General Physics II: Tutorial Material 5

1) Periodic table gives:

Atomic number	Element	Mass number	Atomic mass
13	Al	27	26.981539
26	Fe	56	55.934938

Which has more atoms: 1 kg of iron (Fe) or 1 kg of aluminium (Al)?

Atomic mass of aluminium is less than that of iron, i.e. one atom of Al has less mass than one atom of Fe. Therefore, there are more atoms in 1 kg of aluminium than in 1 kg of iron.

2) How many atoms are there in a 3g of aluminium?

From the atomic mass of Al, 26.981539, the mass of one aluminium atom is given by

$$26.981539 \text{ u} \times 1.66 \times 10^{-27} \text{ kg/u} = 4.48 \times 10^{-26} \text{ kg}$$

Therefore, the number of atoms in 3 g aluminium is given by

$$\frac{3 \times 10^{-3} \text{ kg}}{4.48 \times 10^{-26} \text{ kg}} = 6.70 \times 10^{22}$$

3) Suppose system C is not in equilibrium with system A nor with system B. Does this imply that A and B are not in equilibrium? What can be said about the temperatures of A, B, and C?

No, we do not know whether A and B are in equilibrium or not. The temperature of C is equal neither to A nor B. No information on the temperature relation between A and B.

4) In an alcohol-in-glass thermometer, the alcohol column has length 11.82 cm at 0.0° C and length 21.85 cm at 100.0° C. What is the temperature if the column length has:
a) 18.70 cm
b) 14.60 cm?

The change of the temperature for 1 cm increase in the length of the alcohol

$$\frac{100^\circ - 0^\circ}{21.85 \text{ cm} - 11.82 \text{ cm}} = 9.97^\circ/\text{cm}$$

Therefore, for
a) $9.97^\circ/\text{cm} \times (18.70 - 11.82) \text{ cm} + 0^\circ = 68.6^\circ \text{ C}$
b) $9.97^\circ/\text{cm} \times (14.60 - 11.82) \text{ cm} + 0^\circ = 27.7^\circ \text{ C}$

5) A flat bimetal strip consists of a strip of aluminium riveted to a strip of iron. When heated, the strip will bend. Which metal will be on the outside of the curve? Why?

The aluminium has a larger thermal coefficient of linear expansion than that for iron. Therefore, the aluminium will be outside of the curve when heated.

6) The density of water at 4° C is $1.00 \times 10^3 \ \text{kg/m}^3$. What is water's density at 94° C, assuming a constant coefficient of volume expansion, $210 \times 10^{-6} \ (°\text{C})^{-1}$?

The water with a volume of 1 m³ at 4° C changes its volume at 94° C by

$$210 \times 10^{-6} \ (\text{C}^\circ)^{-1} \times 1 \ \text{m}^3 \times (94-4)\text{C}^\circ = 0.0189 \ \text{m}^3$$

thus the volume is 1.0189 m³. The density of the water at 94° C is then given by

$$\frac{1.00 \times 10^3 \ \text{kg/m}^3}{1.0189} = 0.981 \times 10^3 \ \text{kg/m}^3$$

7) There is a 10 cm long bar made by a material with a coefficient of linear expansion to be $10^{-1}/\text{C}^\circ$ at $0°\text{C}$. If we warm up the bar to $5°\text{C}$, how long will be the bar? If we warm the bar by **another** $5°$ ($5°\text{C}$ to $10°\text{C}$), how long will be the bar? If we warm the 10 cm long bar at $0°\text{C}$ to $10°\text{C}$ directly, how long it will be? How do we understand the result?

Warming up that bar from $0°\text{C}$ *to* $5°\text{C}$, *the length of the bar will become*

$$l_5 = 10 \times \left[1 + 10^{-1} \times (5-0)\right] = 15 \ \text{cm}$$

and warming up by another $5°$,

$$l_{10} = 15 \times \left[1 + 10^{-1} \times (5-0)\right] = 22.5 \ \text{cm}$$

By heating up the bar directly from 0 to $10°\text{C}$, *we obtain*

$$l_{10}^{\text{direct}} = 10 \times \left[1 + 10^{-1} \times (10-0)\right] = 20 \ \text{cm}$$

i.e., two processes to warm up the bar to the same temperature gives different length which is a paradox. This indicates the expression using a linear coefficient is an approximation to calculate the elongation. This can be understood by reformulating the first case as

$$l = l_0 \lim_{n \to \infty} \left[1 + c\frac{(T_f - T_i)}{n}\right]^n \approx l_0 \left[1 + c(T_f - T_i) + \text{higher order}\right]$$

where c is the linear expansion coefficient and T_i *and* T_f *are the initial and final temperature, then the linear approximation can be valid only for* $c(T_f - T_i) << 1$. *For this particular case,* $c(T_f - T_i) = 1$, *thus the linear approximation does not really work.*

