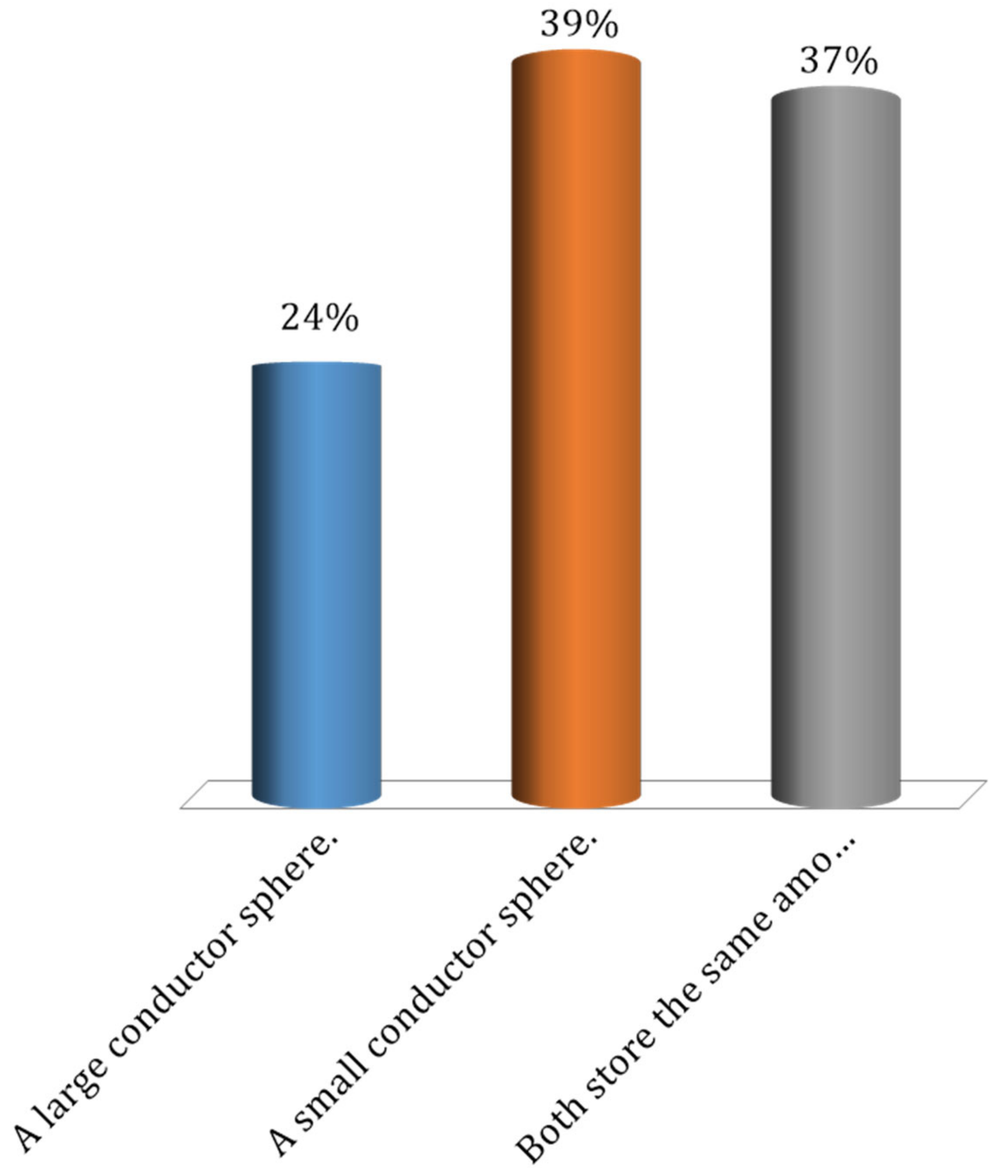
Which of the following bodies store(s) more energy when it holds the same amount Q of charges?

A. A large conductor sphere.

B. A small conductor sphere.

C. Both store the same amount of energy.

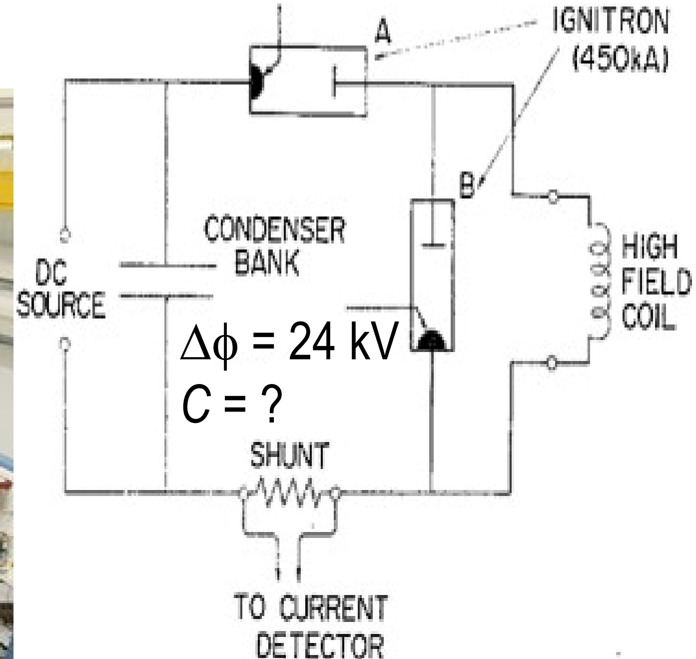
Think in terms of work needed to
to put the charges on the sphere.
This work represents the “stored energy”.
If the sphere is smaller and one brings charges
of the same polarity from infinity to the sphere,
one needs to apply a force against the
repelling Coulomb force on a longer
path length, hence more work is done and
stored in the small sphere.



