Today’s agenda:

Displacement Current and Maxwell’s Equations.

Displacement currents explain how current can flow “through” a capacitor, and how a time-varying electric field can induce a magnetic field.

Electromagnetic Waves.

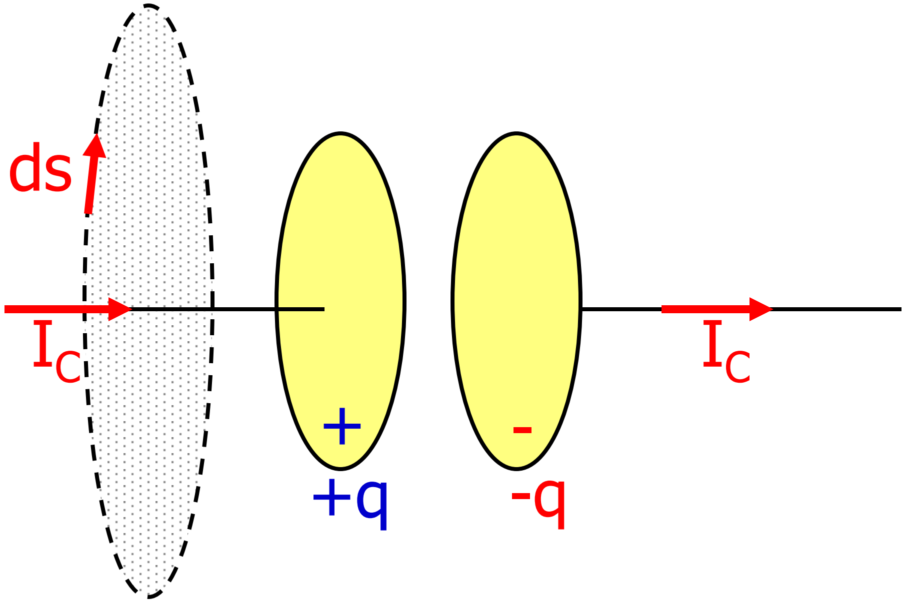
Energy Carried by Electromagnetic Waves.

Momentum and Radiation Pressure of an Electromagnetic Wave.

Displacement Current

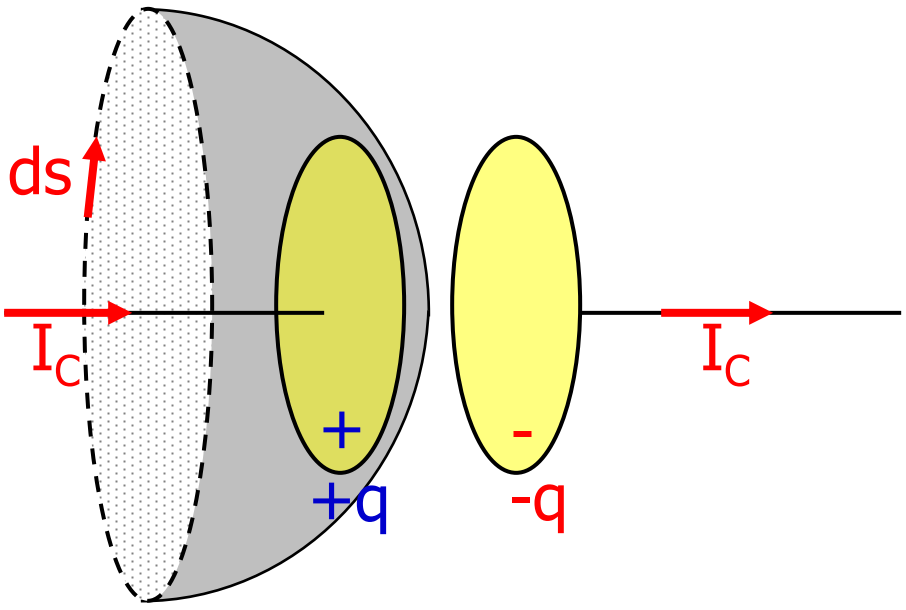
Apply Ampere's Law to a charging capacitor.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_C$$



Ampere's law is **universal**:
Shape of surface shouldn't matter, as long as "path" is the same

"Soup bowl" surface, with the + plate resting near the bottom of the bowl.



Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = I_{encl} = 0$$

two different surfaces give:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_C \qquad \oint \vec{B} \cdot d\vec{s} = 0$$

Contradiction! (The equation on the right is actually incorrect, and the equation on the left is incomplete.)

How to fix this?

