General Physics II: Tutorial Material 4

1) In an inertial frame *S*, two objects, *a* with its rest mass m_a and *b* with its rest mass m_b, are moving with velocity vectors $\vec{v}_a$ and $\vec{v}_b$, respectively, with θ being the angle between the two vectors. After colliding each other, *a* and *b* got stack each other and became one object *x* with its rest mass M and velocity $\vec{V}$.

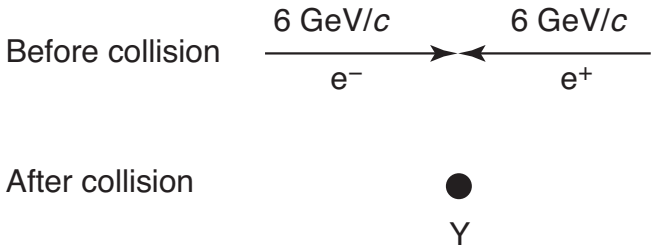
   a) Write down the relativistic energy-momentum relations among *a*, *b* and *x*.

   b) Calculate *M* as a function of m_a, m_b, $|\vec{v}_a|$, $|\vec{v}_b|$ and θ.

   c) Show that when $\vec{v}_a = \vec{v}_b$, we have $M = m_a + m_b$.

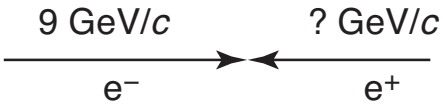
   d) Explain the result obtained for 3), from the principle of relativity.

2) An electron, e^-, and its anti-particle, a positron, e^+, collide head-on with an equal momentum $p_0 = 6$ GeV/c. The rest masses of the electron and positron are equal and about 0.0005 GeV/c^2. Therefore, they are totally negligible in the energy $E = \sqrt{m_0^2c^4 + p^2c^2}$, leading to $E^- = p_0c = 6$ GeV and $E^+ = p_0c = 6$ GeV for e^- and e^+ energies, respectively.



   a) When they collide, the electron and positron annihilate and one new particle, Y, is produced. Using the energy-momentum conservation law, calculate the momentum and rest mass of Y in the GeV/c and GeV/c^2 units, respectively.

   b) We collide an e^- with a momentum of 9 GeV/c head-on with an e^+. What is the momentum of e^+ in order to produce after the collision only one Y particle, identical to that in a)? In which direction does the particle Y move and how large is $\beta_u = u/c$, where *u* is the velocity of Y? The rest mass of e^- and e^+ can be neglected in the calculations.



3) A certain galaxy has a Doppler shift given by $f_0 - f = 0.0987\, f_0$. Estimate how fast it is moving away from us.

4) Show that when $u \ll c$, the Doppler shift in wavelength is

$$\frac{\Delta\lambda}{\lambda_0} = \frac{u}{c}$$