World's largest capacitor bank

World Record in 2011: Highest Magnetic Fields Created in Dresden (91.4 Tesla, 10 ms) by **discharging the world's largest capacitor (bank)**

The stored energy is 50 MJ
=> world's largest capacitor bank

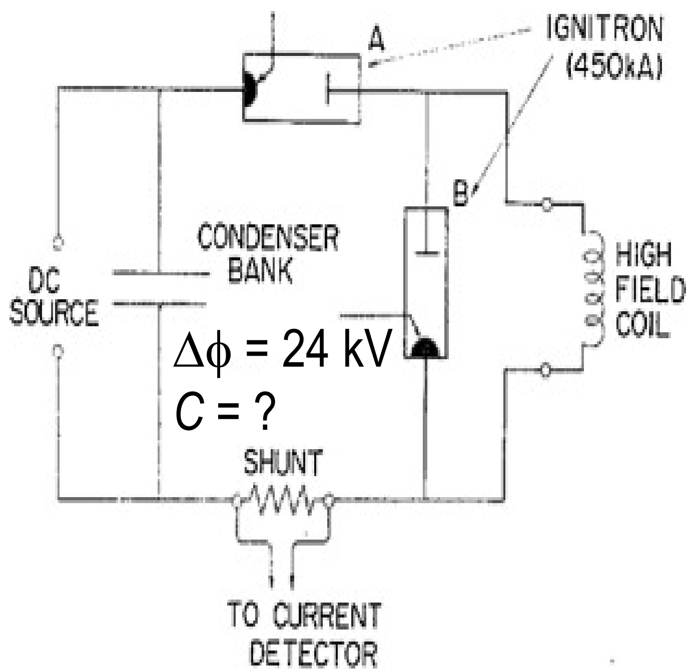


$\Delta\phi$ = 24 kV
C = ?

https://doi.org/10.1016/j.aej.2014.10.001

https://www.hzdr.de/db/Cms?pOid=33768&pNid=0

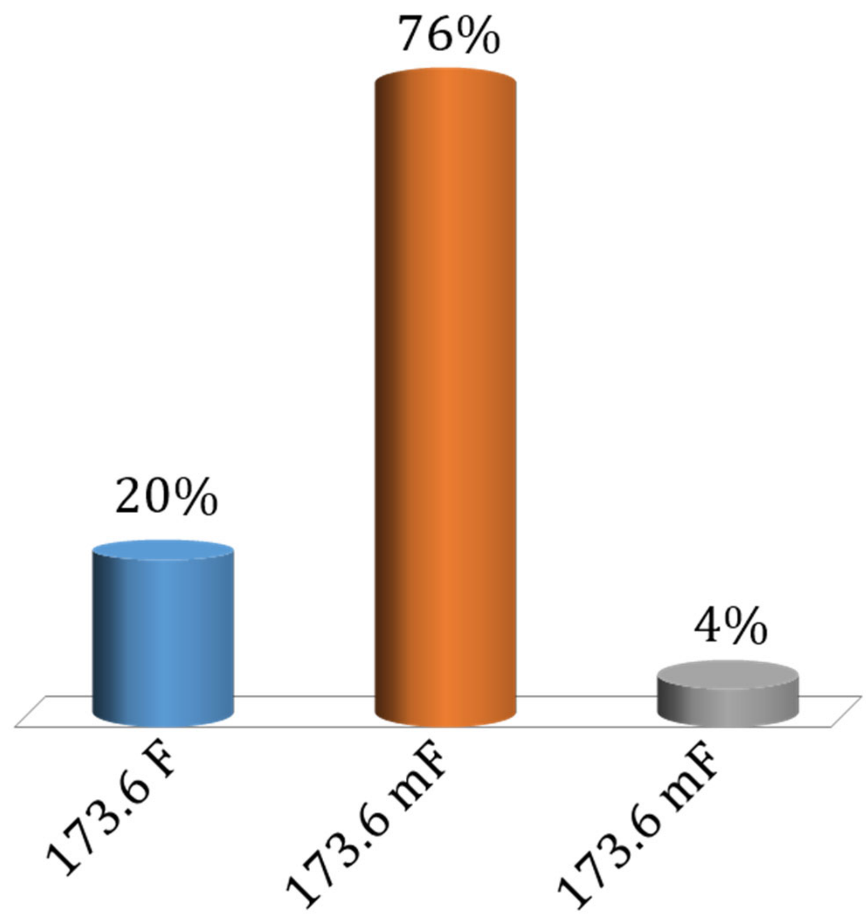
How large is the capacitance *C* of the world's largest capacitor bank (2011) with energy storage of *U*=50 MJ ?



A. 173.6 F

B. 173.6 mF

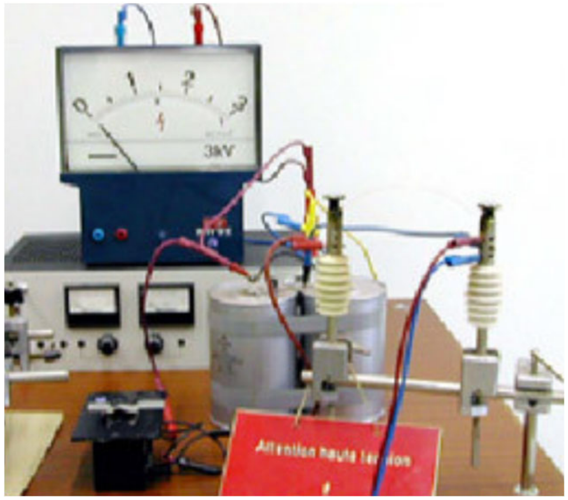
C. 173.6 μF



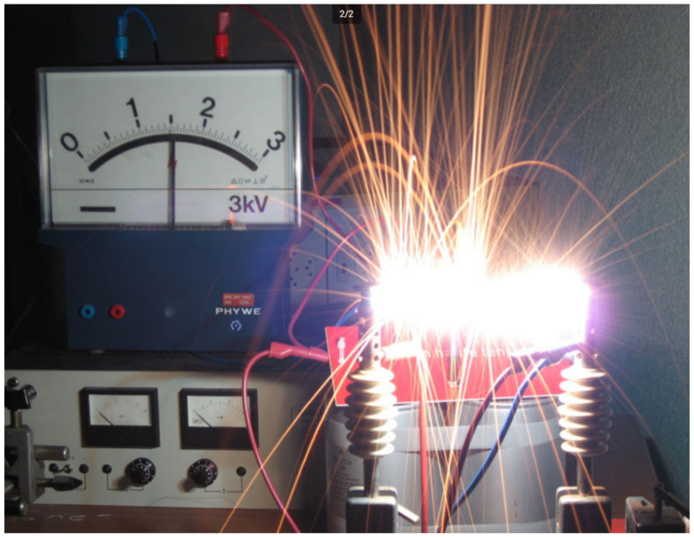
Conclusion: 1 Farad represents a giant capacitance, even larger than the world's largest one.

Energy stored in a charged capacitor

(1)



(2)



(1) We charged a capacitor such that 2 kV was reached as the potential difference.
(2) We disconnected the "battery" from the capacitor and then allowed the charges to flow across a thin Cu wire. The energy stored in the capacitor was "consumed" by enormous Joule heating of the thin Cu wire and its evaporation, i.e., the wire reached its melting and sublimation temperature. The charged capacitor performed work in that the Cu was evaporated.

