Série 10

Friday, 26 March 2021 16:33

Formules

- **Equation de Bernoulli** $(p = \frac{E}{V})$

$$P + \rho \cdot g \cdot h + \frac{1}{2} \cdot \rho \cdot v^2 = \text{cste.} \quad \Rightarrow \text{Conservation d'énergie}$$

Pression [Pa] — Energie potentielle — Energie cinétique — masse vol. $[\frac{kg}{m^3}]$ — cste de gravitation $9{,}81\ [\frac{m}{s^2}]$ — hauteur [m] — masse vol. $[\frac{kg}{m^3}]$ — vitesse $[\frac{m}{s}]$

- **Masse volumique de l'air :** (A 2.1)

$$\rho(p,\theta) = \rho(p_0,\theta_0) \cdot \frac{p}{p_0} \cdot \frac{T_0}{T} \qquad (p_0, \theta_0 : \text{conditions atmosphériques standards})$$

$$= 1{,}2929 \cdot \frac{p}{101\,325\ Pa} \cdot \frac{273\ K}{T\ [K]}$$

Conversions d'unités :

- $1\ atm = 101\,325\ Pa = 760\ Torr$ ←
- $1\ N = 1\ kg\ \frac{m}{s^2}$ ⇐
- $1\ Pa = 1\ \frac{N}{m^2}$

Q1 $\underset{①}{[P]} + \underset{②}{[\rho \cdot g \cdot h]} + \underset{③}{[\frac{1}{2} \cdot \rho \cdot v^2]} = \text{cste.}$

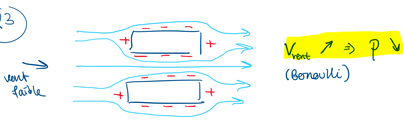
① $[Pa] = [\frac{N}{m^2}] \quad ([N] = [kg\ \frac{m}{s^2}])$

② $[\frac{kg}{m^3}] \cdot [\frac{m}{s^2}] \cdot [m] = [kg\ \frac{m}{s^2}]\ [\frac{1}{m^2}] = [\frac{N}{m^2}]$

③ $[\frac{kg}{m^3}] \cdot [\frac{m}{s^2}] = [kg\ \frac{m}{s^2}]\ [\frac{1}{m^2}] = [\frac{N}{m^2}]$

Q2

Q3



$v_{vent} \nearrow \Rightarrow p \searrow$ (Bernoulli)

P1

$h_0 = 0$

$h_{12} = 12 \cdot 3{,}4\ m = 40{,}8\ m$

$p_0 = 4\ atm = 4 \cdot 101\,300\ Pa = 405\,200\ Pa$

$V = 0 \Rightarrow$ Bernoulli : $p + \rho \cdot g \cdot h = \text{cste.}$ ↳ Hydrostatique

Solution

$$p_0 + \rho \cdot g \cdot h_0 = p_{12} + \rho \cdot g \cdot h_{12}$$

$$\Rightarrow p_{12} = p_0 + \cancel{\rho} \cdot g \cdot h_0 - \rho \cdot g \cdot h_{12} \qquad (\rho_{eau})$$

$$= p_0 - \rho \cdot g\ (h_{12} - h_0) = 405\,200 - 1000 \cdot 9{,}81 \cdot 40{,}8$$

$$= 49\,52\ Pa$$

Règle de 3

$1\ atm = 101\,325\ Pa$
$x\ atm = 4952\ Pa$

$$p\ [atm] = 4952\ Pa \cdot \frac{1\ atm}{101\,325\ Pa} = 0{,}05\ atm$$

P2

① $p_i = p_e$ @ $h = 12m$

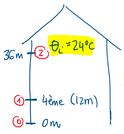
$V = 0$ ↱ Hydrostatique

$\Rightarrow$ Bernoulli : $p + \rho \cdot g \cdot h = \text{cste.}$

$$\rho_i(20°C, p_0) = 1{,}2929 \cdot \frac{p}{p_0} \cdot \frac{273}{273+24}$$

$$= 1{,}1885\ \frac{kg}{m^3}$$

$$\rho_e(32°C, p_0) = 1{,}1573\ \frac{kg}{m^3}$$



$p_{i,1} - p_{e,1}$?

Bernoulli (intérieur) : $p_{i,0} + \rho_i \cdot g \cdot h_0 = p_{i,1} + \rho_i \cdot g \cdot h_1$

$$\Rightarrow p_{i,1} = p_{i,0} + \rho_i \cdot g \cdot (h_0 - h_1)$$

$(h_0 - h_1 < 0)$

Bernoulli (extérieur) : $p_{e,1} = p_{e,0} + \rho_e \cdot g \cdot (h_0 - h_1)$

$p_{i,1} = p_{e,1} \Rightarrow p_{i,0} + \rho_i \cdot g \cdot \Delta h_0 = p_{e,0} + \rho_e \cdot g \cdot \Delta h_0$

① $\Rightarrow p_{i,0} - p_{e,0} = g \cdot \Delta h_0 \cdot (\rho_e - \rho_i)$

$$= 9{,}81\ \frac{m}{s^2} \cdot (0-12)\ m \cdot (1{,}1573 - 1{,}885)\ \frac{kg}{m^3}$$

$$= 3{,}7\ Pa \qquad [\tfrac{kg}{m \cdot s^2}] = [\tfrac{kg\ \frac{m}{s^2}}{m^2 J}] = [\tfrac{N}{m^2}] = [Pa]$$

$(p_{i,1} > p_{e,1})$

② $\Rightarrow p_{i,2} - p_{e,2} = g \cdot \Delta h_2 \cdot (\rho_e - \rho_i)$

$$= 9{,}81\ \frac{m}{s^2} \cdot (36-12)\ m \cdot (1{,}1573 - 1{,}885)\ \frac{kg}{m^3}$$

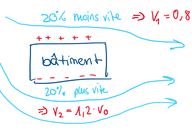
$$= -7{,}4\ Pa$$

$(p_{i,1} < p_{e,1})$

P3

$V_0 = 72\ \frac{km}{h} = 20\ \frac{m}{s}$

$1\ \frac{km}{h} = \frac{1000\ m}{3600\ s} = \frac{1}{3{,}6}\ \frac{m}{s}$



20% moins vite $\Rightarrow V_1 = 0{,}8\ V_0$

20% plus vite $\Rightarrow V_2 = 1{,}2 \cdot V_0$

$\Delta h = 0 \Rightarrow$ Bernoulli : $p + \frac{1}{2}\rho v^2 = \text{cste.}$ → Hydrodynamique

$$p_1 + \frac{1}{2}\rho \cdot V_1^2 = p_2 + \frac{1}{2}\rho \cdot V_2^2$$

Sous-pression : $p_1 - p_2 \qquad (p_1 > p_2 \text{ car } V_1 < V_2)$

$$\Rightarrow p_1 - p_2 = \frac{1}{2} \cdot \rho \cdot (V_2^2 - V_1^2)$$

Sans autres données : $\rho_{air} = 1{,}2\ \frac{kg}{m^3}$

$V_1 = 0{,}8\ V_0 = 16\ \frac{m}{s}$

$V_2 = 1{,}2\ V_0 = 24\ \frac{m}{s}$

$$\Rightarrow p_1 - p_2 = \frac{1}{2} \cdot 1{,}2\ \frac{kg}{m^3} \cdot (24^2 - 16^2)\ \frac{m^2}{s^2}$$

$$= 192\ Pa$$