General Physics II: Tutorial Material 9

1) There are 20 kg of ice at −10°C and 50 kg of water at 10°C.

   a) How much heat is needed to increase the temperature of the ice to 0°C and how much heat is needed to melt all the ice at 0°C? How much heat is needed to decrease the temperature of water to 0°C?

   b) What is the thermal equilibrium state when the 20 kg of ice at −10°C is put into the 50 kg of water at 10°C?

   Note that the specific heats of water and ice are, 1 kcal/kg·C° and 0.5 kcal/kg·C°, respectively. Further more, heat of fusion for water is 80 kcal/kg with a melting point at 0°C.

   a) Heat, Q, needed to increase the temperature of a material by ΔT°C is given by

$$Q = m\cdot c\cdot \Delta T$$

   where m and c are the mass and specific heat of the material, respectively. For 20kg of ice to increase its temperature from −10°C to 0°C, it requires heat of

$$Q = 20\text{kg} \times 0.5\frac{\text{kcal}}{\text{kg}\cdot \text{C}^\circ} \times (0+10)^\circ\text{C} = 100\text{kcal}$$

   Heat, Q', needed to melt a material is given by

$$Q' = m\cdot L$$

   where L is latent heat of the material. For 20kg of ice at 0°C to melt,

$$Q' = 20\text{kg} \times 80\frac{\text{kcal}}{\text{kg}} = 1600\text{kcal}$$

   of heat is needed. In order to cool down 50 kg of water from 10°C to 0°C, heat required, Q'' is given by

$$Q'' = 50\text{kg} \times 1\frac{\text{kcal}}{\text{kg}\cdot \text{C}^\circ} \times (0-10)^\circ\text{C} = -500\text{kcal}$$

   b) If we put the ice and water together, $Q < |Q''|$ tells that the water does not cool down to 0°C when the ice reaches at 0°C. Since $Q' > |Q''| - Q$, not all the ice can melt after reaching at 0°C with the remaining heat of the water cooling down to 0°C. Therefore, the equilibrium state will be a mixture of some ice and some water at 0°C. After the ice reached at 0°C, the water still has heat in order to reach to 0°C,

$$Q + Q'' = 100\text{kcal} - 500\text{kcal} = -400\text{kcal}$$

   which should be cancelled by the heat of melting of ice at 0°C. Therefore,

$$\frac{400\text{kcal}}{80\dfrac{\text{kcal}}{\text{kg}}} = 5\text{kg}$$

of the ice must melt. We finally have the 15kg of ice and 55kg of water at 0°C in a thermal equilibrium state.

2) Suppose 2.60 mol of an ideal gas with a volume and temperature, $V_1 = 3.50\ \mathrm{m}^3$ and $T_1 = 290\ \mathrm{K}$, respectively, is allowed to expand isothermally to $V_2 = 7.00\ \mathrm{m}^3$ at $T_2 = 290\ \mathrm{K}$. Determine (a) the work done by the gas, (b) the heat into the gas, and (c) the change in the internal energy of the gas.

For the isothermal process of an ideal gas, the work is given by

$$W = nRT\ln\frac{V_2}{V_1} = 2.6\ \mathrm{mol} \times 8.314\ \frac{\mathrm{J}}{\mathrm{mol\cdot K}} \times 290\ \mathrm{K} \times \ln\frac{7}{3.5} = 4345\ \mathrm{J}$$

Since the temperature does not change, there is no change in the internal energy. From the first law of thermodynamics, $\Delta E_{\mathrm{int}} = Q - W$ with $\Delta E_{\mathrm{int}} = 0$, the heat is given by

$$Q = W = 4345\ \mathrm{J}.$$

3) An audience of 1800 people fills a concert hall with a volume of $22000\ \mathrm{m}^3$. In average, each person emits 70 W of energy due to metabolism. The concert hall is well isolated and the air can be considered as an ideal gas of diatomic molecules. At the beginning of the concert, the temperature and pressure of the air in the hall are 20° C and 1 atm, respectively. With no ventilation, by how much the temperature of the hall will raise after two hours?

Since the room is thermally isolated, there is no heat exchange with outside. Furthermore, the air is inside of the room, thus kept in a constant volume and no work is done. From the first law of dynamics, $Q = nC_{\mathrm{V}}\Delta T = \Delta E_{\mathrm{int}}$, where Q must be generated by the people inside. It follows that

$$\Delta T = \frac{Q}{nC_{\mathrm{V}}}$$

The heat generated by the people in the hall in two hours is given by

$$Q = 2\ \mathrm{hour} \times 70\ \mathrm{W} \times 1800 = 9.07 \times 10^8\ \mathrm{J}$$