Experiment: Plate capacitor

See video at: https://youtu.be/pI-F7OsJ2bE

Observation: After inducing a (constant)
charge on the plates, the potential is found to increase when we increase the separation s . This variation
is explained by a constant electric field due to a fixed surface charge density σ.

$$E = const = \frac{\phi_1 - \phi_2}{s}$$
$$\Rightarrow \ \phi_1 - \phi_2 = E\ s$$

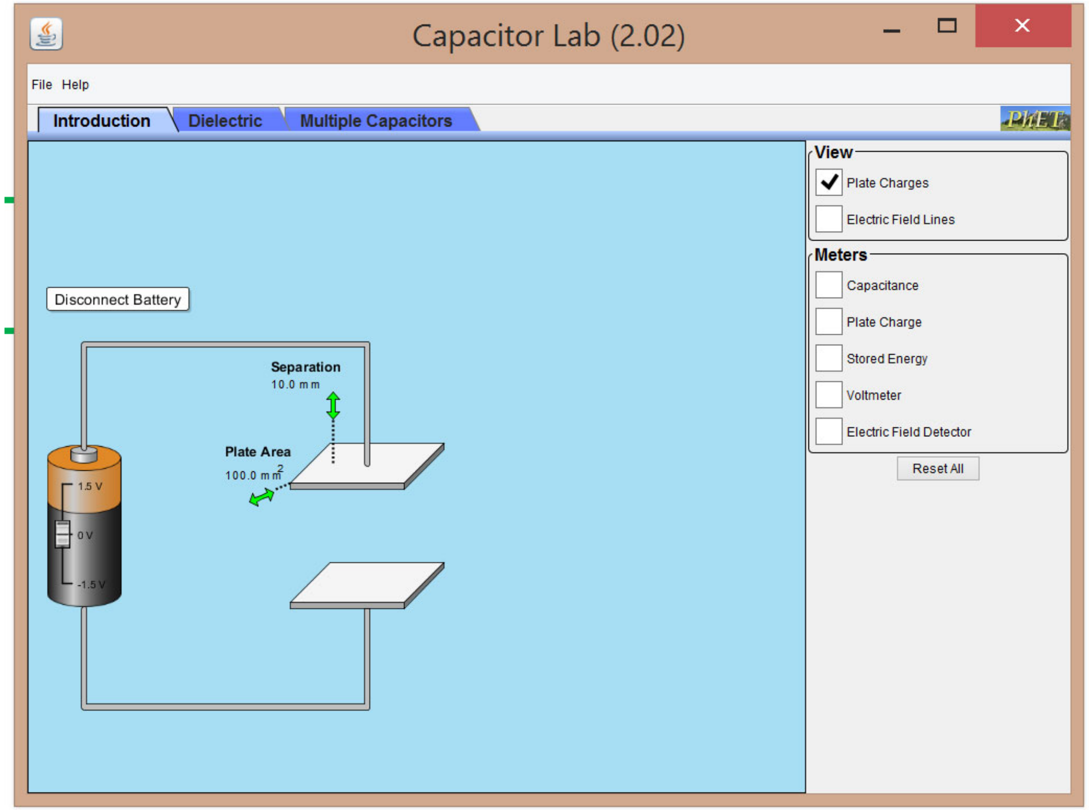
Experiment

$$C = \frac{Q}{\phi_1 - \phi_2} = \frac{Q}{\Delta\phi}$$

Observations: When inserting an insulating perspex between plates with fixed charges Q and fixed separation, the potential difference $\Delta\phi$ reduced. This means that the electric field between the plates was reduced. Later in the lecture it will be shown that:
inside Perspex a counter-electric field is developed which partially reduces the electric field coming from the plates (consider the superposition rule).
=> Polarization of material leads to so-called dipole moments inducing an additional electric field

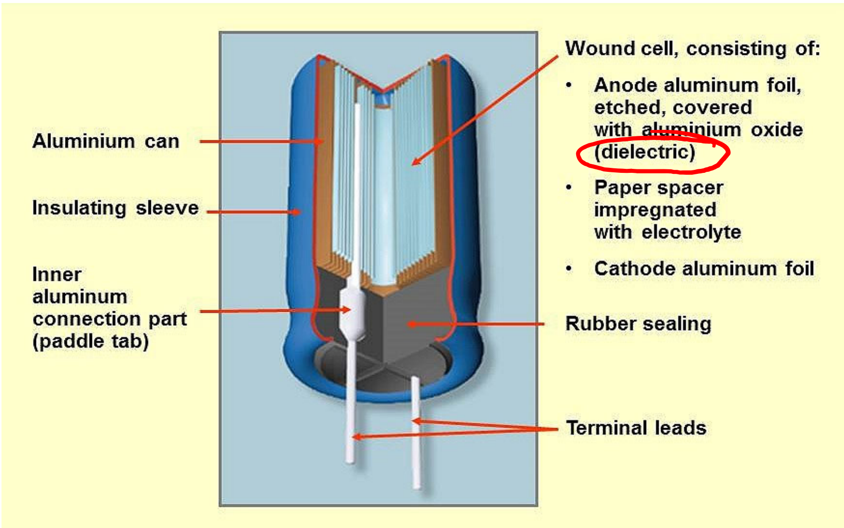
Interactive capacitors for further training

https://phet.colorado.edu/en/simulation/capacitor-lab

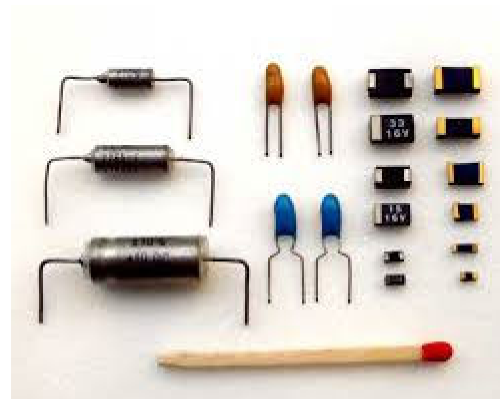


2.3 Polarization of (insulating) matter

Technical capacitors contain a dielectric (for performance enhancement and mechanical stability):



https://pt.wikipedia.org/wiki/Ficheiro:Al-e-cap-construction.jpg



https://en.wikipedia.org/wiki/Tantalum_capacitor#/media/File:Tantal-P1100196c.jpg