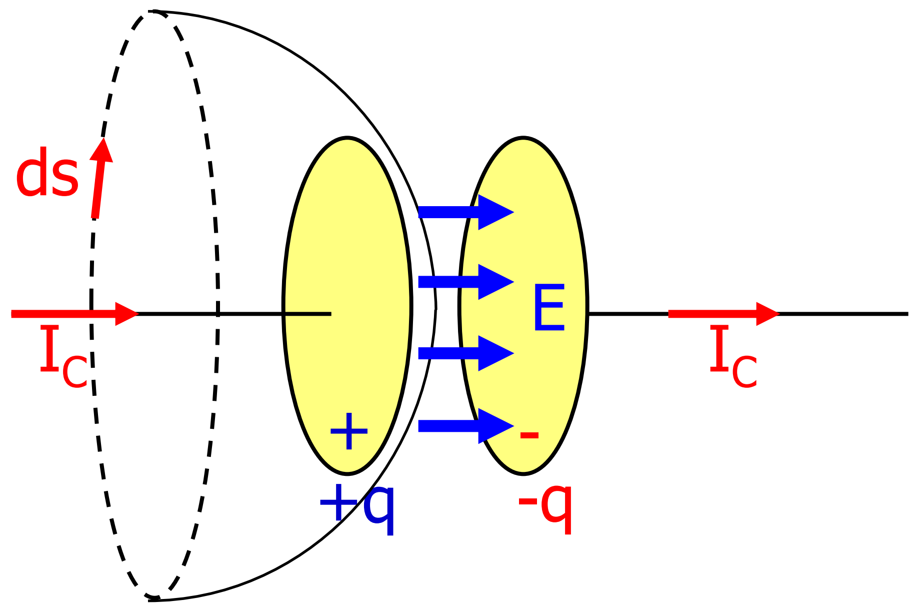
- Ampere's law (as used so far) must be incomplete
- charging capacitor produces changing electric flux between plates

$$q = C\Delta V = \left(\kappa\varepsilon_0 \frac{A}{d}\right)(Ed) = \kappa\varepsilon_0 EA = \kappa\varepsilon_0 \Phi_E$$

Changing electric flux acts like current

$$\frac{dq}{dt} = \frac{d}{dt}(\kappa\varepsilon_0\Phi_E) = \kappa\varepsilon_0 \underbrace{\frac{d}{dt}(\Phi_E)}$$

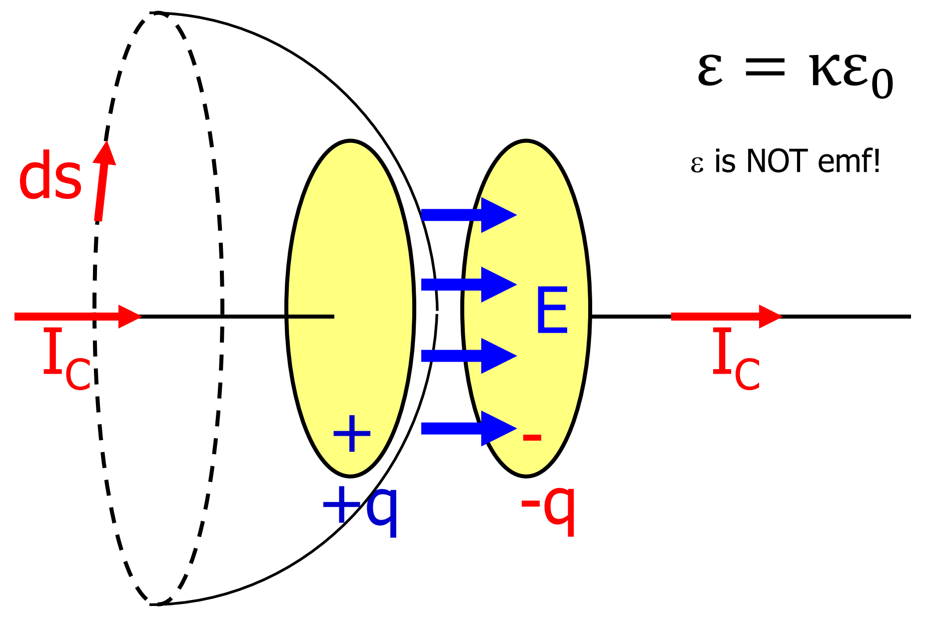
This term has units of current.



κ Is the dielectric constant of the medium in between the capacitor plates. In the diagram, with an air-filled capacitor, $\kappa = 1$.

Define a virtual current:
displacement current

$$I_D = \kappa\varepsilon_0 \frac{d}{dt}(\Phi_E).$$



changing electric flux through "bowl" surface is equivalent to current I_C through flat surface

- include displacement current in Ampere's law
- complete form of Ampere's law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0(I_C + I_D)_{encl} = \mu_0 I_{encl} + \mu_0 \varepsilon \frac{d\Phi_E}{dt}.$$

Magnetic fields are produced by both conduction currents and time varying electric fields.

The Big Picture

Gauss's Law for both electricity and magnetism,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law of Induction, and Ampere's Law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

- these four equations are famous **Maxwell equations** of electromagnetism
- govern all of electromagnetism

These four equations provide a complete description of electromagnetism.

