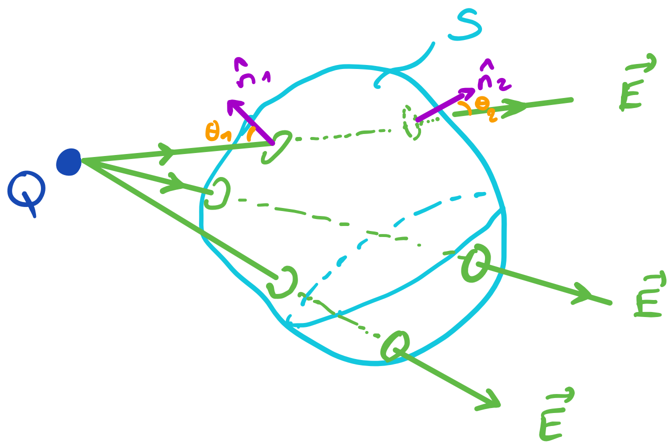
Preuve :

(1) Charge Q à l'extérieur de S.

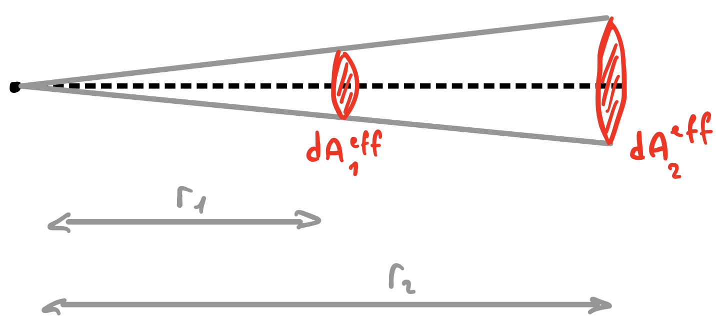


$$Q^S_{enf.} = 0 \quad , \quad \Phi^S_E = \ ?$$

Une ligne va percer la surface deux fois, en

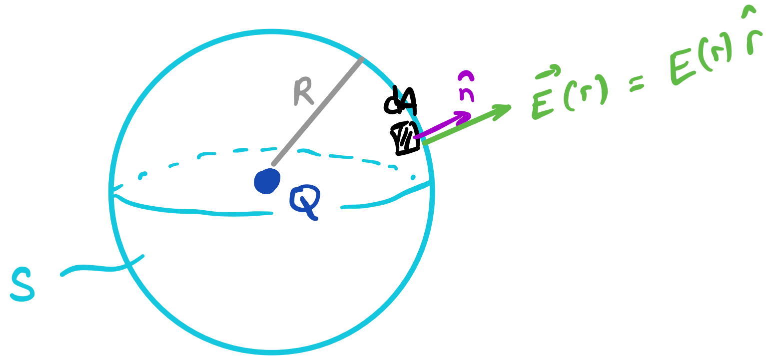
entrant : $d\Phi_E^{dA_1} = \vec{E}_1 \cdot \hat{n}_1 dA_1 = E_1\, dA_1 \cos\theta_1 < 0$

sortant : $d\Phi_E^{dA_2} = \vec{E}_2 \cdot \hat{n}_2\, dA_2 = E_2 \underbrace{dA_2 \cos\theta_2}_{\text{surface efficace}} > 0$



Et $\begin{cases} dA_1^{eff} / dA_2^{eff} = r_1^2 / r_2^2 \\ E_1 / E_2 = r_2^2 / r_1^2 \end{cases} \Rightarrow \Phi^S_E = 0$ ✓

(2) Charge ponctuelle au centre d'une sphère S de rayon R.



