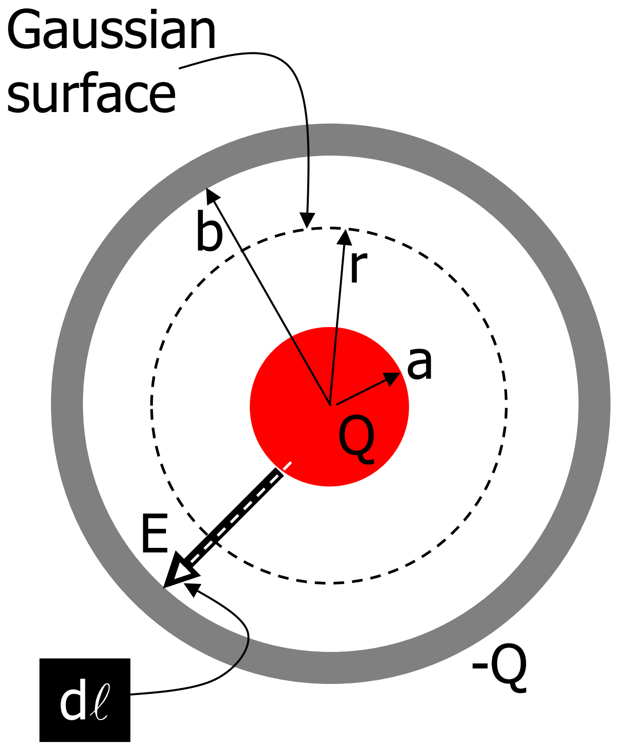
$$\Delta V = V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{\ell} = -\int_a^b E_r\, dr$$

from Gauss law: $E = \dfrac{2k\lambda}{r}$

$$\Delta V = -2k\,\lambda \int_a^b \frac{dr}{r} = -2k\,\lambda \ln\left(\frac{b}{a}\right)$$

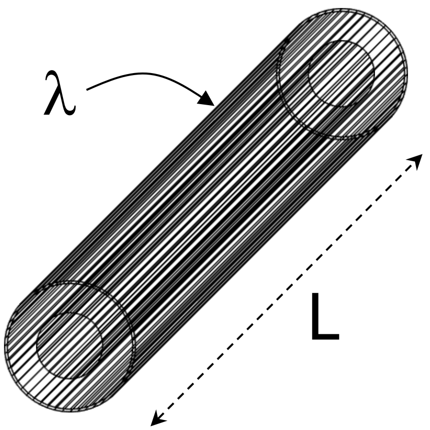
$$C = \frac{Q}{|\Delta V|} = \frac{\lambda L}{|\Delta V|} = \frac{\lambda L}{2k\,\lambda \ln\left(\frac{b}{a}\right)}$$

$$C = \frac{L}{2k \ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}$$

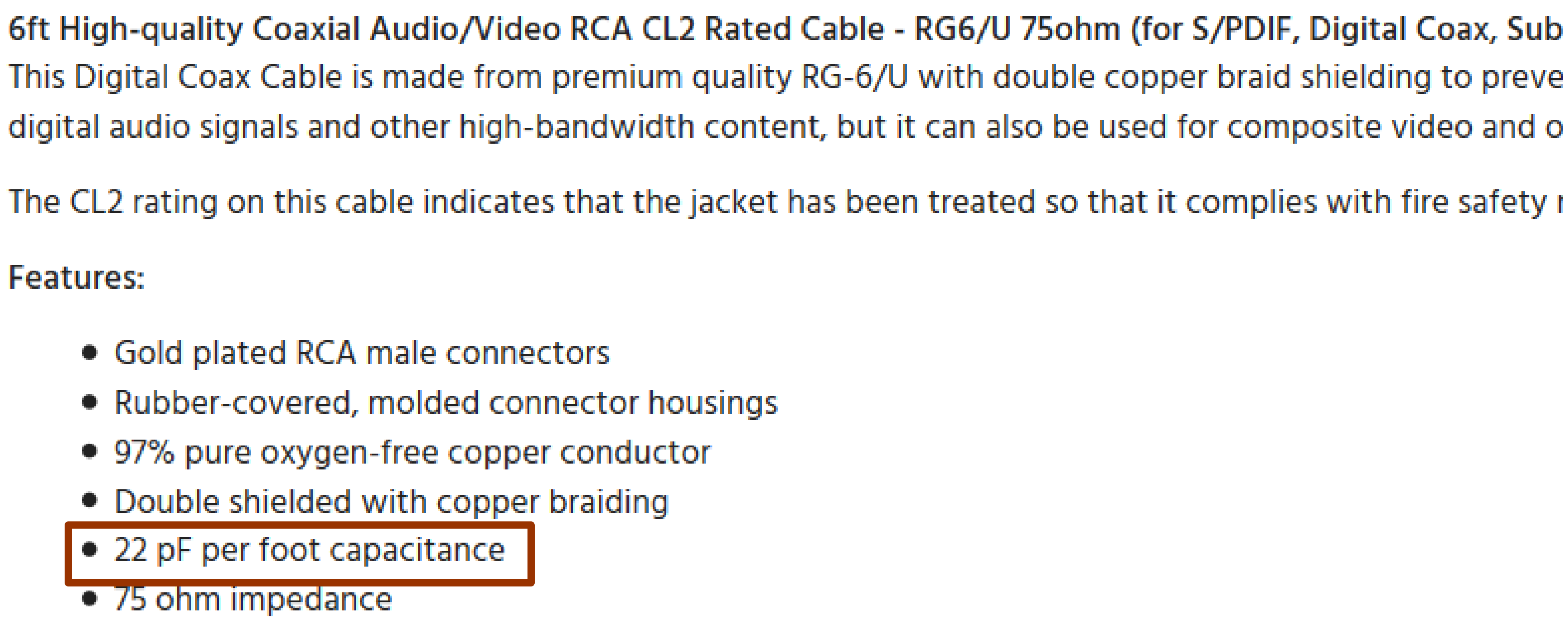


capacitance per unit length: $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}$

Example application:



coaxial cables, capacitance per length is a critical part of the specifications.



6ft High-quality Coaxial Audio/Video RCA CL2 Rated Cable - RG6/U 75ohm (for S/PDIF, Digital Coax, Sub

This Digital Coax Cable is made from premium quality RG-6/U with double copper braid shielding to preve digital audio signals and other high-bandwidth content, but it can also be used for composite video and o

The CL2 rating on this cable indicates that the jacket has been treated so that it complies with fire safety r

Features:

- Gold plated RCA male connectors
- Rubber-covered, molded connector housings
- 97% pure oxygen-free copper conductor
- Double shielded with copper braiding
- 22 pF per foot capacitance
- 75 ohm impedance

Isolated Sphere Capacitance

isolated sphere can be thought of as concentric spheres with the outer sphere at an infinite distance and zero potential.

We already know the potential outside a conducting sphere:

$$V = \frac{Q}{4\pi\varepsilon_0 r} \ .$$

The potential at the surface of a charged sphere of radius R is

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

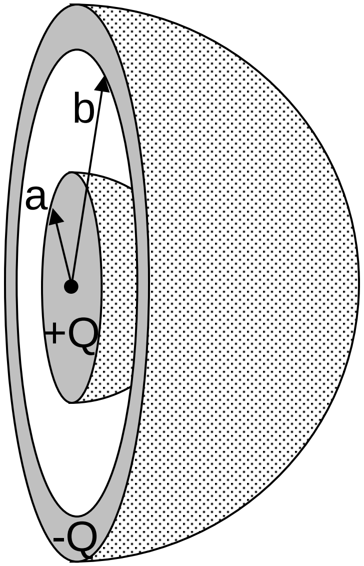
so the capacitance at the surface of an isolated sphere is

$$C = \frac{Q}{V} = 4\pi\varepsilon_0 R.$$

Capacitance of Concentric Spheres

If you have to calculate the capacitance of a concentric spherical capacitor of charge Q…

In between the spheres (Gauss' Law)



$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$|\Delta V| = \frac{Q}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$C = \frac{Q}{|\Delta V|} = \frac{4\pi\varepsilon_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$$

You need to do this derivation ***if*** you have a problem on spherical capacitors!