The Big Picture

Maxwell equations can also be written in differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{d\vec{E}}{dt} + \mu_0 \vec{J}$$

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Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

- oscillating electric and magnetic fields can **"sustain each other"** away from source charges and fields

Faraday's law $\frac{d}{dt} B \rightarrow E$

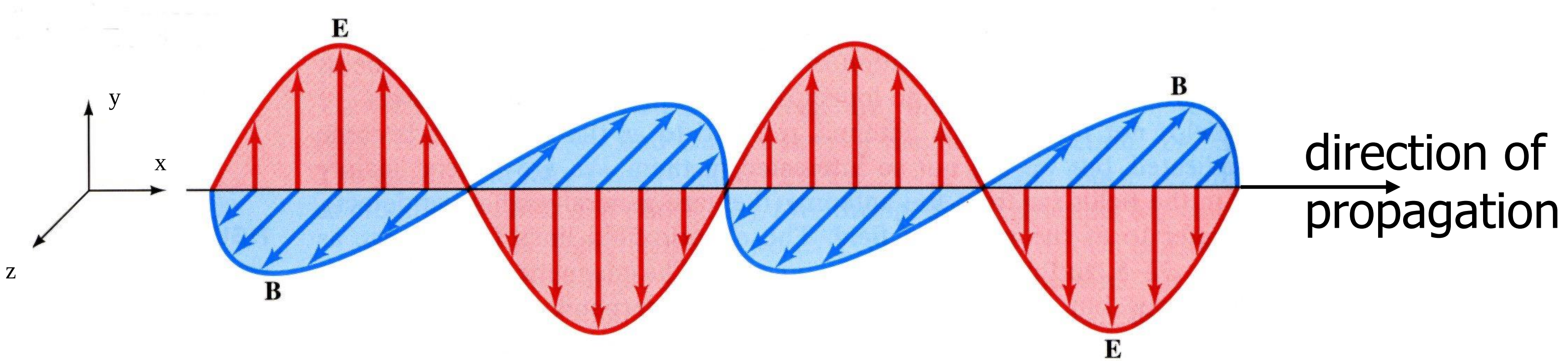
Ampere's law $\frac{d}{dt} E \rightarrow B$

- result: electromagnetic waves that propagates through space

- electromagnetic waves always involve both $\vec{E}$ and $\vec{B}$ fields
- propagation direction, $\vec{E}$ field and $\vec{B}$ field form **right-handed triple** of vectors

Example:

wave propagating in x-direction
$\vec{E}$ field in y-direction
$\vec{B}$ field in z-direction
values of E and B depend only upon x and t



Wave equation

- combine Faraday's law and Ampere's law
- for wave traveling in x-direction with $\vec{E}$ in y-direction and $\vec{B}$ in z direction:

Wave equation:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

$$\frac{\partial^2 B_z}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B_z(x,t)}{\partial t^2}$$

- E and B are not independent: $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

Solutions of the wave equation

$$E_y = E_{max}\sin(kx - \omega t)$$

$$B_z = B_{max}\sin(kx - \omega t)$$

E_{max} and B_{max} are the electric and magnetic field amplitudes

Wave number k, wave length λ

$$k = \frac{2\pi}{\lambda}$$

Angular frequency ω, frequency f

$$\omega = 2\pi f$$

Wave speed

$$f\lambda = \frac{\omega}{k} = c$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

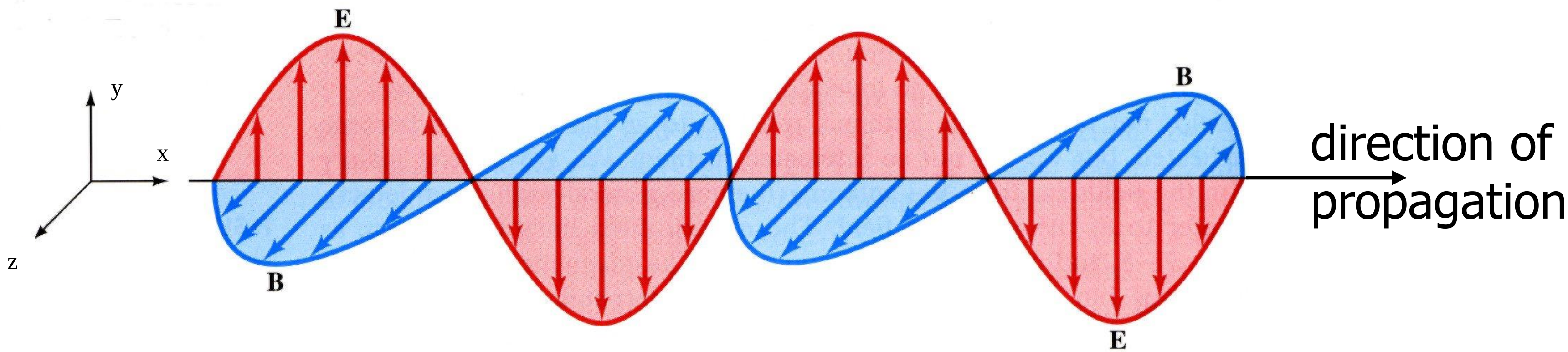
$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$\frac{\partial[E_{max}\sin(kx-\omega t)]}{\partial x} = -\frac{\partial[B_{max}\sin(kx-\omega t)]}{\partial t}$$

