Summary of the last lecture

Relativity

The law of physics have the same form in all inertial frames

A frame moving with a constant velocity respect to an inertial frame is also an inertial frame

- Galilean-Newtonian version

Newton's laws are valid in any inertial frame

⇒Spatial length and time interval do not depends on the inertial frame, i.e. same for all the inertial frames

⇒For an object moving with a constant velocity $\vec{u}$ in one inertial frame, its velocity in another inertial frame $\vec{u}'$ is given by $\vec{u}'=\vec{u}+\vec{v}$, where $\vec{v}$ is the relative velocity between the two frames

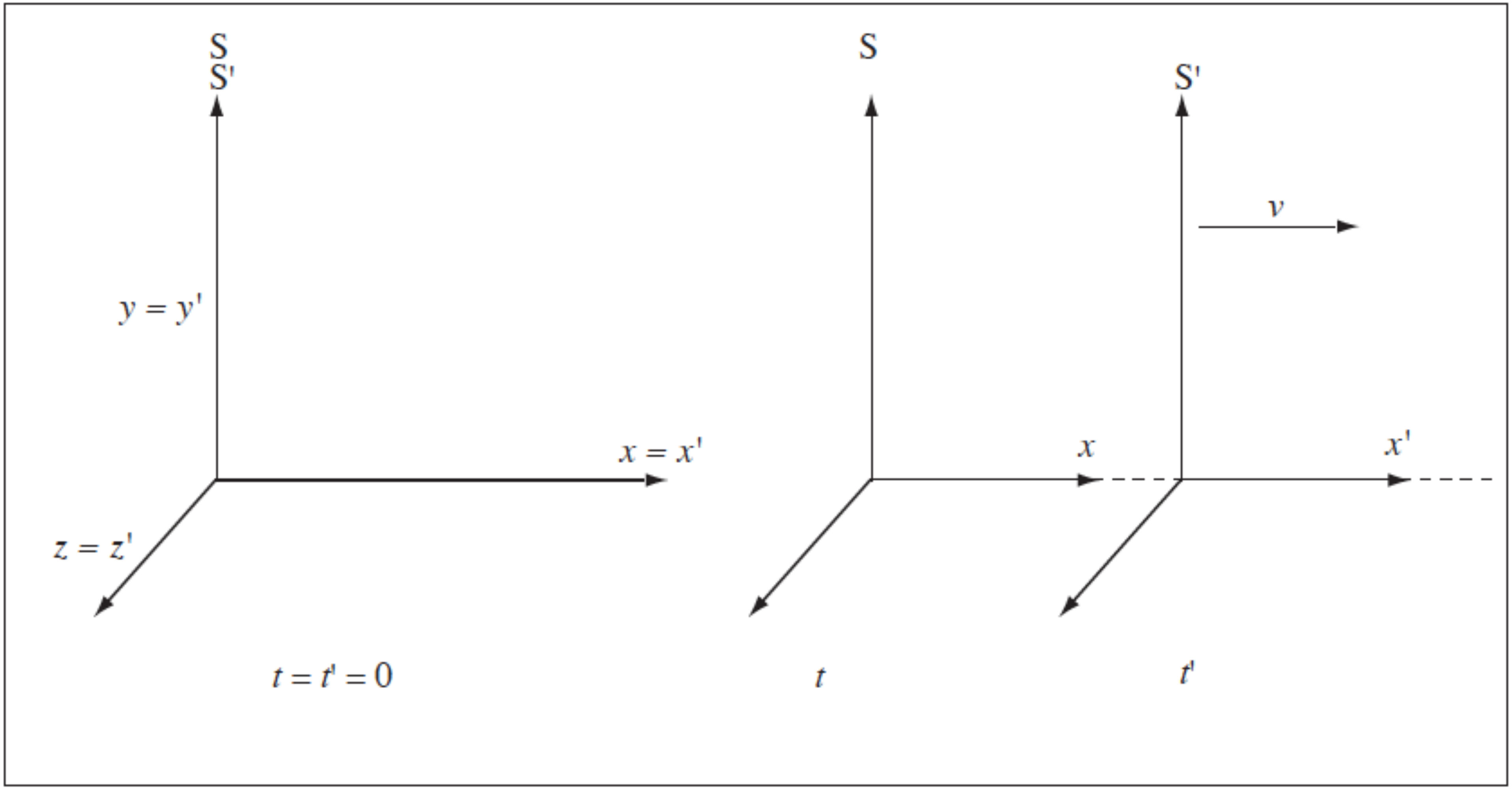
- Einstein's version (Special Relativity)

Speed of light in vacuum, *c*, is independent of the speed of source or that of observer, i.e. speed of light does not depends on the inertial frame.

⇒ neither spatial length nor time interval are invariant

Summary of the last lecture

S’ moves along the *x*’ axis with a constant velocity *v*



S and S’ overlap at $t = t' = 0$ S and S’ after some time

Summary of the last lecture

The same space-time point seen by S and S'

S' (moving with a velocity *v* along *x* respect to *S*)

Galilean Transformation

$t = t'$	$t' = t$
$x = x' + vt'$	$x' = x - vt$
$y = y'$	$y' = y$
$z = z'$	$z' = z$

Lorentz transformation

$ct = \gamma ct' + \gamma\beta x'$	$ct' = \gamma ct - \gamma\beta x$
$x = \gamma x' + \gamma\beta ct'$	$x' = \gamma x - \gamma\beta ct$
$y = y'$	$y' = y$
$z = z'$	$z' = z$

$\beta = v/c$ $\qquad$ $\gamma = \dfrac{1}{\sqrt{1 - \beta^2}}$

Summary of the last lecture

The same space-time point seen by S and S’

S’ (moving with a velocity *v* along *x* respect to *S*)

In a linear algebra!

Galilean Transformation

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} \qquad \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -v & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

Lorentz transformation

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} \qquad \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$\beta = v/c \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

Summary of the last lecture

Lorentz transformation → Velocity transformation

$$u'_{x,\,y,\,z} = \frac{d'x}{dt'} \quad \leftrightarrow \quad u_{x,\,y,\,z} = \frac{dx}{dt}$$

Lorentz transformation gives t' ↔ t and x', y', z' ↔ x ,y , z

$$u_x = \frac{u'_x + v}{1 + vu'_x/c^2} \qquad u_{y,\,z} = \frac{u'_{y,\,z}\sqrt{1 - (v/c)^2}}{1 + vu'_x/c^2}$$

$$u'_x = \frac{u_x - v}{1 - vu_x/c^2} \qquad u'_{y,\,z} = \frac{u_{y,\,z}\sqrt{1 - (v/c)^2}}{1 - vu_x/c^2}$$

(S' moving along the x direction with a velocity *v*)