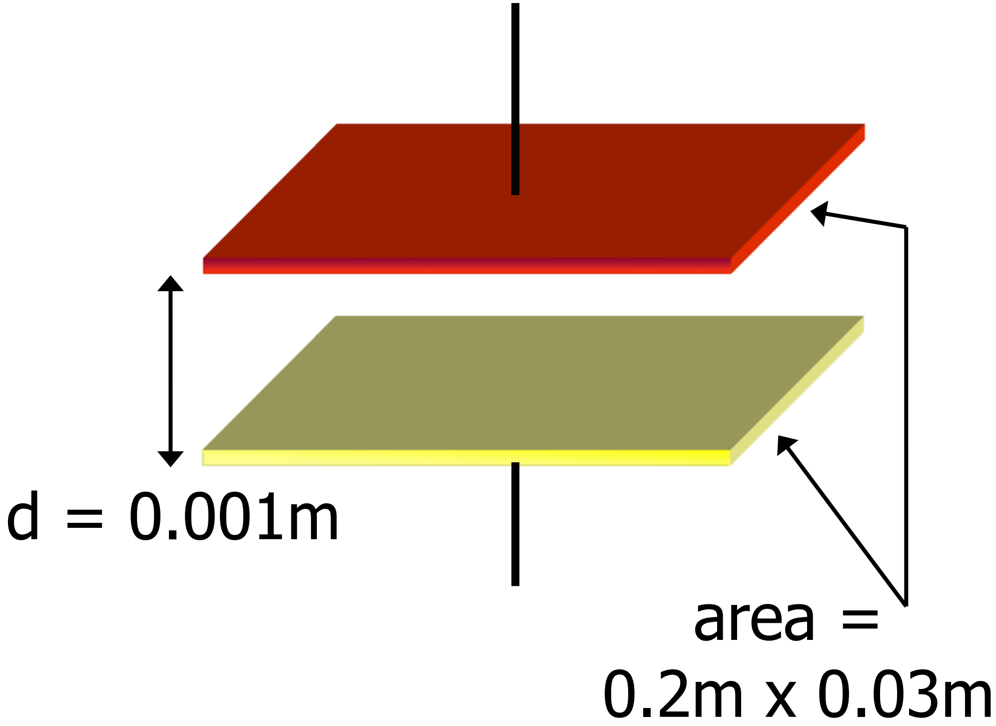
Example: calculate the capacitance of a capacitor whose plates are 20 cm x 3 cm and are separated by a 1.0 mm air gap.

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\left(8.85\times10^{-12}\right)\left(0.2\times0.03\right)}{0.001}\ \text{F}$$

$$C = 53\times10^{-12}\ \text{F}$$

$$\boxed{C = 53\ \text{pF}}$$



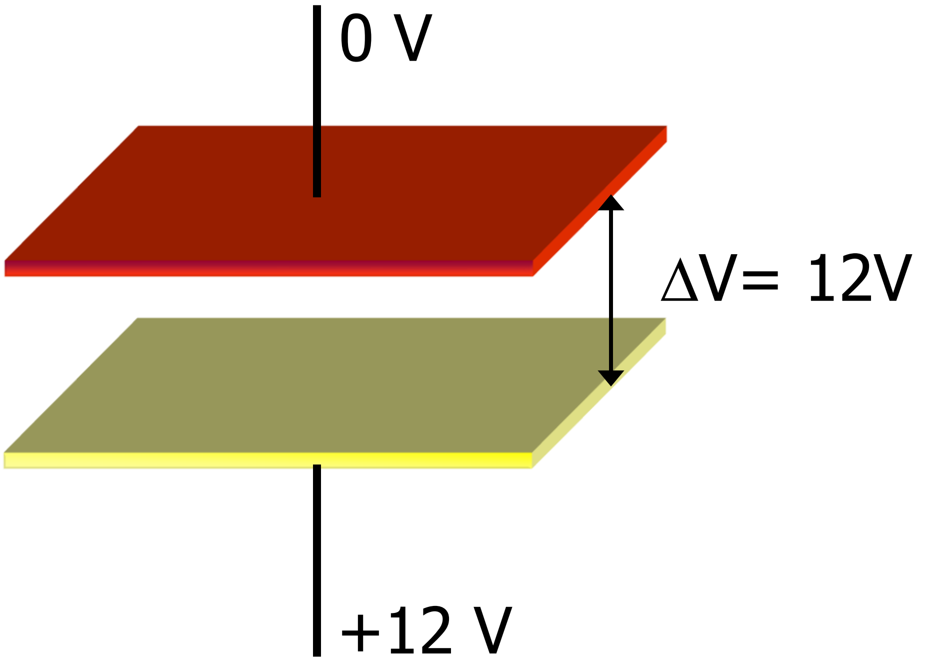
If you keep everything in SI (mks) units, the result is "automatically" in SI units.

Example: what is the charge on each plate if the capacitor is connected to a 12 volt* battery?

$$Q = CV$$

$$Q = \left(53\times10^{-12}\right)\left(12\right)\text{C}$$

$$\boxed{Q = 6.4\times10^{-10}\ \text{C}}$$



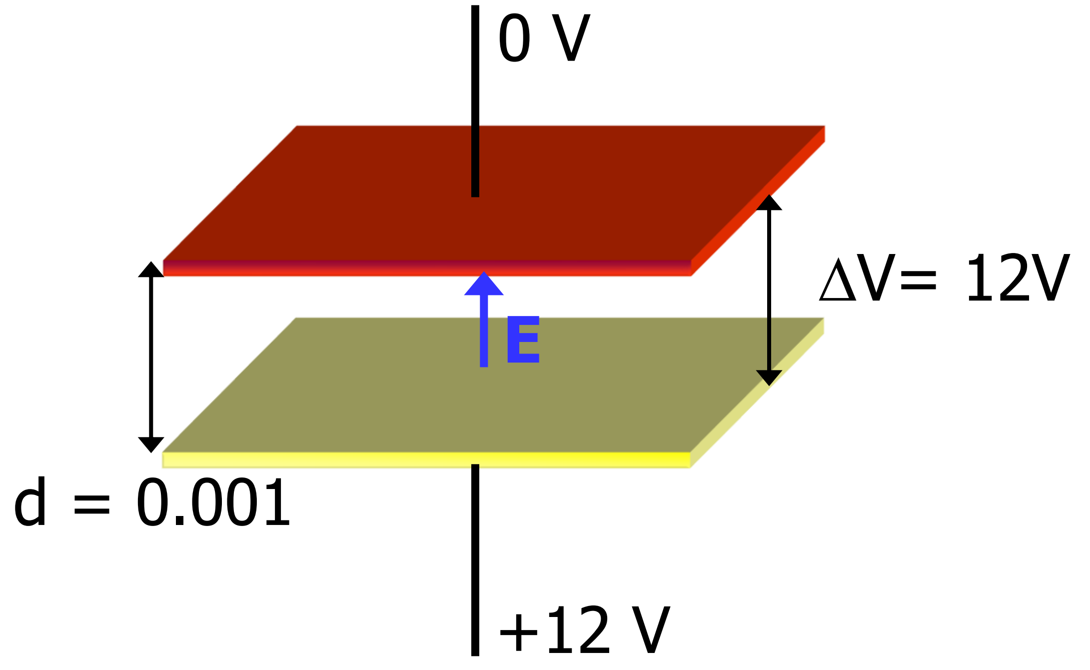
*Remember, it's the potential difference that matters.

Example: what is the electric field between the plates?

$$E = \frac{\Delta V}{d}$$

$$E = \frac{12\text{V}}{0.001\text{ m}}$$

$$\vec{E} = 12000\ \frac{\text{V}}{\text{m}}\ ,\text{"up."}$$



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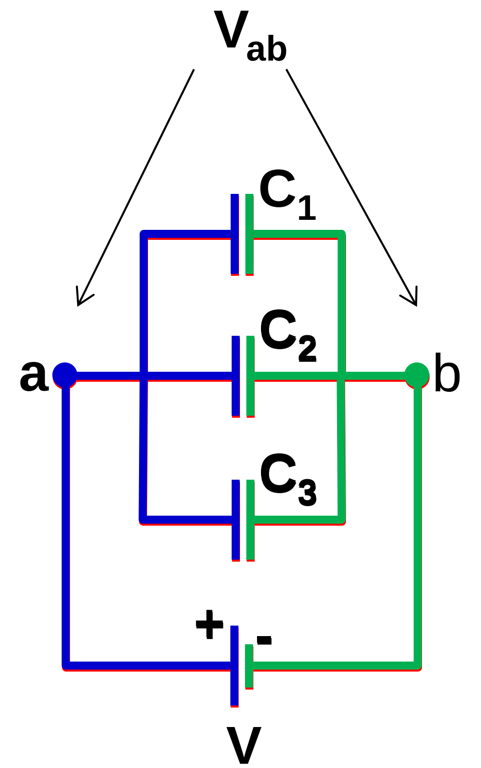
Circuits Containing Capacitors in Parallel

Capacitors connected in parallel:



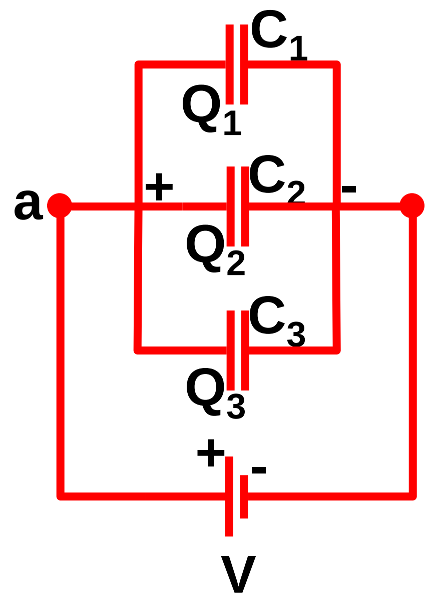
all three capacitors must have the same potential difference (voltage drop) $V_{ab} = V$

General concept: When circuit components are connected in parallel, then the voltage drops across these components are all the same.