$$Q^{S}_{enf.} = Q$$

$$\Phi^{S}_{E} = \int_S \vec{E}\cdot d\vec{A} = \int_S \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\,\hat{r}\cdot d\vec{A}$$

$$= \frac{Q}{4\pi\varepsilon_0}\int_S \frac{dA}{r^2}$$

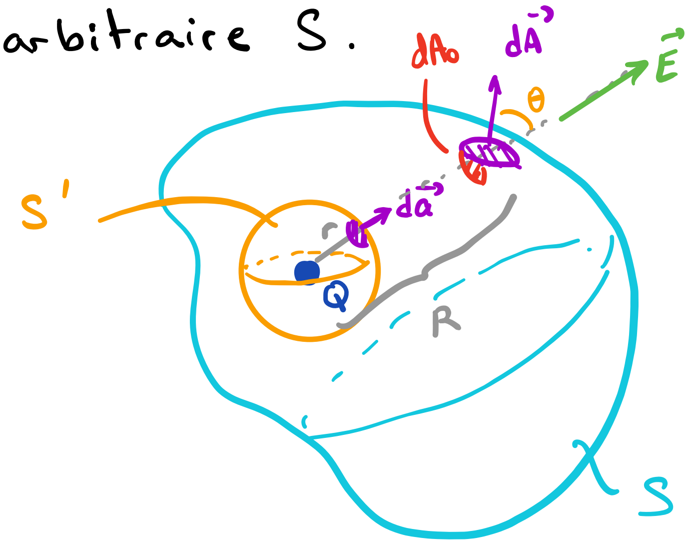
$$= \frac{Q}{4\pi\varepsilon_0 r^2}\underbrace{\int_S dA}_{4\pi r^2}$$

$$= \frac{Q}{\varepsilon_0}$$

✓

⚠ cela ne marchera pas (r^2/r^2 qui se compense) si Coulomb n'allait pas exactement comme $\frac{1}{r^2}$!

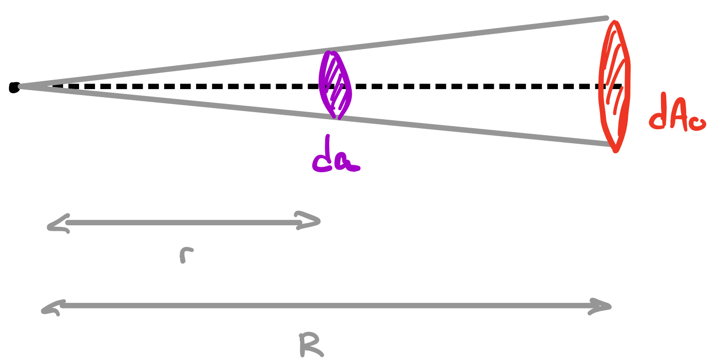
(3) charge ponctuelle dans surface arbitraire S.



$$d\Phi_E^{dA} = \vec{E} \cdot d\vec{A} = E(R)\, \underbrace{dA \cos\theta}_{dA_0}$$

$$= \frac{Q}{4\pi\varepsilon_0 R^2}\, dA_0$$

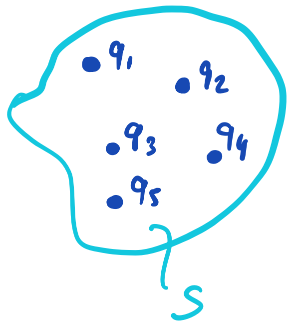
$$d\Phi_E^{da} = \vec{E} \cdot d\vec{a} = E(r)\, da = \frac{Q}{4\pi\varepsilon_0 r^2}\, da$$



et $\quad \dfrac{da}{r^2} = \dfrac{dA_0}{R^2}$ (même angle solide) donc $$d\Phi_E^{da} = \frac{Q}{4\pi\varepsilon_0}\frac{da}{r^2} = \frac{Q}{4\pi\varepsilon_0}\frac{dA_0}{R^2} = d\Phi_E^{dA}$$

$$\Rightarrow \quad \Phi_E^{S} = \Phi_E^{S'} = \frac{Q}{\varepsilon_0} \quad \checkmark$$

(4) Plusieures charges en S.



$$\Phi_E^{S} = \int \vec{E} \cdot d\vec{A}$$

principe de superposition

$$= \sum_j \int \vec{E}_j \cdot d\vec{A}$$

étape (3)

$$= \sum_j q_j/\varepsilon_0$$

$$= Q_{enf.}^{S}/\varepsilon_0 \quad \checkmark$$

Q.E.D.

Maintenant, exploitons la loi de Gauss.