The mole number, n, can be obtained from the ideal gas law as

$$n = \frac{P_0V_0}{RT_0}$$

where P_0, V_0, and T_0 are the initial pressure, volume and temperature of the room respectively. The molar specific heat under constant volume, C_{V}, for a diatomic gas is given by $C_{\mathrm{V}} = 5R/2$. By taking 293 K and $1\ \mathrm{atm} = 1.013 \times 10^5\ \mathrm{N/m^2}$ as initial temperature and pressure of the room, nC_{V} is given by

$$nC_{\mathrm{V}} = \frac{PV}{RT}\frac{5}{2}R = \frac{5}{2} \times \frac{1.013 \times 10^5\ \mathrm{N/m^2} \times 22000\ \mathrm{m^3}}{293\ \mathrm{K}} = 1.90 \times 10^7\ \mathrm{J/K}$$

Combining them, it leads to

$$\Delta T = \frac{Q}{nC_\mathrm{V}} = \frac{9.06\times 10^8 \text{ J}}{1.90\times 10^7 \text{ J/K}} = 47.7 \text{ K}$$

Therefore, the temperature of the room will increase by 47.7 °C.

4) An n mole of ideal gas expands adiabatically from a volume of V_i to V_f. Initially, the pressure was P_i. Determine (a) the work of the gas, (b) the heat of the gas, (c) the change of internal energy from the first law of thermal dynamics and (d) show that the internal energy obtained in (c) is identical to $\Delta E_\mathrm{int} = nC_V\Delta T$.

(a) From the definition, work is given by

$$W = \int_{V_\mathrm{i}}^{V_\mathrm{f}} P\,dV$$

Using a relation valid for the adiabatic process, $PV^\gamma = a$, where $a = P_\mathrm{i}V_\mathrm{i}^\gamma = P_\mathrm{f}V_\mathrm{f}^\gamma$ and $\gamma = C_\mathrm{P}/C_\mathrm{V}$, it follows that

$$W = a\int_{V_\mathrm{i}}^{V_\mathrm{f}} V^{-\gamma}\,dV = \frac{a}{1-\gamma}V^{1-\gamma}\Big]_{V_\mathrm{i}}^{V_\mathrm{f}} = \frac{a}{1-\gamma}\left(V_\mathrm{f}^{1-\gamma} - V_\mathrm{i}^{1-\gamma}\right)$$

(b) Since this is an adiabatic process, there is no heat, i.e. $Q = 0$.

(c) From the first law of the thermodynamics, the work done by the gas is

$$E_\mathrm{int} = Q - W = -W = \frac{-a}{1-\gamma}\left(V_\mathrm{f}^{1-\gamma} - V_\mathrm{i}^{1-\gamma}\right)$$

(d) Using the definition of a and γ, W can be written as

$$\Delta E_\mathrm{int} = \frac{-a}{1-\gamma}\left(V_\mathrm{f}^{1-\gamma} - V_\mathrm{i}^{1-\gamma}\right) = \frac{C_\mathrm{V}}{C_\mathrm{P} - C_\mathrm{V}}\left(P_\mathrm{f}V_\mathrm{f} - P_\mathrm{i}V_\mathrm{i}\right) = \frac{C_\mathrm{V}}{R}\left(T_\mathrm{f} - T_\mathrm{i}\right)nR = nC_\mathrm{V}\Delta T$$

5) An ideal gas is at the state A: $(V_\mathrm{a}, P_\mathrm{a}, T_0)$. By expanding the volume to V_b through isothermal process, the state changes to B: $(V_\mathrm{b}, P_\mathrm{b}, T_0)$, while with the adiabatic process to C: $(V_\mathrm{b}, P_\mathrm{c}, T_\mathrm{c})$. Which pressure is higher, P_b or P_c? Which temperature is higher, T_0 or T_c? Why the temperature changes in the adiabatic process?

For the adiabatic process, A: $(V_\mathrm{a}, P_\mathrm{a}, T_0)$ to C: $(V_\mathrm{b}, P_\mathrm{c}, T_\mathrm{c})$, we have,

$$P_\mathrm{c} = \left(\frac{V_\mathrm{a}}{V_\mathrm{b}}\right)^\gamma P_\mathrm{a}$$

while for the isothermal process, A: $(V_\mathrm{a}, P_\mathrm{a}, T_0)$ to B: $(V_\mathrm{b}, P_\mathrm{b}, T_0)$,

$$P_\mathrm{b} = \left(\frac{V_\mathrm{a}}{V_\mathrm{b}}\right) P_\mathrm{a}$$

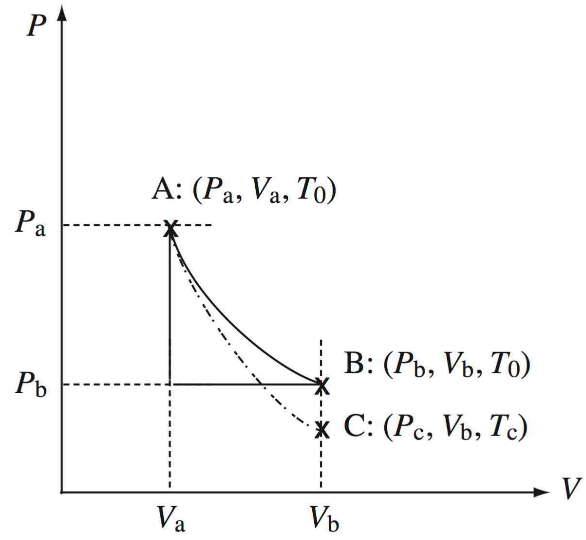
Since $V_\mathrm{a}/V_\mathrm{b} < 1$ and $\gamma > 1$, we have