$\Rightarrow Q_1 = C_1 V$

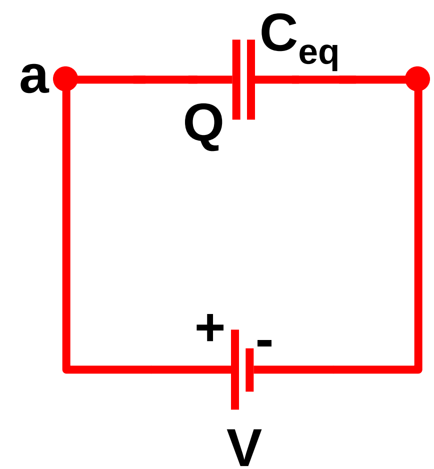
& $Q_2 = C_2 V$

& $Q_3 = C_3 V$



Imagine replacing the parallel combination of capacitors by a single **equivalent capacitor**

"equivalent" means "stores the same total charge if the voltage is the same."



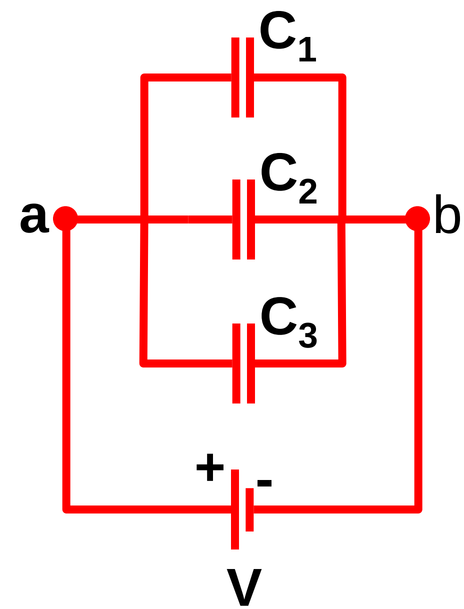
$$Q_{total} = C_{eq} V = Q_1 + Q_2 + Q_3$$

Important!

Summarizing the equations on the last slide:

$$Q_1 = C_1 V \qquad Q_2 = C_2 V \qquad Q_3 = C_3 V$$

$$Q_1 + Q_2 + Q_3 = C_{eq} V$$



Using $Q_1 = C_1V$, etc., gives

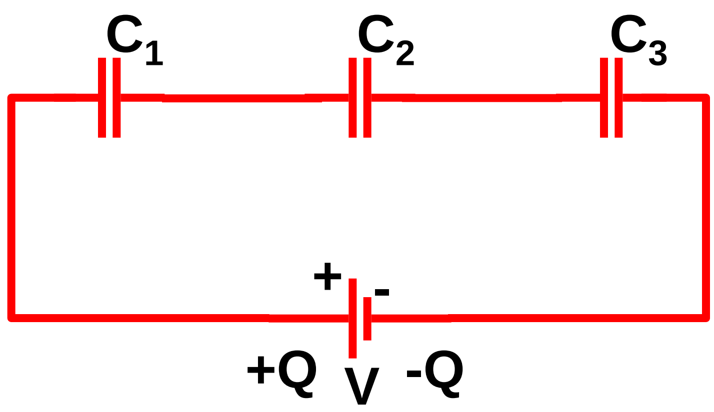
$$C_1V + C_2V + C_3V = C_{eq} V$$

$$C_1 + C_2 + C_3 = C_{eq}$$ (after dividing both sides by V)

Generalizing: $C_{eq} = \sum_i C_i$ (capacitances in parallel add up)

Circuits Containing Capacitors in Series

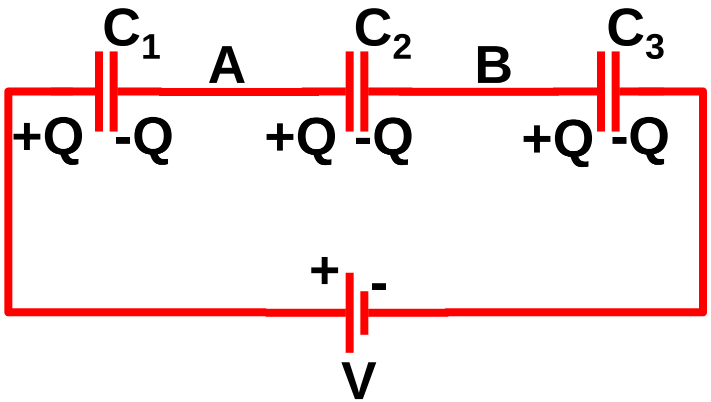
Capacitors connected in series:



charge +Q flows from the battery to the left plate of C_1

charge -Q flows from the battery to the right plate of C_3
(+Q and –Q: the same in magnitude but of opposite sign)

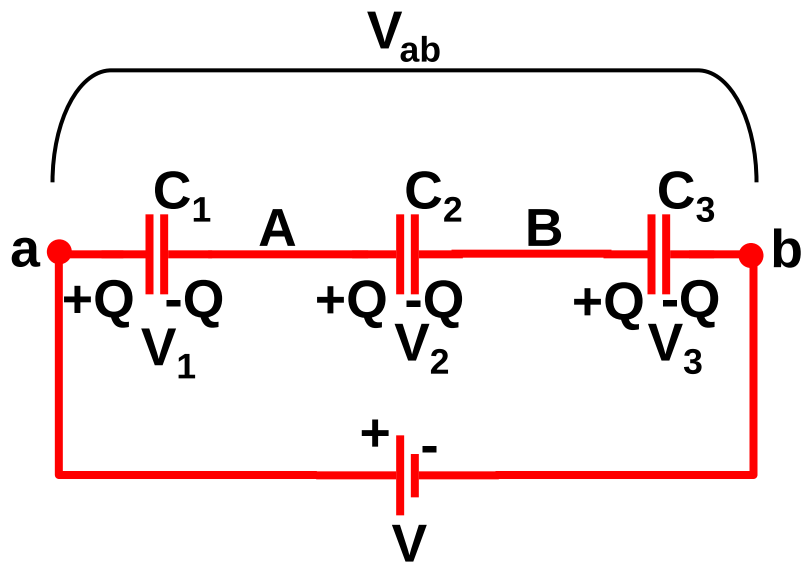
Charges +Q and –Q attract equal and opposite charges to the other plates of their respective capacitors:



These equal and opposite charges came from the originally neutral circuit regions A and B.

Because region A must be neutral, there must be a charge +Q on the left plate of C_2.

Because region B must be neutral, there must be a charge -Q on the right plate of C_2.



The charges on C_1, C_2, and C_3 are the same, and are

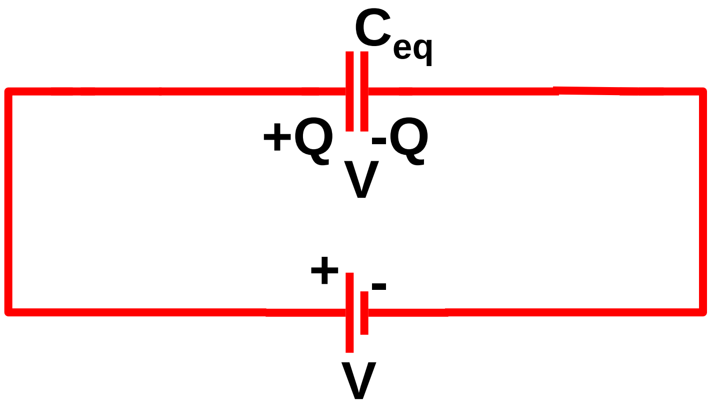
$$Q = C_1 V_1 \qquad Q = C_2 V_2 \qquad Q = C_3 V_3$$

The voltage drops across C_1, C_2, and C_3 add up

$$V_{\mathbf{ab}} = V_1 + V_2 + V_3.$$

General concept: When circuit components are connected in series, then the voltage drops across these components add up to the total voltage drop.

replace the three capacitors by a single **equivalent capacitor**



"equivalent" means it has the same charge Q and the same voltage drop V as the three capacitors

$Q = C_{eq} V$

Collecting equations:

$$Q = C_1 V_1 \qquad Q = C_2 V_2 \qquad Q = C_3 V_3$$

Important!

$$V_{\mathbf{ab}} = V = V_1 + V_2 + V_3.$$

$$Q = C_{eq} V$$

Substituting for V_1, V_2, and V_3:

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Substituting for V:

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

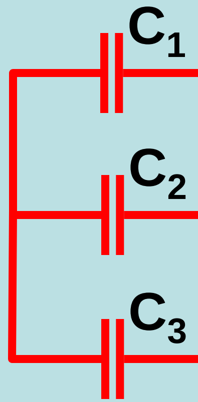
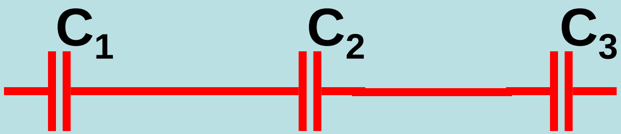
Dividing both sides by Q:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

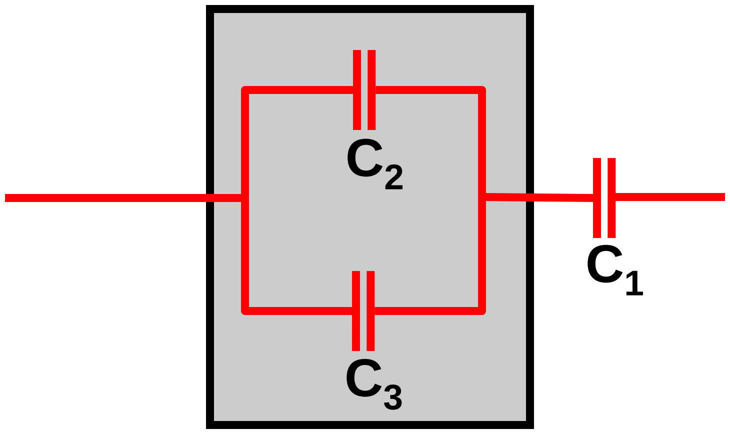
Generalizing:

OSE: $$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$$ (capacitors in series)

Summary

	Parallel  	**Series** 
equivalent capacitance	$C_{eq} = \sum_i C_i$	$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i}$
charge	Q's add	V's add
voltage	same V	same Q

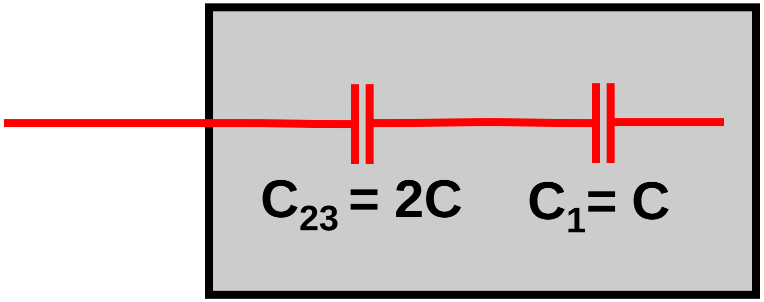
Example: determine the capacitance of a single capacitor that will have the same effect as the combination shown. Use $C_1 = C_2 = C_3 = C$.



Start by combining parallel combination of C_2 and C_3

$$C_{23} = C_2 + C_3 = C + C = 2C$$

Now I see a series combination.

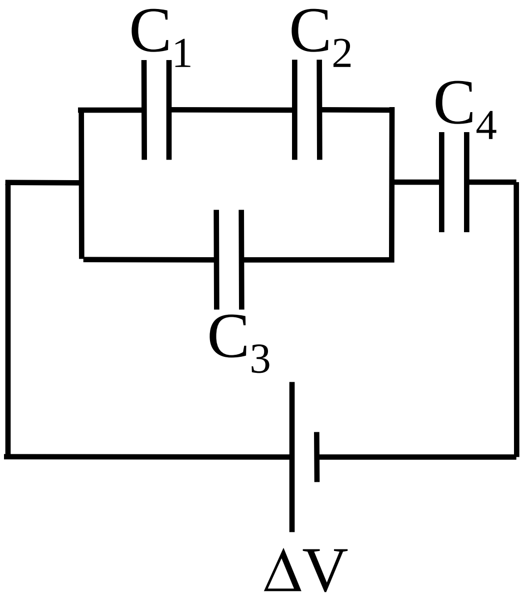


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}}$$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{2C} = \frac{2}{2C} + \frac{1}{2C} = \frac{3}{2C}$$

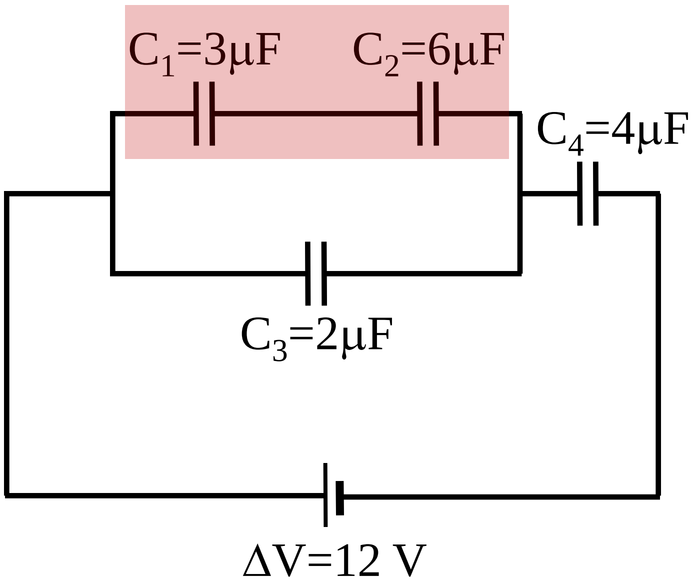
$$\boxed{C_{eq} = \frac{2}{3}C}$$

Example: for the capacitor circuit shown, C_1 = 3μF, C_2 = 6μF, C_3 = 2μF, and C_4 =4μF. (a) Find the equivalent capacitance. (b) if ΔV=12 V, find the potential difference across C_4.



Hint: each capacitor has associated with it a Q, C, and V. If you don't know what to do next, near each capacitor, write down Q= , C= , and V= . Next to the = sign record the known value or a "?" if you don't know the value. As soon as you know any two of Q, C, and V, you can determine the third. This technique often provides visual clues about what to do next.

(a) Find C_{eq}. (b) if $\Delta V = 12$ V, find V_4.



C_1 and C_3 are not in parallel. Make sure you understand why!

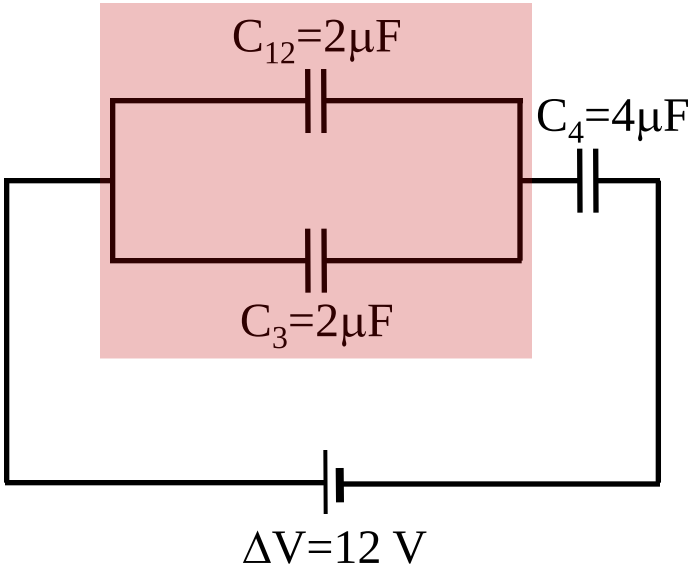
C_2 and C_4 are not in series. Make sure you understand why!

C_1 and C_2 are in series. Make sure you use the correct equation!

$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Don't forget to invert: $C_{12} = 2\ \mu\text{F}$.

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.

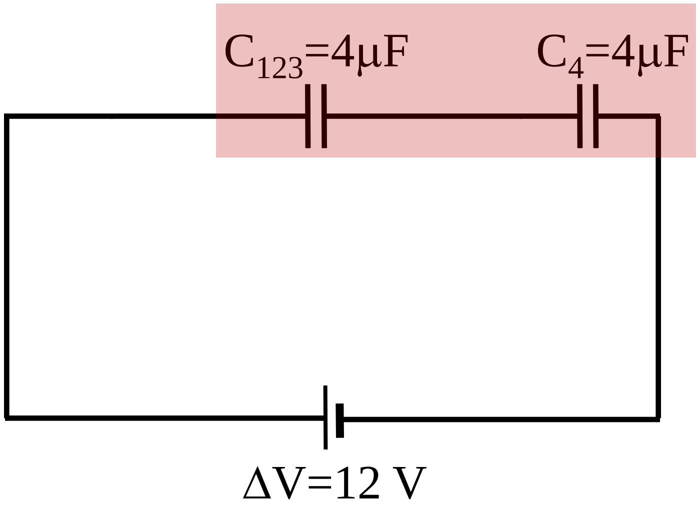


C_{12} and C_4 are not in series. Make sure you understand why!