$$E_{max}\,k\cos(kx-\omega t) = B_{max}\,\omega\cos(kx-\omega t)$$

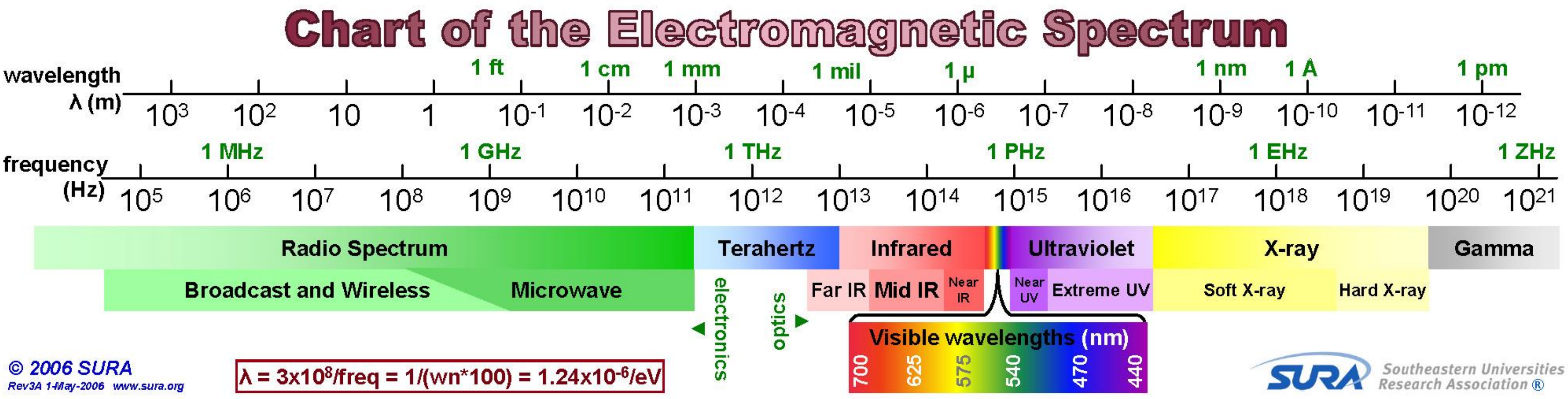
$$\frac{E_{max}}{B_{max}} = \frac{E}{B} = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}.$$

Ratio of electric field magnitude to magnetic field magnitude in an electromagnetic wave equals the speed of light.



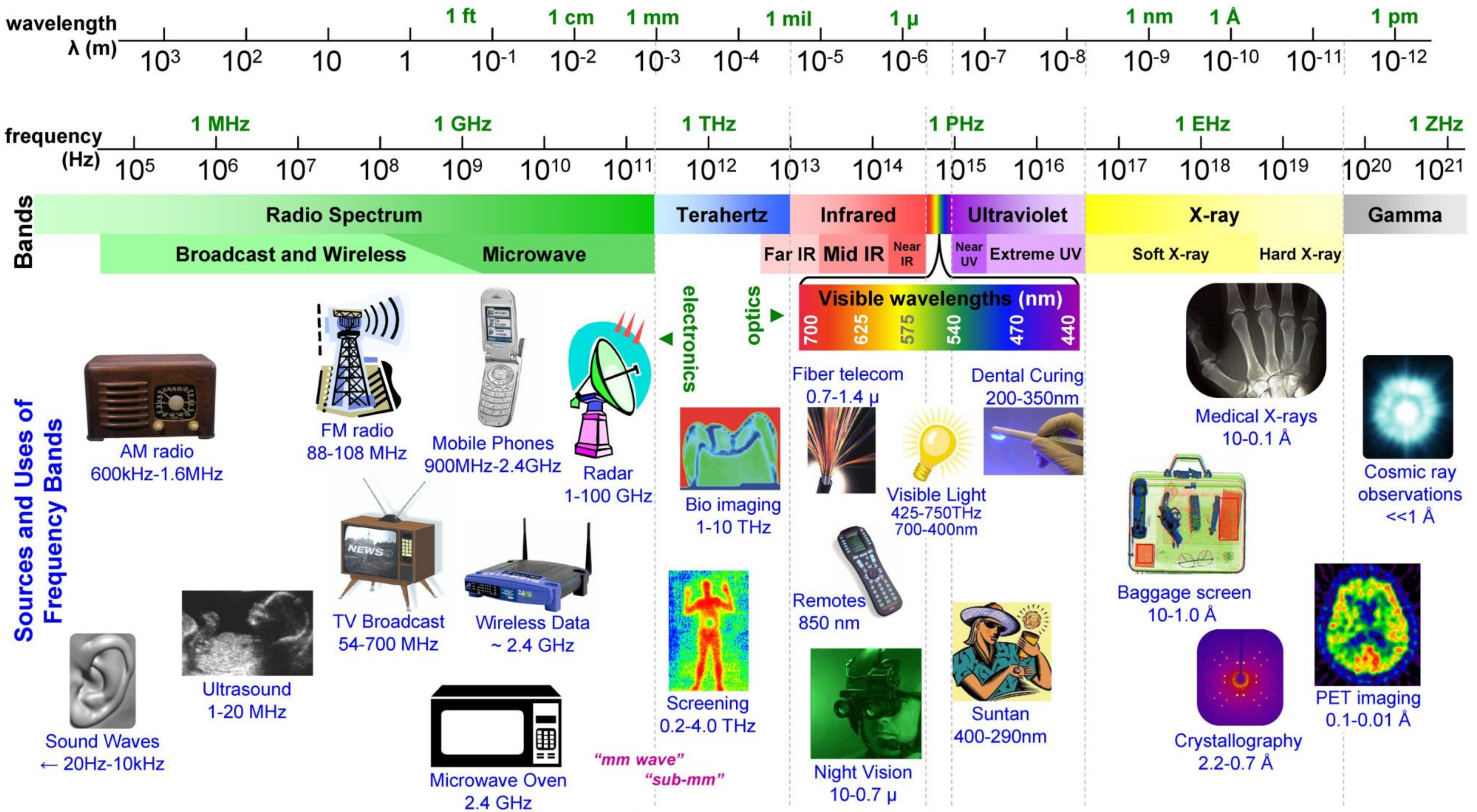
This static image doesn't show how the wave propagates.