8) A ruler was calibrated to the correct length measurement at temperature, T_0. With this ruler, the two sides of a rectangular metal sheet are measured to be a'_1 and b'_1 at temperature, T_1.
a) What is the true surface area of the metal sheet at temperature T_1?
b) What will be the measured surface area of the metal sheet at temperature T_2, with the same ruler?
Note that the coefficient of linear expansion for the material used for the ruler is α_r and that for the metal sheet, α_s, and both are very small.

a) Length of a bar made of the same metal as the ruler, l_0 at temperature T_0, becomes $l_1 = l_0[1 + \alpha_r(T_1 - T_0)]$ at temperature T_1. By denoting a_1 to be the true length at temperature T_1, the measured length by the ruler at temperature T_1 can be expressed as

$$a'_1 = \frac{a_1}{l_1} l_0$$

Therefore, the true length is

$$a_1 = a'_1[1 + \alpha_r(T_1 - T_0)],$$

Similarly,

$$b_1 = b'_1[1 + \alpha_r(T_1 - T_0)]$$

and the true surface area of the metal sheet at temperature T_1 is given by

$$S_1 = a_1 b_1 \approx a'_1 b'_1[1 + 2\alpha_r(T_1 - T_0)]$$

b) The true length of the two sides of the metal sheet at temperature, T_2, is given by

$$a_2 = a_1[1 + \alpha_S(T_2 - T_1)] \approx a'_1[1 + \alpha_r(T_1 - T_0) + \alpha_s(T_2 - T_1)]$$

and

$$b_2 \approx b'_1[1 + \alpha_r(T_1 - T_0) + \alpha_s(T_2 - T_1)]$$

The ruler at temperature T_2 will measure them to be

$$a'_2 = a_2[1 + \alpha_r(T_0 - T_2)] \approx a'_1[1 + \alpha_r(T_1 - T_2) + \alpha_s(T_2 - T_1)],$$

and

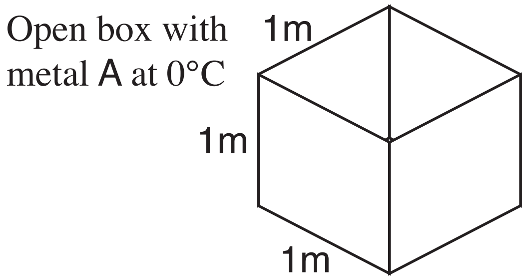
$$b'_2 \approx b'_2[1 + \alpha_r(T_1 - T_2) + \alpha_s(T_2 - T_1)]$$

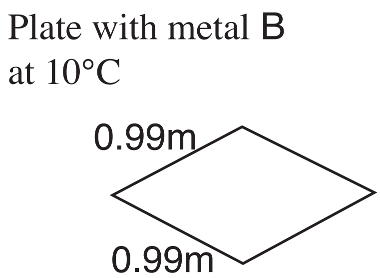
Therefore, the measure surface area of the metal sheet at temperature T_2 is given by

$$S'_2 = a'_2 b'_2 \approx a'_1 b'_1[1 + 2\alpha_r(T_1 - T_2) + 2\alpha_s(T_2 - T_1)].$$

9) Coefficients of linear expansion for the metal A and metal B are $10^{-5}/{}^\circ\mathrm{C}$ and $5 \times 10^{-5}/{}^\circ\mathrm{C}$, respectively. A box with a dimension of 1 m × 1 m × 1 m at 0°C is made of the five sheets of metal A without a top. There is a plate with a

dimension of 0.99 m × 0.99 m at 10°C made of the metal B. When both the box and plate are kept at the same temperature, what is the minimum temperature at which the metal B plate can be used to close the box completely? Note that the thickness of the metal plates can be neglected.





Let us denote the temperature of the room to be T°C*. The opening of the box at this temperature is given by* $l_{\text{box}} \times l_{\text{box}}$*, where*

$$l_{\text{box}} = 1 \times \left[1 + 10^{-5} \times (T - 0)\right]$$

With the same temperature, the side of the plate, l_{plate}*, becomes*

$$l_{\text{plate}} = 0.99 \times \left[1 + 5 \times 10^{-5} \times (T - 10)\right].$$

In order the metal B plate to close the box completely, $l_{\text{box}} \leq l_{\text{plate}}$*. Therefore, the minimum temperature can be obtained by solving*

$$1 \times \left[1 + 10^{-5} \times (T - 0)\right] = 0.99 \times \left[1 + 5 \times 10^{-5}(T - 10)\right]$$

It follows that

$$T \approx 265.70^{\circ}\text{C}.$$