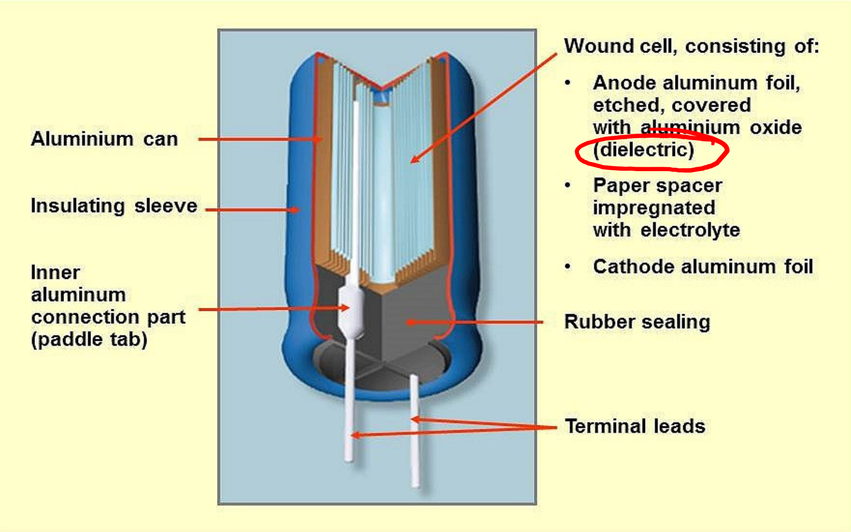
The observations on **capacitors with dielectrics** suggest that more charges are needed on the metal plates to produce the same electrical field $E = \Delta\phi/s$ between them.

This observation would be **in conflict with the laws of Coulomb and Gauss** presented in Ch. 1**.**

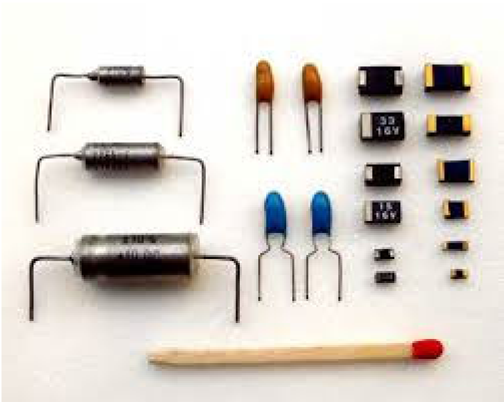
Solution: the neutral dielectric produces an **electrical field**. The field occurs due to the alignment of **electrical dipole moments *p*** which will be explained in the following.

2.3 Polarization of (insulating) matter

Technical capacitors contain a dielectric (for performance enhancement and mechanical stability):



https://pt.wikipedia.org/wiki/Ficheiro:Al-e-cap-construction.jpg



https://en.wikipedia.org/wiki/Tantalum_capacitor#/media/File:Tantal-P1100196c.jpg

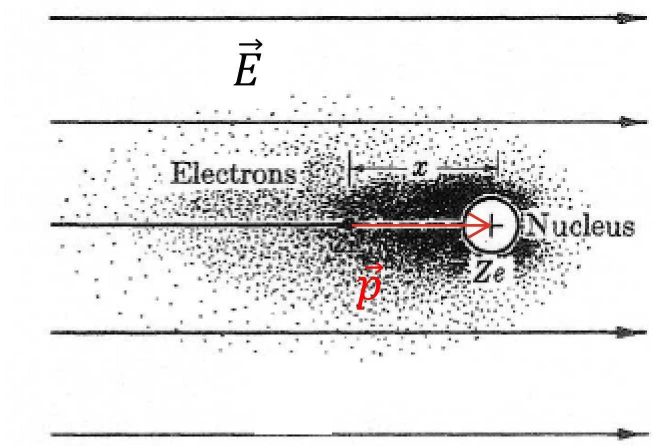
At the end of section 2.3 you will

- know that an applied electric field $\vec{E}$ polarizes insulators, as their charges are partially displaced.
- You can apply the concept of an electric dipole moment *p* aligned by $\vec{E}$ and define a dielectric.
- You can evaluate the potential energy of a dipole moment.

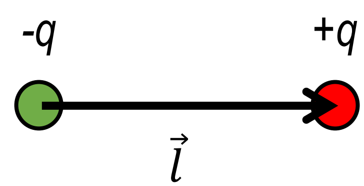
Why are metals not discussed in this section “Polarization of matter”? Because the electric field is zero inside the metal and cannot polarize atoms.

2.3.1 Definition of an electric dipole

- Dipole $\vec{p}$: a system of opposite charges; together the system is neutral; however, the charges do not have the "same center of gravity"
- Assume **an atom in an electric field**:



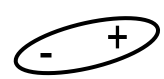
Definition of **dipole moment vector $\vec{p}$**:



$\vec{p} = q \cdot \vec{l}$

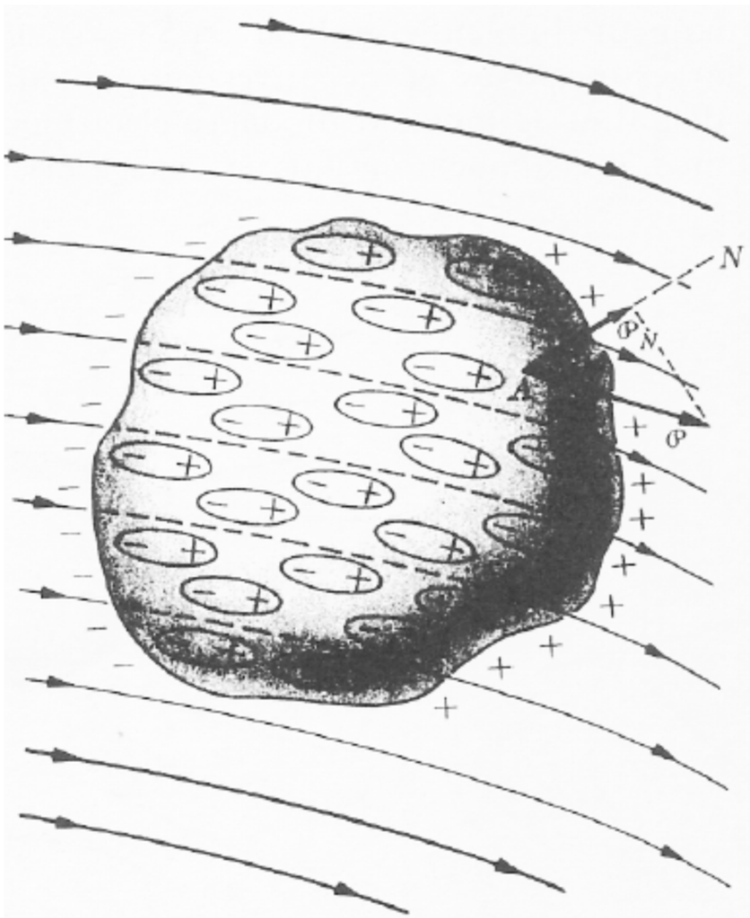
$\vec{p}$: points from negative charge to positive charge!

