General Physics II: Tutorial Material 2

1) Your spaceship travelling at $0.90c$, needs to launch a probe out the forward hatch so that its speed relative to the planet that you are approaching is $0.95c$. With what speed must it leave your ship?

We take the positive direction in the direction the ship is travelling.
The reference S is put on the spaceship and S' on the planet. The S' is moving with a velocity $v = -0.90c$ respect to S. The velocity of the probe in S' is $u = 0.95c$. In S, this velocity is given by

$$u = \frac{u' + v}{1 + vu'/c^2} = 0.34c\ .$$

2) If you were travelling away from Earth at speed $0.5c$, would you notice a change in your heart beat? Would your height or waistline change? What would observers on Earth using telescopes say about you?

No change would be noticed by you. The observer from the Earth would see your hear beats slower. If you are standing up in the space ship such that your height is perpendicular to the direction of the rocket movement, your waistline shrinks. If you are lying down, your height appears shorter.

3) When it is at rest, a box has the form of a cube 2.0 m on a side. This box is loaded onto the flat floor of a spaceship, which then flies and passes us with a horizontal speed of 0.80c. What is the volume of the box as we observe it?

Only the dimension along the direction of motion is contracted:

$$l_0 \times l_0 \times l_0\sqrt{1-(v/c)^2} = 4.8\ m^3$$

4) Let us define two event seen in S frame to be $E_1(t_1, x, 0, 0)$ and $E_2(t_2, x, 0, 0)$, i.e. the two events happened at the same space coordinate. S' frame is moving with a constant velocity, v, in the positive x direction and the same events seen in S' frame can be denoted as $E_1'(t_1', x_1', 0, 0)$ and $E_2'(t_2', x_2', 0, 0)$. Using the Lorentz transformation, show that $\Delta t' = \gamma \Delta t_0$ where $\Delta t_0 = t_2 - t_1$ and $\Delta t = t_2' - t_1'$.

Using the Lorentz transformation, $t_{1,2}'$ and $x_{1,2}'$ are related to $t_{1,2}$ and x as

$$t_1 = \gamma\left(t_1' + \frac{v}{c^2}x_1'\right),\ t_2 = \gamma\left(t_2' + \frac{v}{c^2}x_2'\right)$$

thus

$$t_2 - t_1 = \gamma(t_2' - t_1') + \gamma\frac{v}{c^2}(x_2' - x_1').$$

The inverse Lorentz transformation gives

$$x_1' = \gamma(x - vt_1),\ x_2' = \gamma(x - vt_2)$$

and

$$\gamma\frac{v}{c^2}\left(x_2'-x_1'\right)=\gamma^2\left(\frac{v}{c}\right)^2\left(t_1-t_2\right)=-\gamma^2\left(\frac{v}{c}\right)^2\Delta t_0$$

By introducing $\Delta t = t_2' - t_1'$,

$$\Delta t_0 = \gamma\Delta t - \gamma^2\left(\frac{v}{c}\right)^2\Delta t_0$$

i.e.

$$\gamma\Delta t = \left[1+\gamma^2\left(\frac{v}{c}\right)^2\right]\Delta t_0$$

By noting that

$$1+\gamma^2\left(\frac{v}{c}\right)^2 = 1+\frac{\left(v/c\right)^2}{1-\left(v/c\right)^2} = \frac{1-\left(v/c\right)^2+\left(v/c\right)^2}{1-\left(v/c\right)^2} = \gamma^2$$

it follows that

$$\Delta t' = \gamma\Delta t_0$$

5) If a particle moves in the x-y plane of system S with a velocity u in a direction that makes an angle θ with respect to the x axis, show that it makes an angle θ' in S' given by $\tan\theta' = \sin\theta\sqrt{1-\left(v/c\right)^2}\Big/\left(\cos\theta - v/u\right)$, where $v < u$, and S and S' are equal to what defined during the lecture.

The velocity components in S are $u_x = u\cos\theta$ and $u_y = u\sin\theta$. In the S' frame, they are given by

$$u_x' = \frac{u_x - v}{1 - vu_x/c^2} \quad \text{and} \quad u_y' = \frac{u_y\sqrt{1-\left(v/c\right)^2}}{1 - vu_x/c^2}$$

It follows that

$$\tan\theta' = \frac{u_y'}{u_x'} = \frac{u_y\sqrt{1-\left(v/c\right)^2}}{u_x - v} = \frac{\sin\theta\sqrt{1-\left(v/c\right)^2}}{\cos\theta - v/u}$$

6) Can a particle with non-zero rest mass attain the speed of light?

No. It requires an infinite amount of energy to reach the speed of light.

7) An electron is limited to travel at a speed less than c. Does this put an upper limit of the momentum of an electron?

No. The momentum given by

$$p = \frac{m_0 u}{\sqrt{1-\left(u/c\right)^2}}$$

has no limit for $u \rightarrow c$.

8) A particle travels at $u = 0.10c$. By what percentage will a calculation of momentum be wrong if you use the Newtonian formula. Repeat this for $u = 0.60c$.

The classic and relativistic momenta are given by

$p_{\text{classic}} = m_0 u$ and $p_{\text{classic}} = m_0 u \big/ \sqrt{1-(u/c)^2}$, respectively.

For $u = 0.10c$, $\dfrac{p_{\text{classic}}}{p_{\text{relativisitc}}} = 0.995$ and for $u = 0.60c$, $\dfrac{p_{\text{classic}}}{p_{\text{relativisitc}}} = 0.80$