Types of electromagnetic waves



- enormous range of wave lengths and frequencies
- spans more than 15 orders of magnitude

Applications of electromagnetic waves



$\lambda = 3x10^8/freq = 1/(wn*100) = 1.24x10^{-6}/eV$

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rate of energy flow:

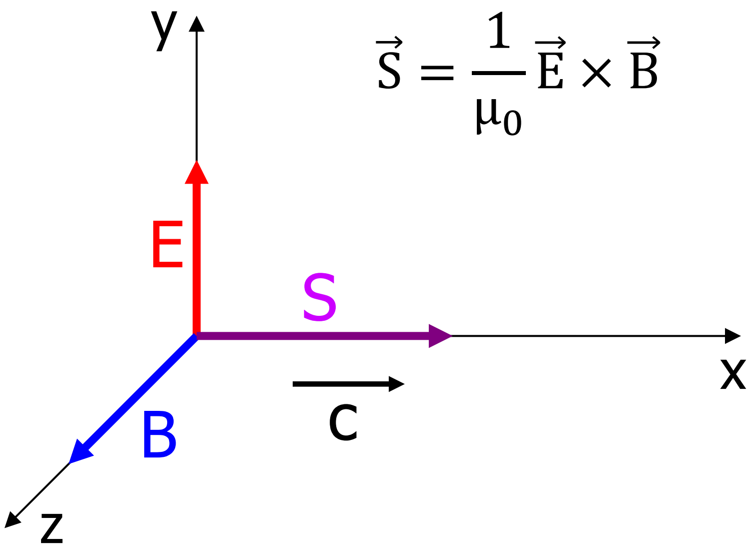
- **Poynting vector*** $\vec{S}$

*J. H. Poynting, 1884.

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

This is derived from Maxwell's equations.

- $\vec{S}$ represents **energy current density**, i.e., energy per time per area or **power per area** (units $J/(s \cdot m^2) = W/m^2$)
- direction of $\vec{S}$ is along the direction of wave propagation



$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

for EM wave: $|\vec{E} \times \vec{B}| = EB$

so $S = \frac{EB}{\mu_0}$.

because B = E/c

$$S = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}.$$

These equations for S apply at any instant of time and represent the instantaneous rate at which energy is passing through a unit area.

$$S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$$

EM waves are sinusoidal.

$$E_y = E_{max}\sin(kx - \omega t)$$

$$B_z = B_{max}\sin(kx - \omega t)$$

EM wave propagating along x-direction

The **average of S** over one or more cycles is called the **wave intensity I**.

The time average of $\sin^2(kx - \omega t)$ is ½, so

$$I = S_{average} = \langle S \rangle = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{cB_{max}^2}{2\mu_0}$$

Notice the 2's in this equation.