C_{12} and C_3 are in parallel. Make sure you use the correct equation!

$$C_{123} = C_{12} + C_3 = 2 + 2 = 4\mu F$$

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.



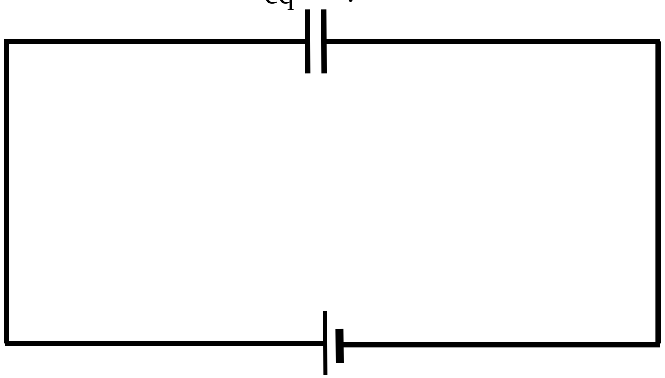
C_{123} and C_4 are in series. Make sure you understand why! Combined, they make give C_{eq}.

Make sure you use the correct equation!

$$\frac{1}{C_{eq}} = \frac{1}{C_{123}} + \frac{1}{C_{24}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

Don't forget to invert: $C_{eq} = 2\ \mu F$.

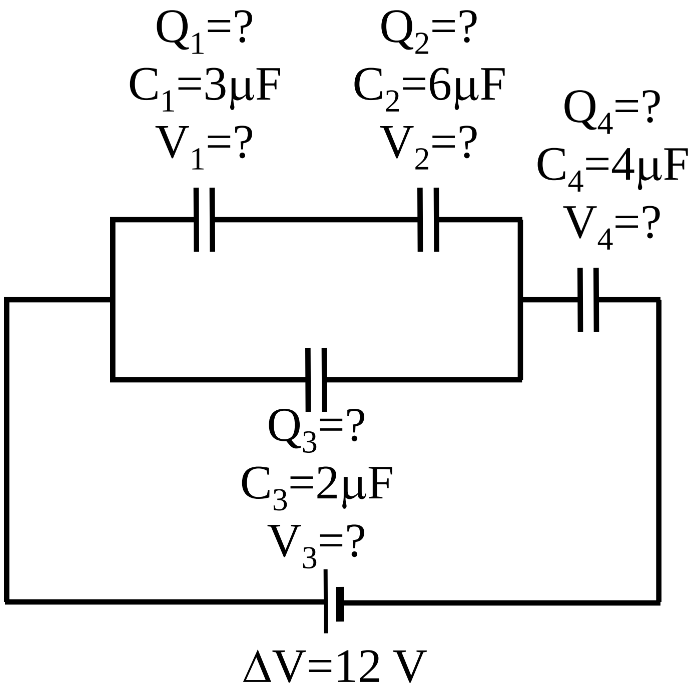
(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.



$C_{eq} = 2\ \mu F$.

If you see a capacitor circuit on the test, read the problem first. Don't go rushing off to calculate C_{eq}. Sometimes you are asked to do other things.

(a) Find C_{eq}. (b) if ΔV=12 V, find V_4.



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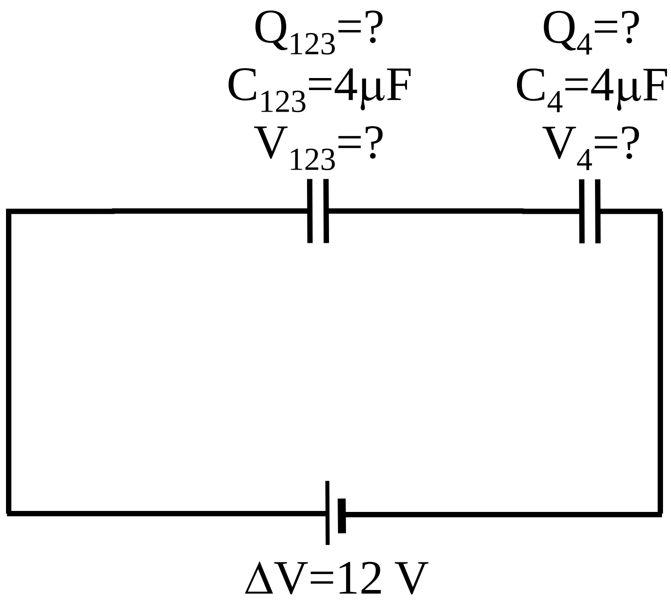
We know C_4 and want to find V_4. If we know Q_4 we can calculate V_4. Maybe that is a good way to proceed.

(a) Find C_{eq}. (b) if $\Delta V=12$ V, find V_4.



C_4 is in series with C_{123} and together they form C_{eq}.

Therefore $Q_4 = Q_{123} = Q_{eq}$.

$$Q_{eq} = C_{eq}\Delta V = (2)(12) = 24\mu C = Q_4$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} \Rightarrow V_4 = \frac{Q_4}{C_4} = \frac{24}{4} = 6V$$

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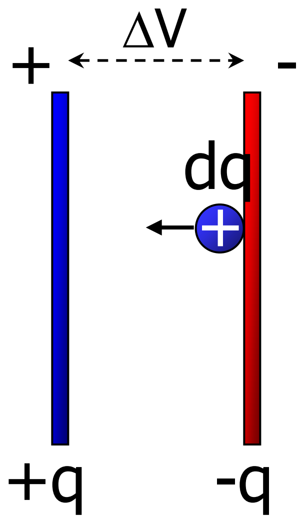
Energy Storage in Capacitors

work to charge a capacitor:

- capacitor already has charge q, voltage (difference) ΔV
- move extra charge element dq from one plate to the other
- external work required: $dW = dq\ \Delta V$.



$$dW = \Delta V\ dq = \frac{q}{C} dq$$

from $q = C\Delta V$

- start with zero charge, end up with Q:

$$W = \int_0^Q dW = \int_0^Q \frac{q}{C} dq = \left.\frac{q^2}{2C}\right|_0^Q = \frac{Q^2}{2C}\ .$$

- work required to charge the capacitor = change in potential energy

$$U_f - U_i = W_{ext}$$

- when starting from empty capacitor: $U_i = 0$

potential energy stored in capacitor: $U = \frac{Q^2}{2C}$.

Using Q=CV, three equivalent expressions:

$$U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2} .$$

All three equations are valid; use the one most convenient for the problem at hand.

- four quantities for a capacitor: C, Q, V, and U
- if you know any two of them, you can find the other two

Example: a camera flash unit stores energy in a 150 μF capacitor at 200 V. How much electric energy can be stored?

$$U = \frac{CV^2}{2}$$

$$U = \frac{\left(150\times10^{-6}\right)\left(200^2\right)}{2}\ \mathrm{J}$$

$$\boxed{U = 3\ \mathrm{J}}$$

If you keep everything in SI (mks) units, the result is "automatically" in SI units.

Energy stored in capacitor vs. energy stored in battery

12 V, 100 Ah car battery

- charge: 3.6×10^5 C, energy: 4.3×10^6 J

100 μF capacitor at 12 V

- charge: $Q=CV= 1.2 \times 10^{-3}$ C, energy: $U=CV^2/2=7.2 \times 10^{-3}$ J

If batteries store so much more energy, why use capacitors?

- capacitor stores charge physically, battery stores charge chemically
- capacitor can **release** stored charge and energy **much faster**

Where does the stored energy reside?

