Summary of the last lectures

Four dimensional space-time defined by (ct, x, y, z):

Space-time coordinate is transformed under Lorentz transformation between different inertial frames moving with constant speeds to each other

$s^2 \equiv (ct)^2 - x^2 - y^2 - z^2$ invariant under Lorentz transformation

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}$$

$$\beta = v/c \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

moving along the x-axis with a velocity v

$s^2 = (ct_0)^2$ for $s^2 > 0$: t_0 is proper time

$s^2 = -l_0^2$ for $s^2 < 0$: l_0 is proper length

Two events (ct, x, y, z) and $(ct+c\Delta t, x+\Delta x, y+\Delta y, z+\Delta z)$ are causally connected if $s^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 > 0$

Summary of the last lectures

If S' moves only in x-direction with a velocity v:

Lorentz transformation

(ct, x, y, z) is a Lorentz four vector (dimension of length)

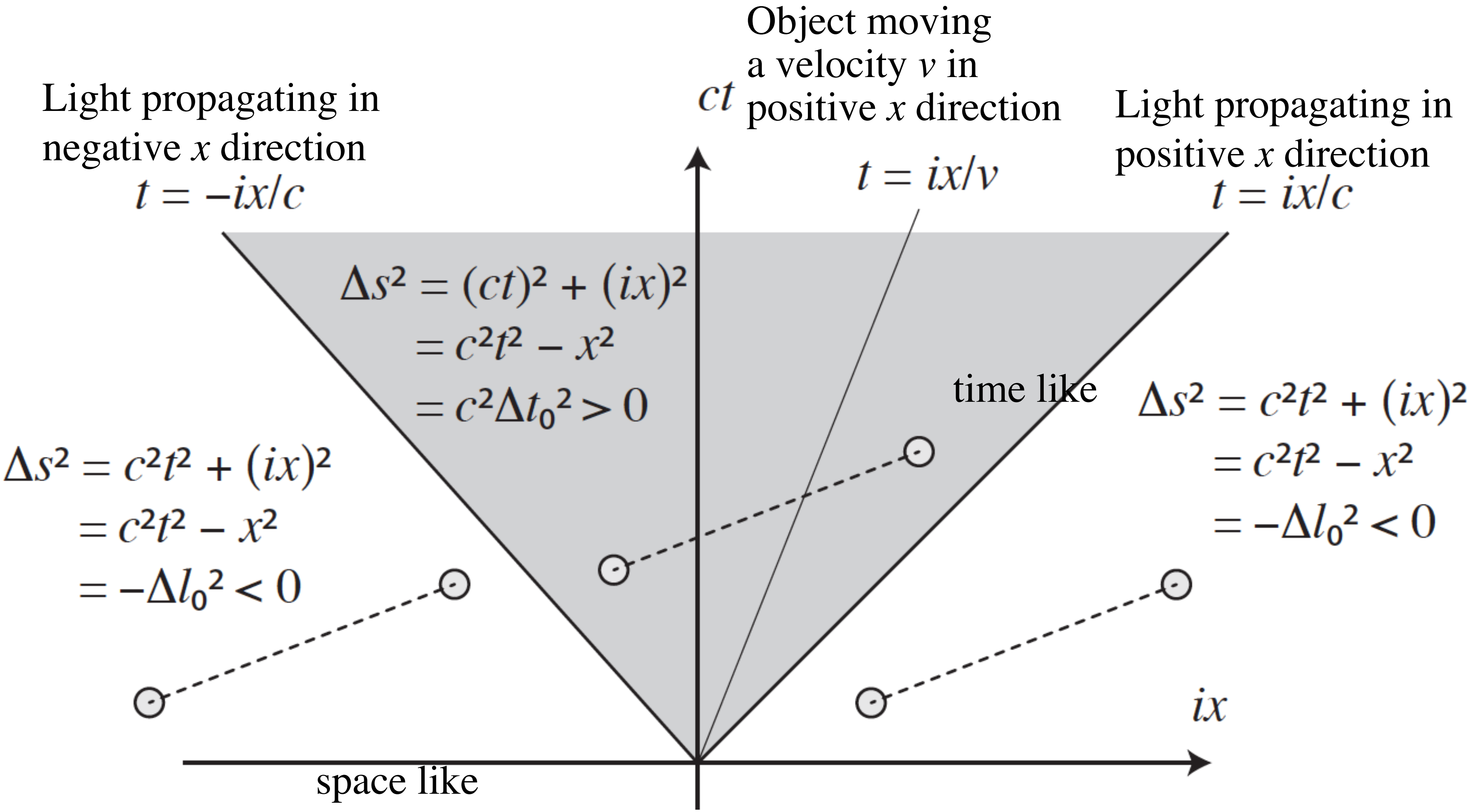
$s^2 \equiv (ct)^2 - x^2 - y^2 - z^2$ invariant under Lorentz transformation

(E/c, p_x, p_y, p_z) also a Lorentz four vector (dimension of momentum)

$$\begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E'/c \\ p'_x \\ p'_y \\ p'_z \end{pmatrix}$$

$(E/c)^2 - p_x^2 - p_y^2 - p_z^2 = m_0^2 c^2$ invariant under Lorentz transformation

Summary of the last lectures



Δt_0: proper time interval

Δl_0: proper distance interval (length)

Summary of the Special Relativity

Doppler effect f: frequency at rest, f': observed

The sound wave propagates through the air

velocity depends on the inertial frame

v_{sound} = defined in the rest frame of the air

$$f' = f \frac{v_{\text{sound}}}{v_{\text{sound}} - v_{\text{source}}} \qquad f' = f \frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}}}$$

sound source approaching to observer at rest respect to the air

observer approaching to the sound source at rest respect to the air

The speed of the light wave is always c independent of the inertial frame

$$f' = f \sqrt{\frac{c + v_{\text{relative}}}{c - v_{\text{relative}}}}$$

observer and source coming closer with a relative velocity v_{relative}