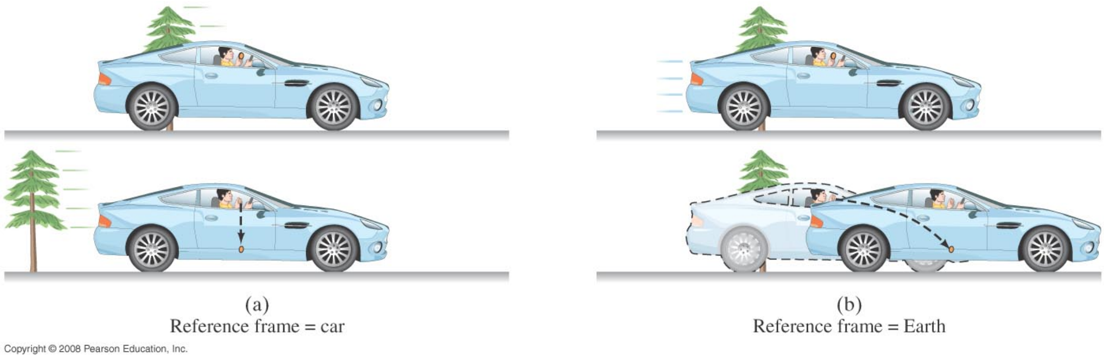
General Physics II: Tutorial Material 1

1) Obtain the trajectory of the object with a mass m, for the two cases. The car is moving with a constant speed of v.



For the observer in the car, the object falls along the straight line perpendicular to the ground. For the observer, the object follows a parabolic line, where the vertical position follows the motion of the car.

2) Does the Earth really go around the Sun? Or is it also valid to say that the Sun goes around the Earth? Are the both description equivalent?

The Galilean relativity principle applies to the frames which are moving with constant velocities, without acceleration. Rotation involves a change of direction, i.e. acceleration. Therefore, the two descriptions are not equivalent.

3) You are standing inside of a lift in a building with a metal ball in your hand. You drop the ball and at the exactly same time the cable of the lift is cut and the lift starts to fall:

A) what do you see for the movement of the ball?

If the air resistance can be neglected, the ball does not move.

(It appears that there is no gravity for you.)

B) what does a person standing on of the floors of the building see?

The lift, the ball and you are falling down with the same acceleration.

4) Using the Taylor expansion, demonstrate that the first order approximation in x is valid for $F(x)=(1-x)/(1+x)$ and $F(x)=1/\sqrt{1-x^2}$ if $|x|<<1$.

As discussed in the lecture note,

$$F(x)=F(0)+\left.\frac{dF(x)}{dx}\right]_{x=0}x+\frac{1}{2}\left.\frac{d^2F(x)}{dx^2}\right]_{x=0}x^2+\cdots$$

For $F(x)=(1-x)/(1+x)$ we have

$$F(0)=1,\ \left.\frac{dF(x)}{dx}\right]_{x=0}=\left.\frac{-2}{(1+x)^2}\right]_{x=0}=-2,\ \text{thus } F(x)=1-2x+\cdots$$

giving, $F(0.001)=0.998002$ and $1-2x=0.998$, i.e. good approximation.

For $F(x)=1/\sqrt{1-x^2}$

$$F(0) = \left.\frac{1}{\left(1-x^2\right)^{1/2}}\right]_{x=0} = 1,$$

$$\left.\frac{dF(x)}{dx}\right]_{x=0} = \left.\frac{x}{\left(1-x^2\right)^{3/2}}\right]_{x=0} = 0,$$

$$\left.\frac{d^2F(x)}{dx^2}\right]_{x=0} = \left.\frac{1}{\left(1-x^2\right)^{3/2}} + \frac{3x^2}{\left(1-x^2\right)^{5/2}}\right]_{x=0} = 1$$

thus $F(x) = 1 + x^2/2 + \cdots$

with x=0.001, $F(0.001) = 1.000000500$ and $1 + x^2/2 = 1.0000005$. Note that for the we need consider the term proportional to x^2.

5) Reference frame S' moves at speed $v = 0.92c$ in the positive x direction with respect to the reference frame S. The origins of S and S' overlap at $t = t' = 0$. An object is stationary in S' at position $x' = 100$ m. What is the position of the object in S when the clock in S reads 1.00 μs, according to the a) Galilean and b) Lorentz transformation?

a) Galilean transformation is $t = t'$ and $x = x' + vt' = x' + vt = 376\ m$

b) Lorentz transformation gives $t' = \gamma\left(t - \beta x/c\right)$ and $x = \gamma\left(x' + vt'\right)$.

which leads to

$$t' = \gamma t - \frac{\beta\gamma x}{c} = \gamma t - \frac{\beta\gamma^2 x' + \beta\gamma^2 vt'}{c} = \gamma t - \beta\gamma^2\frac{x'}{c} - \beta^2\gamma^2 t'$$

$$\left(1 + \beta^2\gamma^2\right)t' = \gamma^2 t' = \gamma t - \beta\gamma^2\frac{x'}{c}$$

thus, $\gamma vt' = vt - \beta^2\gamma x'$. Then x is given by

$$x = \gamma x' + \gamma vt' = \gamma x' - \beta^2\gamma x' + vt = \gamma\left(1 - \beta^2\right)x' + vt = \frac{x'}{\gamma} + vt$$

This equation can also be understood in the following way. After time t, the frame S' moved by vt respect to the frame S. Since object is stationary in S', x is equivalent to a length. In the frame S, this length is contracted by $1/\gamma$. Thus the coordinate of the object in S is $x = vt + x'/\gamma$.

Using $v = 0.92c$, $t = 1 \times 10^{-6}\text{s}$, $x' = 100\text{m}$, $c = 3 \times 10^8\text{ms}^{-1}$

$x = 315\ m$

6) Two space ships are flying toward Earth on the same path; one with 0.60c and the other 0.90c. What is the relative speed of one vessel as seen by the others?

The positive direction is defined as the direction of the sips. Take the frame where slow vessel is at rest as S and earth at rest as S'. In the S' frame, the speed of the slow vessel is 0.60c and the faster one $u' = 0.90c$. The S' frame is moving with a velocity $v = -0.60c$ respect to S. Using the relativistic addition of velocities, the velocity of the fast vessel seen S, which is the relative speed between the two vessels, is given by

$$u = \frac{u' + v}{1 + vu'/c^2} = 0.65c\,.$$

The same result can be obtained if the faster vessel to be the frame S. In the S' frame, the speed of the fast vessel is $0.90c$ and the slower one $u' = 0.60c$. The S' frame is moving with a velocity $v = -0.90c$ respect to S. Using the relativistic addition of velocities, the velocity of the fast vessel seen S, which is the relative speed between the two vessels, is given by

$$u = \frac{u' + v}{1 + vu'/c^2} = -0.65c\,.$$

The sign difference is $0.65c$ the relative velocity of the faster vessel respect to the slower one, while $-0.65c$ is the relative velocity of the slower vessel respect to the faster one.

We can also take the slow vessel S and the faster vessel S'. In S, the Earth is moving with a velocity, $u = -0.60c$ and in S', $u' = -0.90c$. By denoting the relative velocity between the two frames, v, which is the relative velocity between the two vessels,

$$u = \frac{u' + v}{1 + u'v/c^2} \quad \rightarrow \quad v = 0.65c$$

If we take the slow vessel S and the faster vessel S', $v = -0.65c$.