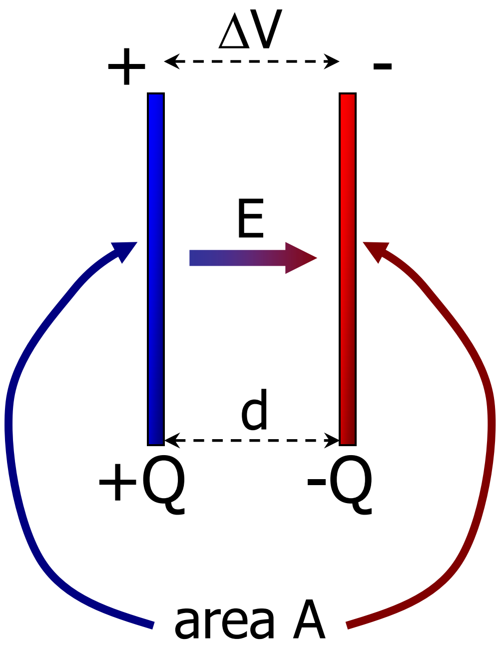
Energy is stored in the capacitor:

$$U = \frac{1}{2}C(\Delta V)^2$$

$$C = \frac{\varepsilon_0 A}{d} \quad \text{and} \quad \Delta V = Ed$$

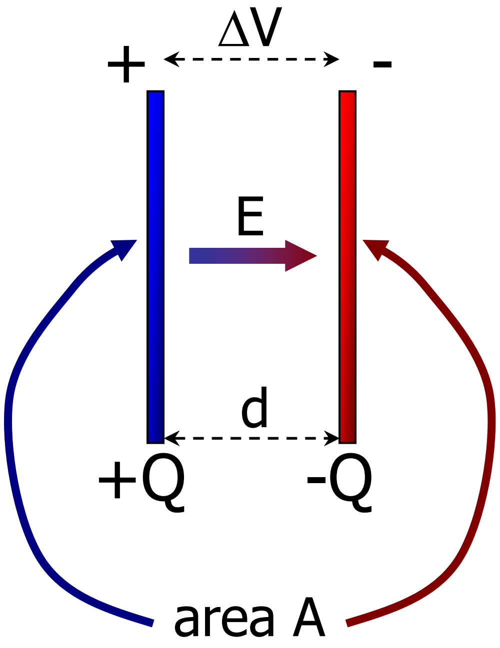
$$U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right)(Ed)^2$$

$$U = \frac{1}{2}(\varepsilon_0 Ad)E^2$$



The "volume of the capacitor" is Volume=Ad energy density u (energy per unit volume):

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}\left(\varepsilon_0 Ad\right)E^2}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$



energy resides in the electric field between the plates

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Energy Density

Today's agenda:

Capacitors and Capacitance.

You must be able to apply the equation C=Q/V.

Capacitors: parallel plate, cylindrical, spherical.

You must be able to calculate the capacitance of capacitors having these geometries, and you must be able to use the equation C=Q/V to calculate parameters of capacitors.

Circuits containing capacitors in series and parallel.

You must understand the differences between, and be able to calculate the "equivalent capacitance" of, capacitors connected in series and parallel.

Energy Storage in Capacitors.

You must be able to calculate the energy stored in a capacitor, and apply the energy storage equations to situations where capacitor configurations are altered.

Dielectrics.

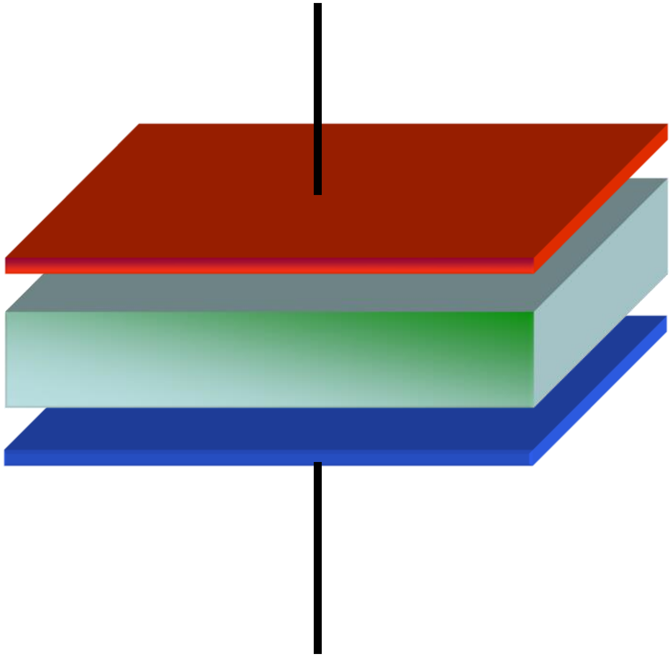
You must understand why dielectrics are used, and be able include dielectric constants in capacitor calculations.

Dielectrics

- if insulating material ("dielectric") is placed between capacitor plates capacitance increases by factor κ

- κ (greek letter kappa) is the **dielectric constant**

$$C = \frac{\kappa \varepsilon_0 A}{d}.$$

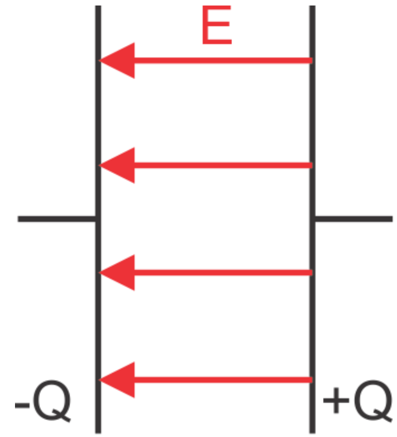


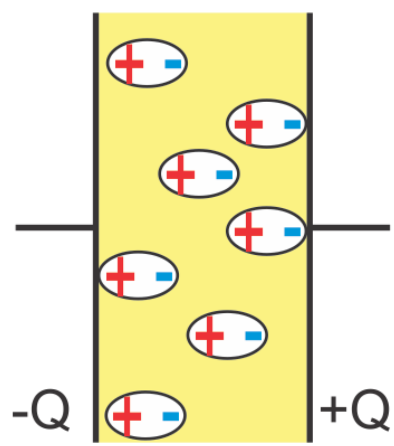
- κ depends on the material

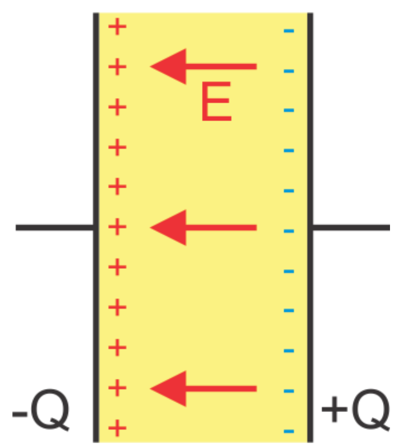
vacuum	$\kappa = 1$
air	$\kappa = 1.0006$
PVC	$\kappa = 3.18$
$SrTiO_3$	$\kappa = 310$

How does a dielectric work?

- dielectric material contains **dipoles**
- dipoles align in electric field
- **induced charges** partially compensate charges on plates
- electric field and voltage in capacitor reduced $(\Delta V = Ed)$
- $C = \dfrac{Q}{\Delta V}$ capacitance increases







Homework hints:

- if you charge a capacitor and then **disconnect the battery,** **Q stays the same** (charge cannot leave the plates!)
  C, V, and U may change

- if you charge a capacitor and **keep the battery connected**, **V stays the same** (voltage is fixed by the battery)
  C, Q, and U may change

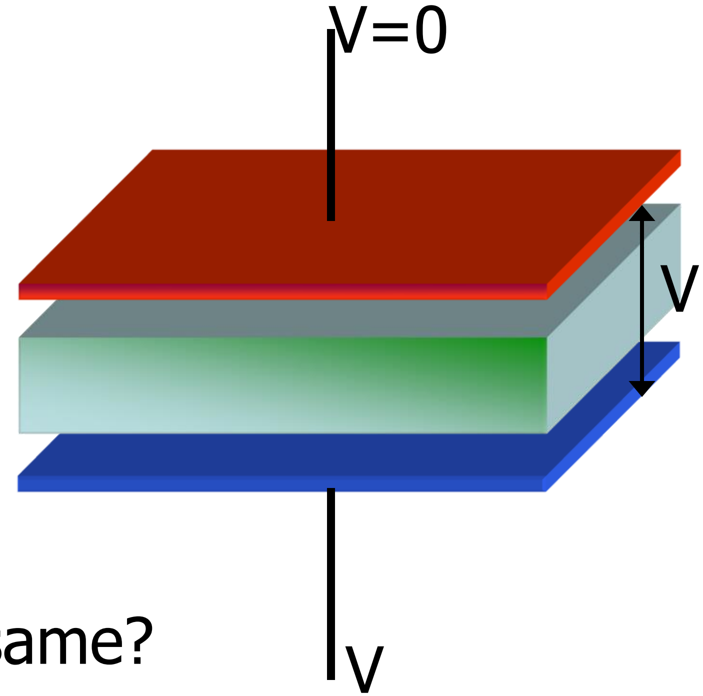
Conceptual Example

A capacitor connected to a voltage source as shown acquires a charge Q.

While the capacitor is still connected to the battery, a dielectric material is inserted.

Will Q increase, decrease, or stay the same?

Why?



Example: a parallel plate capacitor has an area of 10 cm^2 and plate separation 5 mm. A 300 V battery is connected to its plates. If neoprene is inserted between its plates, how much charge does the capacitor hold.