Here, an **induced** dipole moment is shown. It scales as:
$\vec{p} = \hat{\alpha}\vec{E}$, where $\hat{\alpha}$ is the **tensor of polarizability**

- A hydrogen atom in a field of 1MV/m develops a dipole moment of 10^{-34} Cm which is a **small number**. The lateral displacement of centers of gravity is on the order of only 6 x 10^{-16} m.
- But in solids (crystals), the **extremely large number of dip**oles makes the polarization effect relevan
- Often the polarizability is taken as a **scalar** (assuming isotropic characteristics).
- In illustrations, a **dipole is often sketched as follows**: 

2.3.2 Types of polarization of matter

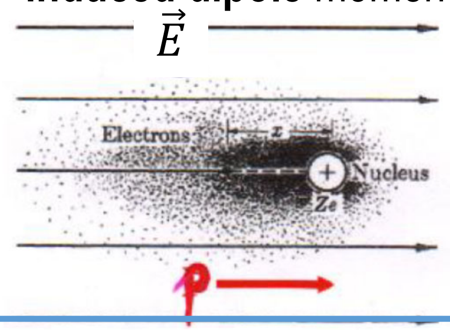
Assume a material into which the electric field $\vec{E}$ can penetrate and polarizes the material:



Alonso/Finn

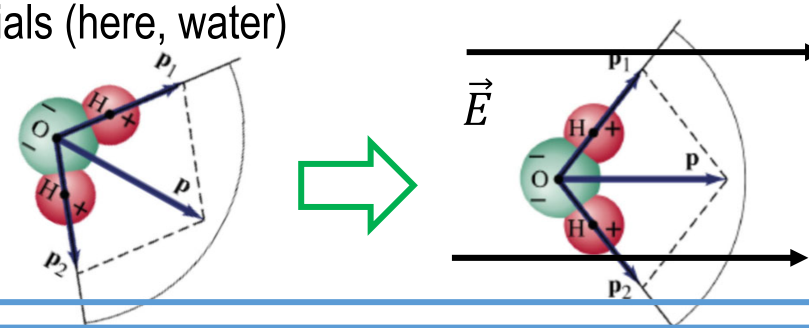
Polarization takes place via different mechanisms:

(i) **induced dipole** moment (see above)



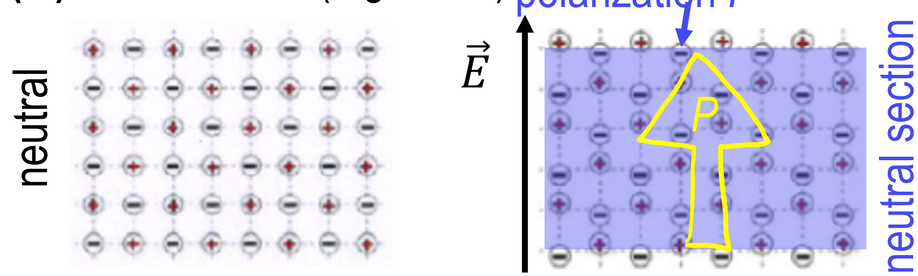
e.g. non-polar covalently bonded materials (glass, SiO_2)

(ii) **alignment of permanent dipoles** in polar materials (here, water)

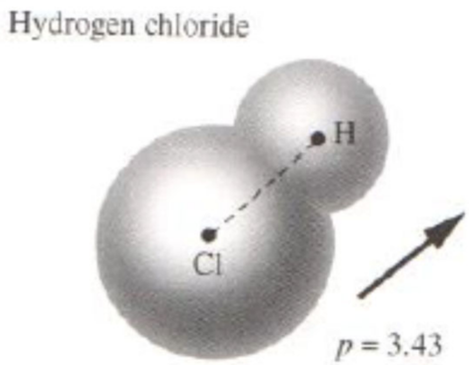


(iii) **Shift of ions** (e.g. NaCl)

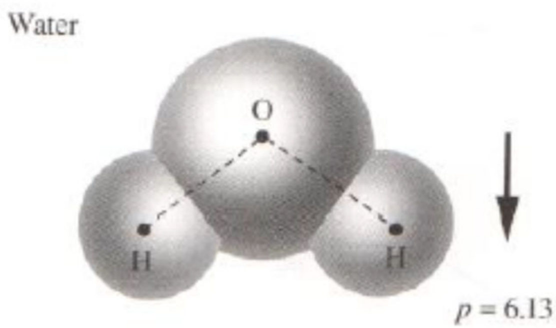
regroup ions to motivate polarization *P*

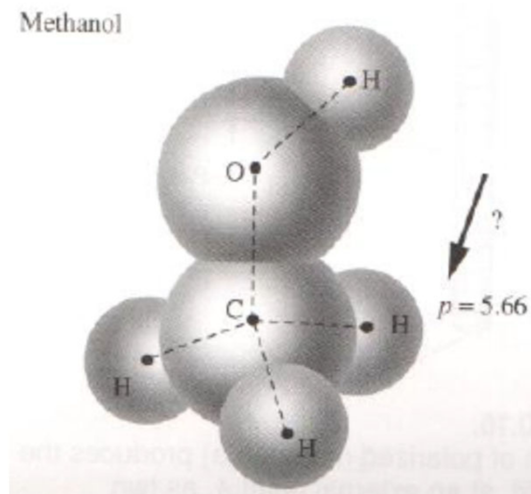


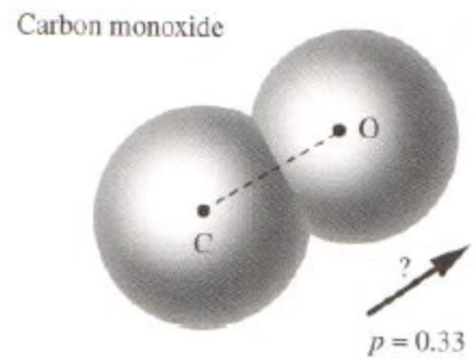
Examples for (ii): permanent electric dipole moments

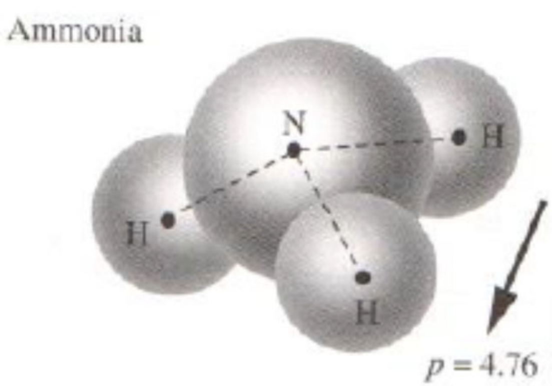


p is given in units of 10^{-30} coulomb-meters.