Energy Density

- so far: energy transported by EM wave
- now: **energy stored in the field** in some volume of space

energy densities (energy per volume)

$$u_E = \frac{1}{2}\varepsilon_0 E^2 \qquad\qquad u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Using B = E/c and $c = 1/(\mu_0\varepsilon_0)^{1/2}$:

$$u_B = \frac{1}{2}\frac{B^2}{\mu_0} = \frac{1}{2}\frac{\left(\frac{E}{c}\right)^2}{\mu_0} = \frac{1}{2}\frac{\mu_0\varepsilon_0 E^2}{\mu_0} = \frac{1}{2}\varepsilon_0 E^2$$

$$\boxed{u_B = u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}}$$

remember: E and B are sinusoidal functions of time

total energy density:

$$u = u_B + u_E = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}$$

instantaneous energy densities
(E and B vary with time)

- average over one or more cycles of electromagnetic wave gives factor ½ from average of $\sin^2(kx - \omega t)$.

$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_{max}^2, \qquad \langle u_B \rangle = \frac{1}{4}\frac{B_{max}^2}{\mu_0}, \qquad \text{and} \qquad \boxed{\langle u \rangle = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{1}{2}\frac{B_{max}^2}{\mu_0}}$$

Recall: intensity of an EM wave

$$S_{average} = \langle S \rangle = \frac{1}{2}\frac{E_{max}^2}{\mu_0 c} = \frac{1}{2}\frac{cB_{max}^2}{\mu_0} = c\langle u \rangle$$

Help!

E or B individually:

At time t:

$$u_B(t) = u_E(t) = \frac{1}{2}\varepsilon_0 E^2(t) = \frac{1}{2}\frac{B^2(t)}{\mu_0}$$

Average:

$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_{max}^2 \qquad \langle u_B \rangle = \frac{1}{4}\frac{B_{max}^2}{\mu_0},$$

Total:

At time t:

$$u(t) = \varepsilon_0 E^2(t) = \frac{B^2(t)}{\mu_0}$$

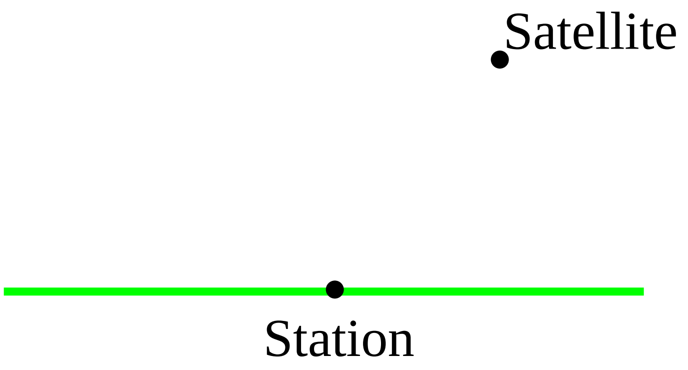
Average:

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{1}{2}\frac{B_{max}^2}{\mu_0}$$

Example: a radio station on the surface of the earth radiates a sinusoidal wave with an average total power of 50 kW.* Assuming the wave is radiated equally in all directions above the ground, find the amplitude of the electric and magnetic fields detected by a satellite 100 km from the antenna.

Strategy: we want E_{max}, B_{max}. We are given average power.

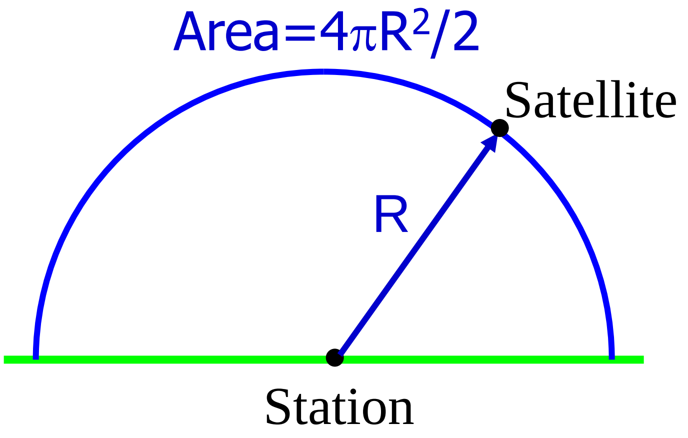
From the average power we can calculate intensity, and from intensity we can calculate E_{max} and B_{max}.



*In problems like this you need to ask whether the power is radiated into all space or into just part of space.

Example: a radio station on the surface of the earth radiates a sinusoidal wave with an average total power of 50 kW.* Assuming the wave is radiated equally in all directions above the ground, find the amplitude of the electric and magnetic fields detected by a satellite 100 km from the antenna.