$$P_\mathrm{b} > P_\mathrm{c},$$

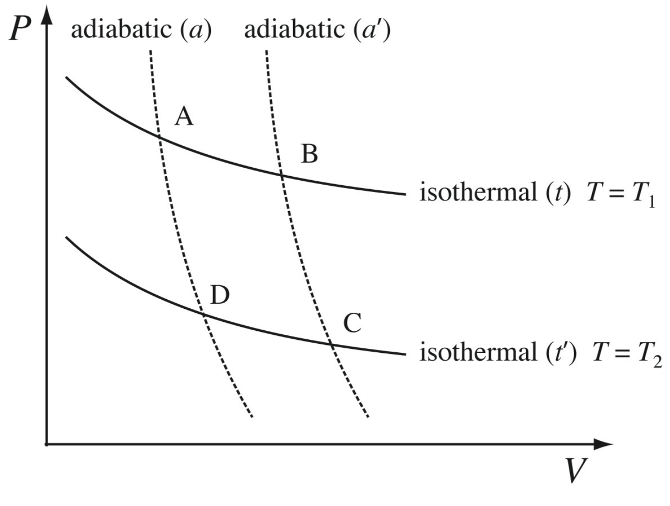
i.e. the pressure difference needed for the adiabatic process is higher than that for the isothermal process for the same volume changes. This leads to

$$T_\mathrm{c} = \frac{V_\mathrm{b}P_\mathrm{c}}{nR} < \frac{V_\mathrm{b}P_\mathrm{b}}{nR} = T_0,$$

i.e. $T_0 > T_c$ and the temperature decreases when the volume increases adiabatically. This is because there is no heat in the adiabatic process and the work due to the expansion of the volume has to be taken out from the internal energy. This results in the decrease of the temperature. This is related to the formation of the cloud, for example.



6) For an n-mole of ideal gas, two isothermal lines, t and t', at temperatures T_1 and T_2 in Kelvin respectively, and two adiabatic lines, a and a', are crossing at the four points, A, B, C and D, in the volume (V) versus pressure (P) diagram, as shown in the plot below. The heat in the process A→B is denoted by Q_{ab} and in the process C→D Q_{cd}. Show that the ratio, Q_{ab}/Q_{cd}, depends only on T_1 and T_2.



Volumes and pressures of the four crossing points, A, B, C and D are given by V_a, V_b, V_c, V_d and P_a, P_b, P_c, P_d, respectively. Along the isothermal line, t, we have $PV = nRT_1$ and since A and B are on the line $t, P_a V_a = nRT_1 = P_b V_b$. The work in the process A→B is then given by

$$W_{ab} = \int_{V_a}^{V_b} PdV = nRT_1 \int_{V_a}^{V_b} \frac{dV}{V} = nRT_1 \ln \frac{V_b}{V_a}$$

Using the first law of thermodynamics, $\Delta E_{\mathrm{int}} = Q - W$, and knowing that there is no change in the internal energy along the isothermal line, we have

$$Q_{\mathrm{ab}} = W_{\mathrm{ab}} = nRT_1 \ln \frac{V_b}{V_a}.$$

Similarly for the C→D process, we obtain

$$Q_{\mathrm{cd}} = W_{\mathrm{cd}} = nRT_2 \ln \frac{V_d}{V_c}.$$

Along the adiabatic line, a, we have $PV^{\gamma} = \mathrm{constant}$, where $\gamma = C_{\mathrm{P}}/C_{\mathrm{V}}$. With a help of the ideal gas law, $PV = nRT$, this can be written as $TV^{\gamma-1} = \mathrm{constant}$. Since D and A are on the line a, it follows that $T_1 V_a^{\gamma-1} = T_2 V_d^{\gamma-1}$. Equally for B and C on line a', we have $T_1 V_b^{\gamma-1} = T_2 V_c^{\gamma-1}$, thus $V_b/V_a = V_c/V_d$. Finally, the ratio of the two heats is given by

$$\frac{Q_{\mathrm{ab}}}{Q_{\mathrm{cd}}} = \frac{T_1}{T_2} \frac{\ln V_b/V_a}{\ln V_d/V_c} = -\frac{T_1}{T_2}$$

thus it depends only on the two temperatures.