- ❑ These molecules have permanent $\vec{p}$, even for $\vec{E} = 0$
- ❑ When $\vec{E} \neq 0$, the permanent moments become aligned with $\vec{E}$
- ❑ **Induced moments** can further augment $\vec{p}$

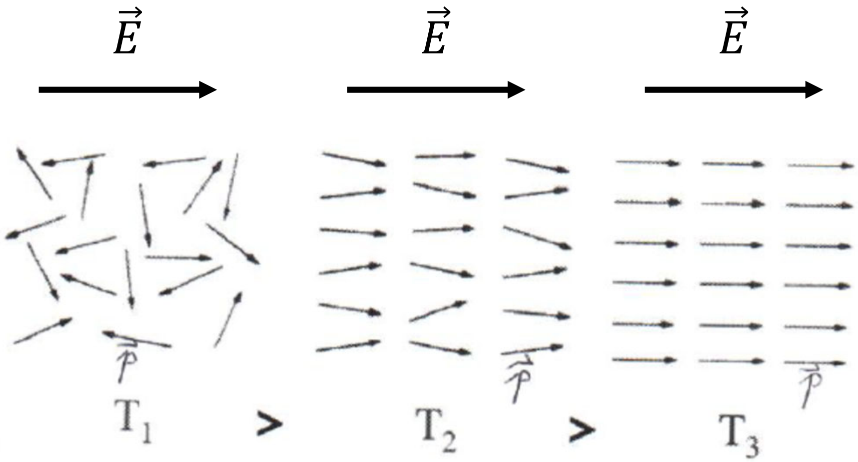
2.3.3 Definition of a dielectric

A dielectric material ("dielectric") is a (neutral) electric insulator that can be polarized by an applied field $\vec{E}$

Examples: glass, Teflon, water, insulating ceramics, semiconductors, NaCl, ... are "dielectrics"

Overall one assumes: $\vec{p} = \hat{\alpha}\vec{E}$

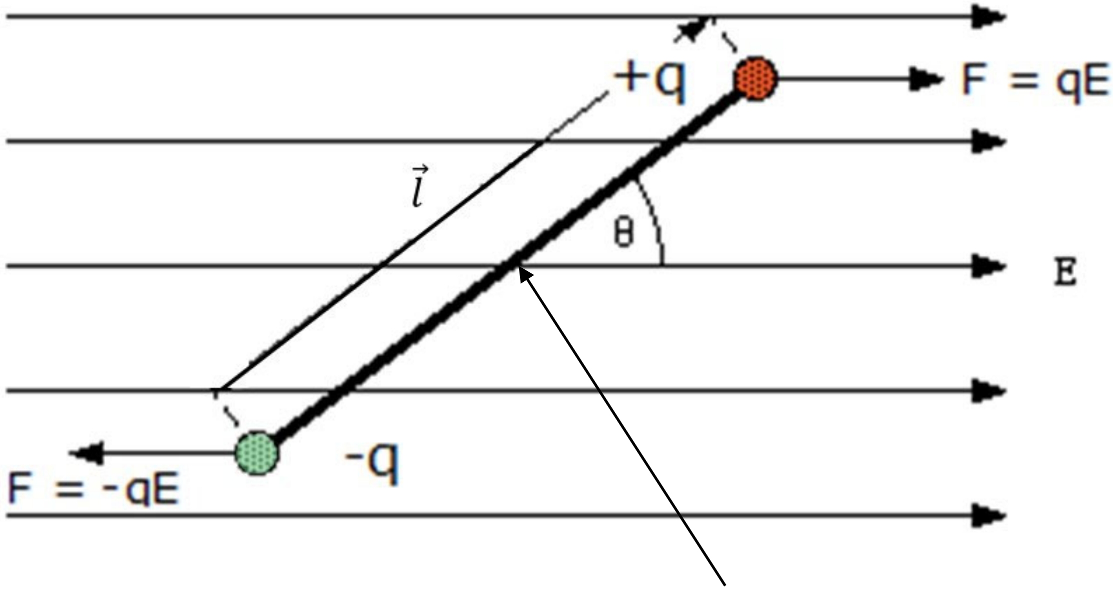
Typically the polarizability α (, i.e., the efficiency of alignment) **depends on temperature:** $\hat{\alpha} = \hat{\alpha}(T)$



Note: **in capacitors**, only the dipole moment "stabilized" by an electric field is relevant for ε_r.

2.3.4 Why do dipole moments align?

Assume a dipole in a **homogeneous field**:



How do equipotential lines look like
in such an external field *E*? (next slide)

Torque on dipole:

$$\vec{\tau} = \frac{1}{2}\vec{l} \times \vec{F}_{q1} + \left(-\frac{1}{2}\vec{l} \times (\vec{F}_{q2})\right)$$
$$= \frac{1}{2}\vec{l} \times \vec{F}_{q1} + \left(-\frac{1}{2}\vec{l} \times (-\vec{F}_{q1})\right)$$
$$= \vec{l} \times \vec{F}_{q1}$$
$$= q\vec{l} \times \vec{E}$$
$$= \vec{p} \times \vec{E}$$