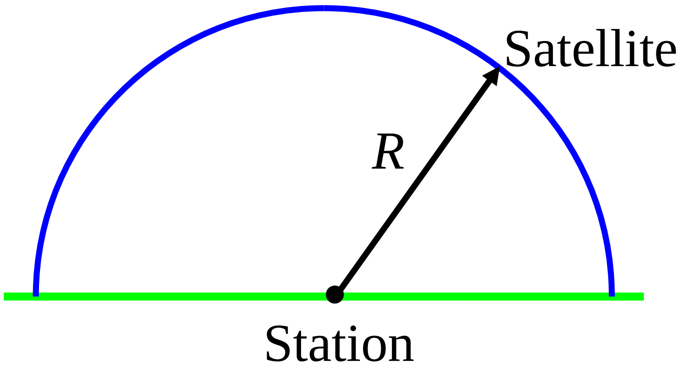
All the radiated power passes through the **hemispherical surface*** so the average power per unit area (the intensity) is



$$I = \left(\frac{\text{power}}{\text{area}}\right)_{\text{average}} = \frac{P}{2\pi R^2} = \frac{\left(5.00\times10^{4}\ \text{W}\right)}{2\pi\left(1.00\times10^{5}\ \text{m}\right)^2} = 7.96\times10^{-7}\ \text{W/m}^2$$

*In problems like this you need to ask whether the power is radiated into all space or into just part of space.

$$I = \langle S \rangle = \frac{1}{2}\frac{E_{max}^2}{\mu_0 c}$$



$$E_{max} = \sqrt{2\mu_0 c I}$$

$$= \sqrt{2(4\pi \times 10^{-7})(3 \times 10^{8})(7.96 \times 10^{-7})}\,\frac{V}{m}$$

$$= 2.45 \times 10^{-2}\,\frac{V}{m}$$

$$B_{max} = \frac{E_{max}}{c} = \frac{\left(2.45 \times 10^{-2}\,\frac{V}{m}\right)}{\left(3 \times 10^{8}\,\frac{m}{s}\right)} = 8.17 \times 10^{-11} T$$

You could get Bmax from $I = c\,B_{max}^2/2\mu_0$, but that's a lot more work

Example: for the radio station in the previous example, calculate the average energy densities associated with the electric and magnetic field at the location of the satelite.

$$\langle u_E \rangle = \frac{1}{4}\varepsilon_0 E_{max}^2$$

$$\langle u_B \rangle = \frac{1}{4}\frac{B_{max}^2}{\mu_0}$$

$$\langle u_E \rangle = \frac{1}{4}(8.85 \times 10^{-12})(2.45 \times 10^{-2})^2 \frac{J}{m^3}$$

$$\langle u_B \rangle = \frac{1}{4}\frac{(8.17 \times 10^{-11})^2}{(4\pi \times 10^{-7})}\frac{J}{m^3}$$

$$\langle u_E \rangle = 1.33 \times 10^{-15} \frac{J}{m^3}$$

$$\langle u_B \rangle = 1.33 \times 10^{-15} \frac{J}{m^3}$$

If you are smart, you will write $<u_B> = <u_E> = 1.33 \times 10^{-15}$ J/m^3 and be done with it.

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Light Mill (Crookes radiometer)

- airtight glass bulb, containing a partial vacuum
- vanes mounted on a spindle (one side black, one silver)
- vanes rotate when exposed to light

This is **NOT** caused by radiation pressure!!

(if vacuum is too good, mill does not turn)

Mill is heat engine: black surface heats up, detailed mechanism leading to motion is complicated, research papers are written about this!

Momentum of electromagnetic wave

- EM waves carry linear momentum as well as energy

momentum stored in wave in some volume of space

- momentum density (momentum per volume):

$$\frac{d\langle p\rangle}{dV} = \frac{\langle S\rangle}{c^2} = \frac{I}{c^2}$$

dp is momentum carried in volume dV

momentum transported by EM wave:

- momentum current density (momentum per area and time)

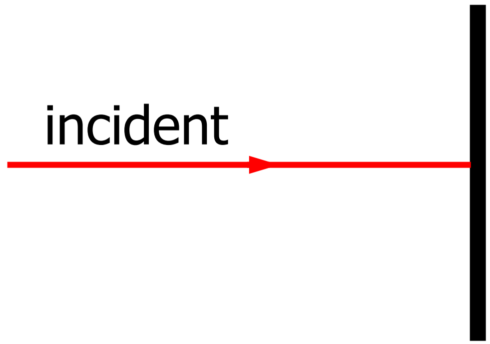
$$c\frac{d\langle p\rangle}{dV} = \frac{\langle S\rangle}{c} = \frac{I}{c}$$

Radiation Pressure

- if EM radiation is incident on an object for a time dt and if radiation is entirely **absorbed**:

object gains momentum

$$d\langle p\rangle = \frac{\langle S\rangle}{c} A\, dt$$



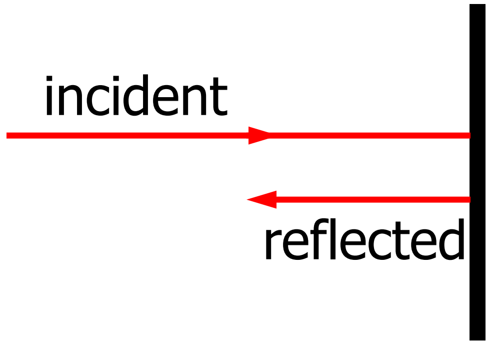
- Newton's 2nd Law (F = dp/dt): **force** $\langle F\rangle = \frac{\langle S\rangle}{c} A$