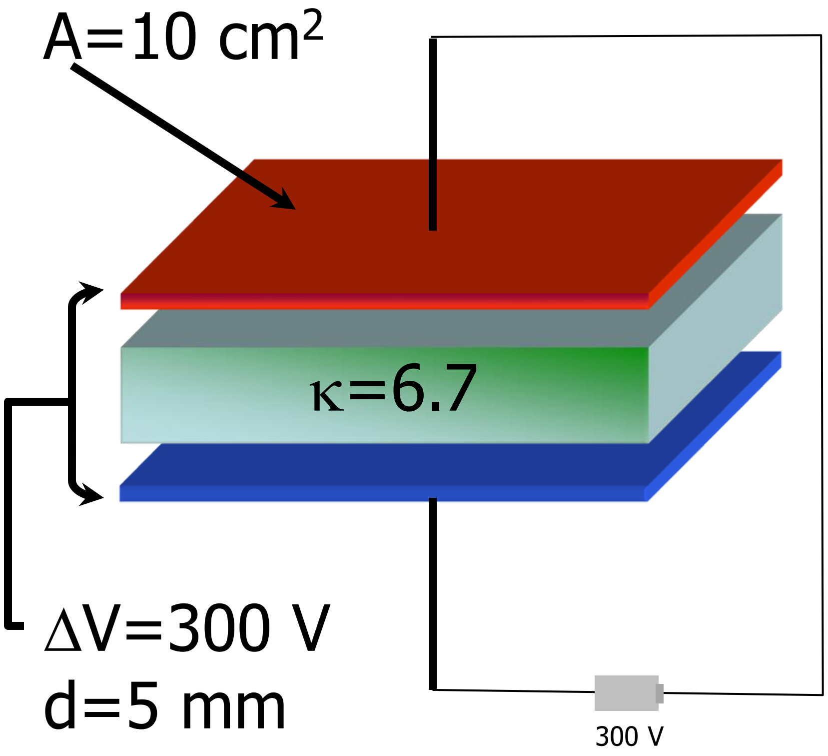
$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$C = \frac{(6.7)(8.85\times10^{-12})(10\times10^{-4})}{5\times10^{-3}}\,F$$

$$C = 1.19\times10^{-11} F$$

$$Q = CV$$

$$Q = (1.19\times10^{-11})(300)\,C = (3.56\times10^{-9}\ C) = 3.56\ nC$$

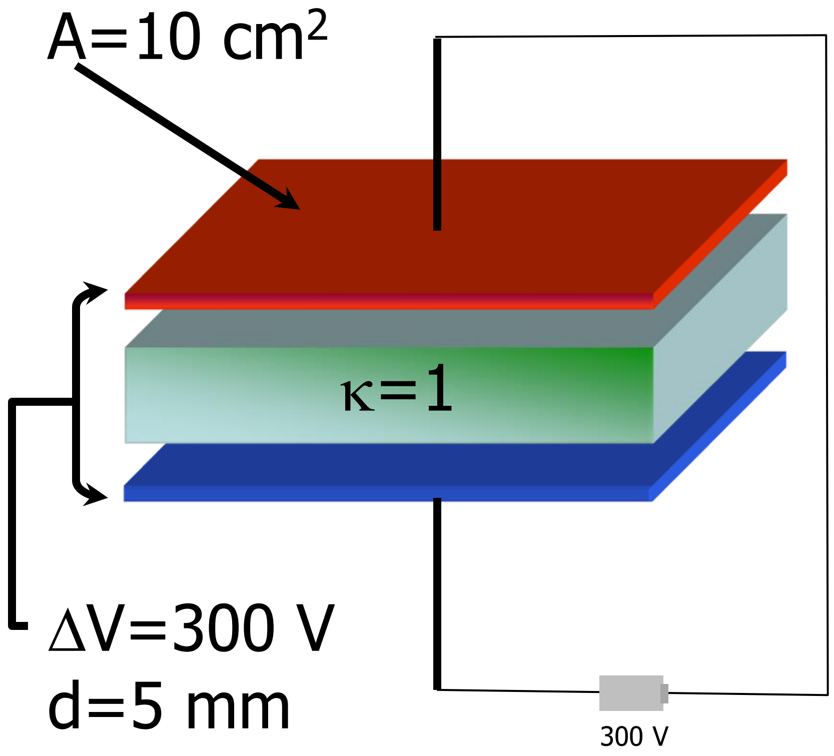


Example: how much charge would the capacitor on the previous slide hold if the dielectric were air?

The calculation is the same, except replace 6.7 by 1.

Or just divide the charge on the previous page by 6.7 to get…

$$Q = 0.53\,\text{nC}$$



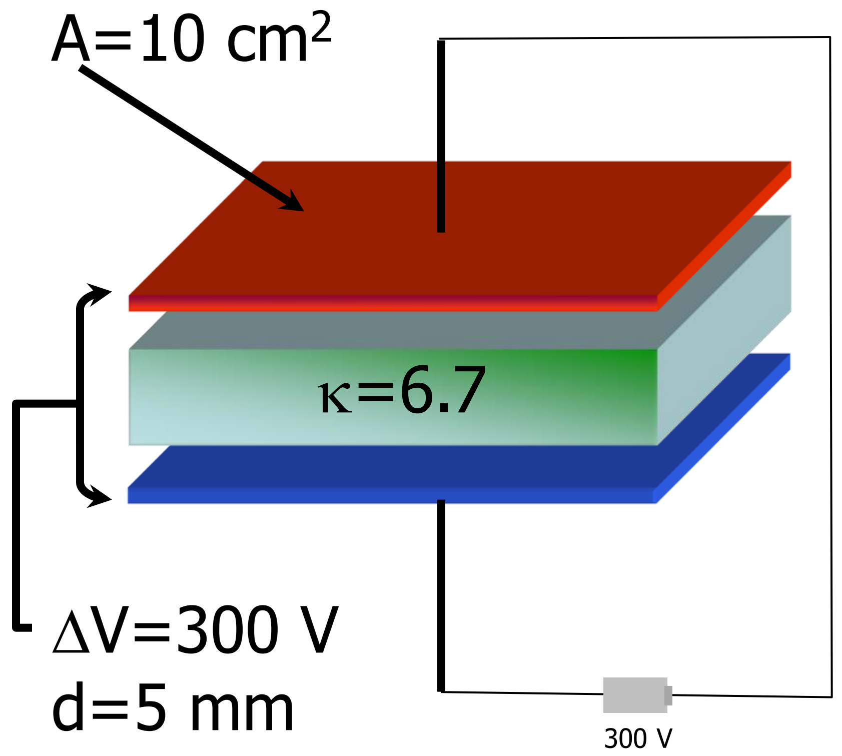
Example: find the energy stored in the capacitor.

$$C = 1.19 \times 10^{-11} F$$

$$U = \frac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2} (1.19 \times 10^{-11})(300)^2 \ J$$

$$U = 5.36 \times 10^{-7} J$$



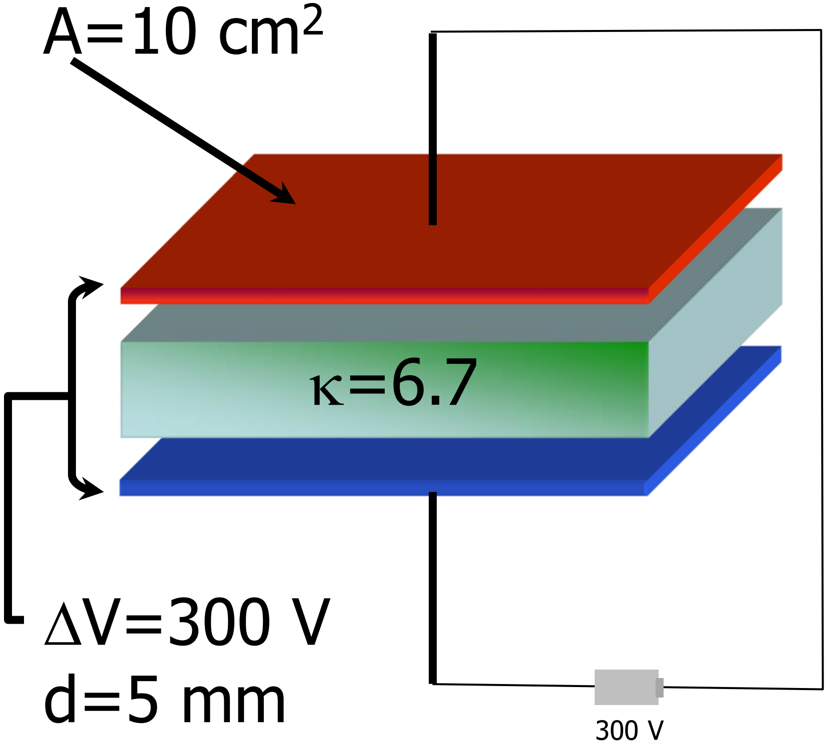
Example: the battery is now disconnected. What are the charge, capacitance, and energy stored in the capacitor?