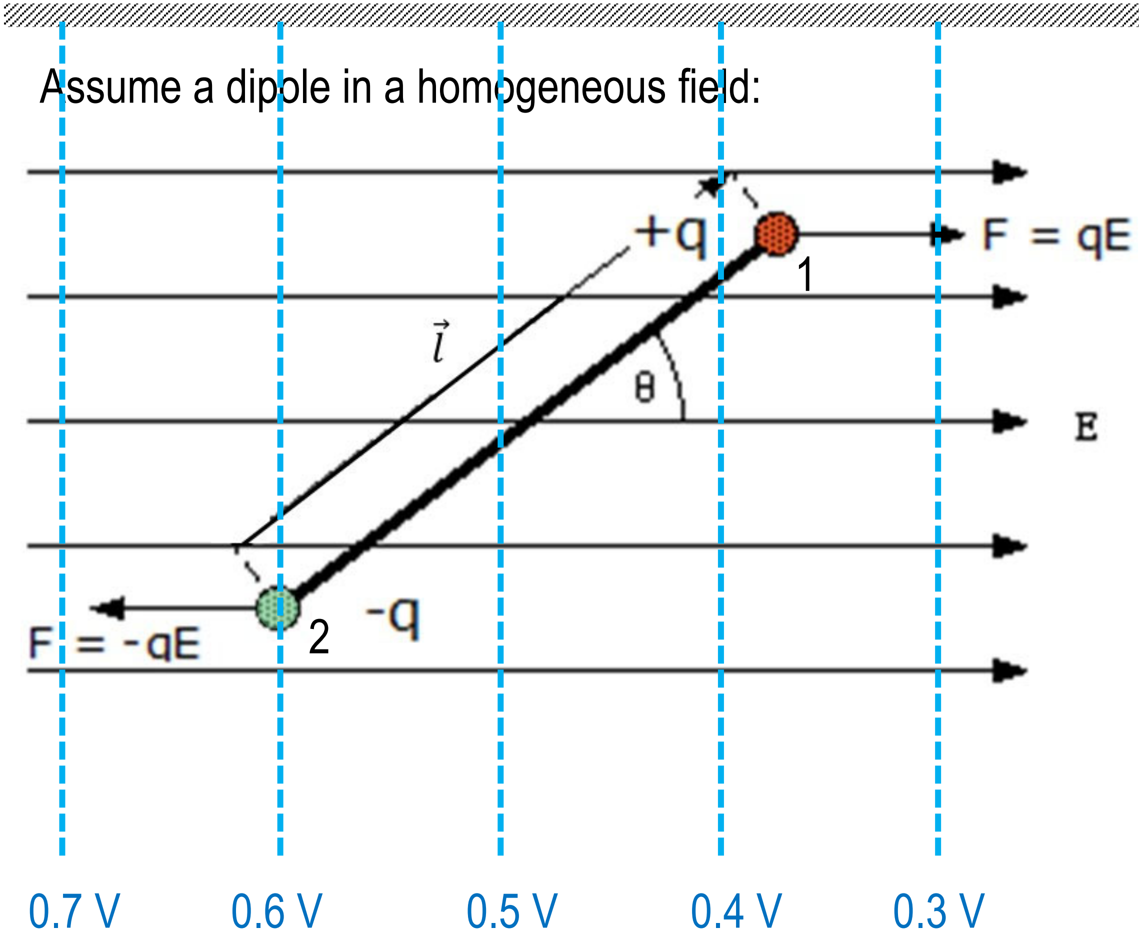
$$|\vec{\tau}| = \left|\vec{p} \times \vec{E}\right| = |\vec{p}|\left|\vec{E}\right| \sin\theta$$

- Maximum torque for $\theta = 90°$
- Dipole gets aligned with $\vec{E}$ (then torque is zero)

"torque" : couple de rotation ou moment de rotation (Fr.)

2.3.5 Potential energy of a dipole

Assume a dipole in a homogeneous field:

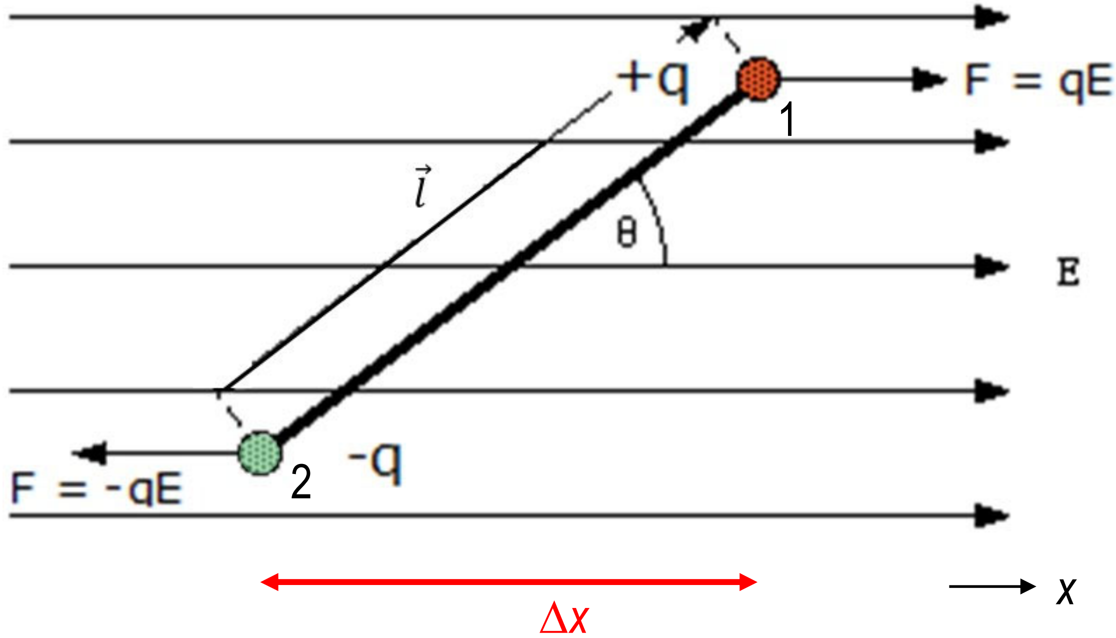


How do equipotential lines look like in such an external field *E*?

In blue color we show an example:
the **sketched charges are hence at different values of** ϕ!

2.3.5 Potential energy of a dipole

Assume a dipole $p = ql$ in a homogeneous field:



Dipole: $q_1 = -q_2 = q$

potential due to external field

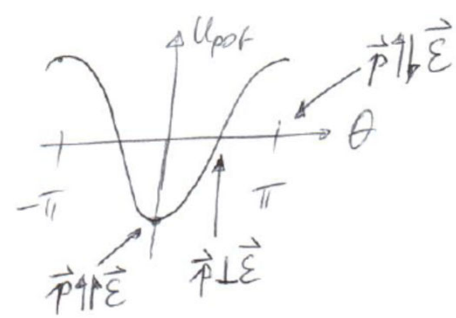
$$U_{pot} = q_1\phi(\vec{r_1}) + q_2\phi(\vec{r_2})$$
$$= q\phi(\vec{r_1}) - q\phi(\vec{r_2})$$
$$= q\big(\phi(\vec{r_1}) - \phi(\vec{r_2})\big) = q\Delta\phi$$