- Radiation exerts **pressure** $\langle P_{rad}\rangle = \frac{\langle F\rangle}{A} = \frac{\langle S\rangle}{c} = \frac{I}{c}$

(for total **absorption**)

- if radiation is totally **reflected** by object, then magnitude of momentum change of the object is **twice** that for total absorption.

$$d\langle p\rangle = 2\frac{\langle S\rangle}{c}A\,dt$$

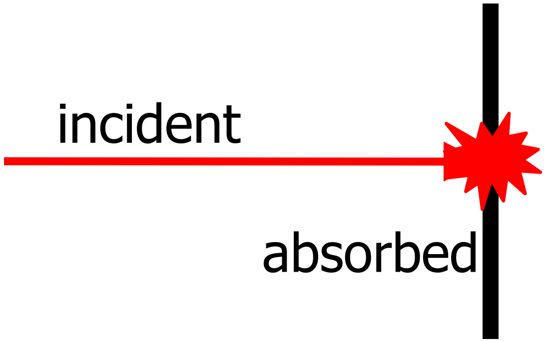


- Newton's 2nd Law (F = dp/dt): **force** $\langle F\rangle = 2\frac{\langle S\rangle}{c}A$

- Radiation exerts **pressure** $\langle P_{rad}\rangle = \frac{\langle F\rangle}{A} = 2\frac{\langle S\rangle}{c} = 2\frac{I}{c}$

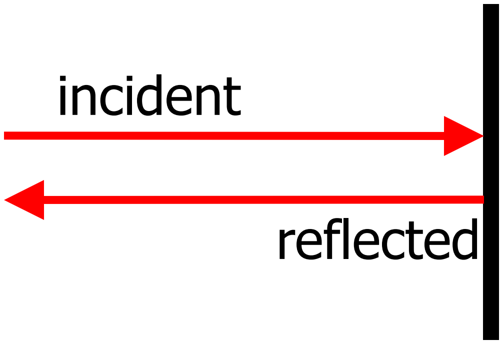
(for total **reflection**)

$$\langle P_{rad} \rangle = \frac{I}{c}$$ (total absorption)



Using the arguments above it can also be shown that:

$$\langle P_{rad} \rangle = \frac{2I}{c}$$ (total reflection)



If an electromagnetic wave does not strike a surface, it still carries momentum away from its emitter, and exerts $P_{rad}=I/c$ on the emitter.

Example: a satellite orbiting the earth has solar energy collection panels with a total area of 4.0 m^2. If the sun's radiation is incident perpendicular to the panels and is completely absorbed, find the average solar power absorbed and the average force associated with the radiation pressure. The intensity (I or $S_{average}$) of sunlight prior to passing through the earth's atmosphere is 1.4 kW/m^2.

$$\text{Power} = \text{IA} = \left(1.4 \times 10^3 \frac{\text{W}}{\text{m}^2}\right)(4.0\ \text{m}^2) = 5.6 \times 10^3\text{W} = 5.6\ \text{kW}$$

Assuming total absorption of the radiation:

$$\text{P}_{\text{rad}} = \frac{\text{S}_{\text{average}}}{\text{c}} = \frac{\text{I}}{\text{c}} = \frac{\left(1.4 \times 10^3 \frac{\text{W}}{\text{m}^2}\right)}{\left(3 \times 10^8 \frac{\text{m}}{\text{s}}\right)} = 4.7 \times 10^{-6}\text{Pa}$$

Caution! The letter P (or p) has been used in this lecture for power, pressure, and momentum!

$$\text{F} = \text{P}_{\text{rad}}\text{A} = \left(4.7 \times 10^{-6} \frac{\text{N}}{\text{m}^2}\right)(4.0\ \text{m}^2) = 1.9 \times 10^{-5}\text{N}$$

New starting equations from this lecture:

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$S_{average} = \frac{1}{2}\frac{E_{max}^2}{\mu_0 c} = \frac{1}{2}\frac{cB_{max}^2}{\mu_0}$$

$$\frac{E_{max}}{B_{max}} = \frac{E}{B} = c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

$$u_B = u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\frac{B^2}{\mu_0}$$

$$k = \frac{2\pi}{\lambda},\quad \omega = 2\pi f,\quad f\lambda = \frac{\omega}{k} = c$$

$$\langle u\rangle = \frac{1}{2}\varepsilon_0 E_{max}^2 = \frac{1}{2}\frac{B_{max}^2}{\mu_0}$$

$$|\Delta p| = \frac{\Delta U}{c} \text{ or } \frac{2\Delta U}{c}$$

$$P_{rad} = \frac{I}{c} \text{ or } \frac{2I}{c}$$