The charge and capacitance are unchanged, so the voltage drop and energy stored are unchanged.

$$Q = 3.56\ \text{nC}$$

$$C = 1.19 \times 10^{-11}\ \text{F}$$

$$U = 5.36 \times 10^{-7}\ \text{J}$$

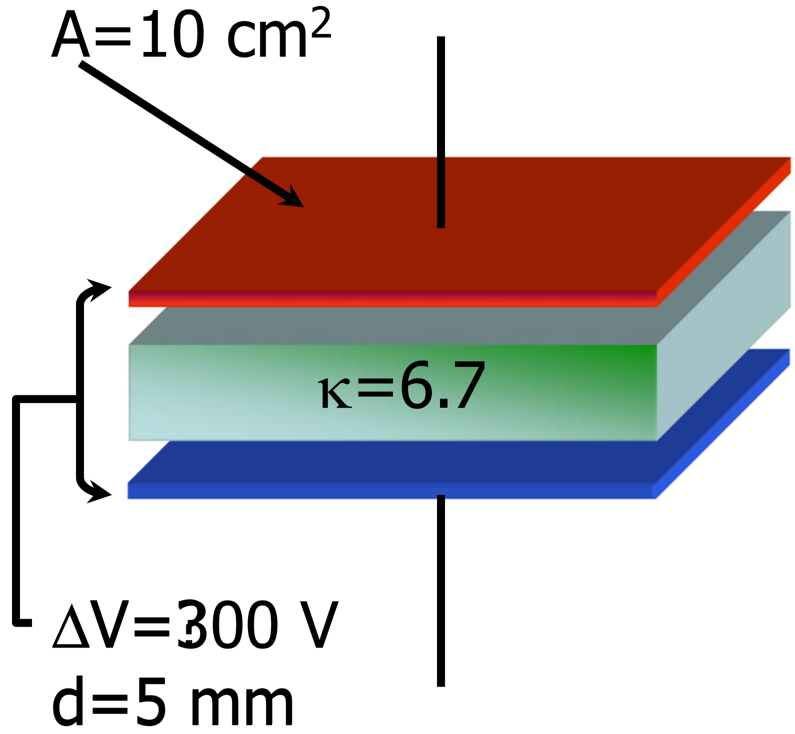


Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\left(8.85\times10^{-12}\right)\left(10\times10^{-4}\right)}{5\times10^{-3}}\,\text{F}$$

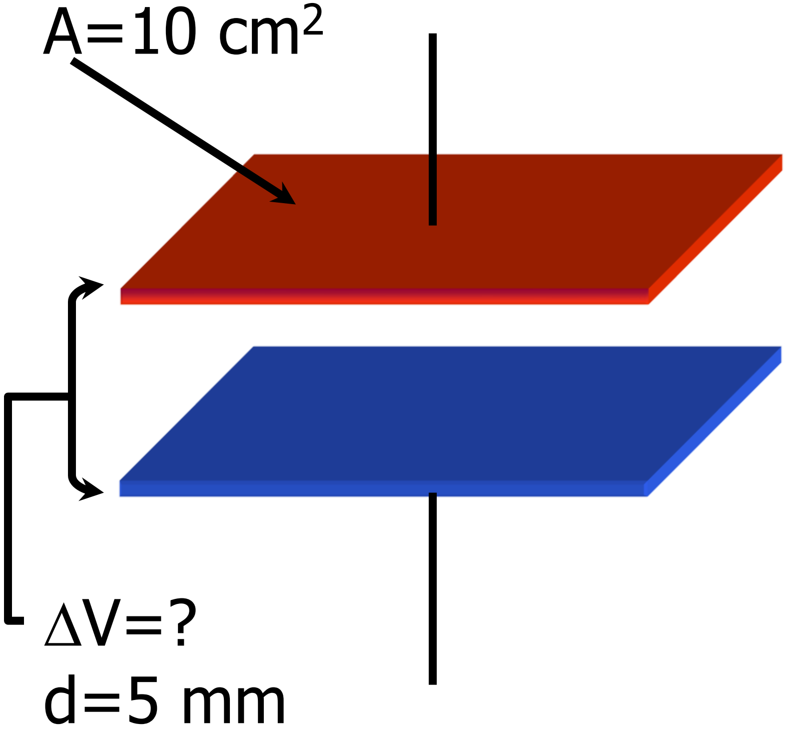
$$C = 1.78\times10^{-12}\,\text{F}$$



Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

The charge remains unchanged, because there is nowhere for it to go.

$$Q = 3.56\ \text{nC}$$

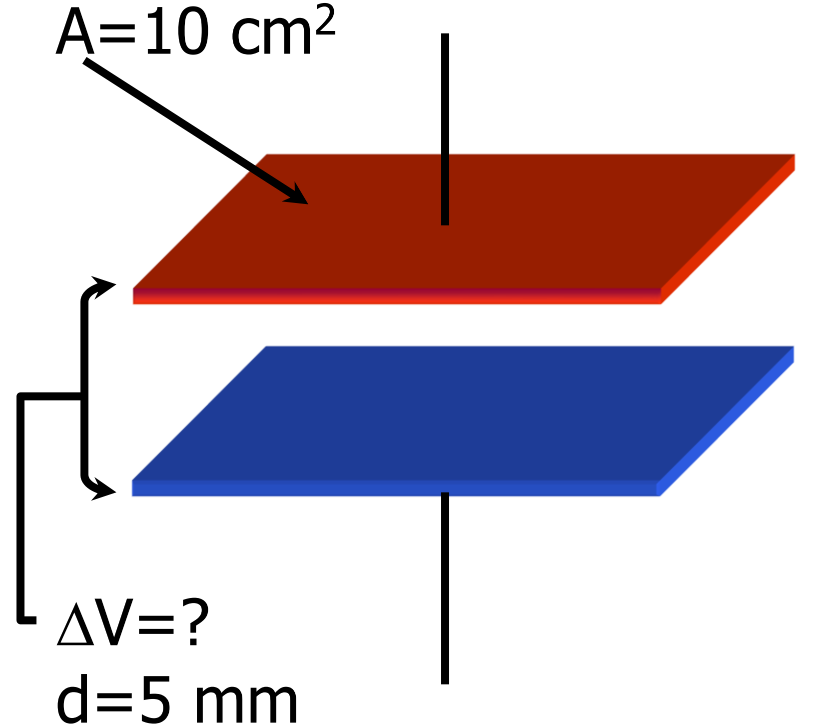


Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

Knowing C and Q we can calculate the new potential difference.

$$\Delta V = \frac{Q}{C} = \frac{\left(3.56\times10^{-9}\right)}{\left(1.78\times10^{-12}\right)}\,V$$

$$\Delta V = 2020\ V$$

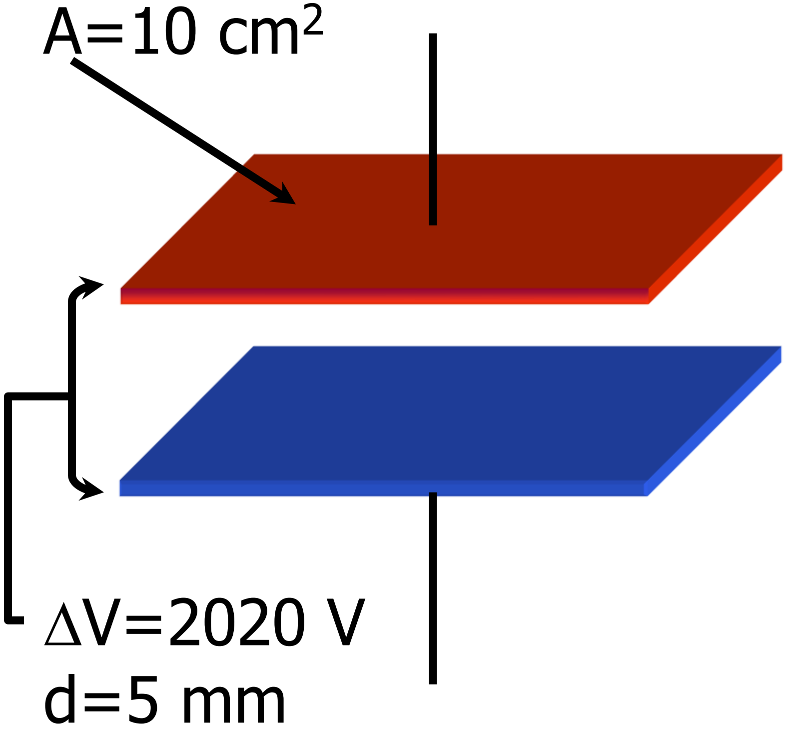


Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

$$U = \frac{1}{2} C \left(\Delta V\right)^2$$

$$U = \frac{1}{2}\left(1.78\times10^{-12}\right)\left(2020\right)^2 \text{ J}$$

$$U = 3.63\times10^{-6} \text{ J}$$



$$U_{before} = 5.36 \times 10^{-7}\ J$$

$$U_{after} = 3.63 \times 10^{-6}\ J$$

$$\frac{U_{after}}{U_{before}} = 6.7$$

Huh?? The energy stored increases by a factor of κ??

Sure. It took work to remove the dielectric. The stored energy increased by the amount of work done.

$$\Delta U = W_{external}$$