Use: $\vec{E} = -\nabla\phi$;
for the homogeneous field,
considering $E = -\frac{\partial\phi}{\partial x}$ one finds:

$$E = \frac{-\Delta\phi}{\Delta x} = \frac{-\Delta\phi}{l \cdot cos\theta}$$

$$\Rightarrow \; U_{pot} = q \cdot (-lEcos\theta)$$
$$= -p \cdot Ecos\theta$$

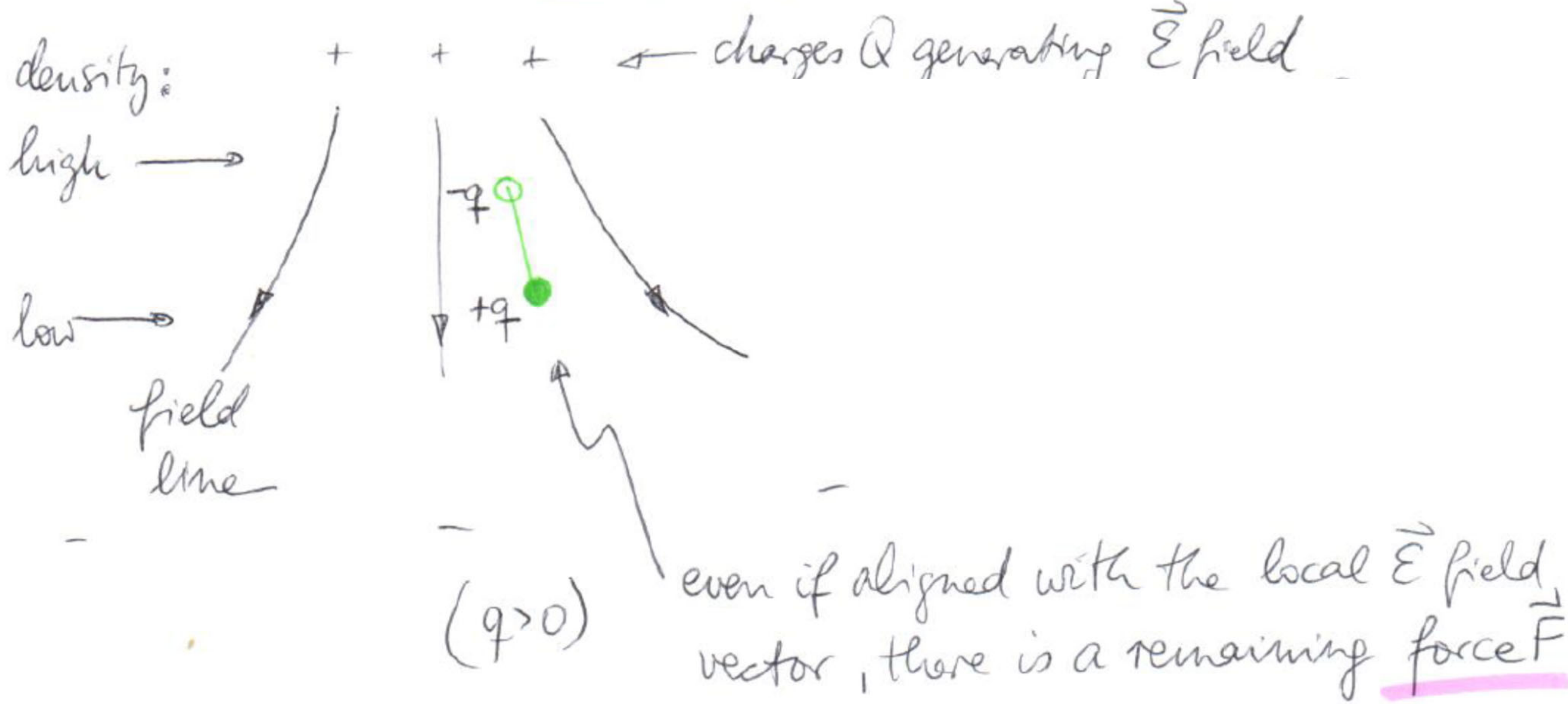
$$\Rightarrow U_{pot} = -\vec{p} \cdot \vec{E}$$



Alignment ensures energy minimization

2.3.6 Force on a dipole moment

In an **inhomogeneous electric field**, there is a **force** on a dipole moment:

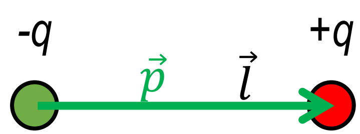


=> A **net force** pulls the dipole moment **into the region of higher field line density**

Full description of force $\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$

Summary 2.3

A dipole $\vec{p}$ is formed by two charges of opposite polarity: $\vec{p} = q \cdot \vec{l}$



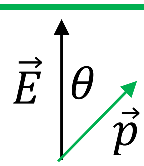
$\vec{l}$ and $\vec{p}$ point from negative to positive charge!

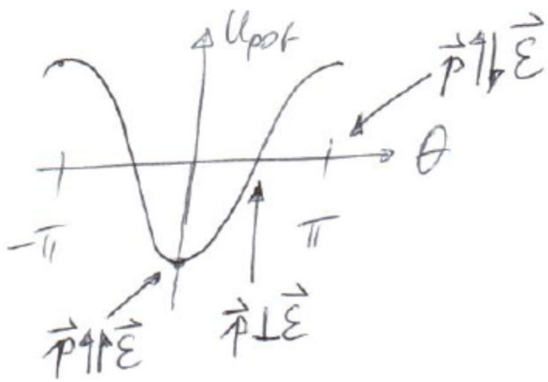
Homogeneous Field:

The Torque $|\vec{\tau}| = |\vec{p} \times \vec{E}| = |\vec{p}||\vec{E}| \sin\theta$

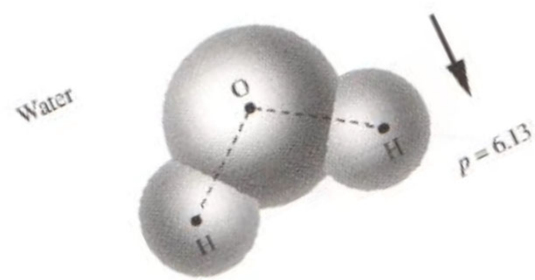
aligns dipoles with $\vec{E}$.

Potential energy $U_{pot} = -\vec{p} \cdot \vec{E}$





Example of a **permanent dipole**:



Inhomogeneous Field:
There is also a
Force $\vec{F} = (\vec{p} \cdot \vec{\nabla})\vec{E}$

Efficiency of alignment/inducing of dipoles = "polarizability" $\hat{\alpha} = \hat{\alpha}(T)$: $\vec{p} = \hat{\alpha}\vec{E}$

EPFL

In the most general case, α is a tensor.